Original Research Article

ESTIMATION OF STRESS STRENGTH RELIABILITY $P[Y < X < Z]$ OF LOMAX DISTRIBUTION UNDER DIFFERENT SAMPLING SCHEME

Abstract

This paper deals with estimating the stress strength reliability for a component with a strength independent of opposite lower and upper bound stresses when the stresses and strength have Lomax Distribution under different sampling schemes. Shrinkage maximum likelihood estimate and Quasi likelihood estimate are obtained both under complete and right censored data. We have considered the asymptotic confidence interval (CI) based on MLE, and bootstrap CI for R. Monte Carlo simulation experiments were performed to compare the performance of estimates obtained.

Keywords: Lomax distribution; Stress Strength reliability; Maximum likelihood estimator; Quasi likelihood estimator; Confidence interval

2010 Mathematics Subject Classification: 53C25; 83C05; 57N16

1 Introduction

The stress strength model plays an important role in reliability analysis. The term stress strength was first introduced by (8). In the context of reliability, R is defined as the probability that the unit strength is greater than stress, that is $R = P[X > Y]$ where X is the random strength of the unit and Y is the instant stress applied to it. Moreover R provides the probability of system failure. The stress strength model was discussed in the literature from different point of view. Inference for generalized Lomax Distribution based on record statistics was considered by (31). (28) studied the inference for the Lomax Distribution stress- strength model. (2) studied exponentiated Lomax Distribution.. The different stress strength model was considered by (4), (6), (20), (33). Estimation of $R = P\left[X > Y\right]$ for Lomax Distribution with the presence of outliers was discussed by (29). (30) studied the Power of Lomax Distribution with an application to bladder cancer data. The recent developments in stress strength reliability was discussed by (40), (5), (1), (17), (32), (24), (25), (7) and (26). In this paper estimates the stress strength reliability for a component with a strength independent of opposite lower and upper bound stresses when the stresses and strength have Lomax distribution under different sampling schemes. Shrinkage maximum likelihood estimate and Quasi likelihood estimate are obtained under complete and right censored data. We have considered the asymptotic confidence interval (CI) based on MLE and bootstrap CI for R. Monte Carlo simulation experiments were performed to compare the performance of estimates obtained

As a natural extension of the two component stress strength model we consider in the present paper the Maximum Likelihood Estimation (MLE) and Quasi likelihood estimate of stress strength reliability model $R = P\left[Y < X < Z\right]$, where X is the random strength and Y and Z are independent random stress variables follows Lomax Distribution. The stress strength model of $P\left[Y < X < Z\right]$ was studied in many branches of science such as Psychology, Medicine, Pedagogy, Engineering etc. The Estimation of $R = P\left[Y < X < Z\right]$ represents the situation where the strength X should be greater than stress Y and smaller than stress Z. For eg:- Many devices cannot function at high temperatures; neither can very low ones. Similarly a person's blood pressure should lie within two limits i.e systolic and diastolic. For instance many electronic components cannot work at high or low voltage. The estimate and the asymptotic confidence intervals are obtained for R under both complete and censored samples.

The Minimum Variance Unbiased (MVU), Maximum Likelihood and Empirical Estimator of $R = P\left[Y < X < Z\right]$ were discussed by (34). (10) deal with the estimation of R when Y, Z and X are exponential random variables. Maximum Likelihood Estimate and Uniformly Minimum Variance Unbiased Estimate of R when Y, Z and X either uniform or exponential random variable with the unknown location parameter was considered by (15). (14) focused on the estimate of $R = P\left[Y < X < Z\right]$, where Y and Z be a random stresses, and X be a random strength, having Weibull distribution in presence of k outliers. (13) focused on the estimate of $R = P\left[Y < X < Z\right]$,when Y, Z and X are independent and that these stress and strength variable follows Kumaraswamy Distribution. (16) discuss the estimation of Stress–Strength Reliability for $P\left[Y < X < Z\right]$ using Dagum Distribution. (19) the reliability of one strength- four stresses for Lomax Distribution was studied. Shrinkage estimation of stress strength reliability $P\left[Y < X < Z\right]$ for Lomax Distribution based on records was studied by (23).

A shrinkage estimator is a new estimate produced by shrinking a raw estimate. (37), (22) have given shrinkage estimates for population mean. (3) has found the shrinkage estimate of the parameters of exponential distribution. (35), (41), (27) obtain the Shrinkage estimation in the context of exponential distribution. (12) obtained the shrinkage estimator of stress strength reliability $R = P\left[X < Y\right]$ when X and Y are geometric distributions using record values.

The remaining part of this paper is organized in to seven sections. In section 3, we estimate the shrinkage estimate of R under complete sample scheme. In section 4, we estimate the shrinkage estimate of R based on right censored sample. Section 5 and 6 discuss the shrinkage estimate of the quasi likelihood function based on complete and censored sample. In Section 7 we illustrate estimator's performance by a simulation study, and finally, in Section 8, conclusions are made.

2 Preliminary

Let X be the life of a device having an exponential distribution with a failure rate λ. It ia assumed that there could be some variation in λ value because of small fluctuations in the manufacturing tolerance (see (21)). This fluctuation is accommodated by assuming that λ have a gamma distribution with probability density function

$$f(\lambda|\alpha,\sigma) = \frac{\sigma^{\alpha}}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\sigma\lambda}, \lambda \geq 0 \tag{2.1}$$

Then the density of X is obtained as

$$f(x) = \int_0^{\infty} \lambda e^{-\lambda x}\frac{\sigma^{\alpha}}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\sigma\lambda}d\lambda = \frac{\alpha}{\sigma}\left(1+\frac{x}{\sigma}\right)^{-(\alpha+1)}; x,\alpha,\sigma > 0 \tag{2.2}$$

which is Lomax Distribution. In other words, equipment is tested in the laboratory or ideal environment following exponential distribution when worked in the real environment, which is lighter or harsher than the laboratory environment, follows Lomax Distribution. So it is very important to consider the estimation problem of $P(Y < X < Z)$ when the underlying distribution follows Lomax Distribution.

Now let X be the strength of the random variable following Lomax distribution with parameters $L(\alpha_1,\lambda)$, where α_1 is the shape parameter and λ is scale parameter and Y and Z be the stress of the random variable following Lomax distribution with parameter $L(\alpha_2,\lambda)$ and $L(\alpha_3,\lambda)$ corresponding probability density functions are given below.

$$f(x,\alpha_1,\lambda) = \frac{\alpha_1\lambda^{\alpha_1}}{(x+\lambda)^{\alpha_1+1}}; x > 0, \alpha_1 > 0, \lambda > 0 \tag{2.3}$$

$$f(y,\alpha_2,\lambda) = \frac{\alpha_2\lambda^{\alpha_2}}{(y+\lambda)^{\alpha_2+1}}; y > 0, \alpha_2 > 0, \lambda > 0 \tag{2.4}$$

$$f(z,\alpha_3,\lambda) = \frac{\alpha_3\lambda^{\alpha_3}}{(y+\lambda)^{\alpha_3+1}}; z > 0, \alpha_3 > 0, \lambda > 0 \tag{2.5}$$

Under this situation the stress strength reliability

$$\begin{aligned} R = P[Y < X < Z] &= \int_0^{\infty} F_y(x)\overline{F}_z(x)f(x)dx = \int_0^{\infty} F_y(x)[1-F_z(x)]f(x)dx \\ &= \frac{\alpha_1\alpha_2}{(\alpha_1+\alpha_3)(\alpha_1+\alpha_2+\alpha_3)}; 0 < R < 1 \end{aligned} \tag{2.6}$$

In the present paper we assumes that the scale parameter λ which is common for all the three variables is known.

3 Maximum Likelihood Estimation of R based on complete sample

Let $\underline{x} = (x_1, x_2, ..., x_{n_1})$ be the random sample of n_1 observation taken from Lomax distribution $L(\alpha_1,\lambda)$ then its likelihood function is given by

$$L(\underline{x}|\alpha_1,\lambda) = \prod_{i=1}^{n_1}\frac{\alpha_1\lambda^{\alpha_1}}{(x_i+\lambda)^{\alpha_1+1}} = \alpha_1^{n_1}\lambda^{n_1\alpha_1}\prod_{i=1}^{n_1}(x_i+\lambda)^{-(\alpha_1+1)} \tag{3.1}$$

Let $\underline{y} = (y_1, y_2, ..., y_{n_2})$ be the random sample of n_2 observation taken from Lomax Distribution $L(\alpha_2,\lambda)$ then its likelihood function is given by

$$L(\underline{y}|\alpha_2,\lambda) = \prod_{j=1}^{n_2}\frac{\alpha_2\lambda^{\alpha_2}}{(y_j+\lambda)^{\alpha_2+1}} = \alpha_2^{n_2}\lambda^{n_2\alpha_2}\prod_{j=1}^{n_2}(y_j+\lambda)^{-(\alpha_2+1)} \tag{3.2}$$

Let $\underline{z} = (z_1, z_2, ..., z_{n_3})$ be the random sample of n_3 observation taken from Lomax Distribution $L(\alpha_3, \lambda)$ then its likelihood function is given by

$$L(\underline{z}|\alpha_3, \lambda) = \prod_{k=1}^{n_3} \frac{\alpha_3 \lambda^{\alpha_3}}{(z_k + \lambda)^{\alpha_3+1}} = \alpha_3^{n_3} \lambda^{n_3\alpha_3} \prod_{k=1}^{n_3} (z_k + \lambda)^{-(\alpha_3+1)} \tag{3.3}$$

The joint likelihood function is given by

$$\begin{aligned} L\left(\underline{x}, \underline{y}, \underline{z}|\alpha_1, \alpha_2, \alpha_3, \lambda\right) =& \alpha_1^{n_1} \lambda^{n_1\alpha_1} \prod_{i=1}^{n_1} (x_i + \lambda)^{-(\alpha_1+1)} \alpha_2^{n_2} \lambda^{n_2\alpha_2} \prod_{j=1}^{n_2} (y_j + \lambda)^{-(\alpha_2+1)} \\ &\times \alpha_3^{n_3} \lambda^{n_3\alpha_3} \prod_{k=1}^{n_3} (z_k + \lambda)^{-(\alpha_3+1)} \end{aligned} \tag{3.4}$$

Taking Logarithm on both side of (3.4) we get

$$\begin{aligned} logL =& n_1 log\alpha_1 + n_1\alpha_1 log\lambda - (\alpha_1 + 1) \sum_{i=1}^{n_1} log(x_i + \lambda) + n_2 log\alpha_2 + n_2\alpha_2 log\lambda - (\alpha_2 + 1) \sum_{j=1}^{n_2} log(y_j + \lambda) \\ &+ n_3 log\alpha_3 + n_3\alpha_3 log\lambda - (\alpha_3 + 1) \sum_{k=1}^{n_3} log(z_k + \lambda) \end{aligned} \tag{3.5}$$

Differentiating (3.5) partially with respect to α_1, α_2 and α_3 and equating to zero we get the MLE of α_1, α_2 and α_3 as

$$\hat{\alpha_1}_{mle} = \frac{n_1}{\sum_{i=1}^{n_1} log\left(1 + \frac{x_i}{\lambda}\right)} \tag{3.6}$$

$$\hat{\alpha_2}_{mle} = \frac{n_2}{\sum_{j=1}^{n_2} log\left(1 + \frac{y_j}{\lambda}\right)} \tag{3.7}$$

$$\hat{\alpha_3}_{mle} = \frac{n_3}{\sum_{k=1}^{n_3} log\left(1 + \frac{z_k}{\lambda}\right)} \tag{3.8}$$

Substituting this in (2.6) we obtained the MLE of R as

$$\hat{R}_{mle} = \frac{\hat{\alpha_1}\hat{\alpha_2}}{(\hat{\alpha_1} + \hat{\alpha_3})(\hat{\alpha_1} + \hat{\alpha_2} + \hat{\alpha_3})}; 0 < R < 1 \tag{3.9}$$

From the above expression, it is very difficult to find the exact variance and distribution of $\hat{R}_{mle}$. So we use the multivariate delta method (See (38), (36), (9), (18)) to find the approximate estimate of the asymptotic variance of $\hat{R}_{mle}$ which is given as

