__Original Research Article__

MODIFIED ROBUST ESTIMATORS OF POPULATION MEAN IN THE PRESENCE OF NON-RESPONSE AND MEASUREMENT ERROR

ABSTRACT

The use of estimators in statistics, quality assurance, and survey methodology can never be over flogged just as the use of sampling. An estimator is a statistic obtained from values in the sample in order to estimate certain characteristic such as the mean, variance, standard deviation, coefficient of variation etc. of population of interest. Two of the major challenges of statisticians or surveyors due encounter at the course of data collection in the field of medical and social sciences is non-response and measurement errors. This poses serious problem during data compilation, computation and estimation stages. In this dissertation, a robust measure of classes of estimators are proposed in the presence of non-response and measurement errors through the use of call-back and imputation scheme incorporated with measurement errors parameters which are robust against the detrimental effects of non-response and measurement errors on the efficiency of the estimators. The properties of the proposed estimators (Bias &MSE_S) were derived up to the second degree approximation using Taylors's series approach and test for the consistency of the estimators was established. The conditions for the efficiencies of the proposed estimators over the existing estimators was also considered and established in this research. The empirical study conducted using simulated data from normal distribution, exponential distribution, chi-square distribution, uniform distribution, gamma distribution and poison distribution revealed that the modified classes of estimators of the proposed imputation schemes are more efficient and satisfactory than the compared existing estimators. Thus, the proposed modified classes of estimators under imputation scheme were recommended for use in the real life situation especially in the presence of non-response and measurement errors during data analysis and estimation stages.

Keywords: Estimator, Study variable, Auxiliary variables, Non-response, Measurement error

1 Introduction

Non-response is one of the challenges statisticians and other researchers do encountered during the course of data collection in the field of medical and social sciences. This poses problems during data compilation, computation and estimation stages. Non response is the absent, refusal or inaccessibility to some respondents. Hansen and Hurwitz [1], were the first to consider the problem of non-response. The normal procedure is to go back to the field to collect the missing information (call-back). Authors like Audu *et al.*[2], Audu *et al.* [3], had proposed several estimators for estimating population parameters like mean, variance and proportion using call-back approach. However, this approach incurred additional cost, time and logistics. Another approach to tackle non-response is the use of imputation schemes. To solve this problem, imputation technique will be used to handle situations where data is missing. Missing data can be completed with specific substitutes and data analyzed using standard methods. The imputation techniques often used includes mean imputation, hot deck imputation, nearest neighbor imputation, worm deck imputation and mean-cum-nearest neighbor imputation. Many researchers have suggested mean imputation techniques to deal with problems of non-response or missing value(s). Notable among them are Singh and Horn [4], Singh and Deo [5], Wang and Wang [6], Kadilar and Cingi [7], Toutenburg*et al.* [8], Singh [9], Singh *et al.* [10], Daina and Perri [11], Al-omari*et al*. [12], Gira [13], Singh *et al.* [14], Bhushan and Pandey [15], Singh and Gogoi [16], and Audu *et al.*[17].

Another non-random error often associated with data collection is measurement errors. The theory of survey sampling assumed that the observations recorded during data collection are always free from measurement error. However, this assumption does not meet in many life situations and data is contaminated with errors. The mean square error and other properties of the estimator obtained with significant measurement errors may lead to serious fallacious results. Cochran [18],has discussed the source of measurement error in survey data. Many authors such as Shallabh [19],Strivastava and Shallabh[20], Maneesha and Singh [21], Allen *et al.* [22], Shalabh and Tsai [23], Singh and Vishwakarma [24], Audu *et al.* [25], Singh *et al.* [26], Audu *et al*. [27],etc. have studied the impact of measurement errors in the ratio, product and regression methods of estimation under various sampling schemes for different population parameters.

The current study focused on the modification of Zaman *et al*. [28], estimators in the presence of non-response and measurement errors associated with both study and auxiliary variables.

Methods

2. Some Existing Related Estimators in the presence of Non-Response and Measurement Error.

Consider a finite population $\left(U = U_{1,} U_{2,} ... U_{N}\right)$ of size N such that Y be a study variable and X be any auxiliary variable and draw a sample of size n from a population by using simple random sampling without replacement scheme. Suppose that N_1 units respond for the survey questions and N_2 units do not respond. Then by Hansen Hurwitz (1946) sampling plan, a sub-sample of size $k = \frac{r_2}{h}\left(h \succ 1\right)$ from N_2 non-respondents is selected at random and re-contacted for their direct interview. Here, it ia assumed that r units respond to the survey.

