

Fixed Point Results on Interval-Valued Fuzzy Metric Space using notation of Pairwise Compatible Maps and Occasionally Weakly Compatible Maps with Application

Abstract

The purpose of the paper is to obtain common fixed point results on interval-valued fuzzy metric space for occasionally weakly compatible maps (OWC) using contractive conditions. With the concept of τ_{norm} , interval numbers, and some important properties of interval-valued τ_{norm} .

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1 Introduction

Zadeh [22] introduced the IVFS, which is a subset of fuzzy sets and is distinguished by its fuzzy membership function. Li [10] introduced three types of distance between two IVFS (or numbers) are defined real line \mathbb{R} and pointed out that each type of distance is a metric on the associated sets and underlined that the metric space for any IVFS of numbers is complete. Shen, Li, and Wang [20] came up with the concept of IVFMS, which generalizes fuzzy metric space based on the ideas of George and Veeramani [4]. Kramosil et.al. [13] initially suggested fuzzy metrics and instantly Kaleva et.al.[8] and Abu Osman [12] used two distinct ways to create an independent, densely fuzzy metric space. The concept of weakly commuting maps on Probabilistic metric spaces was introduced by Singh et.al. [19]. Kumar and Chung developed some common fixed point theorems in metric space using \mathbb{R} -weakly commutative and reciprocal continuity of mappings. Mihet [11] used a contractual need satisfying an explicit relation to derive a widely used proof theorem. Fixed point results in Fuzzy Menger Space with Common Property and Fixed point results for P-1 Compatible in Fuzzy Menger Space were worked on by Ruchi Singh et.al. [17] and [18]. It is clear from a literature review that no effort has been made to derive fixed point theorems with the requirement of occasionally weakly compatible (OWC) mapping on IVFMS. Sewani et.al. [16] applying the *fuzzy iterated contraction abstraction* to create some new results in *intuitionistic-fuzzy metric space*. Here, we have derived several approximate points for OWC mappings satisfying explicit relationships using IVFMS.

2 Preliminaries

Definition2.1 [20]: Assume that Q is a typical non empty set.

The mapping $\mathfrak{R}_m : Q \rightarrow [I_{iv}]$ is referred to as an interval-valued fuzzy set on Q . $IVF(Q)$ is used to identify all interval-valued fuzzy-set on Q .

If $\mathfrak{R}_m \in IVF(Q)$, let $\mathfrak{R}_m(\wp^*) = [\mathfrak{R}_m^-(\wp^*), \mathfrak{R}_m^+(\wp^*)]$, $\mathfrak{R}_m^-(\wp^*) \leq \mathfrak{R}_m^+(\wp^*)$ for all $\wp^* \in Q$, then the conventional *fuzzy-set* $\mathfrak{R}_m^- : Q \rightarrow [I_{iv}]$ and $\mathfrak{R}_m^+ : Q \rightarrow [I_{iv}]$ are referred to as the *Lower*-fuzzy-set* and *Upper*-fuzzy-set* respectively. In particular, \mathfrak{R}_m is referred to degenerate *fuzzy-set* if $\mathfrak{R}_m^-(\wp^*) = \mathfrak{R}_m^+(\wp^*)$ for any $\wp^* \in Q$.

Definition2.2 [20]: A binary operation of the form is an interval-valued τ_{norm} is $*_{I_{iv}} : [I_{iv}]X[I_{iv}] \rightarrow [I_{iv}]$ on $[I_{iv}]$. All four of the following conditions are satisfied for $\bar{\gamma}_{\mathfrak{X}}, \bar{\delta}_{\mathfrak{X}}, \bar{\eta}_{\mathfrak{X}} \in [I_{iv}]$:

- (1) *Commutativity*: $\bar{\gamma}_{\mathfrak{X}} *_{I_{iv}} \bar{\delta}_{\mathfrak{X}} = \bar{\delta}_{\mathfrak{X}} *_{I_{iv}} \bar{\gamma}_{\mathfrak{X}}$,
- (2) *Associativity*: $\bar{\gamma}_{\mathfrak{X}} *_{I_{iv}} [\bar{\delta}_{\mathfrak{X}} *_{I_{iv}} \bar{\eta}_{\mathfrak{X}}] = [\bar{\gamma}_{\mathfrak{X}} *_{I_{iv}} \bar{\delta}_{\mathfrak{X}}] *_{I_{iv}} \bar{\eta}_{\mathfrak{X}}$,
- (3) *Monotonicity*: $\bar{\gamma}_{\mathfrak{X}} *_{I_{iv}} \bar{\delta}_{\mathfrak{X}} \leq \bar{\delta}_{\mathfrak{X}} *_{I_{iv}} \bar{\eta}_{\mathfrak{X}}$ whenever $\bar{\delta}_{\mathfrak{X}} \leq \bar{\eta}_{\mathfrak{X}}$,
- (4) *Boundary condition*: $\bar{\gamma}_{\mathfrak{X}} *_{I_{iv}} \bar{1} = \bar{\gamma}_{\mathfrak{X}}$, $\bar{\gamma}_{\mathfrak{X}} *_{I_{iv}} \bar{0} = [\bar{\gamma}_{\mathfrak{X}}, \bar{\gamma}_{\mathfrak{X}}^+] *_{I_{iv}} [0, 1] = [0, \bar{\gamma}_{\mathfrak{X}}^+]$

Example 1:(i) $\bar{\gamma}_{\mathfrak{X}} *_{I_{iv}} \bar{\delta}_{\mathfrak{X}} = [\bar{\gamma}_{\mathfrak{X}} \cdot \bar{\delta}_{\mathfrak{X}}, \bar{\gamma}_{\mathfrak{X}}^+ \cdot \bar{\delta}_{\mathfrak{X}}^+]$;
(ii) $\bar{\gamma}_{\mathfrak{X}} *_{I_{iv}} \bar{\delta}_{\mathfrak{X}} = [\bar{\gamma}_{\mathfrak{X}} \wedge \bar{\delta}_{\mathfrak{X}}, \bar{\gamma}_{\mathfrak{X}}^+ \wedge \bar{\delta}_{\mathfrak{X}}^+]$