Let the Fisher Information matrix $\emptyset$

$$\emptyset(\alpha_1, \alpha_2, \alpha_3) = \begin{bmatrix} E\left(\frac{-\partial^2 lnL}{\partial\alpha_1^2}\right) & E\left(\frac{-\partial^2 lnL}{\partial\alpha_1\partial\alpha_2}\right) & E\left(\frac{-\partial^2 lnL}{\partial\alpha_1\partial\alpha_3}\right) \\ E\left(\frac{-\partial^2 lnL}{\partial\alpha_2\partial\alpha_1}\right) & E\left(\frac{-\partial^2 lnL}{\partial\alpha_2^2}\right) & E\left(\frac{-\partial^2 lnL}{\partial\alpha_2\partial\alpha_3}\right) \\ E\left(\frac{-\partial^2 lnL}{\partial\alpha_3\partial\alpha_1}\right) & E\left(\frac{-\partial^2 lnL}{\partial\alpha_3\partial\alpha_2}\right) & E\left(\frac{-\partial^2 lnL}{\partial\alpha_3^2}\right) \end{bmatrix} \tag{3.10}$$

$$B' = \begin{bmatrix} \frac{\partial R}{\partial\alpha_1} & \frac{\partial R}{\partial\alpha_2} & \frac{\partial R}{\partial\alpha_3} \end{bmatrix} = \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix} \tag{3.11}$$

Then $\sigma_R^2 = V(R) = B'\emptyset^{-1}B$. In this case

$$\emptyset(\alpha_1, \alpha_2, \alpha_3) = \begin{bmatrix} \frac{n_1}{\alpha_1^2} & 0 & 0 \\ 0 & \frac{n_2}{\alpha_2^2} & 0 \\ 0 & 0 & \frac{n_3}{\alpha_3^2} \end{bmatrix}$$

So

$$\O^{-1} = \begin{bmatrix} \frac{\alpha_1^2}{n_1} & 0 & 0 \\ 0 & \frac{\alpha_2^2}{n_2} & 0 \\ 0 & 0 & \frac{\alpha_3^2}{n_3} \end{bmatrix}$$

Also

$$b_1 = \frac{\partial R}{\partial \alpha_1} = \frac{-\alpha_2\left(\alpha_1^2 - \alpha_2\alpha_3 - \alpha_2^2\right)}{\left(\alpha_1+\alpha_3\right)^2\left(\alpha_1+\alpha_2+\alpha_3\right)^2} \tag{3.12}$$

$$b_2 = \frac{\partial R}{\partial \alpha_2} = \frac{\alpha_1}{\left(\alpha_1+\alpha_2+\alpha_3\right)^2} \tag{3.13}$$

and

$$b_3 = \frac{\partial R}{\partial \alpha_3} = \frac{-\alpha_1\alpha_2(2\alpha_1+\alpha_2+2\alpha_3)}{\left(\alpha_1+\alpha_3\right)^2\left(\alpha_1+\alpha_2+\alpha_3\right)^2} \tag{3.14}$$

Then

$$\sigma^2_{Rmle} = V\left(R\right) = B^{'}\O^{-1}B = \frac{b_1^2\alpha_1^2}{n_1} + \frac{b_2^2\alpha_2^2}{n_2} + \frac{b_3^2\alpha_3^2}{n_3} \tag{3.15}$$

By replacing the parameters with their maximum likelihood estimate we get the estimate $\hat{\sigma}^2_{Rmle}$ of σ^2_{Rmle}. In this case the asymptotic distribution of $\hat{R}_{mle}$ is $N\left(R, \hat{\sigma}^2_{Rmle}\right)$.
Based on this asymptotic distribution a $100\left(1-\gamma\right)\%$ asymptotic CI for R is $\hat{R}_{mle} \pm Z_{\gamma/2}\hat{\sigma}_{Rmle}$. Where $Z_{\gamma/2}$ denotes the table value corresponding to $\gamma/2$ of N(0,1).

3.1 Shrinkage Estimation with Constant shrinkage factor

In this case we obtain the shrinkage estimate,

$$\hat{\beta}_{sh} = \psi\left(\hat{\beta}\right)\hat{\beta}_{ub} + \left(1-\psi\left(\hat{\beta}\right)\right)\hat{\beta}_0$$

with $\psi\left(\hat{\beta}\right) = 0.01$ the constant shrinkage weight factor suggested by (13) this leads the Shrinkage estimates of α_1, α_2 and α_3 as

$$\hat{\alpha 1}_{sh} = 0.01\hat{\alpha 1}_{ub} + 0.99\hat{\alpha 1}_0 \tag{3.16}$$

$$\hat{\alpha 2}_{sh} = 0.01\hat{\alpha 2}_{ub} + 0.99\hat{\alpha 2}_0 \tag{3.17}$$

and

$$\hat{\alpha 3}_{sh} = 0.01\hat{\alpha 3}_{ub} + 0.99\hat{\alpha 3}_0 \tag{3.18}$$

where $\hat{\alpha 1}_{ub} = \frac{n_1-1}{n_1\bar{x}}$, $\hat{\alpha 2}_{ub} = \frac{n_2-1}{n_2\bar{y}}$ and $\hat{\alpha 3}_{ub} = \frac{n_3-1}{n_3\bar{z}}$. $\hat{\alpha 1}_0$, $\hat{\alpha 2}_0$ and $\hat{\alpha 3}_0$ is taken as the boot strap estimate of α_1, α_2 and α_3.
This leads to the constant shrinkage weight factor of R as

$$\hat{R}_{sh} = \frac{\hat{\alpha 1}_{sh}\hat{\alpha 2}_{sh}}{\left(\hat{\alpha 1}_{sh} + \hat{\alpha 2}_{sh} + \hat{\alpha 3}_{sh}\right)\left(\hat{\alpha 1}_{sh} + \hat{\alpha 3}_{sh}\right)} \tag{3.19}$$

3.2 The modified Thompson type shrinkage estimator

Here we use two type of shrinkage estimate first one the modified Thompson type shrinkage weight factor and Shrinkage weight factor suggested by (22) to find out the shrinkage estimator.
(a) Suggested by (13) here we take the weight factor as

$$\phi\left(\hat{R}\right) = \frac{\hat{R}_{ub} - \hat{R}_0}{\left(\hat{R}_{ub} - \hat{R}_0\right)^2 + var\left(\hat{R}_{ub}\right)}\left(0.001\right) \tag{3.20}$$

where $\hat{R}_{ub} = \frac{\hat{\alpha 1}_{ub}\hat{\alpha 2}_{ub}}{(\hat{\alpha 1}_{ub}+\hat{\alpha 2}_{ub}+\hat{\alpha 3}_{ub})(\hat{\alpha 1}_{ub}+\hat{\alpha 3}_{ub})}$ and $var\left(\hat{R}_{ub}\right)$ is as defined in (3.15). So the modified Thomason type shrinkage estimator will be

$$\hat{R}_{Th} = \phi\left(\hat{R}\right)\hat{R}_{ub} + \left(1-\phi\left(\hat{R}\right)\right)\hat{R}_0 \tag{3.21}$$

(b) Shrinkage weight factor suggested by (22) here we take the weight factor as

$$\varphi\left(\hat{R}\right) = a.exp\left\{-\frac{b\left(\hat{R}_{ub}-\hat{R}_0\right)^2}{var\left(\hat{R}_{ub}\right)}\right\} \tag{3.22}$$

where $0 < a < 1$ and $b > 0$. So the modified Thomason type shrinkage estimator will be

$$\hat{R}_{MS} = \varphi\left(\hat{R}\right)\hat{R}_{ub} + \left(1-\varphi\left(\hat{R}\right)\right)\hat{R}_0 \tag{3.23}$$

4 Maximum Likelihood Estimation of R based on right censored sample

In this section we obtained the maximum likelihood estimate when the data on the stress is only is right censored. Let us consider a right censored sample $\underline{x} = \left(x_1, x_2, ..., x_{(n_1-k)}\right)$ with k observations censored on right from Lomax distribution $L\left(\alpha_1, \lambda\right)$ then its likelihood function is given by

$$L\left(\underline{x}|\alpha_1,\lambda\right) = \left[1-F_{(n_1-k)}\right]^k \prod_{i=1}^{n_1-k} f\left(x_i\right) = \alpha_1^{(n_1-k)}\lambda^{(n_1-k)\alpha_1}\left[1+\frac{x_{(n_1-k)}}{\lambda}\right]^{-\alpha_1 k}\prod_{i=1}^{n_1-k}\frac{1}{\left(x_i+\lambda\right)^{\alpha_1+1}} \tag{4.1}$$

Then using (3.2), (3.3) and (4.1) the joint likelihood function can be written as

$$\begin{aligned} L\left(\underline{x},\underline{y},\underline{z}|\alpha_1,\alpha_2,\alpha_3,\lambda\right) =& \alpha_1^{(n_1-k)}\lambda^{(n_1-k)\alpha_1}\left[1+\frac{x_{(n_1-k)}}{\lambda}\right]^{-\alpha_1 k} \\ & \prod_{i=1}^{n_1-k}\frac{1}{\left(x_i+\lambda\right)^{\alpha_1+1}}\alpha_2^{n_2}\lambda^{n_2\alpha_2}\prod_{j=1}^{n_2}\left(y_j+\lambda\right)^{-(\alpha_2+1)}\alpha_3^{n_3}\lambda^{n_3\alpha_3}\times\prod_{k=1}^{n_3}\left(z_k+\lambda\right)^{-(\alpha_3+1)} \end{aligned} \tag{4.2}$$

Taking Logarithm on both side of (4.2) we get

$$\begin{aligned} logL =& -\alpha_1 k.log\left[1+\frac{x_{(n_1-k)}}{\lambda}\right] + \left(n_1-k\right)log\alpha_1 + \alpha_1\left(n_1-k\right)log\lambda - \left(\alpha_1+1\right)\sum_{i=1}^{n_1-k}log\left(x_i+\lambda\right) \\ & + n_2 log\alpha_2 + n_2\alpha_2 log\lambda - \left(\alpha_2+1\right)\sum_{j=1}^{n_2}log\left(y_j+\lambda\right) \\ & + n_3 log\alpha_3 + n_3\alpha_3 log\lambda - \left(\alpha_3+1\right)\sum_{k=1}^{n_3}log\left(z_k+\lambda\right) \end{aligned} \tag{4.3}$$

From (4.3) we get the MLE of α_1, α_2 and α_3 as

$$\hat{\alpha}_{1mlec} = \frac{n_1-k}{\sum_{i=1}^{n_1-k}log\left(1+\frac{x_i}{\lambda}\right)+klog\left(1+\frac{x_{(n_1-k)}}{\lambda}\right)} \tag{4.4}$$

$$\hat{\alpha}_{2mlec} = \frac{n_2}{\sum_{j=1}^{n_2} log\left(1+\frac{y_j}{\lambda}\right)} \tag{4.5}$$

and

$$\hat{\alpha}_{3mlec} = \frac{n_3}{\sum_{k=1}^{n_3} log\left(1+\frac{z_k}{\lambda}\right)} \tag{4.6}$$

So using (4.4), (4.5) and (4.6) the MLE of R can be written as

$$\hat{R}_{mlec} = \frac{\hat{\alpha_1}\hat{\alpha_2}}{\left(\hat{\alpha_1}+\hat{\alpha_3}\right)\left(\hat{\alpha_1}+\hat{\alpha_2}+\hat{\alpha_3}\right)}; 0<R<1 \tag{4.7}$$

In this case

$$\varnothing^{-1} = \begin{bmatrix} \frac{\alpha_1^2}{n_1-k} & 0 & 0 \\ 0 & \frac{\alpha_2^2}{n_2} & 0 \\ 0 & 0 & \frac{\alpha_3^2}{n_3} \end{bmatrix} \tag{4.8}$$

Now using (3.12), (3.13), (3.14) and (4.8) we have

$$\sigma^2_{Rmlec} = V(R) = B^1\varnothing^{-1}B = \frac{b_1^2\alpha_1^2}{n_1-k} + \frac{b_2^2\alpha_2^2}{n_2} + \frac{b_3^2\alpha_3^2}{n_3} \tag{4.9}$$

By replacing the parameters with their maximum likelihood estimate we get the estimate $\hat{\sigma}^2_{Rmlec}$ of σ^2_{Rmlec}. In this case he asymptotic distribution of $\hat{R}_{mlec}$ is $N(R, \hat{\sigma}^2_{Rmlec})$.Based on this asymptotic distribution a $100\left(1-\gamma\right)\%$ asymptotic CI for R is $\hat{R}_{mlec} \pm Z_{\gamma/2}\hat{\sigma}_{Rmlec}$.

