Original Research Article

Fixed Point Results on Interval-Valued Fuzzy Metric Space using notation of Pairwise Compatible Maps and Occasionally Weakly Compatible Maps with Application

Abstract

The purpose of the paper is to obtain common fixed point results on interval-valued fuzzy metric space for occasionally weakly compatible maps (OWC) using contractive conditions. With the concept of τ_{norm}, interval numbers, and some important properties of interval-valued τ_{norm}.

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Keywords- Interval-valued fuzzy metric space (IVFMS), Contractive condition, Fixed point ,Common fixed point, Occasionally weakly compatible (OWC) mappings.

1 Introduction

Zadeh [17] introduced the IVFS, which is a subset of fuzzy sets and is distinguished by its fuzzy membership function. Li [6] three types of distance between two IVFS (or numbers) are defined real line R and pointed out that each type of distance is a metric on the associated sets and underlined that the metric space for any IVFS of numbers is complete. Shen, Li, and Wang [15]came up with the concept of IVFMS, which generalizes fuzzy metric space based on the ideas of George and Veeramani [3]. kramosil et.al. [9] Initially suggested fuzzy metrics and instantly Kaleva et.al.[4] and Abu Osman [8] used two distinct ways to create an independent, densely fuzzy metric space. The concept of weakly commuting maps on Probabilistic metric spaces was introduced by Singh et.al. [14] . Kumar and Chung developed some common fixed point theorems in metric space using R-weakly commutative and reciprocal continuity of mappings. Mihet [7] used a contractual need satisfying an explicit relation to derive a widely used proof theorem. Fixed Point Results in Fuzzy Menger Space with Common Property and Fixed Point Results for P-1 Compatible in Fuzzy Menger Space were worked on by Ruchi Singh et.al. [12] and [13]. It is clear from a literature review that no effort has been made to derive fixed point theorems with the requirement of occasionally weakly compatible (OWC) mapping on IVFMS. Sewani et.al.[11] applying the $f$uzzy iterated contraction abstraction to create some new results in $i$ntuitionistic-$f$uzzy $m$etric $s$pace. Here, we have derived several approximate points for OWC mappings satisfying explicit relationships using IVFMS.

2 Preliminaries

Definition2.1 : Assume that Q is a typical non empty set.
The mapping $\Re_m : Q \to [I_{iv}]$ is referred to as an interval-valued fuzzy set on Q. IVF(Q) is used to identify all interval-valued fuzzy-set on Q.
if $\Re_m \in IVF(Q)$, Let $\Re_m(\wp^*)$=$[\Re_m^-(\wp^*), \Re_m^+(\wp^*)]$, $\Re_m^-(\wp^*) \leq \Re_m^+(\wp^*)$ for all $\wp^* \in Q$, then the conventional $f$uzzy-set $\Re_m^- : Q \to [I_{iv}]$ and $\Re_m^+ : Q \to [I_{iv}]$ are referred to as the $Lower^\star$-$f$uzzy-set and $Upper^\star$-$f$uzzy-set respectively. In particular, $\Re_m$ is referred to degenerate $f$uzzy-set if $\Re_m^-(\wp^*) = \Re_m^+(\wp^*)$ for any $\wp^* \in Q$.