Let $\left(x_{i}^{*}, y_{i}^{*}\right)$ be the observed values and $\left(X_{i}^{*}, Y_{i}^{*}\right)$ be the true values of the study Y and auxiliary variable X, where (i=1,2,….n) units in the sample. Then measurement error is given is given by:

$u_i^* = y_i^* - Y_i^*$ and $v_i^* = x_i^* - X_i^*$

Where $\left(u_{i}^{*}, v_i^*\right)$ are random in nature and both are uncorrelated with mean zero and variance S_u^2 and S_v^2 are associated with measurement error in Y and X respectively for the responding part of the population. $S_{u(2)}^2$ and $S_{v(2)}^2$ are the variances associated with measurement error in Y and X respectively for the non-responding part of the population.

S_y^2 and S_x^2 are the population variances of Y and X respectively for the responding part of the population. $S_{Y(2)}^2$ and $S_{X(2)}^2$ are the variances of X and Y respectively for the non-responding part of the population.

ρ and $\rho_{(2)}$ are the population correlation coefficients between X and Y for the responding and non-responding parts of the population respectively. C_y and $C_{y(2)}$ are the coefficients of variation of Y for responding and non-responding part of the population respectively. Similarly, C_X and $C_{X(2)}$ are the coefficients of variation of X for responding and non-responding part of the population respectively.

Let $w_Y^* = \sum_{i=1}^n \left(Y_i^* - \bar{Y}\right)$ and $w_X^* = \sum_{i=1}^n \left(X_i^* - \bar{X}\right)$, then

$w_u^* = \sum_{i=1}^n u_i^*$ and $w_v^* = \sum_{i=1}^n v_i^*$

Therefore,

$$\frac{1}{n}\left(w_v^* + w_u^*\right) = \frac{1}{n}\sum_{i=1}^n (Y_i^* - \bar{Y}) + \frac{1}{n}\sum_{i=1}^n (y_i^* - \bar{Y}_i^*)$$

Singh *et al.* (2018) consider the effect of Measurement error and Non-response on estimation of population mean and suggested estimator defined in (2.59)

$$t_{sp} = \left[\frac{1}{2}\left\{\bar{y}^* esp\left(\frac{\bar{X} - \bar{x}^*}{\bar{X} + \bar{x}^*}\right) + \bar{y}^* exp\left(\frac{\bar{x}^* - \bar{X}}{\bar{x}^* + \bar{X}}\right)\right\} + \tau_1\left(\bar{X} - \bar{x}^*\right) + \tau_2 \bar{y}^*\right] exp\left[\frac{\bar{X}^* - \bar{x}^{**}}{\bar{X}^* + \bar{x}^{**}}\right] \quad (2.1)$$

where: $\bar{X}^* = \bar{X}\rho$, $\bar{x}^{**} = \bar{x}^* + \bar{X}\left(p-1\right)$, $p = \dfrac{\rho_{xy}+1}{4}$,

$$\tau_1 = \frac{B_s D_s - C_s E_s}{A_s B_s - C_s^2}$$

$$\tau_2 = \frac{A_s E_s - C_s D_s}{A_s B_s - C_s^2}$$

$$Bias\left(t_{sp}\right) = \tau_2 \bar{Y} + \left(\frac{\bar{Y}}{8\bar{X}^2} + \frac{\tau_1}{2\bar{X}\rho} + \frac{3}{8}\frac{\bar{Y}}{\bar{X}^2\rho^2} + \frac{3}{8}\frac{\tau_2\bar{Y}}{\bar{X}^2\rho^2}\right)\nabla_1^2 - \left(\frac{1}{2\bar{X}\rho} + \frac{\tau_2}{2\bar{X}\rho}\right)\nabla_0\nabla_1 \quad (2.2)$$

where:

$$\nabla_0^2 = E\left(\xi_0^2\right) = \kappa_1\left(S_Y^2 + S_u^2\right) + \kappa_2\left(S_{Y(2)}^2 + S_{u(2)}^2\right)$$

$$\nabla_1^2 = E\left(\xi_1^2\right) = \kappa_1\left(S_X^2 + S_v^2\right) + \kappa_2\left(S_{X(2)}^2 + S_{v(2)}^2\right)$$

$$\nabla_0\nabla_1 = E\left(\xi_0\xi_1\right) = \kappa_1\rho_{YX}S_YS_X + \kappa_2\rho_{YX(2)}S_{Y(2)}S_{X(2)}$$

$$MSE\left(t_{SP}\right) = \tau_1^2A_s + \alpha_2^2B_s - 2\tau_1\tau_2C_s - 2\tau_1D_s + 2\tau_2E_s + F_s \tag{2.3}$$

where $A_s = \nabla_1^2$, $B_s = \left(\bar{Y}^2 + \nabla_0^2 + \frac{\bar{Y}^2}{\bar{X}^2\rho^2}\nabla_1^2 - 2\frac{\bar{Y}}{\bar{X}\rho}\nabla_0\nabla_1\right)$, $C_s = \left(\nabla_0\nabla_1 - \frac{\bar{Y}}{\bar{X}\rho}\nabla_1^2\right)$,