4.1 Shrinkage Estimation with Constant shrinkage factor

In this case we obtain the shrinkage estimate, with $\psi\left(\hat{\beta}\right) = 0.01$ the constant shrinkage weight factor leads the Shrinkage estimates of α_1, α_2 and α_3 as

$$\hat{\alpha 1}_{shc} = 0.01\hat{\alpha 1}_{ub} + 0.99\hat{\alpha 1}_1 \tag{4.10}$$

$$\hat{\alpha 2}_{sh} = 0.01\hat{\alpha 2}_{ub} + 0.99\hat{\alpha 2}_1 \tag{4.11}$$

and

$$\hat{\alpha 3}_{sh} = 0.01\hat{\alpha 3}_{ub} + 0.99\hat{\alpha 3}_1 \tag{4.12}$$

where $\hat{\alpha 1}_{ub} = \frac{n_1-1}{n_1\bar{x}}$, $\hat{\alpha 2}_{ub} = \frac{n_2-1}{n_2\bar{y}}$ and $\hat{\alpha 3}_{ub} = \frac{n_3-1}{n_3\bar{z}}$. $\hat{\alpha 1}_1$, $\hat{\alpha 2}_1$ and $\hat{\alpha 3}_1$ is taken as the boot strap estimate of $\hat{\alpha 1}_{mlec}$, $\hat{\alpha 2}_{mlec}$ and $\hat{\alpha 3}_{mlec}$. This leads to the constant shrinkage weight factor of R as

$$\hat{R}_{shc} = \frac{\hat{\alpha 1}_{shc}\hat{\alpha 2}_{shc}}{\left(\hat{\alpha 1}_{shc}+\hat{\alpha 2}_{shc}+\hat{\alpha 3}_{shc}\right)\left(\hat{\alpha 1}_{shc}+\hat{\alpha 3}_{shc}\right)} \tag{4.13}$$

4.2 The modified Thompson type shrinkage estimator

The modified Thompson type shrinkage weight factor estimates suggested by (13) and (22) are
(a) $\phi\left(\hat{R}\right) = \frac{\hat{R}_{ub}-\hat{R}_{shc}}{\left(\hat{R}_{ub}-\hat{R}_{shc}\right)^2+var\left(\hat{R}_{ub}\right)}\left(0.001\right)$ (4.14)

where $\hat{R}_{ub} = \frac{\hat{\alpha 1}_{ub}\hat{\alpha 2}_{ub}}{\left(\hat{\alpha 1}_{ub}+\hat{\alpha 2}_{ub}+\hat{\alpha 3}_{ub}\right)\left(\hat{\alpha 1}_{ub}+\hat{\alpha 3}_{ub}\right)}$ and $var\left(\hat{R}_{ub}\right)$ is as defined in (4.9). So the modified Thomason type shrinkage estimator will be

$$\hat{R}_{Th} = \phi\left(\hat{R}\right)\hat{R}_{ub} + \left(1-\phi\left(\hat{R}\right)\right)\hat{R}_{shc} \tag{4.15}$$

(b) $\varphi\left(\hat{R}\right) = a.exp\left\{-\frac{b\left(\hat{R}_{ub}-\hat{R}_{shc}\right)^2}{var\left(\hat{R}_{ub}\right)}\right\}$ (4.16)
where $0 < a < 1$ and $b > 0$. So the modified Thomason type shrinkage estimator will be

$$\hat{R}_{MS} = \varphi\left(\hat{R}\right)\hat{R}_{ub} + \left(1-\varphi\left(\hat{R}\right)\right)\hat{R}_{shc} \tag{4.17}$$

5 Quasi Likelihood Estimation of R based on complete sample

In this section, we derived the maximum quasi-likelihood estimates for R. The quasi-likelihood function was introduced by (39) to be used for estimating the unknown parameters in generalized linear models when only the mean-variance relationship is specified. Wedderburn defined the quasi- function as

$$Q\left(x,\mu\right) = \int_{\mu} \frac{x-\mu}{V\left(\mu\right)} d\mu + o\left(x\right) \tag{5.1}$$

where $\mu = E\left(x\right)$, $V\left(\mu\right) = Var\left(x\right)$ and $o\left(x\right)$ is some function of x only. The variance assumption is generalized to $Var\left(x\right) = \phi V\left(\mu\right)$ where the variance function $V\left(.\right)$ is assumed to be known and the parameter ϕ may be unknown. The quasi-likelihood function has properties similar to those of the log-likelihood function. Let $\underline{x} = \left(x_1, x_2, ..., x_{n_1}\right)$ be the random sample of n_1 observation taken from Lomax distribution $L\left(\alpha_1, \lambda\right)$ then its Quasi Likelihood function is given by

$$Q\left(x_i, \alpha_1, \lambda\right) = n_1 log\left(\frac{\alpha_1 - 1}{\lambda}\right) - \nu\left(\frac{\alpha_1 - 1}{\lambda}\right) \tag{5.2}$$

where $\nu = \sum_{i=1}^{n_1} x_i$.

The natural exponent of $Q\left(x_i, \alpha_1, \lambda\right)$ as the as taken as the Quasi likelihood function and is given by

$$L\left(\underline{x}|\alpha_1, \lambda\right) = \left(\frac{\alpha_1 - 1}{\lambda}\right)^{n_1} e^{-\left(\frac{\alpha_1-1}{\lambda}\right)\nu}; \alpha_1 > 0, \nu = \sum_{i=1}^{n_1} x_i \tag{5.3}$$

Similar based on the sample $\underline{y} = \left(y_1, y_2, ..., y_{n_2}\right)$ and $\underline{z} = \left(z_1, z_2, ..., z_{n_3}\right)$ the Quasi likelihood function of Y and Z is given by

$$L\left(\underline{y}|\alpha_2, \lambda\right) = \left(\frac{\alpha_2 - 1}{\lambda}\right)^{n_2} e^{-\left(\frac{\alpha_2-1}{\lambda}\right)\zeta}; \alpha_2 > 0, \zeta = \sum_{j=1}^{n_2} y_j \tag{5.4}$$

and

$$L\left(\underline{z}|\alpha_3, \lambda\right) = \left(\frac{\alpha_3 - 1}{\lambda}\right)^{n_3} e^{-\left(\frac{\alpha_3-1}{\lambda}\right)\beta}; \alpha_3 > 0, \beta = \sum_{k=1}^{n_3} z_k \tag{5.5}$$

So the joint quasi likelihood function can be written as

$$L\left(\underline{x}, \underline{y}, \underline{z}|\alpha_1, \alpha_2, \alpha_3, \lambda\right) = \left(\frac{\alpha_1 - 1}{\lambda}\right)^{n_1} e^{-\left(\frac{\alpha_1-1}{\lambda}\right)\nu}. \left(\frac{\alpha_2 - 1}{\lambda}\right)^{n_2} e^{-\left(\frac{\alpha_2-1}{\lambda}\right)\zeta} \left(\frac{\alpha_3 - 1}{\lambda}\right)^{n_3} e^{-\left(\frac{\alpha_3-1}{\lambda}\right)\beta} \tag{5.6}$$

From (5.6) the Quasi Likelihood Estimate of α_1, α_2, α_3 and R are obtained as

$$\hat{\alpha}_{1qmle} = 1 + \left(\frac{n_1\lambda}{\nu}\right) = 1 + \left(\frac{n_1\lambda}{\sum_{i=1}^{n_1} x_i}\right) \tag{5.7}$$

$$\hat{\alpha}_{2qmle} = 1 + \left(\frac{n_2\lambda}{\zeta}\right) = 1 + \left(\frac{n_2\lambda}{\sum_{j=1}^{n_2} y_j}\right) \tag{5.8}$$

$$\hat{\alpha}_{3qmle} = 1 + \left(\frac{n_3\lambda}{\beta}\right) = 1 + \left(\frac{n_3\lambda}{\sum_{k=1}^{n_3} z_k}\right) \tag{5.9}$$

and

$$\hat{R}_{qmle} = \frac{\hat{\alpha_1}\hat{\alpha_2}}{\left(\hat{\alpha_1}+\hat{\alpha_3}\right)\left(\hat{\alpha_1}+\hat{\alpha_2}+\hat{\alpha_3}\right)}; 0 < R < 1 \tag{5.10}$$

In this case

$$\varnothing^{-1} = \begin{bmatrix} \frac{(\alpha_1-1)^2}{n_1} & 0 & 0 \\ 0 & \frac{(\alpha_2-1)^2}{n_1} & 0 \\ 0 & 0 & \frac{(\alpha_3-1)^2}{n_3} \end{bmatrix} \tag{5.11}$$

Now using (3.12), (3.13), (3.14) and (5.11) we have

$$\sigma^2_{Rqmle} = V(R) = B^{'}\phi^{-1}B = \frac{b_1^2(\alpha_1-1)^2}{n_1} + \frac{b_2^2(\alpha_2-1)^2}{n_2} + \frac{b_3^2(\alpha_3-1)^2}{n_3} \tag{5.12}$$

By replacing the parameters with their maximum likelihood estimate we get the estimate $\hat{\sigma}^2_{Rqmle}$ of σ^2_{Rqmle}. In this case the asymptotic distribution of $\hat{R}_{qmle}$ is $N(R, \hat{\sigma}^2_{Rqmle})$. Based on this asymptotic distribution a $100(1-\gamma)\,\%$ asymptotic CI for R is $\hat{R}_{qmle} \pm Z_{\gamma/2}\hat{\sigma}_{Rqmle}$.

5.1 Shrinkage Estimates

In this case we obtain the different type of shrinkage estimates as,
(a) the constant weight shrinkage estimates with $\psi\left(\hat{\beta}\right) = 0.01$ as

$$\hat{R}_{shq} = \frac{\hat{\alpha 1_{shq}}\hat{\alpha 2_{shq}}}{\left(\hat{\alpha 1_{shq}}+\hat{\alpha 2_{shq}}+\hat{\alpha 3_{shq}}\right)\left(\hat{\alpha 1_{shq}}+\hat{\alpha 3_{shq}}\right)} \tag{5.13}$$

$$\hat{\alpha 1_{shq}} = 0.01\hat{\alpha 1_{ub}} + 0.99\hat{\alpha_{12}} \tag{5.14}$$

$$\hat{\alpha 2_{shq}} = 0.01\hat{\alpha 2_{ub}} + 0.99\hat{\alpha_{22}} \tag{5.15}$$

and

$$\hat{\alpha 3_{shq}} = 0.01\hat{\alpha 3_{ub}} + 0.99\hat{\alpha_{32}} \tag{5.16}$$

where $\hat{\alpha 1_{ub}} = \frac{n_1-1}{n_1\bar{x}}$, $\hat{\alpha 2_{ub}} = \frac{n_2-1}{n_2\bar{y}}$ and $\hat{\alpha 3_{ub}} = \frac{n_3-1}{n_3\bar{z}}$. $\hat{\alpha_{12}}$, $\hat{\alpha_{22}}$ and $\hat{\alpha_{32}}$ is taken as the boot strap estimate of $\hat{\alpha}_{1qmle}$, $\hat{\alpha}_{2qmle}$ and $\hat{\alpha}_{3qmle}$.