Definition2.2 : A binary operation of the form is an interval-valued τ_{norm} is

$*_{I_{iv}}$: $[I_{iv}]X[I_{iv}] \to [I_{iv}]$ on $[I_{iv}]$. such that all four of the following conditions are satisfied $\bar{\gamma_{\bowtie}}$, $\bar{\delta_{\bowtie}}$, $\bar{\eta_{\bowtie}} \in [I_{iv}]$:
(1) *Commutativity*: $\bar{\gamma_{\bowtie}} *_{I_{iv}} \bar{\delta_{\bowtie}} = \bar{\delta_{\bowtie}} *_{I_{iv}} \bar{\gamma_{\bowtie}}$,
(2) *Associativity* : $\bar{\gamma_{\bowtie}} *_{I_{iv}} [\bar{\delta_{\bowtie}} *_{I_{iv}} \bar{\eta_{\bowtie}}] = [\bar{\gamma_{\bowtie}} *_{I_{iv}} \bar{\delta_{\bowtie}}] *_{I_{iv}} \bar{\eta_{\bowtie}}$,
(3) *Monotonicity* : $\bar{\gamma_{\bowtie}} *_{I_{iv}} \bar{\delta_{\bowtie}} \leq \bar{\delta_{\bowtie}} *_{I_{iv}} \bar{\eta_{\bowtie}}$ whenever $\bar{\delta_{\bowtie}} \leq \bar{\eta_{\bowtie}}$,
(4) *Boundary condition*; $\bar{\gamma_{\bowtie}} *_{I_{iv}} \bar{1} = \bar{\gamma_{\bowtie}}$, $\bar{\gamma_{\bowtie}} *_{I_{iv}} \bar{0} = [\gamma_{\bowtie}^-, \gamma_{\bowtie}^+] *_I [0,1] = [0, \gamma_{\bowtie}^+]$
Example 1:(i) $\bar{\gamma_{\bowtie}} *_{I_{iv}} \bar{\delta_{\bowtie}} = [\gamma_{\bowtie}^-.\delta_{\bowtie}^-, \gamma_{\bowtie}^+.\delta_{\bowtie}^+]$;
(ii) $\bar{\gamma_{\bowtie}} *_{I_{iv}} \bar{\delta_{\bowtie}} = [\gamma_{\bowtie}^- \wedge \delta_{\bowtie}^-, \gamma_{\bowtie}^+ \wedge \delta_{\bowtie}^+]$
Definition2.3 : Let $\{\bar{\alpha_{\bowtie\eta_a}}\} = \{[\alpha_{\bowtie}^-, \alpha_{\bowtie}^+]\}$, $\eta_a \in \aleph^+$ be a Ⓢequence of interval-numbers in $[I_{iv}]$, $\bar{\alpha_{\bowtie}} = [\alpha_{\bowtie}^-, \alpha_{\bowtie}^+] \in [I_{iv}]$, if $\lim_{\eta_a\to\infty} \alpha_{\bowtie\eta_a}^- = \alpha_{\bowtie}^-$ and $\lim_{\eta_a\to\infty} \alpha_{\bowtie\eta_a}^+ = \alpha_{\bowtie}^+$,then we say that the Ⓢequence $\{\bar{\alpha_{\eta_a}}\}$ is Convergent to $\bar{\alpha_{\bowtie}}$, and which is denoted by $\lim_{\eta_a\to\infty}\bar{\alpha_{\bowtie\eta_a}} = \bar{\alpha_{\bowtie}}$.
Definition2.4 : An interval-valued τ_{norm} $*_{I_{iv}}$ is Continuous if and only if it is Continuous in its first component,
i.e., for each $\bar{\beta_{\bowtie}} \in [I_{iv}]$,if $\lim_{\eta_a\to\infty} \bar{\alpha}_{\bowtie\eta_a} = \bar{\alpha_{\bowtie}}$
then $\lim_{\eta_a\to\infty} (\bar{\alpha_{\bowtie\eta_a}} *_1 \bar{\beta_{\bowtie}}) = (\lim_{\eta_a\to\infty} \bar{\alpha}_{\bowtie\eta_a} *_{I_{iv}} \bar{\beta_{\bowtie}}) = \bar{\alpha_{\bowtie}} *_{I_{iv}} \bar{\beta_{\bowtie}}$, where $\{\bar{\alpha}_{\bowtie\eta_a}\} \subseteq [I_{iv}]$, $\bar{\alpha_{\bowtie}} \in [I_{iv}]$.
Definition2.5 :If Q is a temporary set, $*_{I_{iv}}$ is a Continuous interval-valued τ_{norm} on $[I_{iv}]$, and is an IVF(Q) on $Q^2 \times (0,\infty)$ meets the following conditions, the triple (Q,$\Bbbk$, $*_I$) is know as IVFMS on Q:
following requirements:
(1) $\Bbbk(\wp^*, \varrho^*, \tau_{norm}) > \bar{0}$;
(2) $\Bbbk(\wp^*, \varrho^*, \tau_{norm}) = \bar{1}$ if and only if $\wp^* = \varrho^*$;
(3) $\Bbbk(\wp^*, \varrho^*, \tau_{norm}) = \Bbbk(\varrho^*, \wp^*, \tau_{norm})$;
(4) $\Bbbk(\wp^*, \varrho^*, \tau_{norm}) *_I \Bbbk(\wp^*, r, s) \leq \Bbbk(\wp^*, r, \tau_{norm} + s)$;
(5) $\Bbbk(\wp^*, \varrho^*, .)$:$(0,\infty) \to$ [I] is continuous;
(6) $\lim_{\eta\to\infty} \Bbbk(\wp^*, \varrho^*, \tau_{norm}) = \bar{1}$, where $\wp^*$, ϱ^*,r $\in$ Q and τ_{norm},s$> \bar{0}$.
Definition2.6 : Let (Q,$\Bbbk$, $*_I$) is an IVFMS,
(1) For all $\wp^*, \varrho^* \in$ Q, if s > τ_{norm}> 0 then $\Bbbk(\wp^*, \varrho^*, \tau_{norm}) \leq \Bbbk(\wp^*, \varrho^*, s)$.
(2) A Ⓢequence $\{\wp_{\eta_a}\}$ in Q is referred to as a Cauchy-Ⓢequence if for all $\bar{\varepsilon}$ > $\bar{0}$ and τ_{norm}>0, there is an exits a $\eta_{a0} \in \aleph$ such that $\Bbbk(\wp_{\eta_a}, \wp_{\ell_a}, \tau_{norm}) > \bar{1}$ - $\bar{\varepsilon}$ for all $\eta_a, \ell_a \geq \eta_{a0}$.
(3) An IVFMS in which every Cauchy Ⓢequence is Convergent, is said to be a Complete IVFMS .
Definition2.7 : In interval-valued-metric-space, a function is continuous iff $\wp_{\eta_a} \to \wp^*$, $\varrho_{\eta_a} \to \varrho^* \Rightarrow \lim_{\eta_a\to\infty} \Bbbk(\wp_{\eta_a}, \varrho_{\eta_a}, \tau_{norm}) \to \Bbbk(\wp^*, \varrho^*, \tau_{norm})$.
Definition2.8 Assume that Q is a non-empty set and that Y and Z are the self-maps of Q. A point $\wp^*$ in Q is said to be coincidence point of Y and Z

iff $Y\wp^* = Z\wp^*$.
Definition2.9 : Two mapping Y and Z on IVFMS (Q,$\Bbbk$,$*_I$) are weakly compatible by the expression iff $\Bbbk$(YZ$\wp^*$,ZY$\wp^*$,τ_{norm}) $\geq$ $\Bbbk$(Y$\wp^*$,Z$\wp^*$,τ_{norm}) for τ_{norm} ¿0 and $\wp^* \in$ Q.
Definition2.10 : Two mappings Y and Z on IVFMS (Q,$\Bbbk$, $*_I$) are compatible iff for sequence $\{\wp_{\eta_a}\}$ in Q, $\Bbbk$(YZ$\{\wp_{\eta_a}\}$,ZY$\{\wp_{\eta_a}\}$,t)$\to \bar{1}$ whenever F$\{\wp_{\eta_a}\} \to$ v, G$\{\wp_{\eta_a}\} \to$ v for some v $\in$ Q.
Definition2.11 : Two mappings Y and Z on IVFMS (Q,$\Bbbk$, $*_I$) are said to be OWC iff there exits a coincidence point $\wp^*$ of Y and Z in Q such that Y and Z commutes at the point.
Definition2.12 : Two mappings Y and Z on IVFMS (Q,$\Bbbk$, $*_I$) then they are said to satisfy E.A property if there exists a $\{\wp_{\eta_a}\}$ in Q such that $\lim_{\eta_a\to\infty}$F$\{\wp_{\eta_a}\}$ = $\lim_{\eta_a\to\infty}$G$\{\wp_{\eta_a}\}$ = v, v $\in$ Q.
Lemma2.13 : IVFMS (Q,$\Bbbk$, $*_I$) is non-decreasing for all $\wp^*, \varrho^* \in$ Q.
Proof: Proof is followed by the definition of Interval-valued fuzzy metric space .
Lemma2.14 : If for all $\wp^*, \varrho^* \in$ Q, $\tau_{norm} > 0$ and for a number h$\in$(0,1), $\Bbbk$($\wp^*$, ϱ^*, hτ_{norm}) $\geq$ $\Bbbk$($\wp^*$, ϱ^*, τ_{norm}) then $\wp^* = \varrho^*$.