$$D_s = \left(\nabla_0\nabla_1 - \frac{\bar{Y}}{2\bar{X}\rho}\nabla_1^2\right),\quad E_s = \left(\frac{\bar{Y}}{8\bar{X}}\nabla_1^2 - \frac{3}{2}\frac{\bar{Y}}{\bar{X}\rho}\nabla_0\nabla_1 + \nabla_1^2 + \frac{5}{8}\frac{\bar{Y}^2}{\bar{X}^2}\frac{\nabla_1^2}{\rho^2}\right)$$

$$F_s = \nabla_0^2 - \frac{\bar{Y}}{\bar{X}\rho}\nabla_0\nabla_1 + \frac{\bar{Y}^2}{4\bar{X}^2}\frac{\nabla_1^2}{\rho^2},\quad \tau_1 = \frac{B_sD_s - C_sE_s}{A_sB_s - C_s^2},\quad \tau_2 = \frac{A_sE_s - C_sD_s}{A_sB_s - C_s^2}$$

$$MSE\left(t_{SP}\right)_{\min} = \bar{Y}^2 + \left[\frac{2C_sD_sE_s - B_sD_s^2 - A_sE_s^2}{A_sB_s - C_s}\right] \tag{2.4}$$

Munneer et al. (2018) proposed exponential type estimators of population mean in the presence of non-response as in (2.5) – (2.70)

$$\bar{y}^*_{prop(1)} = \bar{y}^*_{(\exp)}\left(\frac{\left(\bar{X} - \bar{x}^*\right)}{\left(\bar{X} + \bar{x}^*\right)}\right)\exp\left(\frac{\left(\bar{Z} - \bar{z}^*\right)}{\bar{Z} + \bar{z}^*}\right) \tag{2.5}$$

$$\bar{y}^*_{prop(2)} = \bar{y}^*_{(\exp)}\left(\frac{\left(\bar{X} - \bar{x}^*\right)}{\left(\bar{X} + \bar{x}^*\right)}\right)\exp\left(\frac{\left(\bar{z}^* - \bar{Z}\right)}{\bar{z}^* + \bar{Z}}\right) \tag{2.6}$$

$$\bar{y}^*_{prop(3)} = \bar{y}^*_{(\exp)}\left(\frac{\left(\bar{X} - \bar{x}^*\right)}{\left(\bar{X} + \bar{x}^*\right)}\right)\exp\left(\frac{n\left(\bar{Z} - \bar{z}^*\right)}{2N\bar{Z} - n\left(\bar{z}^* + \bar{Z}\right)}\right) \tag{2.7}$$

$$\bar{y}^*_{prop(4)} = \bar{y}^*_{(\exp)}\left(\frac{\left(\bar{x}^* - \bar{X}\right)}{\left(\bar{x}^* + \bar{X}\right)}\right)\exp\left(\frac{\left(\bar{Z} - \bar{z}^*\right)}{\left(\bar{Z} + \bar{z}^*\right)}\right) \tag{2.8}$$

$$\bar{y}^*_{prop(5)} = \bar{y}^*_{(\exp)}\left(\frac{\bar{x}^* - \bar{X}}{x^* + \bar{X}}\right)\exp\left(\frac{\bar{z}^* - \bar{Z}}{\bar{z}^* + \bar{Z}}\right) \tag{2.9}$$

$$\bar{y}^*_{prop(6)} = \bar{y}^*_{(\exp)}\left(\frac{\left(\bar{x}^* - \bar{X}\right)}{x^* + \bar{X}}\right)\exp\left(\frac{n\left(\bar{Z} - \bar{z}^*\right)}{2N\bar{Z} - n\left(\bar{z}^* + \bar{Z}\right)}\right) \tag{2.10}$$

$$\bar{y}^*_{prop(7)} = \bar{y}^*_{(\exp)}\left(\frac{n\left(\bar{X} - \bar{x}^*\right)}{2N\bar{X} - n\left(\bar{x}^* + \bar{X}\right)}\right)\exp\left(\frac{\left(\bar{Z} - \bar{z}^*\right)}{\bar{Z} + \bar{z}^*}\right) \tag{2.11}$$

$$\bar{y}^*_{prop(8)} = \bar{y}^*_{(\exp)}\left(\frac{n\left(\bar{X} - \bar{x}^*\right)}{2NZ\bar{X} - n\left(\bar{x}^* + \bar{X}\right)}\right)\exp\left(\frac{\left(\bar{z}^* - \bar{Z}\right)}{\bar{z}^* + \bar{Z}}\right) \tag{2.12}$$

$$\bar{y}^*_{prop(9)} = \bar{y}^*_{(\exp)}\left(\frac{n\left(\bar{X} - \bar{x}^*\right)}{2N\bar{X} - n\left(\bar{x}^* + \bar{X}\right)}\right)\exp\left(\frac{\left(\bar{Z} - \bar{z}^*\right)}{\bar{z}^* + \bar{Z}}\right) \tag{2.13}$$

