Annual mean temperature and rain precipitation in North America using NHPP to detect climate changes

Original Research Article

Abstract

In this study, non-homogeneous Poisson processes (NHPP) are assumed to analyze annual average temperatures and rain precipitations, considering climate data for some regions of North America reported for a long period. A power law process (PLP) is assumed for the intensity function (derivative of the mean value function) or rate $\lambda(t), t \geq 0$ of the NHPP which the Poisson events occur considering data (accumulated number of years in a given time interval [0,t) where the climate measure is above a threshould given by the overal average in the assumed period) in presence or not of a change-point. The parameters of the assumed model are estimated under a Bayesian approach and using MCMC (Markov Chain Monte Carlo) methods. Alternatively to the use of a PLP process, we also assume a polynomial parametrical form for the mean value function of the NHPP process where a simple Bayesian inference approach is proposed to get better fit for the intensity and mean value functions of the NHPP process. From the fitted models it was possible to to detect the years where climate changes occurred.

Keywords: climate change, non-homogeneous Poisson processes, PLP process, polynomial mean value function, Bayesian inference, change-point, MCMC methods.

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1 Introduction

Climate changes (precipitation, temperature, ocean levels, among many others) have been observed since the end of the 19^{th} century all over the world. These climate changes can be due to different causes, among which we can mention the increase in carbon dioxide and other anthropogenic emissions in the atmosphere (https://www.ncdc.noaa.gov/monitoring-references/faq/indicators.php). A significant increasing in the temperature could be observed especially in the last decades worldwide (https://climate.nasa.gov/evidence/). To study the world climate change, different statistical models have been used by statisticians or climate experts in the analysis of climatic data to obtain inferences of interest (precipitation, temperature, level of the oceans among many others) and its implications (see [1–19]). In some studies it is considered the analysis of the years where these anomalies occurred [19–22].

This situation can occur with continuous responses, count responses or other kind of data assuming different parametric or non-parametric models. Considering count data under homogeneous Poisson processes (HPP) or non-homogeneous Poisson processes (NHPP) the literature presents different inference approaches for models in presence of change-points especially under a Bayesian approach. Bayesian inference for HPP or NHPP processes has been discussed by many authors in the literature (see for example [23], [24], [25], [26], [27]). Those processes have also been used by different authors to get inference for change-point models (see for example [28], [29], [30], [31], [24]). [32] consider a Bayesian analysis for homogeneous Poisson processes in the presence of a change-point. [33] introduce a Bayesian analysis for change-points in NHPPs considering PLP (power law processes) processes and dealing with a random number of change-points.

In this study, Bayesian methods are used in the data analysis. Under a Bayesian approach, Markov chain Monte Carlo (MCMC) methods (see for example [34], [35] or [36]) are used to develop a Bayesian analysis assuming special parametric structures for the rates in non-homogeneous Poisson processes in presence of a change-point for the climate times series using the OpenBugs software [37]. Assuming annual temperature and rain precipitation averages collected from five climate stations in North America for a long period of time, the main goal of this study is to verify the behavior of mean annual temperatures and precipitation over the last decades in this region of the world from statistical data analysis,

The paper is organized as follows: section 2 introduces the climate data sets used in the study; section 3 introduces the use of Non-Homogeneous Poisson processes (NHPP) models in two situations: presence or non-presence of a change-point; section 4 introduces a polynomial regression model for the mean value function (accumulated number of violations); section 5 presents the obtained results assuming two parametrical models, the PLP process and the polynomial regression model for the mean value function; section 6 presents the interpretation of the obtained results; finally section 7 ends the paper with some concluding remarks.

2 Data Set

The data set considered in this study was extracted from the Research Data Archive (RDA), managed by the Data Engineering and Curation Section (DECS) of the Computational and Information Systems Laboratory (CISL) at the National Center for Atmospheric Research, USA, contains a large and diverse collection of meteorological and oceanographic observations, operational and reanalysis model outputs, and remote sensing datasets to support atmospheric and geosciences research, (https://rda,ucar,edu/index,html?hash=data$_u$ser&action=register)(https://rda.ucar.edu /datasets/ds570.0/#!subset.html). The RDA archive has data for over 4700 different stations (2600 in more recent years). Different follow-up periods are given for the diferent climate stations. In this study, as comparative purposes, we assume a fixed follow-up period of 141 years for the five assumed climate stations (1880 to 2020). The original data set consists of monthly average temperature and rain precipitation reported in five climate stations for the last 141 years (1880 to 2020) in North America (all climate stations in USA). The assumed climate stations used in this study are presented in Table 1. Table 1 also shows the latitude(lat) in degrees, longitude(long) in degrees and elevation (elev) of the climate station in meters.

Figure 1 shows the annual mean temperatures and annual mean rain precipitations for the five climate stations.