3 Main Result

Theorem3.1 : Let (Q,$\Bbbk$, $*_I$) be a complete IVFMS and let A,B,C,D,E,F,G and H be self mappings of Q. Let $\{AB, CD\}$ and $\{EF, GH\}$ are pair of OWC mappings. If there exits h$\in$(0,1) such that

$$\Bbbk(AB\rho^*, EF\varrho^*, h\tau_{norm}) \geq min \begin{bmatrix} \Bbbk(CD\rho^*, GH\varrho^*, \tau_{norm}), \Bbbk(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \Bbbk(EF\varrho^*, GH\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, GH\varrho^*, \tau_{norm}), \\ \Bbbk(EF\varrho^*, CD\rho^*, \tau_{norm}) \end{bmatrix} .....(3.1)$$

for all $\wp^*, \varrho^* \in$ Q and τ_{norm} >0, then there exit unique points $w^*_{up}, z^*_{up} \in$ Q such that $ABw^*_{up} = CDw^*_{up} = w^*_{up}$ and $EFz^*_{up} = GHz^*_{up} = z^*_{up}$. Moreover, for $z^*_{up} = w^*_{up}$, A,B,C,D,E,F,G and H has unique common fixed point in Q.
Proof : Let us consider that $\{AB, CD\}$ and $\{EF, GH\}$ are pair of OWC, therefore for $\rho^*, \varrho^*, \in$ Q we have $AB\rho^* = CD\rho^*$ and $EF\varrho^* = GH\varrho^*$.
Now we claim that $AB\rho^*$=$EF\varrho^*$.
If not, then by inequality (3.1) we have

$$\mathbb{k}(AB\rho^*, EF\varrho^*, h\tau_{norm}) \geq min \begin{bmatrix} \mathbb{k}(CD\rho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, GH\varrho^*, \tau_{norm}), \mathbb{k}(AB\rho^*, GH\varrho^*, \tau_{norm}), \\ \mathbb{k}(EF\varrho^*, CD\rho^*, \tau_{norm}) \end{bmatrix}$$

$$= min \begin{bmatrix} \mathbb{k}(AB\rho^*, EF\varrho^*, \tau_{norm}), \mathbb{k}(CD\rho^*, CD\rho^*, \tau_{norm}), \mathbb{k}(EF\varrho^*, EF\varrho^*, \tau_{norm}), \\ \mathbb{k}(AB\rho^*, EF\varrho^*, \tau_{norm}), \mathbb{k}(EF\varrho^*, AB\rho^*, \tau_{norm}) \end{bmatrix}$$

$$\geq min \left[\mathbb{k}(AB\rho^*, EF\varrho^*, \tau_{norm}), \bar{1}, \bar{1}, \mathbb{k}(AB\rho^*, EF\varrho^*, \tau_{norm}), \mathbb{k}(EF\varrho^*, AB\rho^*, \tau_{norm}) \right]$$

$\geq \mathbb{k}(AB\rho^*, EF\varrho^*, \tau_{norm})$
Therefore $AB\rho^* = EF\varrho^*$
i.e. $AB\rho^* = CD\rho^* = EF\varrho^* = GH\varrho^*$.
Suppose that z^*_{up} is another point in Q such that $ABz^*_{up} = CDz^*_{up}$ then by (3.1)
we have $ABz^*_{up}=CDz^*_{up}=EF\varrho^*=GH\varrho^*$,
so $AB\rho^*=ABz^*_{up}$ and $AB\rho^*=CD\rho^*=w^*_{up}$.
Now by Lemma 2.13, w^*_{up} is the common fixed point of AB and CD, and it is unique.
In the same way, there is a unique point $z^*_{up} \in$ Q such that $z^*_{up}=EFz^*_{up}=GHz^*_{up}$.
Assuming that $w^*_{up} \neq z^*_{up}$, we have
$\mathbb{k}(w^*_{up}, z^*_{up}, h\tau_{norm}) = \mathbb{k}(ABw^*_{up}, EFz^*_{up}, h\tau_{norm})$

$$\geq min \begin{bmatrix} \mathbb{k}(CDw^*_{up}, GHz^*_{up}, \tau_{norm}), \mathbb{k}(CDw^*_{up}, ABw^*_{up}, \tau_{norm}), \mathbb{k}(EFz^*_{up}, GHz^*_{up}, \tau_{norm}), \\ \mathbb{k}(ABw^*_{up}, GHz^*_{up}, \tau_{norm}), \mathbb{k}(EFz^*_{up}, CDw^*_{up}, \tau_{norm}) \end{bmatrix}$$

$$= min \begin{bmatrix} (\mathbb{k}(w^*_{up}, z^*_{up}, \tau_{norm}), \mathbb{k}(w^*_{up}, w^*_{up}, \tau_{norm}), \mathbb{k}(z^*_{up}, z^*_{up}, \tau_{norm}), \mathbb{k}(w^*_{up}, z^*_{up}, \tau_{norm}), \\ \mathbb{k}(z^*_{up}, w^*_{up}, \tau_{norm}) \end{bmatrix}$$

$$\geq min \left[(\mathbb{k}(w^*_{up}, z^*_{up}, \tau_{norm}), \bar{1}, \bar{1}, \mathbb{k}(w^*_{up}, z^*_{up}, \tau_{norm}), \mathbb{k}(z^*_{up}, w^*_{up}, \tau_{norm}) \right]$$