The estimators $\bar{y}^*_{prop(i)}, \quad i = 1, 2, ...,9$ can be generally written as in

$$\bar{y}^*_{prop} = \bar{y}^*\left[\exp\left(\frac{G_1 - D_1}{G_1 + D_1}\right)\right]\exp\left(\frac{G_2 - D_2}{G_2 + D_2}\right) \tag{2.14}$$

$$G_1 = \left(A_1 + C_1\right)\bar{X} + fB_1\bar{x}^* \; G_2 = \left(A_2 + C_2\right)\bar{Z} + fB_2\bar{z}^* \; D_1 = \left(A_1 + fB_1\right)\bar{X} + C_1\bar{x}^*$$

$$D_2 = \left(A_2 + fB_2\right)\bar{Z} + C_2\bar{z}^* \; A_i = \left(k_i - 1\right)\left(k_i - 2\right) \; B_i = \left(k_i - 1\right)\left(k_i - 4\right)$$

$$C_i = \left(k_i - 2\right)\left(k_i - 3\right)\left(k_i - 4\right)$$

$i = 1, 2.3, 4, \ldots\ldots 9$

$$MSE\left(\bar{y}_{prop}\right)_{\min} = \bar{Y}^2\left[V^*_y - \frac{V^{*2}_{yx}V^*_z + V^{*2}_{yz}V^*_x - 2V^{*2}_{yx}V^*_{yx}V^*_{xz}}{V^*_xV^*_z - V^{*2}_{xz}}\right] \qquad \text{i= 1,2,….9} \tag{2.15}$$

$$V^*_y = \lambda C^2_y + \theta C^2_{y(2)}, \qquad V^*_x = \lambda C^2_x + \theta C^2_{x(2)} \qquad V^*_z = \lambda C^2_z + \theta C^2_{z(2)}$$

$$V^*_{yx} = \lambda C_{yx} + \theta C_{yx(2)}, V^*_{yz} = \lambda C_{yz} + \theta C_{yz(2)} \qquad V^*_{yz} = \lambda C_{xz} + \theta C_{xz(2)}$$

$$V^*_{yz} = \lambda C_{yz} + \theta C_{yz(2)}, \quad C_{yx} = \rho_{yx}C_yC_x, \quad C_{yz} = \rho_{yz}C_yC_z,$$

$$C_{xz}=\rho_{xz}C_xC_z,\quad C^2_{y(2)}=\frac{S^2_{y(2)}}{\bar{Y}^2},\ C^2_{x(2)}=\frac{S^2_{x(2)}}{\bar{X}^2},\quad C^2_{z(2)}=\frac{S^2_{z(2)}}{\bar{Z}^2}$$

$$C_{yx(2)}=\rho_{yx(2)}C_{y(2)}C_{x(2)},\quad C_{yz(2)}=\rho_{yz(2)}C_{y(2)}C_{z(2)},\quad C_{xz(2)}=\rho_{xz(2)}C_{x(2)}C_{z(2)},$$

$$\lambda=\left(\frac{1-f}{n}\right),\quad f=\frac{n}{N},\quad \theta=W_2\left(\frac{h-1}{n}\right),\quad W_2=\frac{N_2}{N}$$

Salim and Onyango (2022) proposed a modified estimator for single phase sampling in the presence of observational errors as in

$$t_{so}=\left[\left(\bar{y}_e^*+\beta\left(\bar{X}-\bar{x}_e^*\right)\right)\left(\alpha\exp\left(\frac{\bar{Z}-\bar{z}_e^*}{\bar{Z}+\bar{z}_e^*}\right)+(1-\alpha)\exp\left(\frac{\bar{Z}-\bar{z}_e^*}{\bar{Z}+\bar{z}_e^*}\right)\right)\right] \tag{2.16}$$

$$\begin{aligned}MSE\left(t_{so}\right)=\theta\bar{Y}^2\left(C_Y^2+C_U^2\right)+\bar{Y}\left(\frac{1}{2}-\alpha\right)\frac{1}{2}\theta\bar{Y}\,\bar{Z}\rho YZC_YC_Z\\ -\beta\theta\overline{YX}\,\rho XYC_YC_X\end{aligned} \tag{2.17}$$

$$\alpha=\frac{1}{2}\,,\ \beta=\frac{\rho S_Y}{S_X}$$

3 Proposed Estimators

In this paper, four classes of robust estimators of population mean were proposed in the presence non-response and measurement error using imputation scheme as defined in (3.1) – (3.5)

$$T_1^{**}=\bar{y}_{r(e)}\frac{r}{n}+\left(1-\frac{r}{n}\right)\left[\begin{aligned}&\bar{y}_{r(e)}\left(\frac{\bar{X}_1}{\bar{x}_{1r(e)}}\right)^{\lambda_{11}}\left(\frac{\bar{X}_2}{\bar{x}_{2r(e)}}\right)^{\lambda_{21}}+b_{1(LTS)}\left(\bar{X}_1-\bar{x}_{1r(e)}\right)\\&+b_{2(LTS)}\left(\bar{X}_2-\bar{x}_{2r(e)}\right)\end{aligned}\right] \tag{3.1}$$

