Fixed Point Results for Generalized Non-Linear Operators with Convergence Analysis

Abstract:

The purpose of this research article is to introduce a new iteration scheme and to prove convergence and stability results for it. We also claim the newly introduced iterative scheme hasbetter efficiency than some of the existing iterations in the literature. Our claim is supported by numerical example.

Keywords: J-iteration, Suzuki generalized non expansive mapping, stability.

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1. Introduction and Preliminaries

The theory of fixed points has become an interdisciplinary area of research as it has applications in mathematics, economics, game theory etc. In general, the solution of fixedpoint problem is almost impossible therefore need of iterative solution arises. Developing a faster and simpler iterative scheme to obtain the fixed point is an interesting and active area of research. Over the years different iterative schemes for finding the solution of fixed point problems for different operators have been developed by the researchers, for example, see([5,6, 8-10, 16-19])).

In 2017, Ullah and Arshad [15] introduced the following M*-iteration scheme:

$$\begin{cases} \wp_n = (1-\beta_n)\Im_n + \beta_n \ell \Im_n \\ \hbar_n = \ell\big((1-\alpha_n)\,\ell\Im_n + \alpha_n \ell \wp_n\big) \\ \Im_{n+1} = \ell \hbar_{n,} \end{cases} \tag{1.1}$$

where $\{\alpha_n\}, \{\beta_n\}$ sequences such that $\alpha_n, \beta_n \in (0,1)$.

Authors in [15], claimed that their iteration scheme is faster than existing iteration schemes in the literature such that Picard, Mann, Ishikawa, Noor, Agarwal et al. and Abbas et al.

Hussain et al. [7] defined a new iteration scheme and named it as ‘K-iteration scheme’. They proved convergence result for this iterative scheme by considering the class of Suzuki generalized non expansive mapping in uniformly convex Banach space.

$$\begin{cases} \wp_n = (1-\beta_n)\Im_n + \beta_n \ell \Im_n, \\ \hbar_n = \ell((1-\alpha_n)\ell \Im_n + \alpha_n \ell \wp_n, \\ \Im_{n+1} = \ell \hbar_n, \end{cases} \tag{1.2}$$

Again in 2018 Ullah and Arshad [17] introduced the K*-iteration scheme by the method

$$\begin{cases} \wp_n = (1-\beta_n)\Im_n + \beta_n \ell \Im_n \\ \hbar_n = \ell\big((1-\alpha_n)\wp_n + \alpha_n \ell \wp_n\big)) \\ \Im_{n+1} = \ell \hbar_n \end{cases} \tag{1.3}$$

for all $n \in N$,where $\{\alpha_n\}$, $\{\beta_n\}$ sequences such that $\alpha_n, \beta_n \in (0, 1)$.

Again in 2018, Ullah and Arshad [16] introduced the M-iteration scheme by the method

$$\begin{cases} \wp_n = (1-\alpha_n)\Im_n + \alpha_n \ell \Im_n, \\ \hbar_n = \ell \wp_n, \\ \Im_{n+1} = \ell \hbar_{n,} \end{cases} \tag{1.4}$$

for all $n \in N$,where $\{\alpha_n\}$ is a sequence such that $\alpha_n \in (0, 1)$.

In this direction Bhutia and Tiwari [5] defined the J-Iteration scheme as follows:

For some initial approximation $\Im_0$ we have,

$$\begin{cases} \wp_n = \ell\big((1-\beta_n)\Im_n + \beta_n \ell \Im_n\big), \\ \hbar_n = \ell\big((1-\alpha_n)\wp_n + \alpha_n \ell \wp_n\big), \\ \Im_{n+1} = \ell \hbar_n. \end{cases} \tag{1.5}$$

Now we introduce a new iteration scheme with the relation:

For some initial approximation x_0 we have

$$\begin{cases} \wp_n = \ell^n\big((1-\beta_n)\Im_n + \beta_n \ell^n \Im_n\big), \\ \hbar_n = \ell^n\big((1-\alpha_n)\wp_n + \alpha_n \ell^n \wp_n\big), \\ \Im_{n+1} = \ell^n\big((1-\gamma_n)\ell^n \wp_n + \gamma_n \ell^n \hbar_n\big). \end{cases} \tag{1.6}$$

We claim that our newly defined iterative scheme converges faster than the iterative scheme defined by Bhutia and Tiwari [5] and hence some of the existing iterative sequences in the literature.