$\geq \mathbb{k}(w^*_{up}, z^*_{up}, \tau_{norm})$
by Lemma 2.13, we have $w^*_{up} = z^*_{up}$ and z^*_{up} is the common fixed point A,B,C,D,E,F,G and H. The uniqueness holds from (3.1).
Theorem3.2 : Let (Q,$\mathbb{k}$, $*_I$) be a complete IVFMS and let A,B,C,D,E,F,G and H be self mappings of Q. Let $\{AB, CD\}$ and $\{EF, GH\}$ are pair of OWC

mappings. If there exits h∈(0,1) such that

$$\Bbbk(AB\rho^*, EF\varrho^*, h\tau_{norm}) \geq \varphi\left[min\begin{bmatrix}\Bbbk(CD\rho^*, GH\varrho^*, \tau_{norm}), \Bbbk(CD\rho^*, AB\rho^*, \tau_{norm}),\\ \Bbbk(EF\varrho^*, GH\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, GH\varrho^*, \tau_{norm}),\\ \Bbbk(EF\varrho^*, CD\rho^*, \tau_{norm})\end{bmatrix}\right] .....(3.2)$$

for all $\wp^*, \varrho^* \in$ Q and $\varphi \in *_I$ for all $0 < \tau_{norm} < 1$, then A,B,C,D,E,F,G and H has unique common fixed point in Q.

Proof: The proof from theorem 3.1.

Theorem3.3 : Let (Q,$\Bbbk$, $*_I$) be a complete IVFMS and let A,B,C,D,E,F,G and H be self mappings of Q. Let $\{AB, CD\}$ and $\{EF, GH\}$ are pair of OWC mappings. If there exits h∈(0,1) such that

$$\Bbbk(AB\rho^*, EF\varrho^*, h\tau_{norm}) \geq \varphi\begin{bmatrix}\Bbbk(CD\rho^*, GH\varrho^*, \tau_{norm}), \Bbbk(CD\rho^*, AB\rho^*, \tau_{norm}),\\ \Bbbk(EF\varrho^*, GH\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, GH\varrho^*, \tau_{norm}),\\ \Bbbk(EF\varrho^*, CD\rho^*, \tau_{norm})\end{bmatrix} .....(3.3)$$

for all $\wp^*, \varrho^* \in$ Q and $\varphi : [I_{iv}]^5 \to [I_{iv}]$ so that $\varphi(\tau_{norm}, \bar{1}, \bar{1}, \tau_{norm}, \tau_{norm}) > \tau_{norm}$ for all $0 < \tau_{norm} < 1$, then A,B,C,D,E,F,G and H has unique common fixed point in Q.

ProofLet us consider that $\{AB, CD\}$ and $\{EF, GH\}$ are pair of OWC, therefore for $\rho^*, \varrho^*, \in$ Q we have $AB\rho^* = CD\rho^*$ and $EF\varrho^* = GH\varrho^*$.

Now we claim that $AB\rho^*$=$EF\varrho^*$.

If not, then by inequality (3.3) we have

$$\Bbbk(AB\rho^*, EF\varrho^*, h\tau_{norm}) \geq \varphi\begin{bmatrix}\Bbbk(CD\rho^*, GH\varrho^*, \tau_{norm}), \Bbbk(CD\rho^*, AB\rho^*, \tau_{norm}),\\ \Bbbk(EF\varrho^*, GH\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, GH\varrho^*, \tau_{norm}),\\ \Bbbk(EF\varrho^*, CD\rho^*, \tau_{norm})\end{bmatrix}$$

$$= \varphi\begin{bmatrix}\Bbbk(AB\rho^*, EF\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, AB\rho^*, \tau_{norm}), \Bbbk(EF\varrho^*, EF\varrho^*, \tau_{norm}),\\ \Bbbk(AB\rho^*, EF\varrho^*, \tau_{norm}), \Bbbk(EF\varrho^*, AB\rho^*, \tau_{norm})\end{bmatrix}$$

$$\geq \left[\Bbbk(AB\rho^*, EF\varrho^*, \tau_{norm}), \bar{1}, \bar{1}, \Bbbk(AB\rho^*, EF\varrho^*, \tau_{norm}), \Bbbk(EF\varrho^*, AB\rho^*, \tau_{norm})\right]$$

$\geq \Bbbk(AB\rho^*, EF\varrho^*, \tau_{norm})$

Therefore $AB\rho^* = EF\varrho^*$

i.e. $AB\rho^* = CD\rho^* = EF\varrho^* = GH\varrho^*$.

Suppose that z^*_{up} is another point in Q such that $ABz^*_{up} = CDz^*_{up}$ then by

(3.3)
we have ABz^*_{up}=CDz^*_{up}=$EF\varrho^*$=$GH\varrho^*$,
so $AB\rho^*$=ABz^*_{up} and $AB\rho^*$=$CD\rho^*$=w^*_{up} is point of coincidence of AB and CD.
Now by Lemma 2.13, w^*_{up} is the common fixed point of AB and CD, and it is unique.
In the same way, there is a unique point $z^*_{up} \in$ Q such that z^*_{up}=EFz^*_{up}=GHz^*_{up}.
Hence z^*_{up} is the common fixed point A,B,C,D,E,F,G and H.
Example: Let $I = [0, 1]$ with the usual metric $\Bbbk\ (\rho^*, \varrho^*, \tau_{norm}) = |\tau_{norm}\rho^* - \tau_{norm}\varrho^*|$ and set up the following maps:
$AB\rho^* = \frac{1}{2}$ for ρ in [0,1] , $CD\rho^* = \frac{3}{4}$ for ρ in [0,1], $EF\rho^* = 1$ for ρ in [0,1], $GH\rho^* = \frac{1}{4}$ for ρ in [0,1], $\{AB, CD\}$ and $\{EF, GH\}$ are pair of OWC mapps.
Putting φ(§$_1$,§$_2$,§$_3$,§$_4$,§$_5$) = -§$_1$ + $\frac{1}{2}$min (§$_2$,§$_3$)+ $\frac{1}{3}$(§$_4$+§$_5$). we get