$$T_2^{**} = \bar{y}_{r(e)}\frac{r}{n} + \left(1-\frac{r}{n}\right)\left[\begin{array}{l}\bar{y}_{r(e)}\left(\frac{\bar{X}_1}{\bar{x}_{1r(e)}}\right)^{\lambda_{21}}\left(\frac{\bar{X}_2}{\bar{x}_{2r(e)}}\right)^{\lambda_{22}} + b_{1(S)}\left(\bar{X}_1 - \bar{x}_{1r(e)}\right) \\ +b_{2(S)}\left(\bar{X}_2 - \bar{x}_{2r(e)}\right)\end{array}\right] \quad (3.3)$$

$$T_3^{**} = \bar{y}_{r(e)}\frac{r}{n} + \left(1-\frac{r}{n}\right)\left[\begin{array}{l}\bar{y}_{r(e)}\left(\frac{\bar{X}_1}{\bar{x}_{1r(e)}}\right)^{\lambda_{31}}\left(\frac{\bar{X}_2}{\bar{x}_{2r(e)}}\right)^{\lambda_{32}} + b_{1(LMS)}\left(\bar{X}_1 - \bar{x}_{1r(e)}\right) \\ +b_{2(LMS)}\left(\bar{X}_2 - \bar{x}_{2r(e)}\right)\end{array}\right] \quad (3.4)$$

$$T_4^{**} = \bar{y}_{r(e)}\frac{r}{n} + \left(1-\frac{r}{n}\right)\left[\begin{array}{l}\bar{y}_{r(e)}\left(\frac{\bar{X}_1}{\bar{x}_{1r(e)}}\right)^{\lambda_{41}}\left(\frac{\bar{X}_2}{\bar{x}_{2r(e)}}\right)^{\lambda_{42}} + b_{1(HUBM)}\left(\bar{X}_1 - \bar{x}_{1r(e)}\right) + \\ b_{2(HUBM)}\left(\bar{X}_2 - \bar{x}_{2r(e)}\right)\end{array}\right] \quad (3.5)$$

where $\bar{y}_e^* = \dfrac{n_1\bar{y}_{1(e)} + n_2\bar{y}_{h_{2(e)}}}{n_1+n_2}$, $\bar{x}_{1(e)}^* = \dfrac{n_1\bar{x}_{1(e)} + n_2\bar{x}_{1h_2(e)}}{n_1+n_2}$, $\bar{x}_{2(e)}^* = \dfrac{n_1\bar{x}_{2(e)} + n_2\bar{x}_{2h_2(e)}}{n_1+n_2}$

3.1 Bias and MSEs of the proposed estimators $T_i^{**} i = 1,2,3,4$

$$Bias(T_i^{**}) = \left(1-\frac{r}{n}\right)\left(\frac{1}{r}+\frac{1}{N}\right)\left[\begin{array}{l}\frac{\lambda_{i2}\left(\lambda_{i2}-1\right)}{2}C_{x_2}^2 + \frac{\lambda_{i1}\left(\lambda_{i1}-1\right)}{2}C_{x_1}^2 + \lambda_{i1}\lambda_{i2}\rho_{x_1x_2} - \lambda_{i2}\rho_{yx_2}C_yC_{x_2} \\ -\lambda_{i1}\rho_{yx_1}C_yC_{x_1}\end{array}\right] \quad (3.6)$$

$$MSE\left(T_i^{**}\right)_{(\min)} = H^*_{00} + 2A\lambda_{i1(opt)} + 2B\lambda_{i2(opt)} + 2C\lambda_{i1}{}^2_{(opt)} + 2D\lambda_{i2}{}^2_{(opt)} + 2E\lambda_{i1(opt)}\lambda_{i2(opt)} + F \quad (3.7)$$

where $\lambda_{i1}^* = \dfrac{AD-BE}{CD-E^2}$, $\lambda_{i2}^* = \dfrac{BC-AE}{CD-E^2}$

$$A = \left(1-\frac{r}{n}\right)\bar{Y}\left[H_{01}^* - b_1H_{11}^* - b_2H_{12}^*\right], B = \left(1-\frac{r}{n}\right)\bar{Y}\left[H_{02}^* - b_2H_{22}^* - b_1H_{12}^*\right], C = \left(1-\frac{1}{n}\right)\bar{Y}^2H_{11}^*$$

$$D = \left(1-\frac{1}{n}\right)\bar{Y}^2H_{22}^*, E = \bar{Y}^2\left(1-\frac{r}{n}\right)H_{12}^*, F = b_1^2H_{11}^* + b_2^2H_{22}^* - 2b_1H_{01}^* - 2b_2H_{02}^* + 2b_1b_2H_{12}^*$$

