

A SIMPLIFIED MATHEMATICAL EQUATION TO STUDY THE RELATIONSHIP BETWEEN WATER DRIVING FORCES AND WATER LOSSES AMOUNT IN IRRIGATION CANALS IN THE FIELD

Abstract

Two field experiments were conducted at El-Gemmeiza Agricultural Research Station, El-Gharbia Governorate, Egypt, middle of Nile Delta (30° 43' latitude and 31° 47' longitude) on clay soil during two winter seasons (2019/2020 and 2020/2021). The cultivated wheat cultivar was Gemmeiza 11 that planted in two seasons and irrigated three times in addition to planting irrigation. This investigation aimed to study the effect of water driving forces in irrigation canals in the case of using one or two irrigation canals in the field and its impact on water losses amount by deriving a mathematical equation. The results across the two seasons showed that mean water driving forces (WDF) in the case of using one irrigation canal in field (0.26 m³/16.63 minute) was greater than its corresponding value under two irrigation canals (0.24 m³/36.13 minute) and the water losses amount (WLA) resulted from using one irrigation canal (17.78 m³) was less than its corresponding value under the two irrigation canals (36.25 m³) by saving 18.47 m³ of irrigation water. Consequently, there is an inverse relationship between water driving forces in irrigation canals in the field and water losses amount. Grain yield in the case of using one irrigation canal in field was 3.394 ton/fed while in the two irrigation canals recorded 3.276 ton/fed by increasing 118 kg/fed. Accordingly, a quantity of irrigation water can be saved during surface irrigation in order to achieve water abundance can be used to irrigate new lands. The quadratic model was the best statistical model to describe the relationship between Water Driving Force (WDF) and Water Movement Time (WMT) irrigation canal in field. Using the fitted quadratic model, it is clear that the Critical Water Movement Time (CWMT) which reflects the lowest WDF value was 25.55 min. So by studying the relationship between water driving forces and water losses amount by calculating water driving forces rate can be controlled in water driving factors by the optimum irrigation canals planning to achieve water abundance is one of the goals Egypt's agricultural strategy 2030.

Key words: water losses amount, water driving forces, irrigation canals and a new mathematical equation.

INTRODUCTION

As the world population grows, the need for food production has increased, and less water is allocated to the agriculture sector (**Shaheed et al., 2017**). Wheat has managed to increase its share of the winter cropped area ranged from 41 to 47%, and the cultivated area is limited to the narrow strip along the Nile Valley (**FAO, 2015**). Therefore attempts have been made to expand the cultivated area in Egypt by reclaiming new lands using irrigation. The amount of wheat production in Egypt is about of 9 million ton in 2021 with an increase of 1.12% from the previous years (**FAO, 2022**), in a trial to increase wheat cultivation area to decrease dependency on wheat importing. Consequently, supply-oriented management strategies have been used to identify new potential water resources for agricultural purposes. However, climate change and continuous droughts in recent decades in arid regions have created challenges to the provision of a timely and sufficient water supply for agricultural water demand (**Chen et al., 2020**). Meanwhile, the essential need for significant investments in agricultural water supply, conveyance, and distribution infrastructure have limited the application of supply-oriented approaches. Therefore, in order to reduce water losses in the infrastructure and increase water productivity, demand-oriented approaches have become priorities and have been widely employed on the on-farm scale through precision agriculture systems (**Noorisameleh et al., 2020**). In recent years, water resources exposes to an increasing stress due to impacts of climate change, population increase and economic development. Water scarcity is recognized as a main threat particularly in the Mediterranean area. Thus, water utilities should become highly efficient throughout the entire water supply process, to guarantee sufficient quantities of good quality water. Since water is one of the most valuable natural resources, water losses in the Water Distribution System (WDS) represent an urgent problem that needs to be managed (**Kanakoudis and Muhammetoglu, 2014**). Water conveyance and distribution process may be less benefit during irrigation canals due to the effect on water losses, so it is necessary to evaluate the performance of water conveyance and estimation water losses in irrigation canals in field (**Barkhordari and Shahdany 2022**). Water conveyance efficiency is affected by several factors such as seepage, soil type, canal length, canal width and physical damages of the canals (**Sen et al., 2018**). Also, the main factors that influence canal water losses are water level, soil hydraulic properties, groundwater table location, and amount of sediment inside the canal (**Yao et al., 2012**). Consequently, the reduction of seepage helps protect the limited water resources, minimize production of water treatment products and energy consumption (**Ashton and Hope 2001**). The Egyptian country launched a program to lining water canals as part of a major project to reduce water losses in order to achieve water abundance within the