$$\Bbbk(AB\rho^*, EF\varrho^*, h\tau_{norm}) \geq \varphi \begin{bmatrix} \Bbbk(CD\rho^*, GH\varrho^*, \tau_{norm}), \Bbbk(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \Bbbk(EF\varrho^*, GH\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, GH\varrho^*, \tau_{norm}), \\ \Bbbk(EF\varrho^*, CD\rho^*, \tau_{norm}) \end{bmatrix}$$

$\Rightarrow |1 - \frac{1}{2}| = \frac{1}{2} \geq \varphi \left\{\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{4}, \frac{1}{4}\right\}$
$\Rightarrow \frac{1}{2} \geq$ -$\frac{1}{2}$ + $\frac{1}{2}$ max $(\frac{1}{4},\frac{3}{4})$ + $\frac{1}{3}$ $(\frac{1}{2}+\frac{1}{2}) \geq$ -$\frac{1}{2}$ + $\frac{1}{2}$ $(\frac{3}{4})$ + $\frac{1}{3}$ $(\frac{1}{2}) \geq \frac{1}{24}$
The eight maps accept 1 as the unique fixed point between them, and hence, all of the theorem's requirements are satisfied.
Theorem3.4: Let (Q,$\Bbbk$, $*_I$) be a complete IVFMS and let A,B,C,D,E,F,G and H be self mappings of Q. Let $\{AB, CD\}$ and $\{EF, GH\}$ are pair of OWC mappings. If there exits h∈(0,1) such that

$$\Bbbk(AB\rho^*, EF\varrho^*, h\tau_{norm}) \geq min \begin{bmatrix} \Bbbk(CD\rho^*, GH\varrho^*, \tau_{norm}), \Bbbk(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \Bbbk(EF\varrho^*, GH\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, GH\varrho^*, \tau_{norm}) \end{bmatrix} .....(3.4)$$

for all $\wp^*, \varrho^* \in$ Q and τ_{norm} >0, then A,B,C,D,E,F,G and H has unique common fixed point in Q.
Proof:Let us consider that $\{AB, CD\}$ and $\{EF, GH\}$ are pair of OWC, therefore for $\rho^*, \varrho^*, \in$ Q we have $AB\rho^* = CD\rho^*$ and $EF\varrho^* = GH\varrho^*$.
Now we claim that $AB\rho^*$=$EF\varrho^*$.
If not, then by inequality (3.4) we have

$$\Bbbk(AB\rho^*, EF\varrho^*, h\tau_{norm}) \geq min \begin{bmatrix} \Bbbk(CD\rho^*, GH\varrho^*, \tau_{norm}), \Bbbk(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \Bbbk(EF\varrho^*, GH\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, GH\varrho^*, \tau_{norm}) \end{bmatrix}$$

$$\geq \begin{bmatrix} \Bbbk(AB\rho^*, EF\varrho^*, \tau_{norm}), \Bbbk(CD\rho^*, CD\rho^*, \tau_{norm}), \\ \Bbbk(EF\varrho^*, EF\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, EF\varrho^*, \tau_{norm}) \end{bmatrix}$$

$\geq \Bbbk(AB\rho^*, EF\varrho^*, \tau_{norm})$
Therefore $AB\rho^* = EF\varrho^*$
i.e. $AB\rho^* = CD\rho^* = EF\varrho^* = GH\varrho^*$.
Suppose that z^*_{up} is another point in Q such that $ABz^*_{up} = CDz^*_{up}$ then by (3.4)
we have ABz^*_{up}=CDz^*_{up}=$EF\varrho^*$=$GH\varrho^*$,
so $AB\rho^*$=ABz^*_{up} and $AB\rho^*$=$CD\rho^*$=w^*_{up}.
Now by Lemma 2.13, w^*_{up} is the common fixed point of AB and CD, and it is unique.
In the same way, there is a unique point $z^*_{up} \in$ X such that z^*_{up}=EFz^*_{up}=GHz^*_{up}.

Hence $w^*_{up}$is the common fixed point of A,B,C,E,F.G and H. The uniqueness Of w^*_{up}holds from (3.4).
Corollary3.5 : Let (Q,$\Bbbk$, $*_I$) be a complete IVFMS and let A,B,C,D,E,F,G and H be self mappings of Q. Let $\{AB, CD\}$ and $\{EF, GH\}$ are pair of OWC mappings. If there exits h∈(0,1) such that

$$\Bbbk(AB\rho^*, EF\varrho^*, h\tau_{norm}) \geq min \begin{bmatrix} \Bbbk(CD\rho^*, GH\varrho^*, \tau_{norm}), \Bbbk(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \Bbbk(EF\varrho^*, GH\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, GH\varrho^*, \tau_{norm}), \\ \Bbbk(EF\varrho^*, CD\rho^*, 2\tau_{norm}) \end{bmatrix} \quad .....(3.5)$$

for all $\wp^*, \varrho^* \in$ Q and $\tau_{norm} > 0$, then A,B,C,D,E,F,G and H has unique common fixed point in Q.
Proof: We have

$$\Bbbk(AB\rho^*, EF\varrho^*, h\tau_{norm}) \geq min \begin{bmatrix} \Bbbk(CD\rho^*, GH\varrho^*, \tau_{norm}), \Bbbk(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \Bbbk(EF\varrho^*, GH\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, GH\varrho^*, \tau_{norm}), \\ \Bbbk(EF\varrho^*, CD\rho^*, 2\tau_{norm}) \end{bmatrix}$$