3.Efficiency comparisons of the proposed Estimators T_i^*

In this subsection, efficiency conditions of the estimators of the proposed ratio-type over some existing estimators were established.

$$MSE(T_i^*) - MSE(\theta) < 0 \quad (3.8)$$

where θ are the existing estimators

$$MSE(T_i^{**}) - MSE(t_{so}) < 0$$

$$\begin{pmatrix} H_{00}^* - 2A\lambda_{i1} - 2B\lambda_{i2} + 2C\lambda_{i1}^2 + 2D\lambda_{i2}^2 + \\ 2E\lambda_{i1}\lambda_{i2} + F \end{pmatrix} - \begin{bmatrix} \theta\bar{Y}^2\left(C_Y^2 + C_Y^2\right)\bar{Y}\left(\frac{1}{2} - \alpha\right)\theta\bar{Y}\,\bar{Z}\rho_{YZ}C_YC_Z \\ -\beta\theta\bar{Y}\,\bar{X}\rho_{XY}C_YC_X \end{bmatrix} < 0 \quad (3.9)$$

$$F < 2\begin{pmatrix} -H_{00}^* + A\lambda_{i1} + B\lambda_{i2} - C\lambda_{i1}^2 \\ -D\lambda_{i2}^2 - E\lambda_{i1}\lambda_{i2} \end{pmatrix} + \begin{bmatrix} \theta\bar{Y}^2\left(C_Y^2 + C_Y^2\right) + \bar{Y}\left(\frac{1}{2} - \alpha\right)\theta\bar{Y}\,\bar{Z}\rho_{YZ}C_YC_Z \\ -\beta\theta\bar{Y}\,\bar{X}\rho_{XY}C_YC_X \end{bmatrix} \quad (3.10)$$

$$MSE(T_i^{**}) - MSE(\bar{y}_{prop}) < 0$$

$$\begin{pmatrix} H_{00}^* - 2A\lambda_{i1} - 2B\lambda_{i2} + 2C\lambda_{i1}{}^2 + 2D\lambda_{i2}{}^2 + \\ 2E\lambda_{i1}\lambda_{i2} + F \end{pmatrix} - \left[\bar{Y}^2\left(V_y^* - \frac{V_{yx}^{*2}V_x^* + V_{yz}^*V_x^* - V_{yx}^*V_{yz}^*V_{xz}^*}{V_x^*V_z^* - V_{xz}^{*2}}\right)\right] < 0 \quad (3.11)$$

$$F < 2\begin{pmatrix} -H_{00}^* + A\lambda_{i1} + B\lambda_{i2} - C\lambda_{i1}^2 \\ -D\lambda_{i2}^2 - E\lambda_{i1}\lambda_{i2} \end{pmatrix} + \left[\bar{Y}^2\left(V_y^* - \frac{V_{yx}^{*2}V_x^* + V_{yz}^*V_x^* - V_{yx}^*V_{yz}^*V_{xz}^*}{V_x^*V_z^* - V_{xz}^{*2}}\right)\right] \quad (3.12)$$

4 Results and Discussion

In this section, empirical studies were conducted to assess the performance of the estimators of the proposed schemes with respect to existing estimators under study. The Biases, MSEs and PREs of the estimators were computed using (4.1), (4.2) and (4.3) respectively.

$$Bias\left(\hat{\theta}_i\right) = \frac{1}{1000}\sum_{i=1}^{1000}\left(\hat{\theta}_i - \bar{Y}\right), \quad \hat{\theta}_i = \bar{y}_i \quad (4.1)$$

$$MSE\left(\hat{\theta}_i\right) = \frac{1}{1000}\sum_{i=1}^{1000}\left(\hat{\theta}_i - \bar{Y}\right)^2, \quad \hat{\theta}_i = \bar{y}_i \quad (4.2)$$

$$PRE\left(\hat{\theta}_i\right) = \frac{MSE\left(T_i^*\right)}{MSE\left(\hat{\theta}_i\right)} \times 100\% \quad \hat{\theta}_i = \bar{y}_i, T_i^*,\ i = 1,2,3,4 \quad (4.3)$$

Table 1 Population used for Simulation Study

Population	Auxiliary Variable (X)	Study Variable (Y)
1	$X_1 = r\exp(N,2)$ $X_2 = r\exp(N,1)$	$Y = 0.3*X_1 + 5*X_2 + e_i$ Where, e ~ Normal (0, 1) U~ Normal (0, 25) V_1~ Normal (0, 16) V_2~ Normal (0, 9)
2	$X_1 = rchisq(N,2)$ $X_2 = rchisq(N,3)$	
3	$X_1 = rgamma(N,0.5,2.5)$ $X_2 = rgamma(N,1.5,3.5)$	
4	$X_1 = rpois(N,3)$ $X_2 = rpois(N,1.5)$	