Definition2.3 [20] : Let $\{\alpha_{\mathfrak{X}\eta_a}\} = \{[\alpha_{\mathfrak{X}}^-, \alpha_{\mathfrak{X}}^+]\}$, $\eta_a \in \mathbb{N}^+$ be a sequence of interval-numbers in $[I_{iv}]$, $\bar{\alpha}_{\mathfrak{X}} = [\alpha_{\mathfrak{X}}^-, \alpha_{\mathfrak{X}}^+] \in [I_{iv}]$, if $\lim_{\eta_a \rightarrow \infty} \alpha_{\mathfrak{X}\eta_a}^- = \alpha_{\mathfrak{X}}^-$ and $\lim_{\eta_a \rightarrow \infty} \alpha_{\mathfrak{X}\eta_a}^+ = \alpha_{\mathfrak{X}}^+$, then we say that the sequence $\{\alpha_{\mathfrak{X}\eta_a}\}$ is convergent to $\bar{\alpha}_{\mathfrak{X}}$, and which is denoted by $\lim_{\eta_a \rightarrow \infty} \alpha_{\mathfrak{X}\eta_a}^- = \alpha_{\mathfrak{X}}^-$.

Definition2.4 [15] : An interval-valued $\tau_{norm} *_{I_{iv}}$ is continuous if and only if it is continuous in its first component,

i.e., for each $\bar{\beta}_{\mathfrak{X}} \in [I_{iv}]$, if $\lim_{\eta_a \rightarrow \infty} \bar{\alpha}_{\mathfrak{X}\eta_a} = \bar{\alpha}_{\mathfrak{X}}$ then $\lim_{\eta_a \rightarrow \infty} (\bar{\alpha}_{\mathfrak{X}\eta_a} *_{I_{iv}} \bar{\beta}_{\mathfrak{X}}) = (\lim_{\eta_a \rightarrow \infty} \bar{\alpha}_{\mathfrak{X}\eta_a} *_{I_{iv}} \bar{\beta}_{\mathfrak{X}}) = \bar{\alpha}_{\mathfrak{X}} *_{I_{iv}} \bar{\beta}_{\mathfrak{X}}$, where $\{\bar{\alpha}_{\mathfrak{X}\eta_a}\} \subseteq [I_{iv}]$, $\bar{\alpha}_{\mathfrak{X}} \in [I_{iv}]$.

Definition2.5 [4] :If Q is a temporary set, $*_{I_{iv}}$ is a continuous interval-valued τ_{norm} on $[I_{iv}]$, and is an IVF(Q) on $Q^2 \times (0, \infty)$ meets the following conditions, the triple $(Q, \bar{d}, *_{I_{iv}})$ is know as IVFMS on Q :

following requirements:

- (1) $\bar{d}(\wp^*, \varrho^*, \tau_{norm}) > \bar{0}$;
- (2) $\bar{d}(\wp^*, \varrho^*, \tau_{norm}) = \bar{1}$ if and only if $\wp^* = \varrho^*$;
- (3) $\bar{d}(\wp^*, \varrho^*, \tau_{norm}) = \bar{d}(\varrho^*, \wp^*, \tau_{norm})$;
- (4) $\bar{d}(\wp^*, \varrho^*, \tau_{norm}) *_{I_{iv}} \bar{d}(\varrho^*, r, s) \leq \bar{d}(\wp^*, r, \tau_{norm} + s)$;
- (5) $\bar{d}(\wp^*, \varrho^*, .) : (0, \infty) \rightarrow [I]$ is continuous;
- (6) $\lim_{\eta \rightarrow \infty} \bar{d}(\wp^*, \varrho^*, \tau_{norm}) = \bar{1}$, where $\wp^*, \varrho^*, r \in Q$ and $\tau_{norm}, s > \bar{0}$.

Definition2.6 [4] : Let $(Q, \mathbb{k}, *_{I_{iv}})$ is an IVFMS,

- (1) For all $\wp^*, \varrho^* \in Q$, if $s > \tau_{norm} > 0$ then $\mathbb{k}(\wp^*, \varrho^*, \tau_{norm}) \leq \mathbb{k}(\wp^*, \varrho^*, s)$.
- (2) A sequence $\{\wp_{\eta_a}\}$ in Q is referred to as a cauchy sequence if for all $\bar{\varepsilon} > \bar{0}$ and $\tau_{norm} > 0$, there is an exists a $\eta_{a0} \in \mathbb{N}$ such that $\mathbb{k}(\wp_{\eta_a}, \wp_{\ell_a}, \tau_{norm}) > \bar{1} - \bar{\varepsilon}$ for all $\eta_a, \ell_a \geq \eta_{a0}$.
- (3) An IVFMS in which every cauchy sequence is convergent, is said to be a complete IVFMS .

Definition2.7 [5] : In interval-valued-metric-space, a function is continuous iff $\wp_{\eta_a} \rightarrow \wp^*$, $\varrho_{\eta_a} \rightarrow \varrho^* \Rightarrow \lim_{\eta_a \rightarrow \infty} \mathbb{k}(\wp_{\eta_a}, \varrho_{\eta_a}, \tau_{norm}) \rightarrow \mathbb{k}(\wp^*, \varrho^*, \tau_{norm})$.

Definition2.8 [7] : Assume that Q is a non-empty set and that Y and Z are

the self-maps of Q . A point \wp^* in Q is said to be coincidence point of Y and Z iff $Y\wp^* = Z\wp^*$.

Definition2.9 [1] : Two mapping Y and Z on IVFMS $(Q, \mathbb{k}, *_I)$ are weakly compatible by the expression iff $\mathbb{k}(YZ\wp^*, ZY\wp^*, \tau_{norm}) \geq \mathbb{k}(Y\wp^*, Z\wp^*, \tau_{norm})$ for $\tau_{norm} > 0$ and $\wp^* \in Q$.