$$\geq min \begin{bmatrix} \Bbbk(CD\rho^*, GH\varrho^*, \tau_{norm}), \Bbbk(CD\rho^*, AB\rho^*, \tau_{norm}), \Bbbk(EF\varrho^*, GH\varrho^*, \tau_{norm}), \\ \Bbbk(AB\rho^*, GH\varrho^*, \tau_{norm}), \Bbbk(CD\rho^*, GH\varrho^*, \tau_{norm}), \Bbbk(EF\varrho^*, GH\varrho^*, \tau_{norm}) \end{bmatrix}$$

$$\geq min \begin{bmatrix} \Bbbk(CD\rho^*, GH\varrho^*, \tau_{norm}), \Bbbk(CD\rho^*, AB\rho^*, \tau_{norm}), \Bbbk(EF\varrho^*, GH\varrho^*, \tau_{norm}), \\ \Bbbk(AB\rho^*, GH\varrho^*, \tau_{norm}) \end{bmatrix}$$

From Theorem 3.4, unique common fixed point exists for A,B,C,D,E,F,G and H.

Theorem3.6 : Let (Q,$\Bbbk$, $*_I$) be a complete IVFMS and let A,B,C,D,R and V be self mappings of Q. Let $\{R, AB\}$ and $\{V, CD\}$ are pair of OWC mappings. If there exits h$\in$(0,1) such that

$$\Bbbk(R\rho^*, V\varrho^*, h\tau_{norm}) \geq min \begin{bmatrix} \Bbbk(AB\rho^*, CD\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, R\rho^*, \tau_{norm}), \\ \Bbbk(V\varrho^*, CD\varrho^*, \tau_{norm}), \Bbbk(R\rho^*, CD\varrho^*, \tau_{norm}), \\ \Bbbk(V\varrho^*, AB\rho^*, \tau_{norm}) \end{bmatrix} .....(3.6)$$

for all $\wp^*, \varrho^* \in$ Q and τ_{norm} >0, then there exit unique points $w^*_{up}, z^*_{up} \in$ Q such that $ABw^*_{up} = Rw^*_{up} = w^*_{up}$ and $CDz^*_{up} = Vz^*_{up} = z^*_{up}$. Moreover, for $z^*_{up} = w^*_{up}$, A,B,C,D,R and V has unique common fixed point in Q.

Proof: Let us consider that $\{R, AB\}$ and $\{V, CD\}$ are pair of OWC, therefore for $\rho^*, \varrho^*, \in$ Q we have $AB\rho^* = R\rho^*$ and $CD\varrho^* = V\varrho^*$.
Now we claim that $R\rho^* = V\varrho^*$.
if not, then by inequality (3.6) we have

$$\Bbbk(R\rho^*, V\varrho^*, h\tau_{norm}) \geq min \begin{bmatrix} \Bbbk(AB\rho^*, CD\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, R\rho^*, \tau_{norm}), \\ \Bbbk(V\varrho^*, CD\varrho^*, \tau_{norm}), \Bbbk(R\rho^*, CD\varrho^*, \tau_{norm}), \\ \Bbbk(V\varrho^*, AB\rho^*, \tau_{norm}) \end{bmatrix}$$

$$= min \begin{bmatrix} \Bbbk(R\rho^*, V\varrho^*, \tau_{norm}), \Bbbk(R\rho^*, R\rho^*, \tau_{norm}), \Bbbk(V\varrho^*, V\varrho^*, \tau_{norm}), \\ \Bbbk(R\rho^*, V\varrho^*, \tau_{norm}), \Bbbk(V\varrho^*, R\rho^*, \tau_{norm}) \end{bmatrix}$$

$$\geq \left[\Bbbk(R\varrho^*, V\rho^*, \tau_{norm}), \bar{1}, \bar{1}, \Bbbk(R\rho^*, V\varrho^*, \tau_{norm})\Bbbk(V\varrho^*, R\rho^*, \tau_{norm}),\right]$$

$\geq min \left[\Bbbk(R\rho^*, V\varrho^*, \tau_{norm})\right]$
Therefore $AB\rho^* = R\varrho^*$
i.e. $AB\rho^* = R\rho^* = CD\varrho^* = V\varrho^*$.
Suppose that z^*_{up} is another point in Q such that $ABz^*_{up} = Rz^*_{up}$ then by (3.6) we have ABz^*_{up}=Rz^*_{up}=$CD\varrho^*$=$V\varrho^*$,
so $AB\rho^*$=ABz^*_{up} and $AB\rho^*$=$R\rho^*$=w^*_{up}.

Now by Lemma 2.13, w^*_{up} is the common fixed point of AB and R, and it is unique.
In the same way, there is a unique point $z^*_{up} \in$ Q such that z^*_{up}=CDz^*_{up}=Vz^*_{up}.
Assuming that $w^*_{up} \neq z^*_{up}$,
we have $\Bbbk(w^*_{up}, z^*_{up}, h\tau_{norm}) = \Bbbk(Rw^*_{up}, Vz^*_{up}, h\tau_{norm})$

$$\geq min \begin{bmatrix} \Bbbk(ABw^*_{up}, CDz^*_{up}, \tau_{norm}), \Bbbk(ABw^*_{up}, Rw^*_{up}, \tau_{norm}), \Bbbk(Vz^*_{up}, CDz^*_{up}, \tau_{norm}), \\ \Bbbk(Rw^*_{up}, CDz^*_{up}, \tau_{norm}), \Bbbk(Vz^*_{up}, ABw^*_{up}, \tau_{norm}) \end{bmatrix}$$

$$= min \begin{bmatrix} (\Bbbk(w^*_{up}, z^*_{up}, \tau_{norm}), \Bbbk(w^*_{up}, w^*_{up}, \tau_{norm}), \Bbbk(z^*_{up}, z^*_{up}, \tau_{norm}), \Bbbk(w^*_{up}, z^*_{up}, \tau_{norm}), \\ \Bbbk(z^*_{up}, w^*_{up}, \tau_{norm}) \end{bmatrix}$$

$$\geq min \left[\Bbbk(w^*_{up}, z^*_{up}, \tau_{norm}), \bar{1}, \bar{1}, \Bbbk(w^*_{up}, z^*_{up}, \tau_{norm}), \Bbbk(z^*_{up}, w^*_{up}, \tau_{norm})\right]$$