Definition 1.1[6]: Let $\{\wp_n\}_{n=0}^{\infty}$ be the sequence in X. Then the iterative process $\Im_{n+1} = f(\ell, \Im_n)$ which converges to a fixed point q of ℓ is said to be stable with respect to ℓ if for $t_n = \| \wp_{n+1} - f(\ell, \wp_n) \|$, $n = 0, 1, 2, \ldots$, we have $\lim_{n\to\infty} t_n = 0$ if and only if $\lim_{n\to\infty} \wp_n = q$.

Definition 1.2[4]:Let H be a non-empty subset of a Banach space X and$\ell: X \to X$ be a mapping. ℓ is called a generalized contraction mapping if there exists a real number$k < 1$such that for all $x, y \in X$ we have $d(\ell^n x, \ell^n y) \leq kd(x, y)$.

Definition 1.3[15]: An operator $\ell: K \to K$ is said to satisfy the condition (C), if for all$x, y \in K$, we have $\frac{1}{2} d(x, \ell x) \leq d(x, y)$ implies $d(\ell x, \ell y) \leq d(x, y)$. Any mapping satisfies condition (C) is also known as Suzuki generalized non-expansive mapping.

Proposition1.4[15]: Let H be a non-empty subset of a Banach space X and $\ell: X \to X$ be a mapping. Then

1. If ℓ is non-expansive, then ℓ is Suzuki generalized non-expansive mapping.
2. If ℓ is Suzuki generalized non-expansive mapping and has a fixed point, then ℓ is a quasi-non expansive mapping.

Lemma 1.5[14]: Let H be a non-empty subset of a Banach space X and $\ell: X \to X$ be a Suzuki generalized non expansive mapping. Then for all $x, y \in X$, we have

$$\| \ell x - Ty \| \leq 3 \| \ell x - x \| + \| x - y \| \quad (1.7)$$

Lemma 1.6[13]: Suppose X is a uniformly convex Banach space and $\{s_n\}$ be any sequence of real numbers such that $0 < s_n < 1$ for all $n \geq 1$. Let $\{a_n\}$ and $\{b_n\}$ be any two sequences of real numbers in X such that $\limsup_{n\to\infty} \| a_n \| \leq r$, $\limsup_{n\to\infty} \| b_n \| \leq r$ and $\limsup_{n\to\infty} \| s_n a_n + (1 - s_n) b_n \| = r$ holds for some non-negative constant$r$.Then $\lim_{n\to\infty} \| a_n - b_n \| = 0$.

Remark 1.7[14]: Let H be a non-empty subset of a Banach space X and let $\{\mathfrak{I}_n\}_{n=0}^{\infty}$be a bounded sequence in X. For $\mathfrak{I} \in X$, we set $r(\mathfrak{I}, \{\mathfrak{I}_n\}) = \limsup_{n\to\infty} \| \mathfrak{I}_n - \mathfrak{I} \|$. The asymptotic radius of $\{\mathfrak{I}_n\}$ relative to H is given by $r(H, \{\mathfrak{I}_n\}) = \inf\{r(\mathfrak{I}, \{\mathfrak{I}_n\}) : \mathfrak{I} \in H\}$ and the asymptotic center of $\{\mathfrak{I}_n\}$ relative to H is the set $A(H, \{\mathfrak{I}_n\}) = \{\mathfrak{I} \in H : r(\mathfrak{I}, \{\mathfrak{I}_n\}) = r(H, \{\mathfrak{I}_n\})\}$.

2. Main Results

Theorem 2.1: Let H be a non-empty subset of a Banach space X and $\ell: X \to X$ be a generalized contraction mapping. Let $\{\mathfrak{I}_n\}_{n=0}^{\infty}$be the sequence of defined by the iterative scheme (1.6) with real sequences$\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}, \{\gamma_n\}_{n=0}^{\infty}$ in $[0, 1]$ satisfying$\sum_{n=0}^{\infty} \gamma_n = \infty$. Then the sequence $\{\mathfrak{I}_n\}_{n=0}^{\infty}$ converges strongly to unique fixed point of ℓ.