Definition2.10 [7] : Two mappings Y and Z on IVFMS $(Q, \mathbb{k}, *_I)$ are compatible iff for sequence $\{\wp_{\eta_a}\}$ in Q , $\mathbb{k}(YZ\{\wp_{\eta_a}\}, ZY\{\wp_{\eta_a}\}, t) \rightarrow \bar{1}$ whenever $F\{\wp_{\eta_a}\} \rightarrow v$, $G\{\wp_{\eta_a}\} \rightarrow v$ for some $v \in Q$.

Definition2.11 [7] : Two mappings Y and Z on IVFMS $(Q, \mathbb{k}, *_I)$ are said to be OWC iff there exists a coincidence point \wp^* of Y and Z in Q such that Y and Z commutes at the point.

Definition2.12 [6] : Two mappings Y and Z on IVFMS $(Q, \mathbb{k}, *_I)$ then they are said to satisfy E.A property if there exists a $\{\wp_{\eta_a}\}$ in Q such that $\lim_{\eta_a \rightarrow \infty} F\{\wp_{\eta_a}\} = \lim_{\eta_a \rightarrow \infty} G\{\wp_{\eta_a}\} = v$, $v \in Q$.

Lemma2.13 [4] : IVFMS $(Q, \mathbb{k}, *_I)$ is non-decreasing for all $\wp^*, \varrho^* \in Q$.

Proof: Proof is followed by the definition of Interval-valued fuzzy metric space .

Lemma2.14 : If for all $\wp^*, \varrho^* \in Q$, $\tau_{norm} > 0$ and for a number $h \in (0, 1)$, $\mathbb{k}(\wp^*, \varrho^*, h\tau_{norm}) \geq \mathbb{k}(\wp^*, \varrho^*, \tau_{norm})$ then $\wp^* = \varrho^*$.

3 Main Result

Theorem3.1 : Let $(Q, \mathbb{k}, *_I)$ be a complete IVFMS and let A, B, C, D, E, F, G and H be self mappings of Q . Let $\{AB, CD\}$ and $\{EF, GH\}$ are pairs of OWC mappings. If there exists $h \in (0, 1)$ such that

$$\mathbb{k}(AB\rho^*, EF\varrho^*, h\tau_{norm}) \geq \min \left[\begin{array}{l} \mathbb{k}(CD\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\varrho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, CD\rho^*, \tau_{norm}) \end{array} \right] \dots (3.1)$$

for all $\wp^*, \varrho^* \in Q$ and $\tau_{norm} > 0$, then there exist unique points $w_{up}^*, z_{up}^* \in Q$ such that $ABw_{up}^* = CDw_{up}^* = w_{up}^*$ and $EFz_{up}^* = GHz_{up}^* = z_{up}^*$. Moreover, for $z_{up}^* = w_{up}^*$, A, B, C, D, E, F, G and H has unique common fixed point in Q .

Proof : Let us consider that $\{AB, CD\}$ and $\{EF, GH\}$ are pairs of OWC, therefore for $\rho^*, \varrho^* \in Q$ we have $AB\rho^* = CD\rho^*$ and $EF\varrho^* = GH\varrho^*$.

Now we claim that $AB\rho^*=EF\varrho^*$.

If not, then by inequality (3.1) we have

$$\begin{aligned} \Rightarrow \mathbb{k}(AB\rho^*, EF\varrho^*, h\tau_{norm}) &\geq \min \left[\begin{array}{c} \mathbb{k}(CD\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\varrho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, CD\rho^*, \tau_{norm}) \end{array} \right] \\ &= \min \left[\begin{array}{c} \mathbb{k}(AB\rho^*, EF\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, CD\rho^*, \tau_{norm}), \mathbb{k}(EF\varrho^*, EF\varrho^*, \tau_{norm}), \\ \mathbb{k}(AB\rho^*, EF\varrho^*, \tau_{norm}), \mathbb{k}(EF\varrho^*, AB\rho^*, \tau_{norm}) \end{array} \right] \\ &\geq \min \left[\mathbb{k}(AB\rho^*, EF\varrho^*, \tau_{norm}), \bar{1}, \bar{1}, \mathbb{k}(AB\rho^*, EF\varrho^*, \tau_{norm}), \mathbb{k}(EF\varrho^*, AB\rho^*, \tau_{norm}) \right] \end{aligned}$$

Therefore $AB\rho^* = EF\varrho^*$, i.e. $AB\rho^* = CD\rho^* = EF\varrho^* = GH\varrho^*$. Suppose that z_{up}^* is another point in Q such that $ABz_{up}^* = CDz_{up}^*$ then by (3.1), we have $ABz_{up}^* = CDz_{up}^* = EF\varrho^* = GH\varrho^*$, so $AB\rho^* = ABz_{up}^*$ and $AB\rho^* = CD\rho^* = w_{up}^*$. Now by Lemma 2.13, w_{up}^* is the common fixed point of AB and CD, and it is unique.

In the same way, there is a unique point $z_{up}^* \in Q$ such that $z_{up}^* = EFz_{up}^* = GHz_{up}^*$.

Assuming that $w_{up}^* \neq z_{up}^*$, we have

$$\begin{aligned} \mathbb{k}(w_{up}^*, z_{up}^*, h\tau_{norm}) &= \mathbb{k}(ABw_{up}^*, EFz_{up}^*, h\tau_{norm}) \\ &\geq \min \left[\begin{array}{c} \mathbb{k}(CDw_{up}^*, GHz_{up}^*, \tau_{norm}), \mathbb{k}(CDw_{up}^*, ABw_{up}^*, \tau_{norm}), \mathbb{k}(EFz_{up}^*, GHz_{up}^*, \tau_{norm}), \\ \mathbb{k}(ABw_{up}^*, GHz_{up}^*, \tau_{norm}), \mathbb{k}(EFz_{up}^*, CDw_{up}^*, \tau_{norm}) \end{array} \right] \\ &= \min \left[\begin{array}{c} (\mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \mathbb{k}(w_{up}^*, w_{up}^*, \tau_{norm}), \mathbb{k}(z_{up}^*, z_{up}^*, \tau_{norm}), \mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \\ \mathbb{k}(z_{up}^*, w_{up}^*, \tau_{norm}) \end{array} \right] \\ &\geq \min \left[(\mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \bar{1}, \bar{1}, \mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \mathbb{k}(z_{up}^*, w_{up}^*, \tau_{norm}) \right] \end{aligned}$$

$$\geq \mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm})$$

by Lemma 2.13, we have $w_{up}^* = z_{up}^*$ and z_{up}^* is the common fixed point A,B,C,D,E,F,G and H. The uniqueness holds from (3.1).

