ON GENERATING MEASURES OF FUZZY CROSS-ENTROPY BY EMPLOYING MEASURES OF FUZZY WEIGHT ENTROPY

Abstract

By utilizing the ideas of fuzzy-weighted entropy and fuzzy-weighted directed divergence and appropriately selecting the weights, several new measures of fuzzy cross-entropy have been generated. Concurrently, some new measures of fuzzy weighted entropy have been suggested based on the new measures of fuzzy cross-entropy generated.

Index: Fuzzy entropy, Fuzzy weighted entropy, Fuzzy cross-entropy etc.

1. INTRODUCTION

Shannon's [13] concept of entropy simply considers the probabilities attached to the events, not their significance. However, other domains deal with random events where it is important to consider both these probabilities and some qualitative [6] aspects of the events. For instance, when playing a two-handed game, it is important to consider both the likelihood of the various game variants that is, the players' random strategies—and the victories associated with each variety. The idea demonstrates the need to relate probability to weight. Let $E_1, E_2, \ldots\ldots, E_n$ symbolize n alternative outcomes with $p_1, p_2, \ldots\ldots, p_n$ as their probabilities and let $w_1, w_2, \ldots\ldots, w_n$ be non-negative real values representing their utilities or weights to illustrate the idea of weighted entropy. Then, Guiasu [4] defined the qualitative-quantitative entropy measure

$$H(P, U; W) = -\sum_{i=1}^{n} w_i p_i \ln p_i \qquad (1.1)$$

and gave it the name weighted entropy.

A control Diverse fields of the mathematical and engineering sciences find $D(P, Q)$ of divergence or cross-entropy or directed divergence [11] to be of great importance. This probabilistic measure gauges the separation between a [8] probability distribution P and Q. Due to Kullback and Leibler [9], the most significant and practical measure of divergence can be found by using the formula

$$D(P, Q) = \sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i} \qquad (1.2)$$

The aforementioned measure (1.2) is transformed into

$$D(P,Q;W) = \sum_{i=1}^{n} w_i p_i \ln \frac{p_i}{q_i} \tag{1.3}$$

where $w_i \geq 0$ are real integers, after giving distinct events weights.

Consider the measure of weighted entropy,

$$H(P:W) = -\sum_{i=1}^{n} w_i \emptyset(p_i) \tag{1.4}$$

and the corresponding measure of fuzzy weighted entropy,

$$H(A:W) = -\sum_{i=1}^{n} w_i \emptyset\left(\mu_A(x_i) + \left(1 - \mu_A(x_i)\right)\right). \tag{1.5}$$

Where, $\emptyset(\,.\,)$ is a continuous convex twice-differential function.Its maximum subject to natural constraint,

$$\sum_{i=1}^{n} \left(\mu_A(x_i) + \left(1 - \mu_A(x_i)\right)\right) = 1 \tag{1.6}$$

occurs when,

$$w_1 \emptyset'(p_1) = w_2 \emptyset'(p_2) = \cdots = w_n \emptyset'(p_n) \tag{1.7}$$

$i.e.$ $$w_1 \Phi\left(\mu_A(x_1)\right) = w_2 \Phi\left(\mu_A(x_2)\right) = \cdots = w_n \Phi\left(\mu_A(x_n)\right) \tag{1.8}$$

where, $$\Phi\left(\mu_A(x_i)\right) = \emptyset'(p_i)\,;\; i = 1, \dots, n.$$

If the maximum occurs when $p_i = q_i$, $i = 1, \dots, n$, then we get

$$\frac{w_1}{[\Phi(\mu_B(x_1))]^{-1}} = \frac{w_2}{[\Phi(\mu_B(x_2))]^{-1}} = \cdots = \frac{w_n}{[\Phi(\mu_B(x_n))]^{-1}} \tag{1.9}$$

So that (1.5) becomes,

$$-\sum_{i=1}^{n} \frac{\emptyset\left(\mu_A(x_i)+\left(1-\mu_A(x_i)\right)\right)}{\Phi\left(\mu_B(x_i)+\left(1-\mu_B(x_i)\right)\right)} \quad \text{or} \quad \sum_{i=1}^{n} \frac{\emptyset\left(\mu_A(x_i)+\left(1-\mu_A(x_i)\right)\right)}{\Phi\left(\mu_B(x_i)+\left(1-\mu_B(x_i)\right)\right)}. \tag{1.10}$$

According as $\Phi\left(\mu_B(x_i) + \left(1 - \mu_B(x_i)\right)\right) = \emptyset'(q_i)$ is positive or negative.