national agricultural strategy 2030 to achieve this goal. This research was carried out with the aim of studying the effect of water driving forces in irrigation canals in case of using one or two irrigation canal in the field and its impact on water losses amount by deriving a simple mathematical equation.



Fig 1. Experimental area

MATERIALS AND METHODS

Two field experiments were carried out at El-Gemmeiza Agricultural Research Station, El-Gharbia Governorate, Egypt, Middle of Nile Delta ($30^{\circ} 43'$ latitude and $31^{\circ} 47'$ longitude) on clay soil during two winter seasons (2019/2020 and 2020/2021). Wheat cultivar, Gemmeiza 11 was planted on 16th and 18th November in two seasons, respectively and irrigated three times at soil field capacity in addition to planting irrigation, where the experiment area was 2064m^2 (48m length \times 43m width). The soil surface was leveled by laser. Wheat plants were harvested on 27th and 30th April for two seasons respectively. The objective of the research is studying the effect of water driving forces within irrigation canals in field and its impact on water losses by deriving a simplified mathematical equation.

Description of the irrigation canal in field

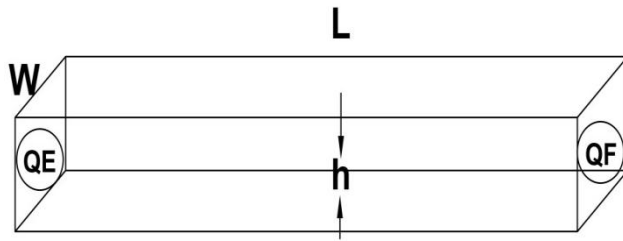


Fig 2. **Description of the irrigation canal**

$$\text{Water driving forces (WDF), m}^3/\text{min} = \frac{L \times W \times H \times G}{T}$$

L: canal length, 48m

w: canal width, one m

H: water height in irrigation canal in field, 0.5m

T: water move time, minute

G: gravity 9.81m/second

Water losses amount (WLA), $\text{m}^3 = \text{QE} - \text{QF}$

QE: Water amount entering in canal irrigation in field, m^3

QF: Water amount flow out from canal irrigation in field, m^3 at water stability

The scientific basis of the equation

Water driving forces determine the direction and rate of water movement. As a result to water movement on soil surface water, physical properties (density and viscosity) change due to dissolved substances increasing in water as a result to water molecules rubbing with soil surface, according to **Reilly and Goodman(1985) and Reilly (1993)** they indicated that water density is important because it is a part of the driving forces for water, moreover water density and viscosity affect the hydraulic transmitting properties (permeability and hydraulic conductivity). Therefore, there is relationship between water driving forces and water losses amount.

The equation importance

- The optimum irrigation canal planning in agricultural soils in the case of surface irrigation to achieve the least water losses.
- By calculating water driving forces by using mathematical equation can be achieving water abundance can be used to irrigate new lands.

- Achieving one of the objectives of agricultural strategy 2030(water abundance) in Egypt.

Table (1):Mechanical analysis of the experimental site.