Theorem3.2 : Let $(Q, \mathbb{k}, *_I)$ be a complete IVFMS and let A,B,C,D,E,F,G and H be self mappings of Q. Let $\{AB, CD\}$ and $\{EF, GH\}$ are pairs of OWC mappings. If there exists $h \in (0,1)$ such that

$$\mathbb{k}(AB\rho^*, EF\rho^*, h\tau_{norm}) \geq \varphi \left[\min \left[\begin{array}{l} \mathbb{k}(CD\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, CD\rho^*, \tau_{norm}) \end{array} \right] \right] \dots (3.2)$$

for all $\rho^*, \rho^* \in Q$ and $\varphi \in *_I$ for all $0 < \tau_{norm} < 1$, then A,B,C,D,E,F,G and H has unique common fixed point in Q.

Proof: As proved in the theorem 3.1.

Theorem3.3 : Let $(Q, \mathbb{k}, *_I)$ be a complete IVFMS and let A,B,C,D,E,F,G and H be self mappings of Q. Let $\{AB, CD\}$ and $\{EF, GH\}$ are pairs of OWC mappings. If there exists $h \in (0,1)$ such that

$$\mathbb{k}(AB\rho^*, EF\rho^*, h\tau_{norm}) \geq \varphi \left[\begin{array}{l} \mathbb{k}(CD\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, CD\rho^*, \tau_{norm}) \end{array} \right] \dots (3.3)$$

for all $\rho^*, \rho^* \in Q$ and $\varphi : [I_{iv}]^5 \rightarrow [I_{iv}]$ so that

$\varphi(\tau_{norm}, \bar{1}, \bar{1}, \tau_{norm}, \tau_{norm}) > \tau_{norm}$ for all $0 < \tau_{norm} < 1$, then A,B,C,D,E,F,G and H has unique common fixed point in Q.

Proof: Let us consider that $\{AB, CD\}$ and $\{EF, GH\}$ are pairs of OWC, therefore for $\rho^*, \rho^* \in Q$, we have $AB\rho^* = CD\rho^*$ and $EF\rho^* = GH\rho^*$. Now we claim that $AB\rho^* = EF\rho^*$.

If not, then by inequality (3.3). We have,

$$\begin{aligned} \mathbb{k}(AB\rho^*, EF\rho^*, h\tau_{norm}) &\geq \varphi \left[\begin{array}{l} \mathbb{k}(CD\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, GH\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\rho^*, CD\rho^*, \tau_{norm}) \end{array} \right] \\ &= \varphi \left[\begin{array}{l} \mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, AB\rho^*, \tau_{norm}), \mathbb{k}(EF\rho^*, EF\rho^*, \tau_{norm}), \\ \mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm}), \mathbb{k}(EF\rho^*, AB\rho^*, \tau_{norm}) \end{array} \right] \end{aligned}$$

$$\geq [\mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm}), \bar{1}, \bar{1}, \mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm}), \mathbb{k}(EF\rho^*, AB\rho^*, \tau_{norm})]$$

$$\geq \mathbb{k}(AB\rho^*, EF\rho^*, \tau_{norm})$$

Therefore $AB\rho^* = EF\rho^*$, i.e. $AB\rho^* = CD\rho^* = EF\rho^* = GH\rho^*$. Suppose that z_{up}^* is another point in Q such that $ABz_{up}^* = CDz_{up}^*$ then by (3.3). We have $ABz_{up}^* = CDz_{up}^* = EF\rho^* = GH\rho^*$, so $AB\rho^* = ABz_{up}^*$ and $AB\rho^* = CD\rho^* = w_{up}^*$ is point of coincidence of AB and CD.

Now by Lemma 2.13, w_{up}^* is the common fixed point of AB and CD, and it is unique.

In the same way, there is a unique point $z_{up}^* \in Q$ such that $z_{up}^* = EFz_{up}^* = GH z_{up}^*$. Hence z_{up}^* is the common fixed point A,B,C,D,E,F,G and H.

Example: Let $I = [0, 1]$ with the usual metric $\mathbb{k}(\rho^*, \varrho^*, \tau_{norm}) = |\tau_{norm}\rho^* - \tau_{norm}\varrho^*|$ and set up the following maps:

$AB\rho^* = \frac{1}{2}$ for ρ in $[0,1]$, $CD\rho^* = \frac{3}{4}$ for ρ in $[0,1]$, $EF\rho^* = 1$ for ρ in $[0,1]$, $GH\rho^* = \frac{1}{4}$ for ρ in $[0,1]$, $\{AB, CD\}$ and $\{EF, GH\}$ are pair of OWC mapps. Putting $\varphi(\S_1, \S_2, \S_3, \S_4, \S_5) = -\S_1 + \frac{1}{2} \min(\S_2, \S_3) + \frac{1}{3}(\S_4 + \S_5)$. we get

$$\mathbb{k}(AB\rho^*, EF\varrho^*, h\tau_{norm}) \geq \varphi \left[\begin{array}{l} \mathbb{k}(CD\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\varrho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, CD\rho^*, \tau_{norm}) \end{array} \right]$$

$$\begin{aligned} \Rightarrow |1 - \frac{1}{2}| &= \frac{1}{2} \geq \varphi \left\{ \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{4}, \frac{1}{4} \right\} \\ \Rightarrow \frac{1}{2} &\geq -\frac{1}{2} + \frac{1}{2} \max\left(\frac{1}{4}, \frac{3}{4}\right) + \frac{1}{3} \left(\frac{1}{2} + \frac{1}{2}\right) \geq -\frac{1}{2} + \frac{1}{2} \left(\frac{3}{4}\right) + \frac{1}{3} \left(\frac{1}{2}\right) \geq \frac{1}{24} \end{aligned}$$

The eight maps accept 1 as the unique fixed point between them, and hence, all of the theorem's requirements are satisfied.