In either case, the measure is a continuous twice-differentiable convex function of $p_1, \dots, p_n$. Which has its minimum value at $q_1, \dots, q_n$. As such, this can be used as a measure of directed-divergence or cross-entropy of P from Q, where

$$P = p_1, \dots, p_n\,;\, Q = q_1, \dots, q_n.$$

If $q_1, \dots, q_n$ is such that either $\emptyset'(q_i) > 0$ or < 0 for all i's is true, then the condition on $\emptyset'(q_i)$ is satisfied for all i. For this purpose, it is sufficient but not necessary that $\emptyset\,(x)$ be monotonically rising or decreasing, and all of the aforementioned conditions hold for the corresponding fuzzy situations.

Fuzzy set theory was proposed by Zadeh, Lotfi A. [16] in 1965 as an extension of the classical notion of a set (Zadeh, 1965). With the proposed methodology, Zadeh introduced a mathematical method with which decision-making using fuzzy descriptions of some information becomes possible. The basis of this theory is the fuzzy set, which is a set that does not have clearly defined limits and can contain elements only at some degree; in other words, elements can have a certain degree of membership. Hence, suitable functions are used namely, membership functions that determine the membership degree of each element in a fuzzy set. If we consider an input variable *x* with a field of definition *S*, the fuzzy set *A* in *S* is defined as:

If A be the subset of universe of discourse $i.e.\ X = \{x_1, \dots\dots, x_n\}$ then, A is defined as,

$$A = \{x_i / \mu_A(x_i) : i = 1,2, \dots\dots\dots\dots, n\}.$$

Where $\mu_A(x_i)$ is a membership function and having the following properties:

1. If $\mu_A(x_i) = 0$, x_i does not belong to A and there is no ambiguity.
2. If $\mu_A(x_i) = 1$, x_i belong to A and there is no ambiguity.

If $\mu_A(x_i) = 0.5$, there is maximum ambiguity whether x_i belong to A or not.

In 1972, De-Luca and Termini [3] proposed the measure of fuzzy entropy

$$H(A) = -\frac{1}{n}\sum_{i=1}^{n}\left[\mu_A(x_i)\ln \mu_A(x_i) + \left(1-\mu_A(x_i)\right)\ln\left(1-\mu_A(x_i)\right)\right]. \quad (3.1.2)$$

This measure serves as a very suitable measure of fuzzy entropy [7] for the finite fuzzy [2] information scheme. Similarly Kapur [10] suggests the fuzzy information measure corresponding to Havrda-Charvats [3] was as follows

$$H^a(A) = \frac{1}{1-a}\sum_{i=1}^{n}\left(\left(\mu_A^a(x_i) + \left(1-\mu_A(x_i)\right)^a\right) - 1\right), \quad a \neq 1, a > 0. \quad (3.1.3)$$

We can obtain a measure of fuzzy cross-entropy that corresponds to each measure of fuzzy weighted entropy of type (1.5). Unfortunately, this metric is only relevant to a particular class of a priori distributions and not to all probability distributions.

2. OUR WORK

1. Guiasu's [4] measure of fuzzy weighted entropy

$$G(A:W) = -\sum_{i=1}^{n} w_i\left[\mu_A(x_i)\ln \mu_A(x_i) + \left(\left(1-\mu_A(x_i)\right)\ln\left(1-\mu_A(x_i)\right)\right]. \quad (2.1)$$

The corresponding measure of fuzzy cross-entropy

$$D_1(A:B) = \frac{\sum_{i=n}^{n}[\mu_B(x_i)\ln\mu_B(x_i)+(1-\mu_B(x_i))\ln(1-\mu_B(x_i))]-[\mu_A(x_i)\ln\mu_A(x_i)+(1-\mu_A(x_i))\ln(1-\mu_A(x_i))]}{\sum_{i=1}^{n}1+\ln\mu_B(x_i)+(1-\mu_B(x_i))\ln(1-\mu_B(x_i))} \quad (2.2)$$

2. Havrda-Charvat [5] measure of entropy

$$H(P) = \frac{1}{\alpha(1-\alpha)}\sum_{i=1}^{n} p_i^{\alpha} - p_i \ , \alpha \neq 1, \alpha > 0. \quad (2.3)$$