Soil depth, cm		0-10	10-30	Soil depth, cm		0-10	10-30
Particle size distribution,%	Coarse sand	6.19	5.70	Settling, %		31.06	31.41
	Fine sand	18.75	18.76	Pore size distribution,%	> 9 μ	21.88	22.42
	Silt	30.96	29.93		9 – 0.2 μ	11.89	12.19
	Clay	44.10	45.61		<0.2 μ	14.16	12.19
Texture class		Clay	Clay				

Settling percentage of the soil aggregates was determined in soil aggregates of 2 – 5 mm size, as the method described by *Williams and Cooke (1961) and Hartge (1969)*.

Pore size distribution was calculated according to *De Leenher and De Boodt (1965)*.

Amount of water applied: the discharge through an orifice was determined from the following equation as described by *Israelson and Hansen (1962)*.

$$Q = CA(2Gy)^{1/2}$$

Where: Q = Discharge rate, m³sec⁻¹, C = discharge coefficient ranges from 0.6 to 0.8, A= area of orifice opening (m²), G= accelerating of gravity (9.8msec⁻¹), Y= the head causing free flow where Y is the upstream head measured from the center of orifice opening.

Climatic Condition The meteorological data including: maximum temperatures (T- max)^oC, minimum temperatures (T- min)^oC, relative humidity(RH): %, wind speed (WS)m/s and rainfall (RF)m.m during the two years of study were recorded and their monthly mean values are presented in Table (2).

Table (2). Meteorological data in 2019/2020 and 2020/2021 growing season

Month	T- ma x		T- min		T- mea n		RH		WS		RF	
	1 st	2 nd	1 st	2 nd	1 st	2 nd	1 st	2 nd	1 st	2 nd	1 st	2 nd
November	21.02	24.64	8.25	10.78	14.64	17.71	65.32	63.38	7.49	8.16	0.11	0.36
December	18.85	22.57	6.17	6.93	12.51	14.75	61.55	60.63	6.48	6.38	0.08	0.02
January	17.63	21.52	5.32	2.03	11.48	11.78	69.18	59.04	5.61	5.62	0.55	0.07
February	20.11	21.78	6.11	5.14	13.11	13.46	66.43	61.52	5.93	5.68	0.67	0.72
March	24.42	23.27	6.83	6.90	15.63	15.09	59.88	62.38	6.36	6.32	2.39	3.36
April	26.88	29.43	8.63	6.68	17.76	18.06	57.24	50.20	7.13	6.29	2.86	0.13
May	32.57	36.84	10.55	7.84	21.56	22.34	50.77	36.64	8.17	6.89	0.0067	0.00

* Source: Water Requirement and Field Irrigation Res., Dept

Response curve analysis

Description of the relationship between Water Driving Force (WDF) and Water Movement Time (WMT) involved fitting three types of models over the data collected under the four irrigations during the two wheat growing seasons, according to **Neter *et al.*, (1990)**. Considering the canal length is constant, the tested models were linear, quadratic and logarithmic model.

(1): The linear model is defined by the equation;

$$WDF = a + b (WMT)$$

(2): The quadratic model is defined by equation;