Theorem3.4: Let $(Q, \mathbb{k}, *_I)$ be a complete IVFMS and let A,B,C,D,E,F,G and H be self mappings of Q. Let $\{AB, CD\}$ and $\{EF, GH\}$ are pairs of OWC mappings. If there exists $h \in (0,1)$ such that

$$\mathbb{k}(AB\rho^*, EF\varrho^*, h\tau_{norm}) \geq \min \left[\begin{array}{l} \mathbb{k}(CD\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\varrho^*, \tau_{norm}) \end{array} \right] \dots (3.4)$$

for all $\varrho^*, \rho^* \in Q$ and $\tau_{norm} > 0$, then A,B,C,D,E,F,G and H has unique common fixed point in Q.

Proof: Let us consider that $\{AB, CD\}$ and $\{EF, GH\}$ are pairs of OWC, therefore for $\rho^*, \varrho^* \in Q$ we have $AB\rho^* = CD\rho^*$ and $EF\varrho^* = GH\varrho^*$.

Now we claim that $AB\rho^* = EF\varrho^*$.

If not, then by inequality (3.4) we have

$$\begin{aligned} \mathbb{k}(AB\rho^*, EF\varrho^*, h\tau_{norm}) &\geq \min \left[\begin{array}{l} \mathbb{k}(CD\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\varrho^*, \tau_{norm}) \end{array} \right] \\ &\geq \left[\begin{array}{l} \mathbb{k}(AB\rho^*, EF\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, CD\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, EF\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, EF\varrho^*, \tau_{norm}) \end{array} \right] \\ &\geq \mathbb{k}(AB\rho^*, EF\varrho^*, \tau_{norm}) \end{aligned}$$

Therefore $AB\rho^* = EF\varrho^*$, i.e. $AB\rho^* = CD\rho^* = EF\varrho^* = GH\varrho^*$. Suppose

that z_{up}^* is another point in Q such that $ABz_{up}^* = CDz_{up}^*$ then by (3.4). We have $ABz_{up}^* = CDz_{up}^* = EF\varrho^* = GH\varrho^*$, so $AB\rho^* = ABz_{up}^*$ and $AB\rho^* = CD\rho^* = w_{up}^*$. Now by Lemma 2.13, w_{up}^* is the common fixed point of AB and CD , and it is unique. In the same way, there is a unique point $z_{up}^* \in X$ such that $z_{up}^* = EFz_{up}^* = GHz_{up}^*$. Hence w_{up}^* is the common fixed point of A, B, C, E, F, G and H . The uniqueness Of w_{up}^* holds from (3.4).

Corollary3.5 : Let $(Q, \mathbb{k}, *_I)$ be a complete IVFMS and let A, B, C, D, E, F, G and H be self mappings of Q . Let $\{AB, CD\}$ and $\{EF, GH\}$ are pairs of OWC mappings. If there exists $h \in (0, 1)$ such that

$$\mathbb{k}(AB\rho^*, EF\varrho^*, h\tau_{norm}) \geq \min \left[\begin{array}{l} \mathbb{k}(CD\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\varrho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, CD\rho^*, 2\tau_{norm}) \end{array} \right] \dots (3.5)$$

for all $\varphi^*, \varrho^* \in Q$ and $\tau_{norm} > 0$, then A, B, C, D, E, F, G and H has unique common fixed point in Q .

Proof: We have

$$\begin{aligned} \mathbb{k}(AB\rho^*, EF\varrho^*, h\tau_{norm}) &\geq \min \left[\begin{array}{l} \mathbb{k}(CD\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\varrho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, CD\rho^*, 2\tau_{norm}) \end{array} \right] \\ &\geq \min \left[\begin{array}{l} \mathbb{k}(CD\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \mathbb{k}(EF\varrho^*, GH\varrho^*, \tau_{norm}), \\ \mathbb{k}(AB\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(EF\varrho^*, GH\varrho^*, \tau_{norm}) \end{array} \right] \\ &\geq \min \left[\begin{array}{l} \mathbb{k}(CD\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \mathbb{k}(EF\varrho^*, GH\varrho^*, \tau_{norm}), \\ \mathbb{k}(AB\rho^*, GH\varrho^*, \tau_{norm}) \end{array} \right] \end{aligned}$$

From Theorem 3.4, unique common fixed point exists for A, B, C, D, E, F, G and H .

Theorem3.6 : Let $(Q, \mathbb{k}, *_I)$ be a complete IVFMS and let A, B, C, D, R and V be self mappings of Q . Let $\{R, AB\}$ and $\{V, CD\}$ are pairs of OWC mappings. If there exists $h \in (0, 1)$ such that

$$\mathbb{k}(R\rho^*, V\varrho^*, h\tau_{norm}) \geq \min \left[\begin{array}{l} \mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, CD\varrho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, AB\rho^*, \tau_{norm}) \end{array} \right] \dots (3.6)$$

for all $\varphi^*, \varrho^* \in Q$ and $\tau_{norm} > 0$, then there exist unique points $w_{up}^*, z_{up}^* \in Q$ such that $ABw_{up}^* = Rw_{up}^* = w_{up}^*$ and $CDz_{up}^* = Vz_{up}^* = z_{up}^*$. Moreover, for $z_{up}^* = w_{up}^*$, A, B, C, D, R and V has unique common fixed point in Q.

Proof: Let us consider that $\{R, AB\}$ and $\{V, CD\}$ are pairs of OWC, therefore for $\rho^*, \varrho^* \in Q$, we have $AB\rho^* = R\rho^*$ and $CD\varrho^* = V\varrho^*$.