$\geq \Bbbk(w^*_{up}, z^*_{up}, \tau_{norm})$
by Lemma 2.13, we have $w^*_{up} = z^*_{up}$ and z^*_{up} is the common fixed point A,B,C,D,R and V. The uniqueness holds from (3.6).
Theorem3.7 : Let (Q,$\Bbbk$, $*_I$) be a complete IVFMS and let A,B,C,D,R and V be self mappings of Q. Let $\{R, AB\}$ and $\{V, CD\}$ are pair of OWC mappings. If there exits h$\in$(0,1) such that

$$\Bbbk(R\rho^*, V\varrho^*, h\tau_{norm}) \geq \varphi \left[min \begin{bmatrix} \Bbbk(AB\rho^*, CD\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, R\rho^*, \tau_{norm}), \\ \Bbbk(V\varrho^*, CD\varrho^*, \tau_{norm}), \Bbbk(R\rho^*, CD\varrho^*, \tau_{norm}), \\ \Bbbk(V\varrho^*, AB\rho^*, \tau_{norm}) \end{bmatrix} \right] .....(3.7)$$

for all $\wp^*, \varrho^* \in$ Q and $\varphi \in *_I$ for all $0 < \tau_{norm} < 1$, then A,B,C,D,R and V has unique common fixed point in Q.
Proof: The proof from theorems 3.6.
Theorem3.8 :Let (Q,$\Bbbk$, $*_I$) be a complete IVFMS and let A,B,C,D,R and V be self mappings of Q. Let $\{R, AB\}$ and $\{V, CD\}$ are pair of OWC mappings. If there exits h$\in$(0,1) such that

$$\Bbbk(R\rho^*, V\varrho^*, h\tau_{norm}) \geq min \begin{bmatrix} \Bbbk(AB\rho^*, CD\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, R\rho^*, \tau_{norm}), \\ \Bbbk(V\varrho^*, CD\varrho^*, \tau_{norm}), \Bbbk(R\rho^*, CD\varrho^*, \tau_{norm}), \end{bmatrix} ....(3.8)$$

for all $\wp^*, \varrho^* \in$ Q and τ_{norm} >0, then A,B,C,D,R and V has unique common fixed point in Q.
Proof: Let us consider that $\{R, AB\}$ and $\{V, CD\}$ are pair of OWC, therefore for $\rho^*, \varrho^*, \in$ Q we have $AB\rho^* = R\rho^*$ and $CD\varrho^* = V\varrho^*$.

Now we claim that $R\rho^* = V\varrho^*$.
if not, then by inequality (3.8) we have

$$\Bbbk(R\rho^*, V\varrho^*, h\tau_{norm}) \geq min\begin{bmatrix}\Bbbk(AB\rho^*, CD\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, R\rho^*, \tau_{norm}),\\ \Bbbk(V\varrho^*, CD\varrho^*, \tau_{norm}), \Bbbk(R\rho^*, CD\varrho^*, \tau_{norm})\end{bmatrix}$$

$$= min\begin{bmatrix}\Bbbk(R\rho^*, V\varrho^*, \tau_{norm}), \Bbbk(R\rho^*, R\rho^*, \tau_{norm}), \Bbbk(V\varrho^*, V\varrho^*, \tau_{norm}),\\ \Bbbk(R\rho^*, V\varrho^*, \tau_{norm})\end{bmatrix}$$

$$\geq \left[\Bbbk(R\varrho^*, V\rho^*, \tau_{norm}), \bar{1}, \bar{1}, \Bbbk(R\rho^*, V\varrho^*, \tau_{norm})\right]$$

$\geq min\left[\Bbbk(R\rho^*, V\varrho^*, \tau_{norm})\right]$
Therefore $AB\rho^* = R\varrho^*$
i.e. $AB\rho^* = R\rho^* = CD\varrho^* = V\varrho^*$.
Suppose that z^*_{up} is another point in Q such that $ABz^*_{up} = Rz^*_{up}$ then by (3.8) we have ABz^*_{up}=Rz^*_{up}=$CD\varrho^*$=$V\varrho^*$,
so $AB\rho^*$=ABz^*_{up} and $AB\rho^*$=$R\rho^*$=w^*_{up}.
Now by Lemma 2.13, w^*_{up} is the common fixed point of AB and R, and it is unique.
In the same way, there is a unique point $z^*_{up} \in$ Q such that z^*_{up}=CDz^*_{up}=Vz^*_{up}.
Hence we get w^*_{up} is a common fixed point of A,B,C,D,R and V.
Corollary3.9 : Let (Q,$\Bbbk$, $*_I$) be a complete IVFMS and let A,B,C,D,R and V be self mappings of Q. Let $\{AB, R\}$ and $\{CD, V\}$ are pair of OWC mappings. If there exits h∈(0,1) such that

$$\Bbbk(R\rho^*, V\varrho^*, h\tau_{norm}) \geq min\begin{bmatrix}\Bbbk(AB\rho^*, CD\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, R\rho^*, \tau_{norm}),\\ \Bbbk(V\varrho^*, CD\varrho^*, \tau_{norm}), \Bbbk(R\rho^*, CD\varrho^*, \tau_{norm}),\\ \Bbbk(V\varrho^*, AB\rho^*, 2\tau_{norm})\end{bmatrix}$$

for all $\wp^*, \varrho^* \in$ Q and τ_{norm} >0, then A,B,C,D,R and V has unique common fixed point in Q.
Proof We have