Proof: Since ℓ is a generalized contraction mapping and hence contraction mapping so by Banach contraction principle it has a unique fixed point. Let q be the unique fixed point of ℓ. Now by the iteration scheme (1.6) we have,

$$\| \wp_n - q \| = \| \ell^n((1-\beta_n)\Im_n + \beta_n \ell^n \Im_n) - q \|$$
$$\leq k \| (1-\beta_n)\Im_n + \beta_n \ell^n \Im_n - q \|$$
$$\leq k[(1-\beta_n) \| \Im_n - q \| + \| \beta_n \ell^n \Im_n - q \|]$$
$$\leq k[(1-\beta_n) \| \Im_n - q \| + \beta_n k \| \beta_n \Im_n - q \|]$$
$$\leq k[1-\beta_n(1-k)] \| \Im_n - q \|.$$

By the hypothesis of theorem, we have $1-\beta_n(1-k) < 1$, so we can write

$$\| \wp_n - q \| \leq k \| \Im_n - q \|. \qquad (2.1)$$

And

$$\| \hbar_n - q \| = \| \ell^n((1-\alpha_n)\wp_n + \alpha_n \ell^n \wp_n) - q \|$$
$$\leq k \| (1-\alpha_n)\wp_n + \alpha_n \ell^n \wp_n - q \|$$
$$\leq k[(1-\alpha_n) \| \wp_n - q \| + \alpha_n \| \ell^n \wp_n - q \|]$$
$$\leq k[(1-\alpha_n) \| \wp_n - q \| + \alpha_n k \| \wp_n - q \|]$$
$$\leq k[1-\alpha_n(1-k)] \| \wp_n - q \|.$$

Again, by the hypothesis of theorem we have $1-\alpha_n(1-k) < 1$ and using (2.1) we have

$$\| \hbar_n - q \| \leq k \| \wp_n - q \|$$
$$\leq k^2 \| \Im_n - q \| \qquad (2.2)$$

And by using (1.6) and (2.1) and (2.2) we have

$$\| \Im_{n+1} - q \| = \| \ell^n((1-\gamma_n)\ell^n z_n + \gamma_n \ell^n \hbar_n) - q \|$$
$$\leq k \| (1-\gamma_n)\ell^n \wp_n + \gamma_n \ell^n \hbar_n - q \|$$
$$\leq k[(1-\gamma_n) \| \ell^n \wp_n - q \| + \gamma_n \| \ell^n \hbar_n - q \|]$$
$$\leq k[(1-\gamma_n)k \| \wp_n - q \| + \gamma_n k \| \hbar_n - q \|]$$
$$\leq k[(1-\gamma_n)k^2 \| \Im_n - q \| + \gamma_n k^3 \| \Im_n - q \|]$$
$$\leq k^3[1-\gamma_n(1-k)] \| \Im_n - q \| \qquad (2.3)$$

By repeating the above arguments, we have

$$\| \Im_n - q \| \leq k^3[1-\gamma_{n-1}(1-k)] \| \Im_{n-1} - q \|$$
$$\| \Im_{n-1} - q \| \leq k^3[1-\gamma_{n-2}(1-k)] \| \Im_{n-2} - q \|$$

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$$\| \Im_1 - q \| \leq k^3[1-\gamma_0(1-k)] \ \| \Im_0 - q \|.$$

Combining all the above inequalities we have

$$\| \Im_{n+1} - q \| \leq k^{3(n+1)} \| \Im_0 - q \| \prod_{i=0}^{n} 1-\gamma_i(1-k).$$

Now $k<1$ so $1-k>0$ and $\gamma_i \leq 1$ for all $n \in N$, hence we have $1-\gamma_i(1-k)<1$. We know that $1-x \leq e^{-x}$ for all $x \in [0,1]$. Hence,

$\| \Im_{n+1} - q \| \leq k^{3(n+1)} \| \Im_0 - q \| e^{-(1-k)\sum_{i=0}^{n}\gamma_i}$ (2.4)

Taking limit as$n \to \infty$ both sides, we have $\lim_{n\to\infty} \| \Im_n - q \| = 0$. This completes the proof.