Table 2: Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from Exponential Distribution

Estimator	Bias	MSE	PRE
Sample mean	-0.009989655	3.670668	100
Singh et al. (2018) t_{sp}	3.023097e+111	1.606164e+224	2.285364e-222
Salim and Onyango (2022) t_{so}	0.1601833	1.593409	378.4095
Muneer et al. (2018) $t_{m(1)}$	0.4332807	10.93393	230.3658
Muneer et al. (2018) $t_{m(2)}$	-0.009989655	3.670668	33.57135
Muneer et al. (2018) $t_{m(3)}$	0.1738815	3.833255	95.75853
Muneer et al. (2018) $t_{m(4)}$	-0.1675946	1.865367	196.7799
Muneer et al. (2018) $t_{m(5)}$	72.44146	6090.717	0.0602666
Muneer et al. (2018) $t_{m(6)}$	-0.1404735	4.09145	89.71559
Muneer et al. (2018) $t_{m(7)}$	-0.06705114	0.9670699	379.566
Muneer et al. (2018) $t_{m(8)}$	0.2097258	9.696611	37.85517
Muneer et al. (2018) $t_{m(9)}$	-0.06705114	0.9670699	379.566
Proposed Estimators under imputation schemes			
T_1^{**}	-0.0123064	0.4026394	911.6515
T_2^{**}	-0.008408729	0.4209914	871.9106
T_3^{**}	-0.03844668	0.2309159	1589.612
T_4^{**}	-0.1847394	4.459426	864.6427

Table 3: Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from Chi-square Distribution

Estimator	Bias	MSE	PRE
Sample mean	-0.08410128	21.12014	100
Singh et al. (2018) t_{sp}	-6.712408e+113	1.103315e+229	1.914243e-226
Salim and Onyango (2022) t_{so}	0.4067392	10.83238	449.5515
Muneer et al. (2018) $t_{m(1)}$	0.9687119	68.01479	194.9722
Muneer et al. (2018) $t_{m(2)}$	-0.08410128	21.12014	31.05228
Muneer et al. (2018) $t_{m(3)}$	0.4246086	25.7296	82.085
Muneer et al. (2018) $t_{m(4)}$	-0.2983239	13.9069	151.868
Muneer et al. (2018) $t_{m(5)}$	217.3016	52068.63	0.04056212
Muneer et al. (2018) $t_{m(6)}$	-0.293289	27.66242	76.34957
Muneer et al. (2018) $t_{m(7)}$	-0.1910325	4.67781	451.4963
Muneer et al. (2018) $t_{m(8)}$	0.3349565	54.84342	38.50989
Muneer et al. (2018) $t_{m(9)}$	-0.1910325	4.67781	451.4963
Proposed Estimators under imputation schemes			
T_1^{**}	-0.05648113	2.141613	986.179
T_2^{**}	-0.05570801	2.175715	970.7216
T_3^{**}	-0.2050515	3.528707	598.5234
T_4^{**}	-1.994144	29.3358	380.305

Table 4: Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from gamma Distribution

Estimator	Bias	MSE	PRE
Sample mean	-0.003964409	0.6811538	100
Singh et al. (2018) t_{sp}	1.41007e+111	7.553066e+222	9.018242e-222
Salim and Onyango (2022) t_{so}	0.1328968	0.55617	221.8051
Muneer et al. (2018) $t_{m(1)}$	0.2428026	2.080598	122.4722
Muneer et al. (2018) $t_{m(2)}$	-0.003964409	0.6811538	32.73836
Muneer et al. (2018) $t_{m(3)}$	0.1419581	0.920482	73.99968
Muneer et al. (2018) $t_{m(4)}$	-0.09282761	0.5294192	128.6606
Muneer et al. (2018) $t_{m(5)}$	30.5277	1088.365	0.06258505
Muneer et al. (2018) $t_{m(6)}$	-0.08356495	0.7941444	85.77203
Muneer et al. (2018) $t_{m(7)}$	-0.02729135	0.3115916	218.6047
Muneer et al. (2018) $t_{m(8)}$	0.07586364	1.525341	44.65584
Muneer et al. (2018) $t_{m(9)}$	-0.02729135	0.3115916	218.6047
Proposed Estimators under imputation schemes			
T_1^{**}	-0.05599152	0.08630086	789.2781
T_2^{**}	-0.05498659	0.08482839	802.9786
T_3^{**}	-0.05425017	0.0593816	1147.079
T_4^{**}	-0.09695563	1.022151	768.975

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Table 5: Biases, MSEs and PREs of the Proposed and Existing Estimators using Data from poison Distribution