The corresponding measure of fuzzy entropy

$$H(A) = \frac{1}{\alpha(1-\alpha)}\sum_{i=1}^{n}\mu_A^{\alpha}(x_i) + (1-\mu_A(x_i))^{\alpha} - \mu_A(x_i) - (1-\mu_A(x_i))$$

$$= \frac{1}{\alpha(1-\alpha)}\sum_{i=1}^{n}\mu_A^{\alpha}(x_i) + (1-\mu_A(x_i))^{\alpha} - 1 \quad (2.4)$$

The corresponding measure of fuzzy weighted entropy

$$H(A:W) = \frac{1}{\alpha(1-\alpha)}\sum_{i=1}^{n} w_i[\mu_A^{\alpha}(x_i) + (1-\mu_A(x_i))^{\alpha} - 1] \quad (2.5)$$

The corresponding measure of fuzzy cross-entropy

$$D_2(A:B) = \frac{1}{\alpha(1-\alpha)}\frac{\sum_{i=1}^{n}\mu_A^{\alpha}(x_i)+(1-\mu_A(x_i))^{\alpha} - \mu_B^{\alpha}(x_i)-(1-\mu_B(x_i))^{\alpha}}{\sum_{i=1}^{n}\alpha\mu_B^{\alpha-1}(x_i)+\alpha(1-\mu_B(x_i))^{\alpha-1}} \quad (2.6)$$

Provided $\alpha\mu_B^{\alpha-1}(x_i) + \alpha(1-\mu_B(x_i))^{\alpha-1} < 0$

3. Burg's [1] measure of entropy

$$B(P) = \sum_{i=1}^{n}\ln p_i \quad (2.7)$$

The corresponding measure of fuzzy entropy

$$B(A) = \sum_{i=1}^{n}\ln\mu_A(x_i) + \ln(1-\mu_A(x_i)) \quad (2.8)$$

and the corresponding measure of fuzzy weighted entropy

$$B(A:W) = \sum_{i=1}^{n} w_i[\ln\mu_A(x_i) + \ln(1-\mu_i(x_i))] \quad (2.9)$$

The corresponding measure of fuzzy cross-entropy

$$D_3(A:B) = \frac{\sum_{i=1}^{n}\ln\mu_A(x_i)+\ln(1-\mu_A(x_i)-\ln\mu_B(x_i)-\ln(1-\mu_B(x_i)))}{1/\sum_{i=1}^{n}\ln\mu_B(x_i)+\ln(1-\mu_B(x_i))} \quad (2.10)$$

$$= \sum_{i=1}^{n}\left(\ln\mu_B(x_i) + \ln\left(1-\mu_B(x_i)\right)\right)$$

$$\left(\ln\mu_A(x_i) + \ln\left(1-\mu_A(x_i)\right) - \ln\mu_B(x_i) - \ln\left(1-\mu_B(x_i)\right)\right)$$

$$D_3(A:B) = \sum_{i=1}^{n}\left(\ln \mu_B(x_i)\ln\frac{\mu_A(x_i)(1-\mu_A(x_i))}{2(1-\mu_B(x_i))^2} + \ln(1-\mu_B(x_i))\ln\frac{\mu_A(x_i)(1-\mu_A(x_i))}{2}\right) \quad (2.11)$$

4. Vajda's [15] measure of entropy

$$V(P) = \sum_{i=1}^{n}(p_i - p_i^2) \quad (2.12)$$

and the corresponding measure of fuzzy entropy

$$V(A) = \sum_{i=1}^{n}\mu_A(x_i) + (1-\mu_A(x_i)) - \mu_A^2(x_i) - \left(1-\mu_A(x_i)\right)^2 \quad (2.13)$$

The corresponding measure of fuzzy weighted entropy

$$V(A:W) = \sum_{i=1}^{n} w_i\left[1 - \mu_A^2(x_i) - \left(1-\mu_A(x_i)\right)^2\right] \quad (2.14)$$

The corresponding measure of fuzzy cross-entropy

$$D_4(A:B) = \frac{\sum_{i=1}^{n}(\mu_A(x_i)+(1-\mu_A(x_i))-\mu_A^2(x_i)-(1-\mu_A(x_i))^2-\mu_B(x_i)-(1-\mu_B(x_i))+\mu_B^2(x_i)+(1-\mu_B(x_i))^2)}{\sum_{i=1}^{n}1-2\mu_B(x_i)-2(1-\mu_B(x_i))}$$

$$= 2\sum_{i=1}^{n}\mu_B(x_i) - \mu_B^2(x_i) - \mu_A(x_i) + \mu_A^2(x_i) \quad (2.15)$$