Theorem 2.2: Let H be a non-empty subset of a Banach space X and$\ell: X \to X$ be a generalized contraction mapping. Let $\{\Im_n\}_{n=0}^{\infty}$be the sequence of defined by the iterative scheme (1.6) with real sequences $\{\alpha_n\}_{n=0}^{\infty}$,$\{\beta_n\}_{n=0}^{\infty}$, $\{\gamma_n\}_{n=0}^{\infty}$ in $[0,1]$ satisfying$\sum_{n=0}^{\infty}\gamma_n = \infty$. Then the sequence $\{\Im_n\}_{n=0}^{\infty}$ is ℓ- stable.

Proof: Let $\{\Im_n\}_{n=0}^{\infty}$ be any sequence in H and let the sequence generated by (1.6) be$t_{n+1} = f(\ell, \Im_n)$ and let it converges to the unique fixed point q of ℓ.

Suppose$\delta_n = \| t_{n+1} - f(\ell, t_n) \|$. Now, we will prove that $\lim_{n\to\infty}\delta_n = 0$ if and only if $\lim_{n\to\infty} t_n = q$. First of all, suppose that $\lim_{n\to\infty} t_n = q$.

Then we have

$$\begin{aligned}\delta_n &= \| t_{n+1} - f(\ell, t_n) \| \\ &\leq \| t_{n+1} - q \| + \| f(\ell, t_n) - q \| \\ &\leq \| t_{n+1} - q \| + k^3[1-\gamma_n(1-k)] \ \| t_n - q \|\end{aligned}$$

Taking limit as$n \to \infty$ both sides of the above inequality we have $\lim_{n\to\infty}\delta_n = 0$.

Conversely suppose that $\lim_{n\to\infty}\delta_n = 0$.

Now, we have

$$\begin{aligned}\| t_{n+1} - q \| &\leq \| t_{n+1} - f(\ell, t_n) \| + \| f(\ell, t_n) - q \| \\ &\leq \delta_n + \| f(\ell, t_n) - q \|.\end{aligned}$$

Using Theorem 2.1, we can write

$$\| t_{n+1} - q \| \leq \delta_n + [1-\gamma_n(1-k)] \ \| t_n - q \|.$$

Now $0 < k < 1$ and $\gamma_i \leq 1$ for all $n \in N$ and $\lim_{n\to\infty} \delta_n = 0$. Then from the above inequality and lemma (1.6) we have, $\lim_{n\to\infty} \| t_n - q \| = 0$.

Hence the sequence $\{x_n\}_{n=0}^{\infty}$is ℓ-stable.

Now we establish some fixed point results related to Suzuki generalized non-expansive mapping.

Lemma 2.3: Let H be a non-empty closed convex subset of a Banach space X and$\ell: X \to X$ be a Suzuki generalized non-expansive mapping with $F(\ell) \neq \emptyset$. Let $\{\mathfrak{I}_n\}_{n=0}^{\infty}$be the sequence of X defined by the iterative scheme (1.6) with real sequences $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}, \{\gamma_n\}_{n=0}^{\infty}$ in $[0,1]$ satisfying $\sum_{n=0}^{\infty} \gamma_n = \infty$. Then $\lim_{n\to\infty} \| \mathfrak{I}_n - q \|$ exists for all$q \in F(\ell)$.

Proof: Let $q \in F(\ell)$. Now,using the convexity of H we have, $(1-\gamma_n)\mathfrak{I}_n + \gamma_n \ell \mathfrak{I}_n \in H$ for all $n \in N$. Since ℓ is Suzuki generalized non-expansive mapping so we can write

$$\frac{1}{2} \| q - \ell q \| = 0 \leq \| q - \big((1-\gamma_n)\mathfrak{I}_n + \gamma_n \ell \mathfrak{I}_n\big) \|,$$

which implies that

$$\| \ell q - \ell\big((1-\gamma_n)\mathfrak{I}_n + \gamma_n \ell \mathfrak{I}_n\big) \| \leq \| q - \big((1-\gamma_n)\mathfrak{I}_n + \gamma_n \ell \mathfrak{I}_n\big) \|.$$