(b) Suggested by (13) here we take the weight factor as

$$\phi\left(\hat{R}\right) = \frac{\hat{R}_{ub} - \hat{R}_0}{\left(\hat{R}_{ub} - \hat{R}_{shq}\right)^2 + var\left(\hat{R}_{ub}\right)}(0.001) \tag{5.17}$$

where $\hat{R}_{ub} = \frac{\hat{\alpha 1_{ub}}\hat{\alpha 2_{ub}}}{\left(\hat{\alpha 1_{ub}}+\hat{\alpha 2_{ub}}+\hat{\alpha 3_{ub}}\right)\left(\hat{\alpha 1_{ub}}+\hat{\alpha 3_{ub}}\right)}$ and $var\left(\hat{R}_{ub}\right)$ is as defined in (5.12). So the modified Thomason type shrinkage estimator will be

$$\hat{R}_{Thq} = \phi\left(\hat{R}\right)\hat{R}_{ub} + \left(1 - \phi\left(\hat{R}\right)\right)\hat{R}_{shq} \tag{5.18}$$

(c) Shrinkage weight factor suggested by (22) here we take the weight factor as

$$\varphi\left(\hat{R}\right) = a.exp\left\{-\frac{b\left(\hat{R}_{ub} - \hat{R}_{shq}\right)^2}{var\left(\hat{R}_{ub}\right)}\right\} \tag{5.19}$$

where $0 < a < 1$ and $b > 0$. So the modified Thomason type shrinkage estimator will be

$$\hat{R}_{MS} = \varphi\left(\hat{R}\right)\hat{R}_{ub} + \left(1 - \varphi\left(\hat{R}\right)\right)\hat{R}_{shq} \tag{5.20}$$

6 Quasi Likelihood Estimation of R based on right censored sample

As in the case of section 4 in this case also we considered a right censoring procedure. Let $\underline{x} = (x_1, x_2, ..., x_{n_1-k})$ be the random sample of $(n_1 - k)$ observation taken from Lomax distribution $L(\alpha_1, \lambda)$ then its Quasi function is given by

$$Q(x_i, \alpha_1, \lambda) = (n_1 - k) \, log\left(\frac{\alpha_1 - 1}{\lambda}\right) - \nu\left(\frac{\alpha_1 - 1}{\lambda}\right) \tag{6.1}$$

where $\nu = \sum_{i=1}^{n_1-k} x_i$. So the quasi likelihood function in this case is given as

$$L(\underline{x}|\alpha_1, \lambda) = \left(\frac{\alpha_1 - 1}{\lambda}\right)^{n_1} e^{-\left(\frac{\alpha_1-1}{\lambda}\right)\nu}; \alpha_1 > 0, \nu = \sum_{i=1}^{n_1-k} x_i \tag{6.2}$$

Now using (6.2), (5.4) an (5.5) the joint likelihood function can be written as

$$L(\underline{x}, \underline{y}, \underline{z}|\alpha_1, \alpha_2, \alpha_3, \lambda) = \left(\frac{\alpha_1 - 1}{\lambda}\right)^{n_1-k} e^{-\left(\frac{\alpha_1-1}{\lambda}\right)\nu}. \left(\frac{\alpha_2 - 1}{\lambda}\right)^{n_2} e^{-\left(\frac{\alpha_2-1}{\lambda}\right)\zeta}. \left(\frac{\alpha_3 - 1}{\lambda}\right)^{n_3} e^{-\left(\frac{\alpha_3-1}{\lambda}\right)\beta} \tag{6.3}$$

Using (6.3) we get the estimate of α_1, α_2, α_3 and R are obtained as

$$\hat{\alpha}_{1qmlec} = 1 + \left(\frac{(n_1 - k)\lambda}{\nu}\right) = 1 + \left(\frac{(n_1 - k)\lambda}{\sum_{i=1}^{(n_1-k)} x_i}\right) \tag{6.4}$$

$$\hat{\alpha}_{2qmlec} = 1 + \left(\frac{n_2\lambda}{\zeta}\right) = 1 + \left(\frac{n_2\lambda}{\sum_{j=1}^{n_2} y_j}\right) \tag{6.5}$$

$$\hat{\alpha}_{3qmlec} = 1 + \left(\frac{n_3\lambda}{\beta}\right) = 1 + \left(\frac{n_3\lambda}{\sum_{k=1}^{n_3} z_k}\right) \tag{6.6}$$

and

$$\hat{R}_{qmlec} = \frac{\hat{\alpha_1}\hat{\alpha_2}}{(\hat{\alpha_1} + \hat{\alpha_2} + \hat{\alpha_3})(\hat{\alpha_1} + \hat{\alpha_3})}; 0 < R < 1 \tag{6.7}$$

In this case

$$\varnothing^{-1} = \begin{bmatrix} \frac{(\alpha_1-1)^2}{n_1-k} & 0 & 0 \\ 0 & \frac{(\alpha_2-1)^2}{n_1} & 0 \\ 0 & 0 & \frac{(\alpha_3-1)^2}{n_3} \end{bmatrix} \tag{6.8}$$

Now using (3.12), (3.13), (3.14) and (6.8) we have

$$\sigma^2_{Rqmlec} = V(R) = B'\phi^{-1}B = \frac{b_1^2(\alpha_1 - 1)^2}{n_1 - k} + \frac{b_2^2(\alpha_2 - 1)^2}{n_2} + \frac{b_3^2(\alpha_3 - 1)^2}{n_3} \tag{6.9}$$

By replacing the parameters with their maximum likelihood estimate we get the estimate $\hat{\sigma}^2_{Rqmlec}$ of σ^2_{Rqmlec}. In this case he asymptotic distribution of $\hat{R}_{qmlec}$ is $N(R, \hat{\sigma}^2_{Rqmlec})$. Based on this asymptotic distribution a $100(1-\gamma)\%$ asymptotic CI for R is $\hat{R}_{qmlec} \pm Z_{\gamma/2}\hat{\sigma}_{Rqmlec}$.

6.1 Shrinkage Estimates

In this case we obtain the shrinkage estimate, with $\psi\left(\hat{\beta}\right) = 0.01$ the constant shrinkage weight factor suggested by (13).

This leads to the constant shrinkage weight factor of R as $\hat{R}_{shqc}$

$$\hat{R}_{shqc} = \frac{\hat{\alpha 1_{shqc}} \hat{\alpha 2_{shqc}}}{\left(\hat{\alpha 1_{shqc}} + \hat{\alpha 2_{shqc}} + \hat{\alpha 3_{shqc}}\right)\left(\hat{\alpha 1_{shqc}} + \hat{\alpha 3_{shqc}}\right)} \tag{6.10}$$

with

$$\hat{\alpha 1_{shqc}} = 0.01\hat{\alpha 1_{ub}} + 0.99\hat{\alpha 13} \tag{6.11}$$

$$\hat{\alpha 2_{shqc}} = 0.01\hat{\alpha 2_{ub}} + 0.99\hat{\alpha 23} \tag{6.12}$$

and

$$\hat{\alpha 3_{shqc}} = 0.01\hat{\alpha 3_{ub}} + 0.99\hat{\alpha 33} \tag{6.13}$$

where $\hat{\alpha 1_{ub}} = \frac{n_1-1}{n_1\bar{x}}$, $\hat{\alpha 2_{ub}} = \frac{n_2-1}{n_2\bar{y}}$ and $\hat{\alpha 3_{ub}} = \frac{n_3-1}{n_3\bar{z}}$. $\hat{\alpha 13}$, $\hat{\alpha 23}$ and $\hat{\alpha 33}$ is taken as the boot strap estimate of $\hat{\alpha 1_{qmlec}}$, $\hat{\alpha 2_{qmlec}}$ and $\hat{\alpha 3_{qmlec}}$.
Also the modified Thompson type shrinkage weight factor and Shrinkage estimate by (22) are

(b) Suggested by (13) here we take the weight factor as

$$\phi\left(\hat{R}\right) = \frac{\hat{R}_{ub} - \hat{R}_{shqc}}{\left(\hat{R}_{ub} - \hat{R}_{shqc}\right)^2 + var\left(\hat{R}_{ub}\right)}(0.001) \tag{6.14}$$

where $\hat{R}_{ub} = \frac{\hat{\alpha 1_{ub}}\hat{\alpha 2_{ub}}}{\left(\hat{\alpha 1_{ub}} + \hat{\alpha 2_{ub}} + \hat{\alpha 3_{ub}}\right)\left(\hat{\alpha 1_{ub}} + \hat{\alpha 3_{ub}}\right)}$ and $var\left(\hat{R}_{ub}\right)$ is as defined in (6.9). So the modified Thomason type shrinkage estimator will be

$$\hat{R}_{Th} = \phi\left(\hat{R}\right)\hat{R}_{ub} + \left(1 - \phi\left(\hat{R}\right)\right)\hat{R}_{shqc} \tag{6.15}$$

(c) Shrinkage weight factor suggested by (22) here we take the weight factor as

$$\varphi\left(\hat{R}\right) = a.exp\left\{-\frac{b\left(\hat{R}_{ub} - \hat{R}_{shqc}\right)^2}{var\left(\hat{R}_{ub}\right)}\right\} \tag{6.16}$$

where $0 < a < 1$ and $b > 0$. So the modified Thomason type shrinkage estimator will be

$$\hat{R}_{MS} = \varphi\left(\hat{R}\right)\hat{R}_{ub} + \left(1 - \varphi\left(\hat{R}\right)\right)\hat{R}_{shqc} \tag{6.17}$$

7 Simulation Study

In this section we obtained the numerical results using simulation data.
Here, we have considered a bootstrap CI for r by using a parametric percentile bootstrap method ((11)). The following algorithm is used to generate the parametric bootstrap estimates of R.
Step-1. Simulate a random sample from Uniform (0,1). Using this simulated value compute random sample for $X \sim L(\alpha_1, \lambda)$, $Y \sim L(\alpha_2, \lambda)$ and $Z \sim L(\alpha_3, \lambda)$ respectively.
Compute the MLE of α_1, α_2, α_3 say $\hat{\alpha}_{1mle}$, $\hat{\alpha}_{2mle}$, $\hat{\alpha}_{3mle}$ given in setion-2.
Step-2. Generate an independent parametric bootstrap sample using $\hat{\alpha}_{1mle}$, $\hat{\alpha}_{2mle}$, $\hat{\alpha}_{3mle}$ instead of α_1, α_2, α_3. Then using these values, calculate $\hat{R}_{mle}$.
Step-3. Calculate the maximum likelihood estimate of $\hat{\alpha}_{1mle}$, $\hat{\alpha}_{2mle}$, $\hat{\alpha}_{3mle}$. and $\hat{R}_{mle}$ obtained in step-2 say $\hat{\alpha'}_{1mle}$, $\hat{\alpha'}_{2mle}$, $\hat{\alpha'}_{3mle}$ and $\hat{R'}_{1mle}$.
Step-4. Repeat the step-2 and step-3 N times to obtained the parametric bootstrap estimates $\hat{R'}_{ML1}, \hat{R'}_{ML2}, ..., \hat{R'}_{MLN}$ of R.
Step-5. Let $H(x) = P\left(\hat{R}_{ML} \leq x\right)$ be the cumulative distribution function of $\hat{R}_{ML}$. Define $\hat{R}_{Boot}(x) = H^{-1}(x)$ for a given x. The approximate $100(1-\gamma)\,\%$ CI of R is given by $\left(\hat{R}_{Boot}(\gamma/2), \hat{R}_{Boot}(1-\gamma/2)\right)$.

In the absence of real data finally, we study the performance of the estimates obtained in the above section using Monte Carlo Simulated data sets. All the computations are done by using R Program. Generate the sample of sizes $n_1 = n_2, n_3 = (10, 10), (10, 25), (10, 50), (25, 10), (25, 25), (25, 50), (50, 10), (50, 25), (50, 50)$ from Lomax Distribution with parameter values 0.5,2,3.5 for α_1, α_2 and α_3. The bias, mean square error, confidence interval and relative efficiency are calculated and are given in the following table.

8 Conclusion

From the numerical study conducted so for we can conclude that

When sample size increases bias and mean square error decreases.

The relative efficiency improvement over MLE of the $\hat{R}_{sh}$ greater than that of $\hat{R}_{Th}$ and $\hat{R}_{Ms}$. So $\hat{R}_{sh}$ is performs better than $\hat{R}_{Th}$ and $\hat{R}_{Ms}$.

In Maximum Likelihood Estimation when sample sizes is large the width of confidence interval of $\hat{R}_{mle}$ is less than that of $\hat{R}_{mlec}$.

In Quasi Likelihood Estimation when sample sizes is large the width of confidence interval $\hat{R}_{qmle}$ is less than that of $\hat{R}_{qmlec}$.