5. Kapur's [10] measure of entropy

$$K(P) = -\sum_{i=1}^{n}p_i\ln p_i + \frac{1}{a}\sum_{i=1}^{n}(1+ap_i)\ln(1+ap_i) - \frac{1}{a}\sum_{i=1}^{n}(1+a)\ln(1+a)p_i \quad (2.16)$$

and the corresponding measure of fuzzy entropy

$$K(A) = -\sum_{i=1}^{n}\mu_A(x_i)\ln\mu_A(x_i) + (1-\mu_A(x_i))\ln(1-\mu_A(x_i))$$

$$+\frac{1}{a}\sum_{i=1}^{n}(1+a\mu_A(x_i))\ln(1+a\mu_A(x_i)) + (1+a-a\mu_A(x_i))\ln(1+a-a\mu_A(x_i))$$

$$-\frac{1}{a}\sum_{i=1}^{n}(1+a)\ln(1+a) + \left(\mu_A(x_i) + \left(1-\mu_A(x_i)\right)\right) \quad (2.17)$$

so the corresponding measure of fuzzy weighted entropy is

$$K(A:W) = -\sum_{i=1}^{n}w_i[\mu_A(x_i)\ln\mu_A(x_i) + (1-\mu_A(x_i))\ln(1-\mu_A(x_i))$$

$$+\frac{1}{a}(1+a\mu_A(x_i))\ln(1+a\mu_A(x_i)) + (1+a-a\mu_A(x_i))\ln(1+a-a\mu_A(x_i))$$

$$-\frac{1}{a}(1+a)\ln(1+a) + \left(\mu_A(x_i) + \left(1-\mu_A(x_i)\right)\right)\Big] \quad (2.18)$$

The corresponding measure of fuzzy cross-entropy

$$D_5(A:B) = \frac{\sum_{i=1}^{n}[\mu_B(x_i)\ln\mu_B(x_i)+(1-\mu_B(x_i))\ln(1-\mu_B(x_i))-\mu_A(x_i)\ln\mu_A(x_i)-(1-\mu_A(x_i))\ln(1-\mu_A(x_i))- \frac{1}{a}\left((1+a\mu_B(x_i))\ln(1-a\mu_B(x_i))+(1+a-a\mu_B(x_i))\ln(1+a-a\mu_B(x_i))\right)+\frac{1}{a}((1+a\mu_A(x_i))\ln(1+a\mu_A(x_i))+ (1+a-a\mu_A(x_i)\ln(1+a-a\mu_A(x_i)))]}{\sum_{i=1}^{n}[\ln\mu_B(x_i)+\ln(1-\mu_B(x_i))-\ln(1+a\mu_B(x_i))-\ln(1+a-a\mu_B(x_i))}$$

$$= \sum_{i=1}^{n}\frac{\mu_B(x_i)\ln\frac{\mu_B(x_i)}{1-\mu_B(x_i)}-\mu_A(x_i)\ln\frac{\mu_A(x_i)}{1-\mu_A(x_i)}+\ln\frac{1-\mu_B(x_i)}{1-\mu_A(x_i)}+\frac{1}{a}\ln\left(\frac{1+a+a^2\mu_A(x_i)-a^2\mu_A^2(x_i)}{1+a+a^2\mu_B(x_i)-a^2\mu_B^2(x_i)}\right) +\mu_A(x_i)\ln\left(\frac{1+a\mu_A(x_i)}{1+a-a\mu_A(x_i)}\right)-\mu_B(x_i)\ln\left(\frac{1+a\mu_B(x_i)}{1+a-a\mu_B(x_i)}\right)+\ln\left(\frac{1+a-a\mu_A(x_i)}{1+a-a\mu_B(x_i)}\right)}{\ln\left(\mu_B(x_i)(1-\mu_B(x_i))\right)-\ln(1+a)}$$

$$D_5(A:B) = \sum_{i=1}^{n}\frac{\mu_B(x_i)\ln\frac{\mu_B(x_i)}{1-\mu_B(x_i)}.\frac{1+a-a\mu_B(x_i)}{1+a\mu_B(x_i)}+\mu_A(x_i)\ln\frac{1+a\mu_A(x_i)}{1+a-a\mu_A(x_i)}.\frac{1-\mu\mu_A(x_i)}{\mu_A(x_i)} +\ln\frac{(1-\mu_B(x_i))}{(1-\mu_A(x_i))}.\frac{(1+a-a\mu_A(x_i))}{(1+a-a\mu_B(x_i))}+\frac{1}{a}\left(\frac{1+a+a^2\mu_A(x_i)-a^2\mu_A^2(x_i)}{1+a+a^2\mu_B(x_i)-a^2\mu_B^2(x_i)}\right)}{\ln(\mu_B(x_i))(1-\mu_B(x_i))-\ln(1+a)} \tag{2.19}$$