Now from the iterative process (1.6) we have

$$\begin{aligned} \| \wp_n - q \| &= \| \ell^n((1-\beta_n)\mathfrak{I}_n + \beta_n \ell^n \mathfrak{I}_n) - \ell^n q \| \\ &\leq \| ((1-\beta_n)\mathfrak{I}_n + \beta_n \ell^n \mathfrak{I}_n) - q \| \\ &\leq (1-\beta_n) \| \mathfrak{I}_n - q \| + \beta_n \| \ell^n \mathfrak{I}_n - q \| \\ &\leq (1-\beta_n) \| \mathfrak{I}_n - q \| + \beta_n \| \mathfrak{I}_n - q \| \end{aligned}$$

$\leq \| \mathfrak{I}_n - q \|$(2.5)

Now

$$\begin{aligned} \| \hbar_n - q \| &= \| \ell^n((1-\alpha_n)\wp_n + \alpha_n \ell^n \wp_n) - \ell^n q \| \\ &\leq \| (1-\alpha_n)\wp_n + \alpha_n \ell^n \wp_n - q \| \\ &\leq (1-\alpha_n) \| \wp_n - q \| + \alpha_n \| \ell^n \wp_n - q \| \\ &\leq (1-\alpha_n) \| \wp_n - q \| + \alpha_n \| \wp_n - q \| \end{aligned}$$

$\leq \| \wp_n - q \|$(2.6)

$\leq \| \mathfrak{I}_n - q \|$.(2.7)

Again using (1.6), (2.5) and (2.7), we get

$$\| \Im_{n+1} - q \| = \| \ell^n((1-\gamma_n)\ell^n \wp_n + \gamma_n \ell^n \hbar_n) - \ell^n q \|$$
$$\leq \| (1-\gamma_n)\ell^n \wp_n + \gamma_n \ell^n \hbar_n - q \|$$
$$\leq (1-\gamma_n) \| \ell^n \wp_n - q \| + \gamma_n \| \ell^n \hbar_n - q \|$$
$$\leq (1-\gamma_n) \| \wp_n - q \| + \gamma_n \| \hbar_n - q \|$$
$$\leq (1-\gamma_n) \| \Im_n - q \| + \gamma_n \| \Im_n - q \|$$

$\leq \| \Im_n - q \|$.

Hence $\{\| \Im_n - q \|\}$ is bounded and non-increasing for all $q \in F(\ell)$.

Hence $\lim_{n\to\infty} \| \Im_n - q \|$ exists for all $q \in F(\ell)$.

Theorem 2.4: Let H be a non-empty closed convex subset of a Banach space X and $\ell: X \to X$ be a Suzuki generalized non-expansive mapping. Let $\{\Im_n\}_{n=0}^{\infty}$ be the sequence of defined by the iterative scheme (1.6) with real sequences $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}, \{\gamma_n\}_{n=0}^{\infty}$ in $[0,1]$ satisfying $\sum_{n=0}^{\infty} \gamma_n = \infty$. Then $F(T) \neq \emptyset$ if and only if $\{\Im_n\}_{n=0}^{\infty}$ is bounded and $\lim_{n\to\infty} \| T\Im_n - \Im_n \| = 0$.

Proof: Firstsuppose that, $F(\ell) \neq \emptyset$. Let $q \in F(\ell)$.

Then by Lemma 2.4, $\lim_{n\to\infty} \| \Im_n - q \|$ exists for all $q \in F(\ell)$ and $\{\Im_n\}_{n=0}^{\infty}$ is a bounded sequence.

Let $\lim_{n\to\infty} \| \Im_n - q \| = \theta$ for some $\theta > 0$.

Now, from (2.5), we have

$$\limsup_{n\to\infty} \| \wp_n - q \| \leq \limsup_{n\to\infty} \| \Im_n - q \| = \theta.$$

By Proposition 1.4, we have

$$\limsup_{n\to\infty} \| \ell\Im_n - q \| \leq \limsup_{n\to\infty} \| \Im_n - q \| = \theta.$$