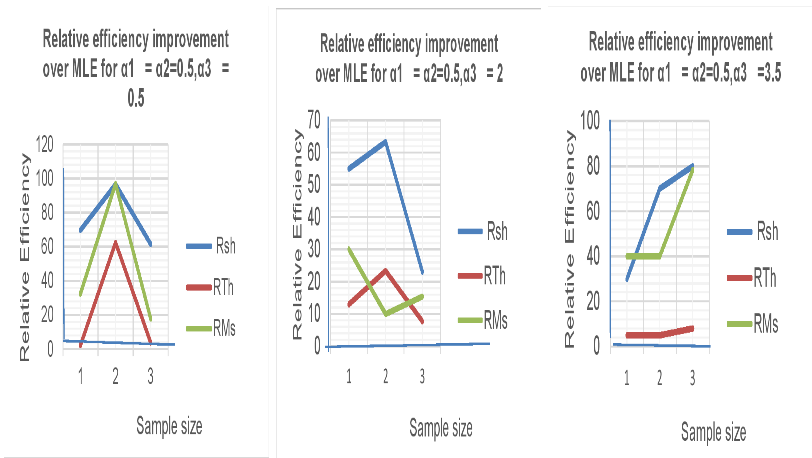
Relative efficiency improvement over MLE is higher in the case of censored sample

Table1: Bias, MSE and Relative Efficiency of the estimates of Reliability functions under complete sample.

$n_1=n_2$	n_3	$\alpha_1 = \alpha_2$	α_3		RMLE	Rsh	RTh	RMs
10	10	0.5	0.5	Bias	0.0278	0.02201	0.0276	0.02552
				MSE	0.0046	0.00139	0.0045	0.00311
				RE		69.78261	2.17391304	32.39130435
25	10			Bias	0.0355	0.0067	0.0354	0.0199
				MSE	0.0032	0.00011	0.0012	0.0001
				RE		96.5625	62.5	96.875
50	10			Bias	0.0127	0.00842	0.0156	0.0229
				MSE	0.0013	0.0005	0.00125	0.00107
				RE		61.53846	3.84615385	17.69230769
10	25	0.5	2	Bias	0.006	0.00186	0.00595	0.00371
				MSE	0.0004	0.00018	0.000348	0.00028
				RE		55	13	30
25	25			Bias	0.0077	0.00598	0.0067	0.0087
				MSE	0.0003	0.00011	0.00023	0.00027
				RE		63.33333	23.3333333	10
50	25			Bias	0.0025	0.0022	0.00249	0.00181
				MSE	0.00013	0.0001	0.00012	0.00011
				RE		23.07692	7.69230769	15.38461538
10	50	0.5	3.5	Bias	0.0063	0.00618	0.00631	0.00228
				MSE	0.0002	0.00014	0.00019	0.00012
				RE		30	5	40
25	50			Bias	0.0108	0.00562	0.01079	0.04648
				MSE	0.0004	0.00012	0.00038	0.00024
				RE		70	5	40
50	50			Bias	0.0081	0.00408	0.00813	0.00432
				MSE	0.0005	0.0001	0.00046	0.000108
				RE		80	8	78.4
10	10	2	0.5	Bias	0.0208	0.01786	0.0201	0.01714
				MSE	0.0088	0.00654	0.0086	0.00719
				RE		25.68182	2.27272727	18.29545455
25	10			Bias	0.0418	0.09197	0.04124	0.0123
				MSE	0.0043	0.0024	0.00424	0.00204
				RE		44.18605	1.39534884	52.55813953
50	10			Bias	0.0013	0.00154	0.00177	0.00119
				MSE	0.0048	0.00395	0.0046	0.00372

				RE		17.70833	4.16666667	22.5
10	25			Bias	0.0064	0.00288	0.00602	0.00117
				MSE	0.002	0.00163	0.00193	0.00125
				RE		18.5	3.5	37.5
25	25			Bias	0.0147	0.0191	0.0146	0.01129
				MSE	0.0021	0.001	0.0011	0.001
				RE		52.38095	47.6190476	52.38095238
50	25			Bias	0.059	0.02413	0.02413	0.02283
				MSE	0.0184	0.00105	0.01489	0.0108
		2	2	RE		94.29348	19.076087	41.30434783
10	50			Bias	0.0189	0.01958	0.0184	0.01204
				MSE	0.0022	0.00106	0.00214	0.00146
				RE		51.81818	2.72727273	33.63636364
25	50			Bias	0.0628	0.04496	0.02156	0.01164
				MSE	0.00089	0.00017	0.0006	0.00025
				RE		80.89888	32.5842697	71.91011236
50	50			Bias	0.0008	0.00046	0.00074	0.00067
				MSE	0.00067	0.0001	0.00052	0.00037
		2	3.5	RE		85.07463	22.3880597	44.7761194
10	10			Bias	0.0634	0.08927	0.06215	0.01124
				MSE	0.00985	0.00121	0.00201	0.00144
				RE		87.71574	79.5939086	85.38071066
25	10			Bias	0.0033	0.0055	0.0034	0.00645
				MSE	0.00178	0.0011	0.00122	0.0012
				RE		38.20225	31.4606742	32.58426966
50	10			Bias	0.0105	0.01538	0.01061	0.01269
				MSE	0.0022	0.00198	0.002	0.00197
		3.5	0.5	RE		10	9.09090909	10.45454545
10	25			Bias	0.05422	0.045	0.04406	0.03653
				MSE	0.00708	0.0026	0.00356	0.00277
				RE		63.27684	49.7175141	60.87570621
25	25			Bias	0.0101	0.00509	0.01171	0.00998
				MSE	0.00564	0.0022	0.00445	0.00243
				RE		60.99291	21.0992908	56.91489362
50	25			Bias	0.0213	0.01085	0.021	0.0119
				MSE	0.0025	0.00179	0.0024	0.002
		3.5	2	RE		28.4	4	20
10	50			Bias	0.0123	0.01168	0.01266	0.01238
				MSE	0.0027	0.00153	0.00262	0.0018
				RE		43.33333	2.96296296	33.33333333
25	50	3.5	3.5	Bias	0.08267	0.07287	0.07861	0.0742

				MSE	0.008	0.00173	0.0072	0.00394
				RE		78.375	10	50.75
				Bias	0.0903	0.01578	0.08922	0.04195
50	50			MSE	0.008	0.00165	0.00607	0.00229
				RE		79.375	24.125	71.375



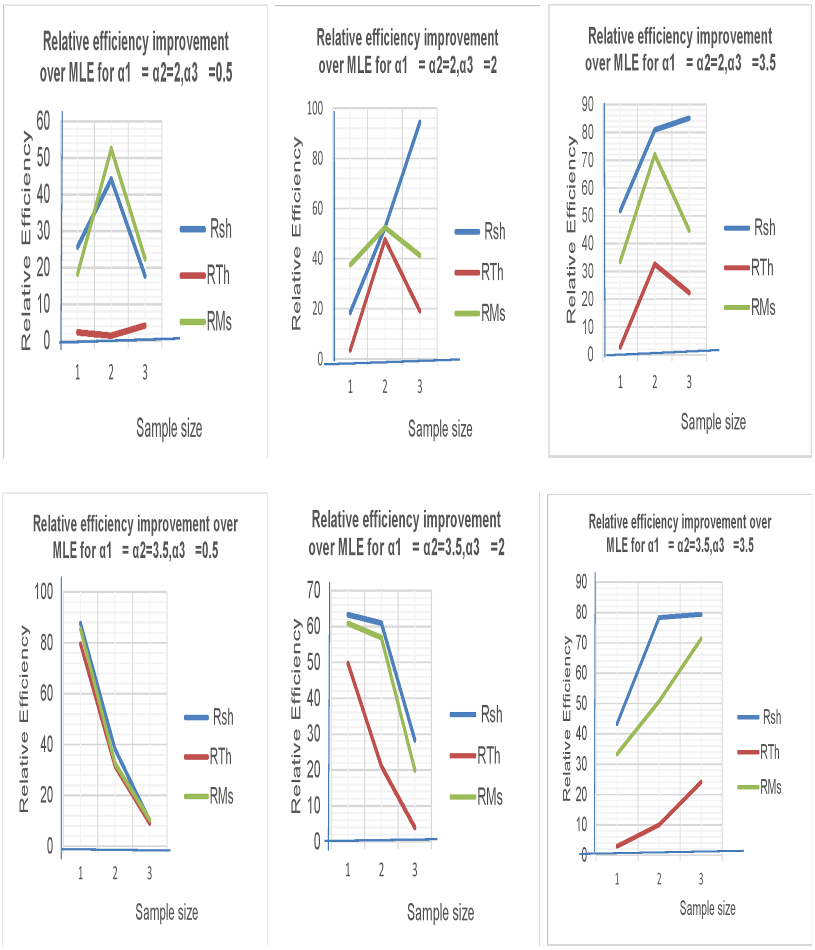


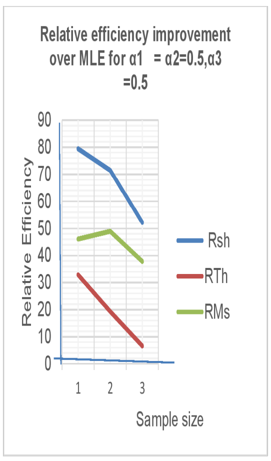
Fig 1.Relative efficiency improvement Over MLE with complete sample

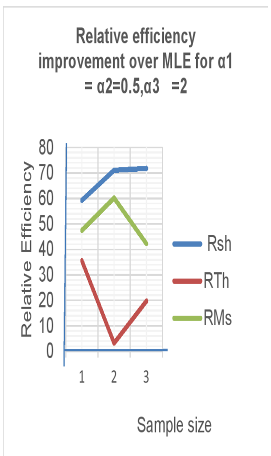
Table2: Bias, MSE and Relative Efficiency of the estimates of Reliability functions under censorsed sample

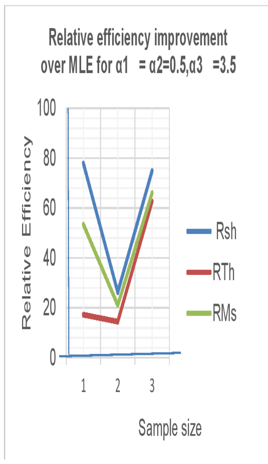
n_1=n_2	n_3	α_1 = α_2	α_3		RMLE	Rsh	RTh	RMs
				Bias	0.0672	0.01263	0.01531	0.01381
				MSE	0.00784	0.00161	0.00526	0.00423
10	10			RE		79.46429	32.90816	46.04592
				Bias	0.00115	0.00153	0.00456	0.00193
				MSE	0.00049	0.00014	0.000395	0.00025
25	10			RE		71.42857	19.38776	48.97959
				Bias	0.08179	0.012184	0.03241	0.015582
				MSE	0.00243	0.00116	0.00227	0.00151
50	10	0.5	0.5	RE		52.26337	6.584362	37.86008
				Bias	0.07281	0.01567	0.0233	0.0166
				MSE	0.0027	0.0011	0.00174	0.00142
10	25			RE		59.25926	35.55556	47.40741
				Bias	0.0889	0.01124	0.0735	0.01604
				MSE	0.00324	0.00101	0.00314	0.00135
25	25			RE		71.01911	3.184713	60.19108
				Bias	0.03532	0.01809	0.03106	0.02208
				MSE	0.00071	0.0002	0.00057	0.00041
50	25	0.5	2	RE		71.83099	19.71831	42.25352
				Bias	0.00893	0.00115	0.00796	0.00254
				MSE	0.00058	0.000128	0.00048	0.00027
10	50			RE		77.93103	17.24138	53.44828
				Bias	0.01895	0.01392	0.01751	0.01697
				MSE	0.00076	0.00056	0.00065	0.0006
25	50			RE		26.31579	14.47368	21.05263
				Bias	0.00793	0.001256	0.001851	0.001285
				MSE	0.00048	0.00012	0.000179	0.000163
50	50	0.5	3.5	RE		75	62.70833	66.04167
				Bias	0.0757	0.01574	0.07567	0.0651
				MSE	0.00144	0.00109	0.00133	0.00123
10	10			RE		24.30556	7.638889	14.58333
				Bias	0.01248	0.01185	0.01261	0.013374
				MSE	0.00546	0.00271	0.0053	0.00513
25	10			RE		50.3663	2.930403	6.043956
				Bias	0.09703	0.018195	0.08456	0.0529
50	10	2	0.5	MSE	0.01688	0.001138	0.014294	0.01393