Estimator	Bias	MSE	PRE
Sample mean	0.02252968	6.508619	100
Singh et al. (2018) t_{sp}	-1.840903e+113	5.938039e+227	1.096089e-225
Salim and Onyango (2022) t_{so}	0.09002835	2.20565	460.7558
Muneer et al. (2018) $t_{m(1)}$	0.4742451	19.57555	295.0884
Muneer et al. (2018) $t_{m(2)}$	0.02252968	6.508619	33.24872
Muneer et al. (2018) $t_{m(3)}$	0.1198424	6.75827	96.306
Muneer et al. (2018) $t_{m(4)}$	-0.06961472	2.750671	236.6193
Muneer et al. (2018) $t_{m(5)}$	121.6848	16301.8	0.03992577
Muneer et al. (2018) $t_{m(6)}$	-0.05585562	6.508619	96.91564
Muneer et al. (2018) $t_{m(7)}$	-0.05232447	6.715757	461.1598
Muneer et al. (2018) $t_{m(8)}$	0.3083664	1.411359	38.16082
Muneer et al. (2018) $t_{m(9)}$	-0.05232447	17.05576	461.1598
Proposed Estimators under imputation schemes			
T_1^{**}	0.271166	7.465962	894.3244
T_2^{**}	0.276815	0.7277694	861.1318
T_3^{**}	0.1049767	0.7558215	1867.092
T_4^{**}	-0.1308369	7.835589	1048.67

Table 1 presents the results of the biases, MSEs and PREs of the reviewed estimators and that of proposed estimators using simulated data from population 2 defined in Table 1. The results shows that all the proposed estimators under imputation schemes has minimum MSEs and higher PREs compared to all the existing estimators considered in this study. Likewise, the proposed estimator (T_3^*) under call-back has minimum MSEs and higher PREs than most of the estimators under study with the exception of Salim and Onyango (2022) t_{so}, Muneer et al. (2018) estimators $t_{m(1)}, t_{m(4)}, t_{m(7)}$, and $t_{m(9)}$ Therefore, all the proposed estimators under imputation scheme are more efficient compared to other related estimators considered in this study, and can perform better and produced better estimate of the population parameters in the presence of non-response and measurement errors, using exponential distribution.

Table 2presents the results of the biases, MSEs and PREs of the reviewed estimators and that of proposed estimators using simulated data from population 3 defined in Table 2. The results shows that all the proposed estimators under imputation schemes has minimum MSEs and higher PREs compared to all the existing estimators considered in this study with the exception Salim and Onyango (2022) estimator (t_{so}) and Maneer et al (2018) estimator ($t_{m(1)}$) which is a little higher than the proposed estimator (T_1^{**}). Also, the proposed estimator (T_4^*) under call-back has minimum MSEs and higher PREs than most of the estimators under study with the exception Salim and Onyango (2022) t_{so}, Maneer et al (2018) estimator ($t_{m(1)}$),($t_{m(4)}$),($t_{m(7)}$) and ($t_{m(9)}$) Therefore, the proposed estimators (T_1^{**}), (T_2^{**}) and (T_3^{**}) under imputation scheme are more efficient compared to other related estimators considered in this study, and can perform better and produced better estimate of the population parameters in the presence of non-response and measurement errors, using chi-square distribution.

Table 3 presents the results of the biases, MSEs and PREs of the reviewed estimators and that of proposed estimators using simulated data from population 5 defined in Table 3. The results shows that all the proposed estimators under imputation schemes has minimum MSEs and higher PREs compared to all the existing estimators considered in this study. However, the proposed estimators under call-back have higher MSEs and minimum PREs than most of the estimators under study. Therefore, all the proposed estimators under imputation scheme are more efficient compared to other related estimators considered in this study, and can perform better and produced better estimate of the population parameters in the presence of non-response and measurement errors, using gamma distribution.

Table 4 presents the results of the biases, MSEs and PREs of the reviewed estimators and that of proposed estimators using simulated data from population 6 defined in Table 4. The results shows that all the proposed estimators under imputation schemes has minimum MSEs and higher PREs compared to all the existing estimators considered in this study. Also, the proposed estimator (T_1^*) under call-back has minimum MSEs and higher PREs than most of the estimators under study with the exception of Salim and Onyango (2022) t_{so}, Maneer et al. (2018) estimators ($t_{m(1)}$), ($t_{m(4)}$), ($t_{m(7)}$) and ($t_{m(9)}$). Therefore, the proposed estimators under imputation scheme are more efficient compared to other related estimators considered in this study, and can perform better and produced better estimate of the population parameters in the presence of non-response and measurement errors, using uniform distribution.

5 Conclusion

From the results of the theoretical and empirical study, it was obtained that the new classes of proposed estimators using imputation scheme have the minimum MSE_S and higher PRE compared to all other competing estimators considered in the literature. Hence, the suggested estimators demonstrated higher level of efficiency over the existing estimators considered in the study. By implication, the suggested estimators have higher chance of producing estimate that is closer to the true value of the population mean in the presence of non-response and measurement errors than the other estimators considered in the literature of this study. Also, the results of the consistency test revealed that the proposed classes of the estimators are consistent.

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