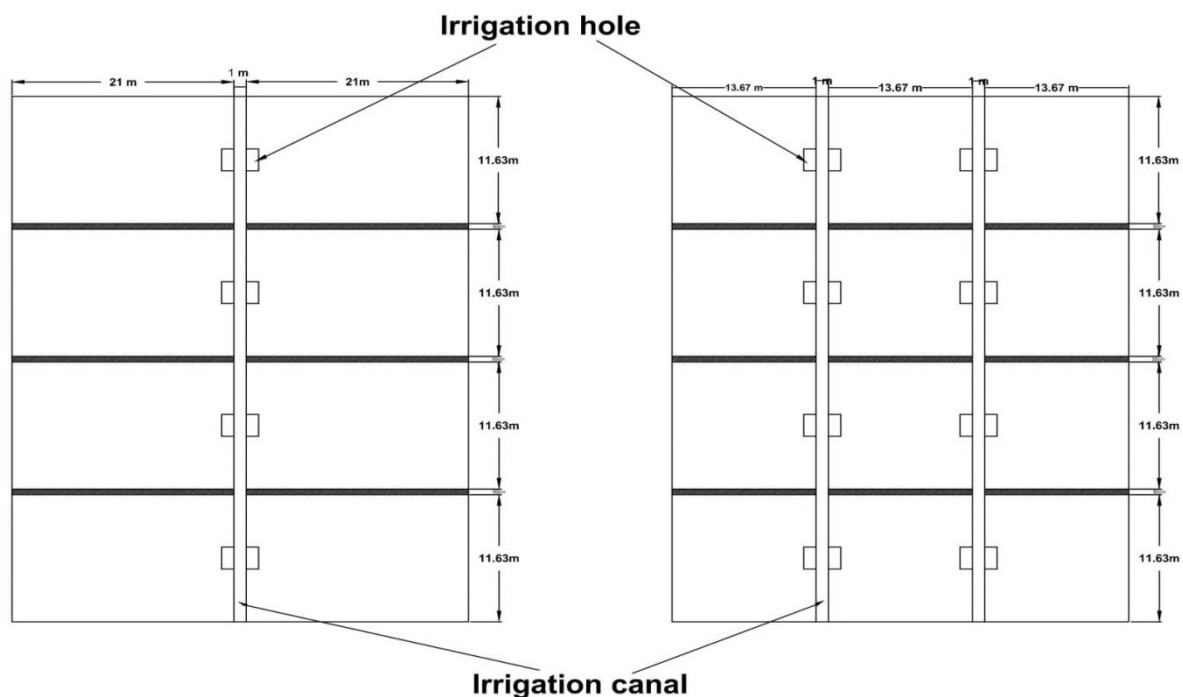
$$WDF = a + b (WMT) + c (WMT)^2$$

(3): The logarithmic model is defined by equation;

$$WDF = a + b \ln (WMT)$$

Where, (a) represents as intercept, b and c are coefficients of the regression equation. Coefficient of determination (R^2), standard error of estimate (SE) and significance of the model were used as the basis for comparing the above-mentioned response models. The model that had the highest (R^2) value and the lowest estimate (SE) was the best model for describing the relationship between Water Driving Force (WDF) and Water Movement Time (WMT). By taking the first derivative of the optimum quadratic equation with respect to WMT and equating that to zero, it gets the Critical Water Movement Time (CWMT) which reflect the lowest WDF. In other words, Water Driving Force (WDF) should not significantly after this critical time period.

Experiments design



One irrigation canal

Two irrigation canal

Fig 3. Irrigation canal

RESULTS AND DISSCUTION

The results in Tables(3 and 4) showed that water losses amount were 17.26 and 18.29 m³ in the two seasons, respectively at the rate of water driving forces 0.25m³ in 16.50 minute in the first season and 0.27 m³ in 16.75minute in the second season by using one irrigation canal. These results maybe attributed to the effect of several factors of water driving forces such as canal length, canal width and change in water density and viscosity. These results arein agreement with those obtained by Yao et al., (2012) and Sen et al., (2018).

Table (3): Water losses amount of one irrigation canal in the first season

planting irrigation		first irrigation		second irrigation		third irrigation	
QE = 34.16m ³	Water movement	QE = 29.37m ³	Water movement	QE = 26.83m ³	Water movement	QE = 24.29m ³	Water movement
QF = 26.27m ³	time(23 minute)	QF = 24.19m ³	time (17 minute)	QF = 23.82m ³	time (14 minute)	QF = 23.11m ³	time (12 minute)

$$\text{Total QE} = 114.65\text{m}^3$$

$$\text{Total QF} = 97.39\text{m}^3$$

$$\text{Water losses amount, Total(QE - QF)} = 17.26\text{m}^3$$

$$\text{Water driving forces} = \frac{L \times W \times h \times g}{T}$$

$$\text{WDF}_{T23} = \frac{48 \times 1 \times 0.5 \times 9.81}{23 \times 60} = 0.17\text{m}^3/\text{min}$$