				RE		93.25829	15.31991	17.4763
				Bias	0.09171	0.011242	0.09075	0.04584
				MSE	0.00855	0.00232	0.00837	0.00436
10	25			RE		72.8655	2.105263	49.00585
				Bias	0.04352	0.0112	0.04312	0.04088
				MSE	0.00418	0.00239	0.00406	0.00312
25	25			RE		42.82297	2.870813	25.35885
				Bias	0.0632	0.01934	0.05783	0.02259
				MSE	0.00505	0.00113	0.00441	0.00383
50	25	2	2	RE		77.62376	12.67327	24.15842
				Bias	0.05729	0.01227	0.05679	0.02656
				MSE	0.00461	0.00101	0.00452	0.00121
10	50			RE		78.09111	1.952278	73.75271
				Bias	0.06276	0.01347	0.0613	0.01415
				MSE	0.00136	0.001009	0.00127	0.00121
25	50			RE		25.80882	6.617647	11.02941
				Bias	0.025	0.013057	0.02405	0.017793
				MSE	0.00143	0.0006	0.00127	0.00116
50	50	2	3.5	RE		58.04196	11.18881	18.88112
				Bias	0.0216	0.01281	0.02035	0.02081
				MSE	0.00471	0.00196	0.00442	0.00431
10	10			RE		58.38641	6.157113	8.492569
				Bias	0.09945	0.06477	0.0858	0.06671
				MSE	0.00846	0.0055	0.00793	0.00651
25	10			RE		34.98818	6.264775	23.04965
				Bias	0.02347	0.01283	0.015907	0.01491
				MSE	0.0057	0.00237	0.00538	0.00475
50	10	3.5	0.5	RE		58.42105	5.614035	16.66667
				Bias	0.02477	0.02156	0.02411	0.02368
				MSE	0.0034	0.00212	0.00331	0.00329
10	25			RE		37.64706	2.647059	3.235294
				Bias	0.07894	0.05772	0.07537	0.06982
				MSE	0.0082	0.00489	0.00772	0.00718
25	25			RE		40.36585	5.853659	12.43902
				Bias	0.02537	0.01372	0.0244	0.02392
				MSE	0.00822	0.00215	0.00735	0.00636
50	25	3.5	2	RE		73.84428	10.58394	22.62774
				Bias	0.03353	0.02413	0.0333	0.024654
				MSE	0.00515	0.00402	0.00508	0.00471
10	50			RE		21.94175	1.359223	8.543689
25	50	3.5	3.5	Bias	0.02439	0.01952	0.02157	0.02107

				MSE	0.01336	0.00127	0.0131	0.01
				RE		90.49401	1.946108	25.1497
				Bias	0.05583	0.0376	0.05133	0.0428
				MSE	0.00981	0.00125	0.00906	0.007737
50	50			RE		87.2579	7.64526	21.1315







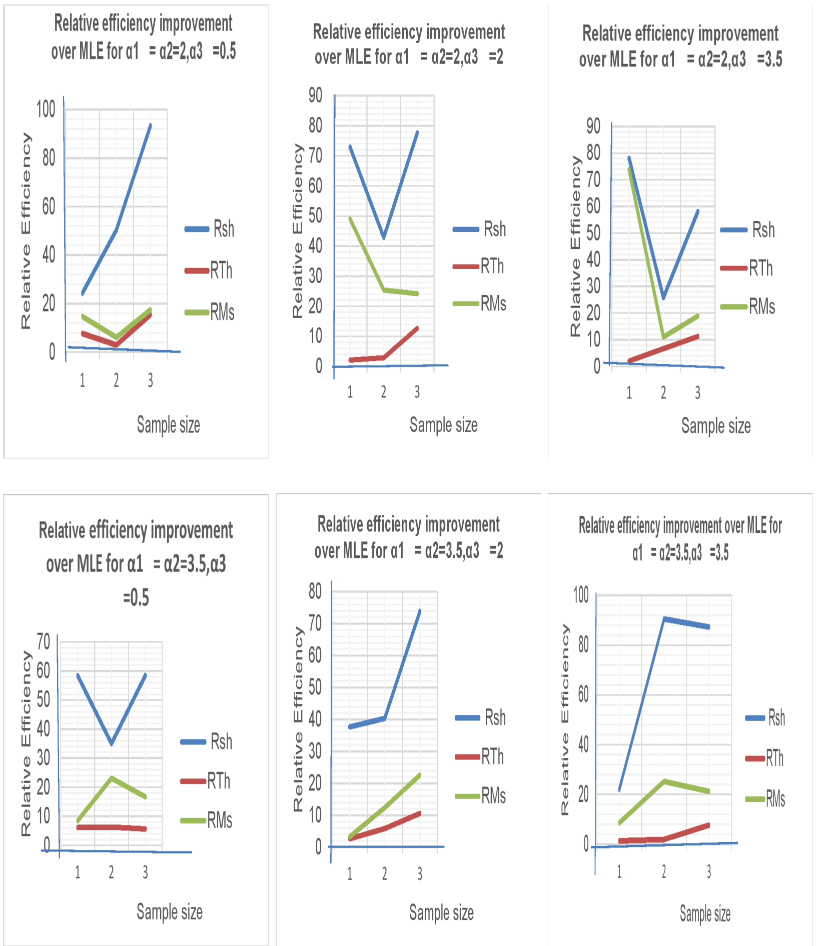


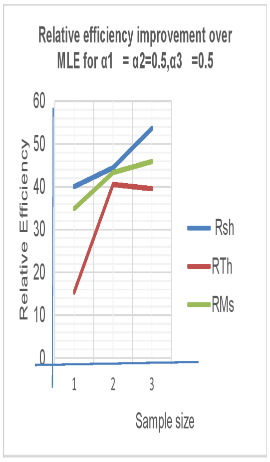
Fig 2.Relative efficiency improvement Over MLE with censored sample

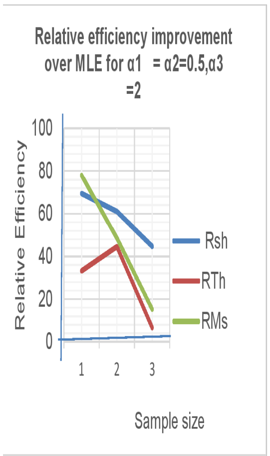
Table3: Bias, MSE and Relative Efficiency of the estimates of Reliability functions under complete sample using Quasi Likelihood Estimation

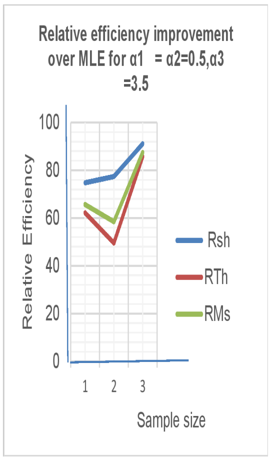
$n_1=n_2$	n_3	$\alpha_1 = \alpha_2$	α_3		RMLE	Rsh	RTh	RMs
				Bias	0.018288	0.01006	0.06929	0.021592
				MSE	0.001748	0.00104	0.00147	0.00113
10	10			RE		40.0457666	15.44622	34.89703
				Bias	0.01599	0.01021	0.01492	0.011712
				MSE	0.0018	0.001	0.00107	0.00102
25	10			RE		44.4444444	40.55556	43.33333
				Bias	0.0861	0.03263	0.0834	0.0536
				MSE	0.0022	0.00102	0.00133	0.00119
50	10	0.5	0.5	RE		53.6363636	39.54545	45.90909
				Bias	0.0588	0.01147	0.05024	0.04383
				MSE	0.00461	0.00141	0.00308	0.00102
10	25			RE		69.4143167	33.18872	77.87419
				Bias	0.08438	0.04895	0.08139	0.079
				MSE	0.00776	0.00303	0.0043	0.00399
25	25			RE		60.9536082	44.58763	48.58247
				Bias	0.0595	0.01476	0.05801	0.05526
				MSE	0.0042	0.00232	0.00392	0.00356
50	25	0.5	2	RE		44.7619048	6.666667	15.2381
				Bias	0.0812	0.0201	0.0381	0.0311
				MSE	0.00611	0.00154	0.00231	0.0021
10	50			RE		74.7954173	62.19313	65.63011
				Bias	0.03186	0.02046	0.02533	0.02066
				MSE	0.00226	0.00051	0.00114	0.00094
25	50			RE		77.4336283	49.55752	58.40708
				Bias	0.02969	0.01624	0.02288	0.01866
				MSE	0.00112	0.0001	0.00016	0.00014
50	50	0.5	3.5	RE		91.0714286	85.71429	87.5
				Bias	0.0813	0.01682	0.08111	0.02348
				MSE	0.001107	0.00014	0.00109	0.00046
10	10			RE		87.3532069	1.535682	58.44625
				Bias	0.08198	0.01061	0.017925	0.01079
				MSE	0.00771	0.00126	0.004517	0.00222
25	10			RE		83.6575875	41.41375	71.20623
				Bias	0.08511	0.0125	0.05476	0.013106
50	10	2	0.5	MSE	0.00735	0.00393	0.00529	0.0041

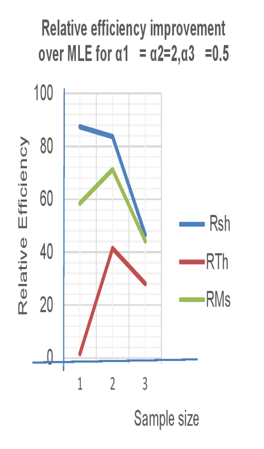
				RE		46.5306122	28.02721	44.21769
				Bias	0.04585	0.02381	0.036282	0.02883
				MSE	0.00463	0.00114	0.00259	0.00174
10	25			RE		75.3779698	44.06048	62.41901
				Bias	0.0488	0.02274	0.04821	0.04693
				MSE	0.00422	0.00135	0.00365	0.00267
25	25			RE		68.0094787	13.50711	36.72986
				Bias	0.0543	0.02012	0.02341	0.02204
				MSE	0.00424	0.00127	0.00268	0.00178
50	25	2	2	RE		70.0471698	36.79245	58.01887
				Bias	0.07654	0.01218	0.02656	0.01496
				MSE	0.0015	0.00049	0.0011	0.0005
10	50			RE		67.3333333	26.66667	66.66667
				Bias	0.09305	0.01339	0.0889	0.07917
				MSE	0.00738	0.00142	0.00725	0.00683
25	50			RE		80.7588076	1.761518	7.452575
				Bias	0.09349	0.07497	0.08526	0.0813
				MSE	0.00815	0.00264	0.00758	0.00619
50	50	2	3.5	RE		67.607362	6.993865	24.04908
10	10			Bias	0.09425	0.04107	0.06125	0.04187
				MSE	0.00872	0.00332	0.00392	0.0034
				RE		61.9266055	55.04587	61.00917
25	10			Bias	0.06694	0.01247	0.06641	0.02858
				MSE	0.00474	0.00176	0.00456	0.00183
				RE		62.8691983	3.797468	61.39241
50	10			Bias	0.05612	0.0122	0.05507	0.01545
				MSE	0.00373	0.00256	0.00368	0.00278
		3.5	0.5	RE		31.3672922	1.340483	25.46917
				Bias	0.05431	0.03443	0.04227	0.03449
				MSE	0.00767	0.00569	0.00621	0.00576
10	25			RE		25.8148631	19.0352	24.90222
				Bias	0.013958	0.01142	0.013209	0.01151
				MSE	0.002163	0.0009	0.00167	0.00105
25	25			RE		58.3911234	22.79242	51.45631
				Bias	0.08737	0.01456	0.03314	0.01478
				MSE	0.004929	0.00167	0.00319	0.00169
50	25	3.5	2	RE		66.1188882	35.28099	65.71313
				Bias	0.06102	0.02032	0.02465	0.02313
				MSE	0.00871	0.00216	0.00835	0.00273
10	50			RE		75.2009185	4.13318	68.65672
25	50	3.5	3.5	Bias	0.0543	0.01212	0.01614	0.01239