Table 1: Climate stations in North America (USA)

Climate Station	Lat. (deg)	Long.(deg)	Elev. (meters)
Charleston	32.9	-80.0	15
New Orleans	30.0	-90.3	9
Washington	38.9	-77.1	20
Chicago	41.9	-87.6	190
Portland	45.6	-122.6	12

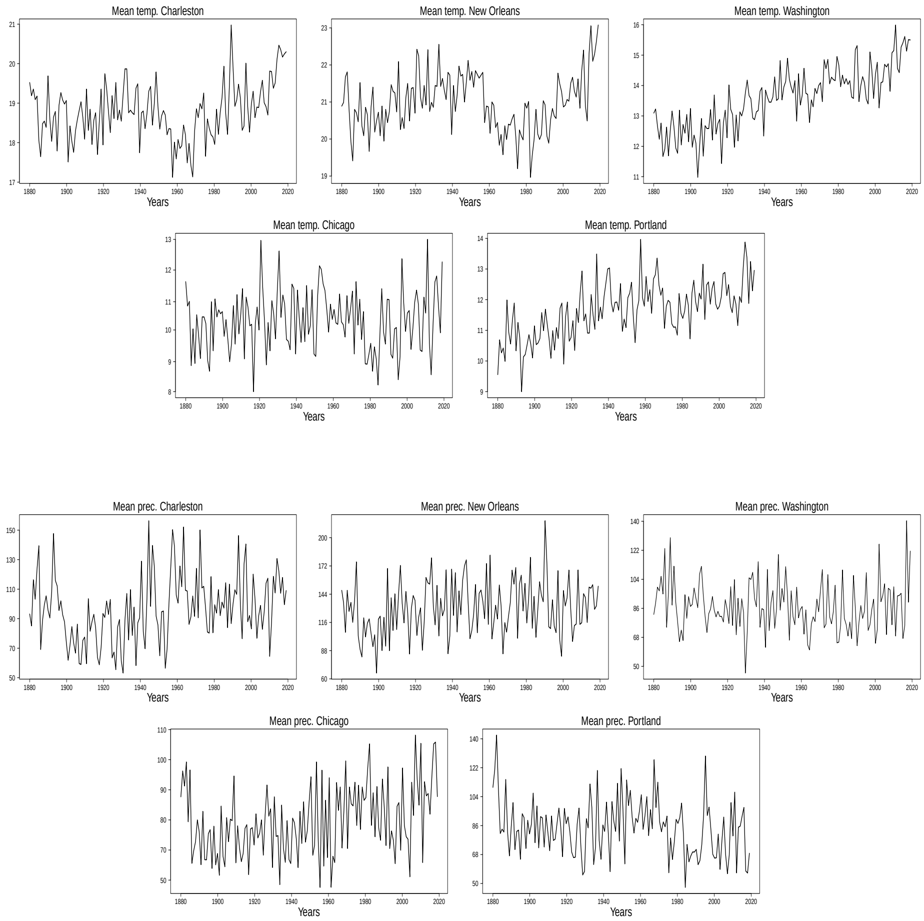


Figure 1: Annual mean temperatures and precipitations

From the plots presented in Figure 1, we see that in all stations there is an increasing behavior in the annual meam temperature at the end of the follow-up period (close to the year 2020). Considering the annual rain precipitation, it is difficult to afirm that there is similar behavior for each one of the climate stations.

In this study, This statement is not clear what the author want to say. We consider the modeling of the accumulated number of years where it is observed the occurrence of a violation of some event of interest in the climate series (here the climate mean to be above the overall observed average during the follow-up period) assuming non-homogeneous Poisson processes (NHPP) with different parametric forms for the intensity function (or equivalently the mean value function).

The main goals of this study are:

- The first goal of this work is to detect the year where it is observed a climate change-point (temperature or rain precipitation). In this way, we consider a special modeling approach assuming PLP (Power Law Process) processes in presence of a change-point in the assumed NHPP considered in the statistical analysis of the climate data sets.
- The second goal of this work is to get a better fit for the mean value function to detect accurate climate standards of each region of interest. Since in practical work, usually it is difficult to get accurate fit for the mean value functions using the different existing parameterized forms of NHPP models introduced in the literature, we also propose, as a second model approach, a simple Bayesian model based on a polynomial structure for the mean value function assuming the responses given by the accumulated number of climate violations until each fixed time t where there is the occurrence of a climate violation, to get accurate estimators for the mean value function. With the obtained fitted models for the mean value functions (or the intensity or mean value functions), we detect the climate change years for each climate station.

3 Use of Non-Homogeneous Poisson processes (NHPP) models

Non-homogeneous Poisson processes (NHPP) are used in many applications (see for example [38], [25], [39] [40] among many others). In this study, we consider the use of non-homogeneous Poisson model to estimate the probability that a climate (precipitation, temperature) standard is exceeded a given number of times in a time interval of interest.