$$\text{WDF}_{T17} = \frac{48 \times 1 \times 0.5 \times 9.81}{17 \times 60} = 0.23\text{m}^3/\text{min}$$

$$\text{WDF}_{T14} = \frac{48 \times 1 \times 0.5 \times 9.81}{14 \times 60} = 0.28\text{m}^3/\text{min}$$

$$\text{WDF}_{T12} = \frac{48 \times 1 \times 0.5 \times 9.81}{12 \times 60} = 0.33\text{m}^3/\text{min}$$

$$\text{Mean WDF}_{16.50} = 0.25\text{m}^3/\text{min}$$

Table (4): Water losses amount of one irrigation canal in the second season

planting irrigation		first irrigation		second irrigation		third irrigation	
QE = 34.72m ³	Water movement	QE = 29.84m ³	Water movement	QE = 27.18m ³	Water movement	QE = 24.68m ³	Water movement
QF = 27.37m ³	time (25 minute)	QF = 24.47m ³	time (19 minute)	QF = 23.56m ³	time (13 minute)	QF = 22.73m ³	time (10 minute)

$$\text{Total QE} = 116.42\text{m}^3$$

$$\text{Total QF} = 98.13\text{m}^3$$

$$\text{Water losses amount, Total (QE - QF)} = 18.29\text{m}^3$$

$$\text{Water driving forces} = \frac{L \times W \times h \times g}{T}$$

$$\text{WDF}_{T25} = \frac{48 \times 1 \times 0.5 \times 9.81}{25 \times 60} = 0.16\text{m}^3/\text{min}$$

$$\text{WDF}_{T19} = \frac{48 \times 1 \times 0.5 \times 9.81}{19 \times 60} = 0.21\text{m}^3/\text{min}$$

$$WDF_{T13} = \frac{48 \times 1 \times 0.5 \times 9.81}{13 \times 60} = 0.30 \text{m}^3/\text{min}$$

$$WDF_{T10} = \frac{48 \times 1 \times 0.5 \times 9.81}{10 \times 60} = 0.39 \text{m}^3/\text{min}$$

$$\text{Mean } WDF_{16.75} = 0.27 \text{m}^3/\text{min}$$

The results in Tables(5 and 6)obtained that water movement in irrigation canal were affected by water driving forces $0.24\text{m}^3/34.75$ minute in the first season and $0.23\text{m}^3/37.50$ minute in the second season led to water losses amount 35.11 and 37.39m^3 in the two seasons, respectively by using two irrigation canal in the field. These results may be attributed to the effect of water driving forces on water movement in irrigation canal, consequently water losses amount were affected. These results are in parallel line with those reported by (Ashton and Hope 2001) and Sen et al., (2018).

Table (5): Water losses amount of two irrigation canal in the first season

planting irrigation		first irrigation		second irrigation		third irrigation	
QE = 70.13m ³	Water movement	QE = 59.64m ³	Water movement	QE = 55.38m ³	Water movement	QE = 51.86m ³	Water movement
QF = 54.72m ³	time (49 minute)	QF = 51.19m ³	time (36 minute)	QF = 48.43m ³	time (29 minute)	QF = 47.56m ³	time (25 minute)

$$\text{Total QE} = 237.01 \text{m}^3$$

$$\text{Total QF} = 201.90 \text{m}^3$$

$$\text{Water losses amount, Total (QE – QF)} = 35.11 \text{m}^3$$

$$\text{Water driving forces} = \frac{L \times W \times h \times g}{T}$$

$$WDF_{T49} = \frac{(48 \times 1 \times 0.5 \times 9.81) \times 2}{49 \times 60} = 0.16 \text{m}^3/\text{min}$$

$$WDF_{T36} = \frac{(48 \times 1 \times 0.5 \times 9.81) \times 2}{36 \times 60} = 0.22 \text{m}^3/\text{min}$$