Now we claim that $R\rho^* = V\varrho^*$.

if not, then by inequality (3.6) we have

$$\begin{aligned} \mathbb{k}(R\rho^*, V\varrho^*, h\tau_{norm}) &\geq \min \left[\begin{array}{l} \mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, CD\varrho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, AB\rho^*, \tau_{norm}) \end{array} \right] \\ &= \min \left[\begin{array}{l} \mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, R\rho^*, \tau_{norm}), \mathbb{k}(V\varrho^*, V\varrho^*, \tau_{norm}), \\ \mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}), \mathbb{k}(V\varrho^*, R\rho^*, \tau_{norm}) \end{array} \right] \\ &\geq [\mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}), \bar{1}, \bar{1}, \mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm})\mathbb{k}(V\varrho^*, R\rho^*, \tau_{norm}),] \end{aligned}$$

$$\geq \min [\mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm})]$$

Therefore $AB\rho^* = R\rho^*$, i.e. $AB\rho^* = R\rho^* = CD\varrho^* = V\varrho^*$. Suppose that z_{up}^* is another point in Q such that $ABz_{up}^* = Rz_{up}^*$ then by (3.6). We have $ABz_{up}^* = Rz_{up}^* = CD\varrho^* = V\varrho^*$, so $AB\rho^* = ABz_{up}^*$ and $AB\rho^* = R\rho^* = w_{up}^*$. Now by Lemma 2.13, w_{up}^* is the common fixed point of AB and R, and it is unique.

In the same way, there is a unique point $z_{up}^* \in Q$ such that $z_{up}^* = CDz_{up}^* = Vz_{up}^*$.

Assuming that $w_{up}^* \neq z_{up}^*$, we have

$$\begin{aligned} \mathbb{k}(w_{up}^*, z_{up}^*, h\tau_{norm}) &= \mathbb{k}(Rw_{up}^*, Vz_{up}^*, h\tau_{norm}) \\ &\geq \min \left[\begin{array}{l} \mathbb{k}(ABw_{up}^*, CDz_{up}^*, \tau_{norm}), \mathbb{k}(ABw_{up}^*, Rw_{up}^*, \tau_{norm}), \mathbb{k}(Vz_{up}^*, CDz_{up}^*, \tau_{norm}), \\ \mathbb{k}(Rw_{up}^*, CDz_{up}^*, \tau_{norm}), \mathbb{k}(Vz_{up}^*, ABw_{up}^*, \tau_{norm}) \end{array} \right] \\ &= \min \left[\begin{array}{l} (\mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \mathbb{k}(w_{up}^*, w_{up}^*, \tau_{norm}), \mathbb{k}(z_{up}^*, z_{up}^*, \tau_{norm}), \mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \\ \mathbb{k}(z_{up}^*, w_{up}^*, \tau_{norm}) \end{array} \right] \\ &\geq \min [\mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \bar{1}, \bar{1}, \mathbb{k}(w_{up}^*, z_{up}^*, \tau_{norm}), \mathbb{k}(z_{up}^*, w_{up}^*, \tau_{norm})] \\ &\geq \mathbb{k}(w_{up}^*, z_{up}^*, h\tau_{norm}) \end{aligned}$$

by Lemma 2.13, we have $w_{up}^* = z_{up}^*$ and z_{up}^* is the common fixed point A,B,C,D,R and V. The uniqueness holds from (3.6).

Theorem3.7 : Let $(Q, \mathbb{k}, *_I)$ be a complete IVFMS and let A,B,C,D,R and V be self mappings of Q. Let $\{R, AB\}$ and $\{V, CD\}$ are pairs of OWC mappings. If there exists $h \in (0,1)$ such that

$$\mathbb{k}(R\rho^*, V\varrho^*, h\tau_{norm}) \geq \varphi \left[\min \left[\begin{array}{l} \mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, CD\varrho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, AB\rho^*, \tau_{norm}) \end{array} \right] \right] \dots (3.7)$$

for all $\varphi^*, \varrho^* \in Q$ and $\varphi \in *_I$ for all $0 < \tau_{norm} < 1$, then A,B,C,D,R and V has unique common fixed point in Q.

Proof: As proved in the theorem 3.6.

Theorem3.8 :Let $(Q, \mathbb{k}, *_I)$ be a complete IVFMS and let A,B,C,D,R and V be self mappings of Q. Let $\{R, AB\}$ and $\{V, CD\}$ are pairs of OWC mappings. If there exists $h \in (0,1)$ such that

$$\mathbb{k}(R\rho^*, V\varrho^*, h\tau_{norm}) \geq \min \left[\begin{array}{l} \mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, CD\varrho^*, \tau_{norm}), \end{array} \right] \dots (3.8)$$

for all $\varphi^*, \varrho^* \in Q$ and $\tau_{norm} > 0$, then A,B,C,D,R and V has unique common fixed point in Q.

Proof: Let us consider that $\{R, AB\}$ and $\{V, CD\}$ are pairs of OWC, therefore for $\rho^*, \varrho^* \in Q$ we have $AB\rho^* = R\rho^*$ and $CD\varrho^* = V\varrho^*$.

Now we claim that $R\rho^* = V\varrho^*$.

if not, then by inequality (3.8) we have

$$\begin{aligned} \mathbb{k}(R\rho^*, V\varrho^*, h\tau_{norm}) &\geq \min \left[\begin{array}{l} \mathbb{k}(AB\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \\ \mathbb{k}(V\varrho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, CD\varrho^*, \tau_{norm}) \end{array} \right] \\ &= \min \left[\begin{array}{l} \mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}), \mathbb{k}(R\rho^*, R\rho^*, \tau_{norm}), \mathbb{k}(V\varrho^*, V\varrho^*, \tau_{norm}), \\ \mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}) \end{array} \right] \\ &\geq \left[\mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}), \bar{1}, \bar{1}, \mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}) \right] \\ &\geq \left[\mathbb{k}(R\rho^*, V\varrho^*, \tau_{norm}) \right] \end{aligned}$$

Therefore $AB\rho^* = R\rho^*$, i.e. $AB\rho^* = R\rho^* = CD\varrho^* = V\varrho^*$. Suppose that z_{up}^* is another point in Q such that $ABz_{up}^* = Rz_{up}^*$ then by (3.8). We have $ABz_{up}^* = Rz_{up}^* = CD\varrho^* = V\varrho^*$, so $AB\rho^* = ABz_{up}^*$ and $AB\rho^* = R\rho^* = w_{up}^*$.