$$\Bbbk(R\rho^*, V\varrho^*, h\tau_{norm}) \geq min\begin{bmatrix}\Bbbk(AB\rho^*, CD\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, R\rho^*, \tau_{norm}),\\ \Bbbk(V\varrho^*, CD\varrho^*, \tau_{norm}), \Bbbk(R\rho^*, CD\varrho^*, \tau_{norm}),\\ \Bbbk(V\varrho^*, AB\rho^*, 2\tau_{norm})\end{bmatrix}$$

$$\geq min \begin{bmatrix} \Bbbk(AB\rho^*, CD\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, R\rho^*, \tau_{norm}), \Bbbk(V\varrho^*, CD\varrho^*, \tau_{norm}), \\ \Bbbk(R\rho^*, CD\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, CD\varrho^*, \tau_{norm}) \Bbbk(V\varrho^*, CD\varrho^*, \tau_{norm}) \end{bmatrix}$$

$$\geq min \begin{bmatrix} \Bbbk(AB\rho^*, CD\varrho^*, \tau_{norm}), \Bbbk(AB\rho^*, R\rho^*, \tau_{norm}) \Bbbk(V\varrho^*, CD\varrho^*, \tau_{norm}), \\ \Bbbk(R\rho^*, CD\varrho^*, \tau_{norm}), \end{bmatrix}$$

From theorem 3.8, unique common fixed point exits for A,B,C,D,R and V.

Theorem3.10 : Let (Q,$\Bbbk$, $*_I$) be a complete IVFMS and let A,B,C,D be self mappings of Q. Let $\{AB\}$ and $\{CD\}$ are pair of OWC mappings. If there exits h∈(0,1) such that

$$\Bbbk(CD\rho^*, CD\varrho^*, h\tau_{norm}) \geq \left[\alpha \Bbbk(AB\rho^*, AB\varrho^*, \tau_{norm})\right] + \beta min \begin{bmatrix} \Bbbk(AB\rho^*, AB\varrho^*, \tau_{norm}), \\ \Bbbk(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \Bbbk(CD\varrho^*, AB\varrho^*, \tau_{norm}) \end{bmatrix}$$

where $\alpha, \beta > 0, \alpha + \beta > 1$. Then A,B,C and D has unique common fixed point in Q.

Proof: Let AB and CD are OWC, therefore for point $\rho^*\epsilon$Q such that $AB\rho^*$=$CD\rho^*$.

Suppose that there exist $\varrho^*\epsilon$Q for which $AB\varrho^*$=$CD\varrho^*$.

On contrary let us assume $AB\rho^*$=$CD\varrho^*$

we have

$$\Bbbk(CD\rho^*, CD\varrho^*, h\tau_{norm}) \geq \left[\alpha \Bbbk(AB\rho^*, AB\varrho^*, \tau_{norm})\right] + \beta min \begin{bmatrix} \Bbbk(AB\rho^*, AB\varrho^*, \tau_{norm}), \\ \Bbbk(CD\rho^*, AB\rho^*, \tau_{norm}), \\ \Bbbk(CD\varrho^*, AB\varrho^*, \tau_{norm}) \end{bmatrix}$$

$$= \left[\alpha \Bbbk(CD\rho^*, CD\varrho^*, \tau_{norm})\right] + \beta min \begin{bmatrix} \Bbbk(CD\rho^*, CD\varrho^*, \tau_{norm}), \Bbbk(CD\rho^*, CD\rho^*, \tau_{norm}), \\ \Bbbk(CD\varrho^*, CD\varrho^*, \tau_{norm}) \end{bmatrix}$$

$$\geq \left[\alpha \Bbbk(CD\rho^*, CD\varrho^*, \tau_{norm})\right] + \beta min \left[\Bbbk(CD\rho^*, CD\varrho^*, \tau_{norm}), \bar{1}, \bar{1}\right]$$

$$\geq \left[\alpha \Bbbk(CD\rho^*, CD\varrho^*, \tau_{norm})\right] + \beta \left[\Bbbk(CD\rho^*, CD\varrho^*, \tau_{norm})\right]$$

$$\geq (\alpha + \beta) \left[\Bbbk(CD\rho^*, CD\varrho^*, \tau_{norm})\right]$$

Which is a contradiction, since $(\alpha + \beta) > 1$.

Therefore $CD\rho^* = CD\varrho^*$ and hence $AB\rho^* = AB\varrho^*$ and $AB\rho^*$ is unique.

Therefore from lemma 2.13 A,B ,C and D have a unique fixed point.

4 Application

Consider the integral equation
ρ(ž,τ) = ğ$_n$ (ρ(ž),τ) + $\int_{\alpha}^{\beta}$y̌(ž,š,τ)ȟ$_n$(š,ŭ(š),τ)ds + $\int_{\alpha}^{\beta}$x̌(ž,š,τ)ǰ$_n$(š,ŭ(š),τ)ds for all ž∈ $[\alpha, \beta]$ where
1. ğ$_n$: $[\alpha, \beta] \to [0, 1]$ are Continuous.
2. y̌(ž,š,τ),x̌(ž,š,τ) : $[\alpha, \beta] \times [\alpha, \beta] \to [0, 1]$ are Continuous.
3. ȟ$_n$,ǰ$_n$: $[\alpha, \beta] \times$ [0,1] → [0,1] are Continuous.
Let Ȩ = C[0, 1] be the set of Continuous function on $[\alpha, \beta]$
$\Bbbk(\rho, \varrho, \tau) = |\tau\rho|_1 + |\tau\varrho|_1$ = max (ρ(ž),τ) + max (ϱ(ž),τ) for all $\rho, \varrho \in$ Ȩ. It is evident that $\Bbbk(\rho, \varrho, \tau)$ is a dislocated fuzzy metric space.

6 Conclusions

We established some fixed point theorems in IVFMS in our research and it can also be extended to analyze the data collected in various streams depending on data availability to find fixed point results with feasibility. Application of this can be used to get the optimized solution or exact solution that is close to the factual value.

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