6. Bose-Einstein [10] measure of entropy

$$E(P) = \sum_{i=1}^{n}-p_i\ln p_i + (1+p_i)\ln(1+p_i) - 2\ln 2p_i \tag{2.20}$$

and the corresponding measure of fuzzy entropy

$$E(A) = \sum_{i=1}^{n}-\left(\mu_A(x_i)\ln\mu_A(x_i) + (1-\mu_A(x_i))\ln\left(1-\mu_A(x_i)\right)\right) + \left(\left(1+\mu_A(x_i)\right)\ln\left(1+\mu_A(x_i)\right) + \left(2-\mu_A(x_i)\right)\ln\left(2-\mu_A(x_i)\right) - 2\ln 2\right) \tag{2.21}$$

so the corresponding measure of fuzzy weighted entropy

$$E(A:W) = -\sum_{i=1}^{n}w_i[-(\mu_A(x_i)\ln\mu_A(x_i) + (1-\mu_A(x_i))\ln(1-\mu_A(x_i))) + \left(\left(1+\mu_A(x_i)\right)\ln\left(1+\mu_A(x_i)\right) + \left(2-\mu_A(x_i)\right)\ln\left(2-\mu_A(x_i)\right) - 2\ln 2\right)]. \tag{2.22}$$

The corresponding measure of fuzzy cross-entropy

$$D_6(A:B) = \sum_{i=1}^{n}\frac{\mu_B(x_i)\ln\mu_B(x_i)+(1-\mu_B(x_i))\ln(1-\mu_B(x_i))-\mu_A(x_i)\ln\mu_A(x_i)-(1-\mu_A(x_i))\ln(1-\mu_A(x_i)) -(1+\mu_B(x_i))\ln(1+\mu_B(x_i))-(2-\mu_B(x_i))\ln(2-\mu_B(x_i))+(1+\mu_A(x_i))\ln(1+\mu_A(x_i))+ (2-\mu_A(x_i))\ln(2-\mu_A(x_i))}{\ln\mu_B(x_i)+\ln(1-\mu_B(x_i))-\ln(1+\mu_B(x_i))-\ln(2-\mu_B(x_i))} \tag{2.23}$$

so that,

$$D_6(A:B) = \sum_{i=1}^{n}\frac{\mu_B(x_i)\ln\frac{\mu_B(x_i)(2-\mu_B(x_i))}{(1+\mu_B(x_i))(1-\mu_B(x_i))}+\mu_A(x_i)\ln\frac{(1+\mu_A(x_i))(1-\mu_A(x_i))}{\mu_A(x_i)(2-\mu_A(x_i))}+\ln\frac{(1+\mu_A(x_i))(1-\mu_B(x_i))(2-\mu_A(x_i))^2}{(1+\mu_B(x_i))(1+\mu_A(x_i))(2-\mu_B(x_i))^2}}{\ln\frac{\mu_B(x_i)(1-\mu_B(x_i))}{(1+\mu_B(x_i))(2-\mu_B(x_i))}}$$

(2.24)

7. Fermi-Dirac [10] measure of entropy

$$F(P) = \sum_{i=1}^{n} -p_i \ln p_i - (1-p_i)\ln(1-p_i) \tag{2.25}$$

so the corresponding measure of fuzzy entropy

$$F(A) = \sum_{i=1}^{n} -2\mu_A(x_i)\ln\mu_A(x_i) - 2\big(1-\mu_A(x_i)\big)\ln\big(1-\mu_A(x_i)\big) \tag{2.26}$$

and the corresponding measure of fuzzy weighted entropy

$$F(A:W) = \sum_{i=1}^{n} w_i\big[-2\mu_A(x_i)\ln\mu_A(x_i) - 2\big(1-\mu_A(x_i)\big)\ln\big(1-\mu_A(x_i)\big)\big] \tag{2.27}$$