Now by Lemma 2.13, w_{up}^* is the common fixed point of AB and R, and it is unique.

In the same way, there is a unique point $z_{up}^* \in Q$ such that $z_{up}^* = CDz_{up}^* = Vz_{up}^*$. Hence we get w_{up}^* is a common fixed point of A,B,C,D,R and V.

Corollary 3.9 : Let $(Q, \mathbb{k}, *_I)$ be a complete IVFMS and let A,B,C,D,R and V be self mappings of Q. Let $\{AB, R\}$ and $\{CD, V\}$ are pairs of OWC mappings. If there exists $h \in (0,1)$ such that

$$\mathbb{k}(R\rho^*, V\rho^*, h\tau_{norm}) \geq \min \left[\begin{array}{l} \mathbb{k}(AB\rho^*, CD\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \\ \mathbb{k}(V\rho^*, CD\rho^*, \tau_{norm}), \mathbb{k}(R\rho^*, CD\rho^*, \tau_{norm}), \\ \mathbb{k}(V\rho^*, AB\rho^*, 2\tau_{norm}) \end{array} \right]$$

for all $\rho^*, \rho^* \in Q$ and $\tau_{norm} > 0$, then A,B,C,D,R and V has unique common fixed point in Q.

Proof We have

$$\begin{aligned} \mathbb{k}(R\rho^*, V\rho^*, h\tau_{norm}) &\geq \min \left[\begin{array}{l} \mathbb{k}(AB\rho^*, CD\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \\ \mathbb{k}(V\rho^*, CD\rho^*, \tau_{norm}), \mathbb{k}(R\rho^*, CD\rho^*, \tau_{norm}), \\ \mathbb{k}(V\rho^*, AB\rho^*, 2\tau_{norm}) \end{array} \right] \\ &\geq \min \left[\begin{array}{l} \mathbb{k}(AB\rho^*, CD\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}), \mathbb{k}(V\rho^*, CD\rho^*, \tau_{norm}), \\ \mathbb{k}(R\rho^*, CD\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, CD\rho^*, \tau_{norm}) \mathbb{k}(V\rho^*, CD\rho^*, \tau_{norm}) \end{array} \right] \\ &\geq \min \left[\begin{array}{l} \mathbb{k}(AB\rho^*, CD\rho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, R\rho^*, \tau_{norm}) \mathbb{k}(V\rho^*, CD\rho^*, \tau_{norm}), \\ \mathbb{k}(R\rho^*, CD\rho^*, \tau_{norm}), \end{array} \right] \end{aligned}$$

From theorem 3.8, unique common fixed point exists for A,B,C,D,R and V.

Theorem 3.10 : Let $(Q, \mathbb{k}, *_I)$ be a complete IVFMS and let A,B,C,D be self mappings of Q. Let $\{AB\}$ and $\{CD\}$ are pairs of OWC mappings. If there exists $h \in (0,1)$ such that

$$\mathbb{k}(CD\rho^*, CD\rho^*, h\tau_{norm}) \geq [\alpha \mathbb{k}(AB\rho^*, AB\rho^*, \tau_{norm})] + \beta \min \left[\begin{array}{l} \mathbb{k}(AB\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}) \end{array} \right]$$

where $\alpha, \beta > 0, \alpha + \beta > 1$. Then A,B,C and D has unique common fixed point in Q.

Proof: Let AB and CD are OWC, therefore for point $\rho^* \in Q$ such that $AB\rho^* = CD\rho^*$. Suppose that there exist $\rho^* \in Q$ for which $AB\rho^* = CD\rho^*$.

On contrary let us assume $AB\rho^*=CD\varrho^*$. We have,

$$\begin{aligned}
\Rightarrow \mathbb{k}(CD\rho^*, CD\varrho^*, h\tau_{norm}) &\geq [\alpha\mathbb{k}(AB\rho^*, AB\varrho^*, \tau_{norm})] + \beta \min \begin{bmatrix} \mathbb{k}(AB\rho^*, AB\varrho^*, \tau_{norm}), \\ \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(CD\varrho^*, AB\varrho^*, \tau_{norm}) \end{bmatrix} \\
&= [\alpha\mathbb{k}(CD\rho^*, CD\varrho^*, \tau_{norm})] + \beta \min \begin{bmatrix} \mathbb{k}(CD\rho^*, CD\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, CD\rho^*, \tau_{norm}), \\ \mathbb{k}(CD\varrho^*, CD\varrho^*, \tau_{norm}) \end{bmatrix} \\
&\geq [\alpha\mathbb{k}(CD\rho^*, CD\varrho^*, \tau_{norm})] + \beta \min [\mathbb{k}(CD\rho^*, CD\varrho^*, \tau_{norm}), \bar{1}, \bar{1}] \\
&\geq [\alpha\mathbb{k}(CD\rho^*, CD\varrho^*, \tau_{norm})] + \beta [\mathbb{k}(CD\rho^*, CD\varrho^*, \tau_{norm})] \\
&\geq (\alpha + \beta) [\mathbb{k}(CD\rho^*, CD\varrho^*, \tau_{norm})]
\end{aligned}$$

Which is a contradiction, since $(\alpha + \beta) > 1$.

Therefore $CD\rho^* = CD\varrho^*$ and hence $AB\rho^* = AB\varrho^*$ and $AB\rho^*$ is unique.

Therefore from lemma 2.13 A,B ,C and D have a unique fixed point.