Now using (1.6) and (2.5) we have

$$\| \Im_{n+1} - q \| = \| \ell^n((1-\gamma_n)\ell^n \wp_n + \gamma_n \ell^n \hbar_n) - \ell^n q \|$$
$$\leq \| (1-\gamma_n)\ell^n \wp_n + \gamma_n \ell^n \hbar_n - q \|$$
$$\leq (1-\gamma_n) \| \ell^n \wp_n - q \| + \gamma_n \| \ell^n \hbar_n - q \|$$
$$\leq (1-\gamma_n) \| \wp_n - q \| + \gamma_n \| \hbar_n - q \|$$
$$\leq (1-\gamma_n) \| \wp_n - q \| + \gamma_n \| \wp_n - q \|$$
$$\leq \| \wp_n - q \|,$$

which implies that $\| \Im_{n+1} - q \| \leq \| \wp_n - q \|$ and hence

$$\theta \leq \liminf_{n\to\infty} \| \wp_n - q \|.$$

Therefore, we can write

$$\theta \leq \liminf_{n\to\infty} \| \wp_n - q \| \leq \limsup_{n\to\infty} \| \wp_n - q \| \leq \theta.$$

Thus, we obtain $\lim_{n\to\infty} \| \wp_n - q \| = \theta$.

Now, we have

$$\begin{aligned} \theta &= \lim_{n\to\infty} \| \ell^n((1-\beta_n)\Im_n + \beta_n \ell^n \Im_n) - q \| \\ &\leq \lim_{n\to\infty} \| (1-\beta_n)\Im_n + \beta_n \ell^n \Im_n - q \| \\ &\leq \lim_{n\to\infty} [(1-\beta_n) \| \Im_n - q \| + \beta_n \| \ell^n \Im_n - q \|] \\ &\leq \lim_{n\to\infty} (1-\beta_n) \| \Im_n - q \| + \beta_n \| \Im_n - q \| \\ &\leq \lim_{n\to\infty} \| \Im_n - q \| \leq \theta. \end{aligned}$$

Hence we can write

$$\theta \leq \lim_{n\to\infty} \| (1-\beta_n)(\Im_n - q) + \beta_n(\ell^n \Im_n - q) \| \leq \theta.$$

Thus $\lim_{n\to\infty} \| (1-\beta_n)(\Im_n - q) + \beta_n\left(\ell^n \Im_n - q\right) \| = \theta$.

Using Lemma 1.6 and the above calculations we have $\lim_{n\to\infty} \| \ell^n \Im_n - \Im_n \| = 0$.

Conversely, let $\{\Im_n\}_{n=0}^{\infty}$ is bounded and $\lim_{n\to\infty} \| \ell^n \Im_n - \Im_n \| = 0$.

Let $q \in A(H, \{\Im_n\})$.

By Lemma 1.5, we have

$$\begin{aligned} r(\ell q, \{\Im_n\}) &= \limsup_{n\to\infty} \| \Im_n - \ell q \| \\ &\leq \limsup_{n\to\infty} (3 \| \ell \Im_n - x_n \| + \| \Im_n - q \|) \\ &\leq \limsup_{n\to\infty} (\| \Im_n - q \|) \end{aligned}$$

which implies that $\ell q \in A(H, \{\Im_n\})$. Since X is uniformly convex Banach space. It follows that $A(H, \{\Im_n\})$ is singleton. Hence $\ell q = q$ implies that $q \in F(\ell)$ and hence $F(\ell) \neq \emptyset$. This completes the proof.

Example 2.3: Consider the mapping $\ell(\Theta) = (\Theta + 2)^{\frac{1}{2}}$. Clearly ℓ is a generalized contraction mapping and $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}, \{\gamma_n\}_{n=0}^{\infty}$ be the sequence defined by $\alpha_n =$

$\beta_n = \gamma_n = \frac{1}{4}$for all $n \in N$. We now compare the rate of convergence of our iterative scheme with J-iteration scheme by considering the example of Bhutia and Tiwari [5] using following Table 1.

Table 1. Iterative scheme with J-iteration scheme by considering the example of Bhutia and Tiwari

Iteration	J-Iteration	Iteration (1.6)
0	4	4
1	2.0183456356079	2.01874653158
2	2.0001845667869	2.00015675043
4	2.0000000200677	2.00000001877
5	2.0000000002057	2.00000000011
6	2.0000000000026	**2**
7	2.0000000000002	**2**
8	**2**	**2**

From the above table, we can see that the iterative scheme (1.6) has higher rate of convergence than the J-iteration process.

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