$$WDF_{T29} = \frac{(48 \times 1 \times 0.5 \times 9.81) \times 2}{29 \times 60} = 0.27 \text{m}^3/\text{min}$$

$$WDF_{T25} = \frac{(48 \times 1 \times 0.5 \times 9.81) \times 2}{25 \times 60} = 0.31 \text{m}^3/\text{min}$$

$$\text{Mean } WDF_{34.75} = 0.24 \text{m}^3/\text{min}$$

Table (6): Water losses amount of two irrigation canal in the second season

planting irrigation		first irrigation		second irrigation		third irrigation	
QE = 71.92m ³	Water movement	QE = 61.86m ³	Water movement	QE = 56.18m ³	Water movement	QE = 49.74m ³	Water movement
QF = 56.46m ³	time (54 minute)	QF = 50.37m ³	time (40 minute)	QF = 48.92m ³	time (29 minute)	QF = 46.56m ³	time (27 minute)

$$\text{Total QE} = 239.70 \text{m}^3$$

$$\text{Total QF} = 202.31 \text{m}^3$$

$$\text{Water losses amount, Total (QE – QF)} = 37.39 \text{m}^3$$

$$\text{Water driving forces} = \frac{L \times W \times h \times g}{T}$$

$$\text{WDF}_{T54} = \frac{(48 \times 1 \times 0.5 \times 9.81) \times 2}{54 \times 60} = 0.15 \text{m}^3/\text{min}$$

$$\text{WDF}_{T40} = \frac{(48 \times 1 \times 0.5 \times 9.81) \times 2}{40 \times 60} = 0.20 \text{m}^3/\text{min}$$

$$\text{WDF}_{T29} = \frac{(48 \times 1 \times 0.5 \times 9.81) \times 2}{29 \times 60} = 0.27 \text{m}^3/\text{min}$$

$$\text{WDF}_{T27} = \frac{(48 \times 1 \times 0.5 \times 9.81) \times 2}{27 \times 60} = 0.29 \text{m}^3/\text{min}$$

$$\text{Mean WDF}_{37.50} = 0.23 \text{m}^3/\text{min}$$

Across the two seasons, the results in Table (7) referred that mean water driving forces (WDF) in the case of using one irrigation canal in field ($0.26 \text{m}^3/16.63 \text{minute}$) was greater than its corresponding value under two irrigation canals ($0.24 \text{m}^3/36.13 \text{minute}$) while water losses amount (WLA) from one irrigation canal (17.78m^3) was less than its value under the two irrigation canals (36.25m^3) by saving 18.47m^3 of water. Consequently, there is an inverse relationship between water driving forces in irrigation canals in the field and water losses amount. On the other hand, grain yield in the case of using one irrigation canal in field was 3.394ton/fed , while the productivity in the two irrigation canals was 3.276ton/fed by increasing 118kg/fed , which may be increase the total production of wheat cultivated area in Egypt by surface irrigation.

Table (7): Mean water driving forces, water losses amount, water saving and grain yield across the two seasons.

Season	In the case of one irrigation canal		In the case of two irrigation canal		Grain yield		Saving water
	Water driving forces	Water losses amount	Water driving forces	Water losses amount	one irrigation canal	two irrigation canal	
First	$0.25 \text{m}^3/16.50 \text{min}$	17.26m^3	$0.24 \text{m}^3/34.75 \text{min}$	35.11m^3	3.271ton/fed	3.145ton/fed	18.47m^3
Second	$0.27 \text{m}^3/16.75 \text{min}$	18.29m^3	$0.23 \text{m}^3/37.50 \text{min}$	37.39m^3	3.517ton/fed	3.407ton/fed	
Mean	$0.26 \text{m}^3/16.63 \text{min}$	17.78m^3	$0.24 \text{m}^3/36.13 \text{min}$	36.25m^3	3.394ton/fed	3.276ton/fed	

Linear, quadratic and Logarithmic models were fitted to describe the relationship between Water Driving Force (WDF) and Water Movement Time (WMT) under the four irrigations during the two wheat growing seasons. Three statistical bases were considered to compare the three models i.e. coefficient of determination (R^2), standard error of estimate (SE) and the significance of the model. The significant model which had the highest R^2 and lowest SE was the best model fitted to the yield data.