4 Application

An essential component of analytic and engineering mathematics research is the examination of the existence, nonexistence, and uniqueness of solutions to differential and integral equations. An important tool created in this field is the use of the fixed-point theorem. Consider the integral equation

$\rho(\check{z}, \tau) = \check{g}_n(\rho(\check{z}), \tau) + \int_{\alpha}^{\beta} \check{y}(\check{z}, \check{s}, \tau) \check{h}_n(\check{s}, \check{u}(\check{s}), \tau) ds + \int_{\alpha}^{\beta} \check{x}(\check{z}, \check{s}, \tau) \check{j}_n(\check{s}, \check{u}(\check{s}), \tau) ds$ for all $\check{z} \in [\alpha, \beta]$ where

1. $\check{g}_n : [\alpha, \beta] \rightarrow [0, 1]$ are Continuous.
2. $\check{y}(\check{z}, \check{s}, \tau), \check{x}(\check{z}, \check{s}, \tau) : [\alpha, \beta] \times [\alpha, \beta] \rightarrow [0, 1]$ are Continuous.
3. $\check{h}_n, \check{j}_n : [\alpha, \beta] \times [0, 1] \rightarrow [0, 1]$ are Continuous.

Let $\mathbb{E} = \mathbb{C}[0, 1]$ be the set of Continuous function on $[\alpha, \beta]$

$\mathbb{k}(\rho, \varrho, \tau) = |\tau\rho|_1 + |\tau\varrho|_1 = \max(\rho(\check{z}), \tau) + \max(\varrho(\check{z}), \tau)$ for all $\rho, \varrho \in \mathbb{E}$. It is evident that $\mathbb{k}(\rho, \varrho, \tau)$ is a dislocated fuzzy metric space.

5 Conflicts interest

According to the authors, there are no conflicts interest.

6 Conclusions

We have shown that there is a unique common fixed point for eight self-mappings, six self-mappings, and four self-mappings in a complete IVFMS using the concepts of contractive conditions and OWC. Our results are useful for theoretical mathematics and computer science.

References

- [1] Al-Thagafi, M. A., and Naseer Shahzad. "A note on occasionally weakly compatible maps." *Int. J. Math. Anal* 3, no. 2 (2009): 55-58.
- [2] AL-Mayahi, Noori F., and Sarim H. Hadi. "On—Contraction in Fuzzy Metric Space and its Application." *Gen* 26, no. 2 (2015): 104-118.
- [3] George, A., and P. Veeramani. "On some results of analysis for fuzzy metric spaces." *Fuzzy sets and systems* 90, no. 3 (1997): 365-368.
- [4] George, A., and P. Veeramani. "On some results in fuzzy metric spaces." *Fuzzy sets and systems* 64, no. 3 (1994): 395-399.
- [5] Jenei, Sándor, and János C. Fodor. "On continuous triangular norms." *Fuzzy Sets and Systems* 100, no. 1-3 (1998): 273-282.
- [6] Jungck, Gerald. "Compatible mappings and common fixed points (2)." *International Journal of Mathematics and Mathematical Sciences* 11 (1988): 285-288.
- [7] Jungck, G., and BE2284600 Rhoades. "Fixed point theorems for occasionally weakly compatible mappings." *Fixed point theory* 7, no. 2 (2006): 287-296.
- [8] Kaleva, Osmo, and Seppo Seikkala. "On fuzzy metric spaces." *Fuzzy sets and systems* 12, no. 3 (1984): 215-229.
- [9] Kumar, Sanjay, and Renu Chugh. "Common fixed points theorem using minimal commutativity and reciprocal continuity conditions in metric space." *Scientiae Mathematicae Japonicae* 56, no. 2 (2002): 269-276.

- [10] Li, Chen. "Distances between interval-valued fuzzy sets." In NAFIPS 2009-2009 Annual Meeting of the North American Fuzzy Information Processing Society, pp. 1-3. IEEE, 2009.
- [11] Miheţ, Dorel. "A generalization of a contraction principle in probabilistic metric spaces. Part II." *International Journal of Mathematics and Mathematical Sciences* 2005, no. 5 (2005): 729-736.
- [12] Osman, MT Abu. "Fuzzy metric spaces and fixed fuzzy set theorem." *Bull. Malaysian Math. Soc* 6, no. 1 (1983): 1-4.
- [13] O.Kramosil and J.Michalek ,Fuzzy metric spaces and statistical metric spaces, *Kybernetika* Vol.11,(1975),326-334.
- [14] Rao, V. Sambasiva, and Uma Dixit. "Common Fixed Point Theorems for OWC Maps Satisfying Property (EA) in S-Metric Spaces Using an Inequality Involving Quadratic Terms." *Communications in Mathematics and Applications* 13, no. 5 (2022): 1393.
- [15] Schweizer, Berthold, and Abe Sklar. "Statistical metric spaces." *Pacific J. Math* 10, no. 1 (1960): 313-334.
- [16] Sewani Godavari, A. D. Singh, Ruchi Singh, and Ramakant Bhardwaj. "Generalized Intuitionistic Fuzzy b-Metric Space." *ECS Transactions* 107, no. 1 (2022): 12415.
- [17] Singh, Ruchi, A. D. Singh, and Anil Goyal. "Fixed point Results in fuzzy Menger space with common property (EA)." *Journal of Engineering Research and applications* 5, no. 8 (2015).
- [18] Singh, Ruchi, A. D. Singh, and Anil Goyal. "Fixed point results for P-1 compatible in fuzzy Menger space." *Adv. Fixed Point Theory* 6, no. 4 (2016): 520-527.
- [19] Singh, S. L., B. D. Pant, and R. Talwar. "Fixed points of weakly commuting mappings on Menger spaces, *Jnanabha* 23 (1993), 115–122." Received: April (2009).
- [20] Shen, Yonghong, Haifeng Li, and Faxing Wang. "On interval-valued fuzzy metric spaces." *International Journal of Fuzzy Systems* 14, no. 1 (2012): 35-44.

- [21] Zadeh, Lotfi A. "Fuzzy sets." *Information and control* 8, no. 3 (1965): 338-353.
- [22] Zadeh, Lotfi Asker. "The concept of a linguistic variable and its application to approximate reasoning—I." *Information sciences* 8, no. 3 (1975): 199-249.