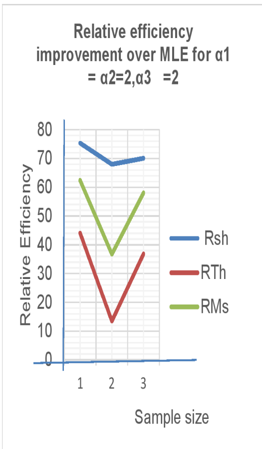
				MSE	0.00387	0.00159	0.00236	0.00163
				RE		58.9147287	39.01809	57.88114
				Bias	0.0717	0.02231	0.06716	0.0268
				MSE	0.00308	0.0003	0.00212	0.00058
50	50			RE		90.2597403	31.16883	81.16883

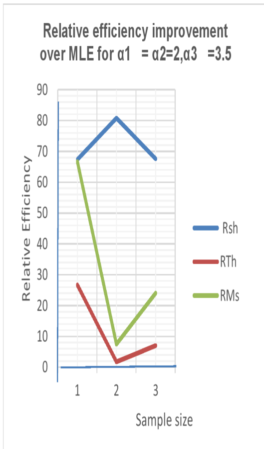


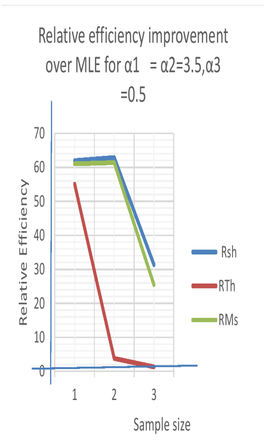


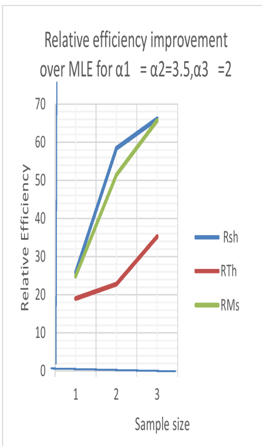












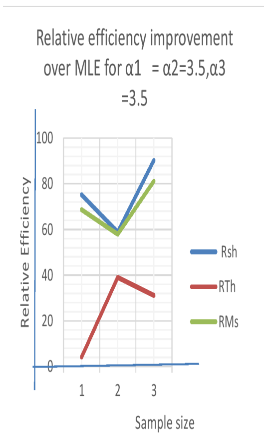


Fig 3.Relative efficiency improvement Over MLE with complete sample using quasi Liklihood Estimation

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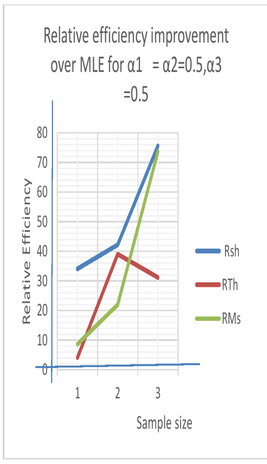
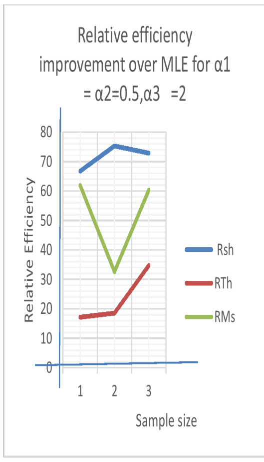
Table4: Bias, MSE and Relative Efficiency of the estimates of Reliability functions under censored sample using Quasi Likelihood Estimation

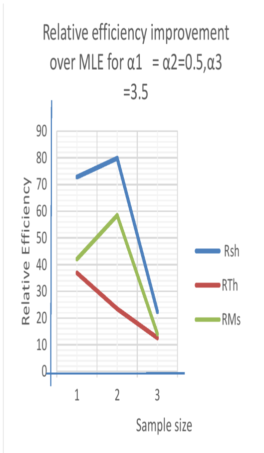
$n_1=n_2$	n_3	$\alpha_1 = \alpha_2$	α_3		RMLE	Rsh	RTh	RMs
	10			Bias	0.08254	0.01433	0.07033	0.018275
				MSE	0.001729	0.00114	0.00169	0.00158
10				RE		34.06593	2.255639	8.617698
	10			Bias	0.05561	0.0102	0.020104	0.013016
				MSE	0.00108	0.000624	0.0009	0.000843
25				RE		42.22222	16.66667	21.94444
				Bias	0.02994	0.02045	0.027	0.02047
				MSE	0.00414	0.00101	0.0038	0.00109
50	10	0.5	0.5	RE		75.60386	8.21256	73.6715
				Bias	0.09784	0.06245	0.08585	0.0826
				MSE	0.0043	0.00143	0.00356	0.001639
10	25			RE		66.74419	17.2093	61.88372
				Bias	0.0861	0.01149	0.08356	0.058
				MSE	0.00586	0.00145	0.00477	0.00395
25	25			RE		75.25597	18.60068	32.59386
				Bias	0.0883	0.012039	0.05738	0.01471
				MSE	0.00582	0.00158	0.0038	0.00231
50	25	0.5	2	RE		72.85223	34.7079	60.30928
				Bias	0.0788	0.0208	0.0473	0.0323
				MSE	0.00195	0.00053	0.00123	0.00113
10	50			RE		72.82051	36.92308	42.05128
				Bias	0.02994	0.02477	0.027	0.02047
				MSE	0.00541	0.00109	0.00414	0.00225
25	50			RE		79.85213	23.47505	58.41035
				Bias	0.03958	0.01632	0.03127	0.02954
				MSE	0.00143	0.00111	0.00125	0.00123
50	50	0.5	3.5	RE		22.37762	12.58741	13.98601
				Bias	0.06867	0.01198	0.04748	0.02108
				MSE	0.00734	0.00101	0.00184	0.00121
10	10			RE		86.23978	74.93188	83.51499
				Bias	0.09425	0.01061	0.017821	0.012828
				MSE	0.00447	0.00114	0.00222	0.0017
25	10			RE		74.49664	50.33557	61.96868
				Bias	0.08468	0.012935	0.073037	0.0529
				MSE	0.00933	0.00399	0.00731	0.00627
50	10	2	0.5	RE		57.23473	21.65059	32.79743

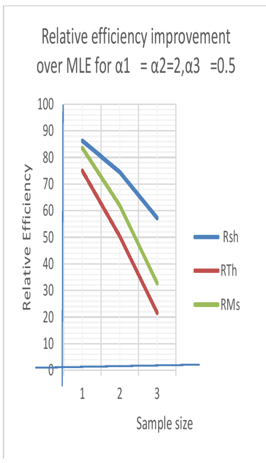
				Bias	0.03628	0.02238	0.03123	0.02348
				MSE	0.00594	0.00357	0.00468	0.00449
10	25			RE		39.89899	21.21212	24.41077
				Bias	0.0488	0.02269	0.04845	0.03935
				MSE	0.0036	0.00135	0.00253	0.00221
25	25			RE		62.5	29.72222	38.61111
				Bias	0.04982	0.02204	0.0233	0.02209
				MSE	0.00393	0.00113	0.002204	0.00147
50	25	2	2	RE		71.24682	43.91858	62.59542
				Bias	0.095	0.01106	0.02499	0.0114
				MSE	0.00166	0.00145	0.00163	0.00156
10	50			RE		12.6506	1.807229	6.024096
				Bias	0.07321	0.0339	0.06533	0.0652
				MSE	0.00875	0.00372	0.00522	0.00475
25	50			RE		57.48571	40.34286	45.71429
				Bias	0.097437	0.03108	0.08647	0.08459
				MSE		0.00158	0.00754	0.0065
50	50	2	3.5	RE		83.77823	22.58727	33.26489
				Bias	0.08646	0.012706	0.050516	0.04036
				MSE	0.00378	0.00127	0.00331	0.00209
10	10			RE		66.40212	12.43386	44.70899
				Bias	0.09946	0.024	0.09245	0.02858
				MSE	0.00899	0.00316	0.0055	0.00456
25	10			RE		64.84983	38.82091	49.27697
				Bias	0.0906	0.015309	0.04924	0.03101
				MSE	0.00339	0.00215	0.00281	0.00229
50	10	3.5	0.5	RE		36.57817	17.10914	32.44838
				Bias	0.05298	0.0234	0.03425	0.02626
				MSE	0.00766	0.00558	0.00722	0.00648
10	25			RE		27.15405	5.744125	15.4047
				Bias	0.0766	0.02974	0.06911	0.06182
				MSE	0.0046	0.00128	0.00382	0.00154
25	25			RE		72.17391	16.95652	66.52174
				Bias	0.038	0.01377	0.03351	0.0175
				MSE	0.00228	0.00129	0.00216	0.00155
50	25	3.5	2	RE		43.42105	5.263158	32.01754
				Bias	0.0223	0.01184	0.01074	0.01899
				MSE	0.00465	0.00078	0.00197	0.00126
10	50			RE		83.22581	57.63441	72.90323
				Bias	0.0674	0.01212	0.02891	0.01276
25	50	3.5	3.5	MSE	0.00762	0.00117	0.00154	0.00123

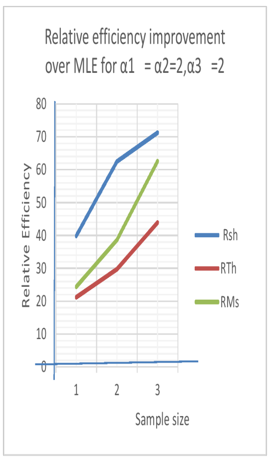
				RE		84.64567	79.79003	83.85827
				Bias	0.02237	0.01009	0.0216	0.017073
				MSE	0.0088	0.000212	0.00578	0.00088
50	50			RE		97.59091	34.31818	90

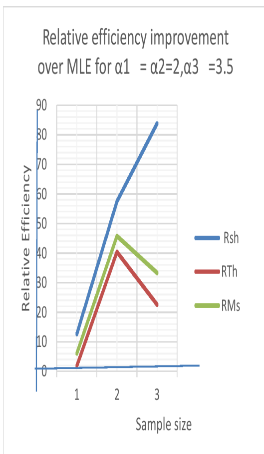


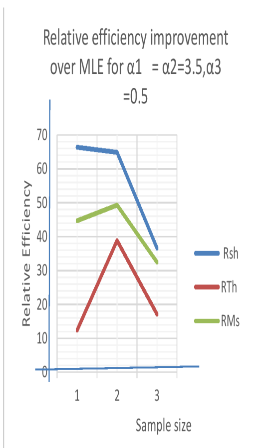


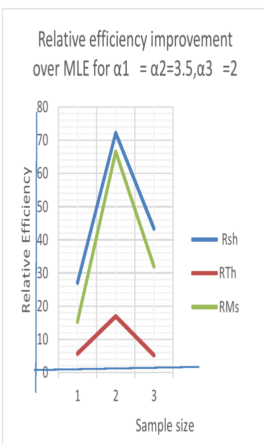












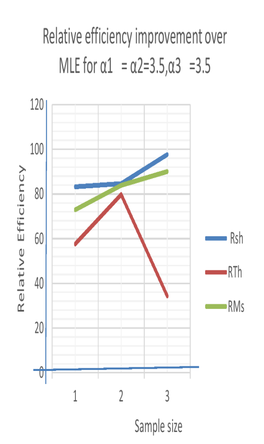


Fig 4.Relative efficiency improvement Over MLE with censored sample using quasi Liklihood Estimation