The results in Table (8) indicated clearly that the highest value of the coefficient of determination (R^2) was in favor of the significant quadratic model

in addition to its lower SE confirmed that the quadratic model is the best model compared to others. Accordingly, the Critical Water Movement Time (CWMT) which reflects the lowest WDF value was calculated as follows:

$$\text{Critical Water Movement Time (CWMT)} = \frac{-b}{2c} = \frac{-(-0.04672)}{2(0.000914)} = 25.55 \text{ min.}$$

This means that Water Driving Force (WDF) did not decreased significantly after 25.55 minutes. On the other hand, the Water Driving Force (WDF) at the moment it enters the irrigation canal (WMT = zero) would equals 0.7596 m³/min.

Table (8): Linear, quadratic and logarithmic models describing the relationship between Water Driving Force (WDF) and Water Movement Time (WMT) under the four irrigations during the two wheat growing seasons.

Regression models	Regression equation	R ² %	SE	Sig.
Linear	WDF = -0.0145 + 0.4995 WMT	93.5	0.022	000
Quadratic	WDF = 0.7596 - 0.0467 WMT + 0.0009 WMT ²	99.6	0.006	000
Logarithmic	WDF = 0.94 - 0.247 ln (WMT)	98.2	0.012	000

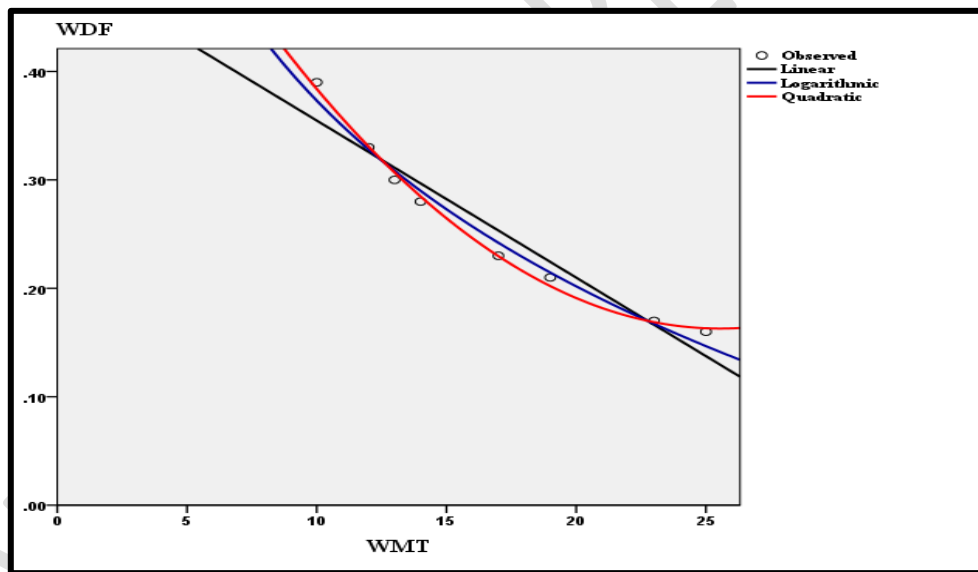


Fig 4. Graphical presentation of Linear, quadratic and logarithmic models for describing the relationship between Water Driving Force (WDF) and Water Movement Time (WMT)

CONCLUSION

The present research showed that there is an inverse relationship between water driving forces in irrigation canals in the field and water losses amount. When water driving forces increased, the water losses amount are reduced which partially saved irrigation water under the case of surface irrigation. This amount of water abundance can be used to irrigate new lands.

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