The corresponding measure of fuzzy cross-entropy

$$D_7(A:B) = \sum_{i=1}^{n}[\mu_B(x_i)\ln\mu_B(x_i) + \big(1-\mu_B(x_i)\big)\ln\big(1-\mu_B(x_i)\big) - \mu_A(x_i)\ln\mu_A(x_i)$$

$$-\big(1-\mu_A(x_i)\big)\ln\big(1-\mu_A(x_i)\big)\big] \tag{2.28}$$

8. Renyi's [12] measure of entropy

$$R(P) = \frac{1}{1-\alpha}\ln\sum_{i=1}^{n} p_i\,,\quad \alpha \neq 1\,,\quad \alpha > 0 \tag{2.29}$$

so the corresponding measure of fuzzy entropy

$$R(A) = \frac{1}{1-\alpha}\ln\sum_{i=1}^{n}(\mu_A^{\alpha}(x_i) + (1-\mu_A(x_i))^{\alpha})\,,\alpha \neq 1\,,\alpha > 0 \tag{2.30}$$

It's corresponding measure of fuzzy weighted entropy

$$R(A:W) = \frac{1}{1-\alpha}\ln\sum_{i=1}^{n} w_i(\mu_A^{\alpha}(x_i) + (1-\mu_A(x_i))^{\alpha})\,,\alpha \neq 1\,,\alpha > 0 \tag{2.31}$$

The corresponding measure of fuzzy cross-entropy

$$D_8(A:B) = \frac{1}{1-\alpha}\ln\sum_{i=1}^{n}\frac{\mu_A^{\alpha}(x_i)+\big(1-\mu_A(x_i)\big)^{\alpha}-\mu_B^{\alpha}(x_i)-\big(1-\mu_B(x_i)\big)^{\alpha}}{\alpha\mu_B^{\alpha-1}(x_i)+\alpha\mu_B^{\alpha-1}(x_i)}\,,\alpha \neq 1\,,\alpha > 0 \tag{2.32}$$

9. Sharma and Taneja's [14] measure of entropy

$$T(P) = \frac{1}{\beta-\alpha}\left(\sum_{i=1}^{n} p_i^{\alpha} - \sum_{i=1}^{n} p_i^{\beta}\right),\alpha \neq \beta \tag{2.33}$$

so the corresponding measure of fuzzy entropy

$$T(A) = \frac{1}{\beta-\alpha}\sum_{i=1}^{n}\left[\mu_A^{\alpha}(x_i) + \big(1-\mu_A(x_i)\big)^{\alpha} - \mu_A^{\beta}(x_i) - \big(1-\mu_A(x_i)\big)^{\beta}\right],\alpha \neq \beta \tag{2.34}$$

and the corresponding measure of fuzzy weighted entropy

$$T(A:W) = \frac{1}{\beta-\alpha}\sum_{i=1}^{n} w_i \left[\mu_A^{\alpha}(x_i) + \left(1-\mu_A(x_i)\right)^{\alpha} - \mu_A^{\beta}(x_i) - \left(1-\mu_A(x_i)\right)^{\beta}\right], \alpha \neq \beta \qquad (2.35)$$

The corresponding measure of fuzzy cross-entropy

$$D_9(A:B) = \frac{1}{\beta-\alpha}\sum_{i=1}^{n} \frac{\mu_A^{\alpha}(x_i)+\left(1-\mu_A(x_i)\right)^{\alpha}-\mu_B^{\alpha}(x_i)-\left(1-\mu_B(x_i)\right)^{\alpha}-\mu_A^{\beta}(x_i)-\left(1-\mu_A(x_i)\right)^{\beta}+\mu_B^{\beta}(x_i)+\left(1-\mu_B(x_i)\right)^{\beta}}{\beta\mu_B^{\beta-1}(x_i)+\beta\left(1-\mu_B(x_i)\right)^{\beta-1}+\alpha\mu_B^{\alpha-1}(x_i)+\alpha\left(1-\mu_B(x_i)\right)^{\alpha-1}}$$

$$, \alpha \neq \beta \qquad (2.36)$$

3. CONCLUSION

There are numerous probabilistic and non-probabilistic ways to calculate entropy. We may create a fuzzy entropy measure and a weighted fuzzy entropy measure for each measure of probabilistic entropy, and vice versa. The literature on fuzzy measurements can be improved by using the understanding of probabilistic measures, which have many similarities to both types of measures. Both types of entropies can be used to gain a deeper understanding of the concept of uncertainty so that it can be controlled for the benefit of mankind, while there are some parallels and differences between the two types of entropy measures.

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