Table 5: Confidence Interval for R

$n_1=n_2$	n_3	$\alpha_1=\alpha_2$	α_3	CI based on Maximum Likelihood		CI based on Quasi Likelihood	
				Complete Sample	Censored Sample	Complete Sample	Censored Sample
10	10	0.5	0.5	(0.28855,0.31636)	(0.15350,0.16600)	(0.09297,0.22206)	(0.10668,0.21231)
	25	0.5	2	(0.07304,0.07904)	(0.03121,0.03312)	(0.01313,0.13004))	(0.00627,0.13539)
	50	0.5	3.5	(0.23386,0.25177)	(0.01306,0.02264)	(0.00535,0.05423))	(0.01295,0.07302)
25	10	0.5	0.5	(0.21772,0.25318)	(0.11332,0.16304)	(0.08964,0.22769)	(0.10724,0.21307)
	25	0.5	2	(0.05207,0.05980)	(0.01741,0.03321)	(0.01625,0.13186)	(0.01450,0.13917)
	50	0.5	3.5	(0.04857,0.05936)	(0.01305,0.02863)	(0.00787,0.05611)	(0.00689,0.06164))
50	10	0.5	0.5	(0.21337,0.22904)	(0.04533,0.16615)	(0.15286,0.17211)	(0.09498,0.22524)
	25	0.5	2	(0.05090,0.05340)	(0.01130,0.03327)	(0.01110,0.13303)	(0.01115,0.13520)
	50	0.5	3.5	(0.05319,0.06132)	(0.14523,0.30152)	(0.01692,0.07439)	(0.01359,0.05937)
10	10	2	0.5	(0.17594,0.55593)	(0.29699,0.33841)	(0.28300,0.34679)	(0.23508,0.35615)
	25	2	2	(0.08069,0.25904)	(0.07523,0.16638)	(0.15948.0.20268)	(0.05799.0.29771)
	50	2	3.5	(0.01291,0.19987)	(0.04228,0.09435)	(0.09419,0.11192)	(0.03385,0.18506)
25	10	2	0.5	(0.25560,0.49728)	(0.34512,0.37807)	(0.24960,0.35355)	(0.27487,0.30795)
	25	2	2	(0.11002,0.23800)	(0.16217,0.21468)	(0.16406,0.21748)	(0.10514.0.26754)
	50	2	3.5	(0.05560,0.15998)	(0.08384,0.17286)	(0.08859,0.16556)	(0.12482,0.14317)
50	10	2	0.5	(0.22044,0.49194)	(0.27315,0.34092)	(0.27066,0.35533)	(0.29460,0.33182)
	25	2	2	(0.18821,0.58189)	(0.10555,0.16458)	(0.16412,0.22350)	(0.10510,0.19858)
	50	2	3.5	(0.04538,0.14877)	(0.09538,0.12355)	(0.09280.0.16555)	(0.12089,0.12784)
10	10	3.5	0.5	(0.20326,0.54997)	(0.28368,0.51137)	(0.32861,0.44617)	(0.20993,0.55621)
	25	3.5	2	(0.16684,0.28309)	(0.14396,0.32620)	(0.11589,0.41648)	(0.08899,0.43220)
	50	3.5	3.5	(0.13660,0.15472)	(0.04230,0.32455)	(0.07480,0.28166)	(0.10906,0.23500)
25	10	3.5	0.5	(0.34019,0.47313)	(0.28030,0.60112)	(0.37280,0.37860)	(0.35622,0.36916)
	25	3.5	2	(0.14997,0.34598)	(0.16010,0.25589)	(0.20404,0.29204)	(0.21082,0.27793)
	50	3.5	3.5	(0.24106,0.30581)	(0.05483,0.36377)	(0.09132,0.22961)	(0.33681,0.44883)
50	10	3.5	0.5	(0.31777,0.48834)	(0.26758,0.52560)	(0.36103,0.39950)	(0.35193,0.44120)
	25	3.5	2	(0.01268,0.11261)	(0.31249,0.83280)	(0.17222,0.33750)	(0.15575,0.31094)
	50	3.5	3.5	(0.25716,0.33736)	(0.01154,0.26594)	(0.13524,0.20525)	(0.12838,0.18257)

References

[1] Abd El-Monsef, M. M. E., Marei, G. A., and Kilany, N. M. (2021). Estimating Stress-Strength Model for Weighted Lomax Distribution. *Asian Journal of Probability and Statistics*, 12(2), 19-31.

[2] Abdul–Moniem, I.B., and Abdel–Hameed, H.F. (2012). On Exponentiated Lomax Distribution. *International Journal of Mathematical Archive*, 5(3), 1-7.

[3] Abu-Salih, M. S., Abu-Salih, M. A. Ali, and M.A.Q. Yousef. (1988). On Some Shrinkage Techniques For Estimating The Parameters Of Exponential Distribution. Journal of Information and Optimization Sciences, 9(20), 201-214.

[4] Ali, H.M. (2013). The System Reliability of Stress Strength Model for Lomax Distribution . M.Sc Thesis, AL- Mustansiriya University.

[5] Almetwally, E. M., and Almongy, H. M. (2020). Parameter Estimation and Stress-Strength Model of Power Lomax Distribution: Classical Methods and Bayesian Estimation. Journal of Data Science, 18(4), 718-738.

[6] Attia, A.F, Shaban, S.A., and Amer, Y.M. (2014). Parameter Estimation for the Bivariate Lomax Distribution Based on Censored Sample. Applied Mathematical Sciences, 8(35), 1711-1721.

[7] Baklizi, A., Saadati Nik, A., and Asgharzadeh, A. (2022). Likelihood and Bayesian Inference in the Lomax Distribution under Progressive Censoring. Mathematics and Statistics, 10(3), 615-623.

[8] Church, J. D., and Harris, B. (1970). The Estimation of Reliability from Stress-Strength Relationships. Technometrics, 12, 49-54.

[9] Dhanya.M., and Jeevavand, E. S. (2018). Stress-Strength Reliability of Power Function Distribution based on Records. J. Stat. Appl. Prob, 7(1), 39-48.

[10] Dutta, K., and Sriwastav, G.L. (1986). An n –Standby System with $P(X < Y < Z)$. IAPQR Transaction, 12, 95-97.

[11] Efron, B. (1982). The Jackknife, the Bootstrap and Other Resembling Plans. SIAM, Philadelphia.

[12] Glifin, F., Anjana, E. J., and Jeevanand, E. S. (2022). Shrinkage Estimation of Strength Reliability for Geometric Distribution Using Record Values. Acta Scientific Computer Sciences, 4(4), 26-30.

[13] Hameed, B.A., Salman, A.N, and Kalaf, B.A. (2020). On Estimation of $P(Y_1 < X < Y_2)$ in cased Inverse Kumaraswamy Swamy Distribution. Iraqi Journal of Science, 61(4), 845-853.

[14] Hassan, A. S., Elsayed, A. E., and Rania, M. S. (2013). On the Estimation of for Weibull Distribution in the Presence of k Outliers. International Journal of Engineering Research and Applications, 3(6), 1727-1733.

[15] Ivshin, V.V. (1998). On the Estimation of Probabilities of a Double Linear Inequality in the Case of Uniform and Two Parameter Exponential Distributions. Journal of Mathematical Science, 88(6), 819-827.

[16] Karam, N.S, and Attia, A.M. (2021). Stress strength Reliability for $P(T < X < Z)$ using Dagum Distribution. Ibn Al-Haitham International Conference for Pure and Applied Sciences(IHICPS), Journal of Physics Conference Series, 1879032004, doi:10.1088/1742-6596/1879/3/032004, 1-11.

[17] Karam, N. S., and Jani, H. H. (2019). Reliability of Multi-Component Stress-Strength System Estimation for Generalized Lomax Distribution. In AIP Conference Proceedings, 2190(1), 020038.

[18] Khan, M. J. S., and Khatoon, B. (2019). Statistical Inferences of $R = P(X < Y)$ for Exponential Distribution based on Generalized Order Statistics. Annals of Data Science, 7(3), 525-545.

[19] Khaleel, A. H. (2021). Reliability of One Strength-Four Stresses for Lomax Distribution. In Journal of Physics: Conference Series, 1879(3), 032015.

[20] Mahmoud, M. A., El-Sagheer, R. M., Soliman, A. A., and Abd Ellah, A. H. (2016). Bayesian Estimation of $P(Y < X)$ Based on Record Values from the Lomax distribution and MCMC technique. Journal of Modern Applied Statistical Methods, 15(1), 488-510.

[21] McNolty, F., Doyle, J., and Hansen, E. (1980). Properties of the Mixed Exponential Failure Process. Technometrics, 22(4), 555-565.

[22] Mehta, J. S., and Srinivasan, R. (1971). Estimation of the Mean by Shrinkage to a Point. Journal of the American Statistical Association, 66(333), 86-90.

[23] Neethu J., and Anjana, E. J. (2023). Shrinkage Estimation of Stress Strength Reliability $P(Y < X < Z)$ for Lomax distribution based on records. International Journal of Statistics and Applied Mathematics, 8(1), 107-123.

[24] Neethu J., and Jeevanand, E.S. (2021). Semi Parametric Estimation of Stress Strength Reliability $P(X > Y)$ of Lomax distribution. Far East Journal of Mathematical Sciences, 61(2), 95-107.

[25] Neethu, J. and Jeevanand, E.S. (2021). Bayesian Estimation of Stress Strength Reliability $P(X > Y)$ of Lomax and Exponential Distribution. Journal of Information Storage and Processing Systems, 20, 182-189.

[26] Neethu, J. and Jeevanand, E.S.,(2021). Bayesian Estimation of Stress Strength Reliability $P(X > Y)$ of Lomax and Exponential Distribution based on right censored sample. International Journal of Statistics and Applied Mathematics, 6(2), 27-30.

[27] Nooghabi, M. J. (2016). Shrinkage estimation of $P(Y < X)$ in the Exponential Distribution Mixing with Exponential Distribution. Communications in Statistics-Theory and Methods, 45(5), 1477-1486.

[28] Panahi, H., and Asadi, S. (2011). Inference of Stress Strength Model for Lomax Distribution. World Academy of Science, Engineering and Technology, International Journal of Mathematical and computational sciences, 5(7), 937-940.

[29] Parviz .N. (2016). Estimation Parameter of $R = P(Y < X)$ for Lomax Distribution with presence of outliers. International Mathematical Forum, 11(5), 239-248.

[30] Rady, E. H. A., Hassanein, W. A., and Elhaddad, T. A. (2016). The Power Lomax Distribution with an Application to Bladder Cancer Data. SpringerPlus, 5(1), 5:1838.

[31] Raqab, M. Z. (2002). Inferences for Generalized Exponential Distribution Based on Record Statistics. Journal of statistical planning and inference, 104(2), 339-350.

[32] Salman, A. N., and Hamad, A. M. (2019). On Estimation of the Stress–Strength Reliability Based on Lomax Distribution. In IOP Conference Series: Materials Science and Engineering, 571(1), 012038. IOP Publishing.

[33] Singh, S. K., Singh, U., and Yadav, A. S. (2016). Reliability Estimation for Inverse Lomax Distribution Under type II Censored Data using Markov chain Monte Carlo method. International Journal of Mathematics and Statistics, 17(1), 128-146.

[34] Singh, N. (1980). On the estimation of $P\left(X_1 < Y < X_2\right)$. Communications in Statistics-Theory and Methods, 9(15), 1551-1561.

[35] Siu-Keung, T., and Geoffrey, T. (1996). Shrinkage Estimation of Reliability for Exponentially Distributed Lifetimes. Communications in Statistics-Simulation and Computation, 25(2), 415-430.

[36] Soliman, A. A., Abd-Ellah, A. H., Abou-Elheggag, N. A., and Ahmed, E. A. (2013). Reliability Estimation in Stress–Strength Models: An MCMC Approach. Statistics, 47(4), 715-728.

[37] Thompson J.R.(1968). Some Shrinkage Techniques for Estimating the Mean JASA, 63,113-122.

[38] Wasserman, L. (2003). All of Statistics: A Concise Course in Statistical Inference, first ed. Springer, New York.

[39] Wedderburn, R. W. (1974). Quasi-Likelihood Functions, Generalized Linear Models and the Gauss—Newton Method. Biometrika, 61(3), 439-447.

[40] Yadav, A. S., Singh, S. K., and Singh, U. (2019). Bayesian Estimation of Stress–Strength Reliability for Lomax Distribution Under Type-II Hybrid Censored Data Using Asymmetric Loss Function. Life Cycle Reliability and Safety Engineering, 8, 257-267.

[41] Zakerzadeh, H., Jafari, A. A., and Karimi, M. (2016). Preliminary Test and Shrinkage Estimations of Scale Parameters for Two Exponential Distributions based on Record Values. J. Statist. Res, 1343-1358.