Let $M(t) \geq 0, t \geq 0$ be the number of times a climate standard is above the usual annual overall average in the time interval $[0, t)$. The climate standard average for each year is obtained from the reported month averages usually available in climate data sets. We assume that the number of times the climate standard is above the overall average follows a non-homogeneous Poisson process where the random variable $M(t)$ has a Poisson distribution with rate function $\lambda(t)$ and mean function $m(t)$ given by,

$$m(t) = \int_0^t \lambda(s)ds \tag{3.1}$$

The rate function $\lambda(t)$ models the behavior of the Poisson process $M = \{M(t) : t \geq 0\}$. Different parametric models could be assumed for the rate function that is parameterized by a parameter vector $\boldsymbol{\theta}$. Thus, we assume a non-homogeneous Poisson process with mean value function $m(t \mid \boldsymbol{\theta})$ where $\boldsymbol{\theta}$ is a vector of parameters. The function $m(t|\boldsymbol{\theta})$ denotes the expected number of events registered by $M(t)$ up to time t. In this study, the events are the climate standards to be above the yearly overall average in a climate station considering the total observed follow-up period. The characterization of a non-homogeneous Poisson process of this type is specified by the functional form of $m(t|\boldsymbol{\theta})$, or equivalently, of its intensity function $\lambda(t|\boldsymbol{\theta})$, given by the first derivative of $m(t|\boldsymbol{\theta})$, that is, $\lambda(t|\boldsymbol{\theta}) = dm(t|\boldsymbol{\theta})/dt$. In applications of climate data, it is interesting to have a rate function $\lambda(t|\boldsymbol{\theta}), t \geq 0$ that presents different behaviors as decreasing or increasing depending of time.

Different formulations of NHPP could be used in the climate data analysis. One of these formulations, usually used in software reliability studies and denoted as NHPP-I, assumes that the mean value function is given by $m(t) = \alpha F(t)$ where $F(t)$ is the cumulative function of a specified probability distribution and α is an unknown parameter that should be estimated (see [39]); another formulation also used in software reliability studies and denoted as NHPP-II is given by $m(t) = -\log(1–F(t))$ where $F(t)$ is the cumulative function of a probability distribution also usually used in reliability software applications [41].

For the statistical analysis of climate data, we could consider some existing parametric structures as the power law process (PLP) [?], [42]; the Musa–Okumoto process (MOP) [43]; the Goel–Okumoto process (GOP) [44]; a generalized form of Goel–Okumoto (GGOP) and the exponentiated-Weibull (GPLP) [45], [46], [39] that generalizes the PLP process.

The power law process (PLP), the Musa–Okumoto process (MOP) and the exponentiated-Weibull process(GPLP) are defined as special cases of the mean function $m(t) = -\log(1–F(t))$, that is, in the class NHPP-II, where $F(t)$ is the cumulative function of a Weibull distribution [47] by $F(t) = \exp-(t/\sigma)^{\alpha}, t > 0$ for the PLP; $F(t)$ is the cumulative function of a Lomax or Pareto type II distribution [48], given by $F(t) = 1–(1–t/\alpha)^{-\beta}, t > 0$ for the MOP and $F(t) = 1 - \exp[-(t/\sigma)^{\alpha}]^{\beta}, t > 0$ is the cumulative distribution of a exponentiated-Weibull distribution for the GPLP that generalizes the PLP process.

The Goel–Okumoto process (GOP) [44] and the generalized form of the Goel-Okumoto process (GGOP) are obtained from formulation of the mean value function given by $m(t) = \alpha F(t)$ where $F(t)$ is the cumulative function of a exponential distribution, that is, $F(t) = 1 - \exp(-\beta t)$ for the GOP model and $F(t) = 1 - \exp(-\beta t^{\gamma})$ is the cumulative distribution of a Weibull distribution for the GGOP model.

The mean value functions for these popular NHPP processes are given by,

$$\begin{aligned} mPLP(t|\boldsymbol{\theta}) &= (t/\sigma)\alpha, \quad \text{where} \quad \boldsymbol{\theta} = (\alpha, \sigma);\ \alpha, \sigma > 0, \\ mMOP(t|\boldsymbol{\theta}) &= \beta \log(1 + t/\alpha), \quad \text{where} \quad \boldsymbol{\theta} = (\alpha, \beta);\ \alpha, \beta > 0, \\ mGOP(t|\boldsymbol{\theta}) &= \alpha[1 - \exp(-\beta t)], \quad \text{where} \quad \boldsymbol{\theta} = (\alpha, \beta);\ \alpha, \beta > 0 \\ mGGOP(t|\boldsymbol{\theta}) &= \alpha[1 - \exp(-\beta t\gamma)], \quad \text{where} \quad \boldsymbol{\theta} = (\alpha, \beta, \gamma); \alpha, \beta, \gamma > 0, \\ mGPLP(t|\boldsymbol{\theta}) &= -\log[1 - F_{EW}(t)], \quad \text{where} \quad \boldsymbol{\theta} = (\alpha, \beta, \sigma); \alpha, \beta, \sigma > 0, \end{aligned} \tag{3.2}$$

and $F_{EW}(t) = \{1 - \exp[-(t/\sigma)^{\alpha}]\}^{\beta}$

The corresponding intensity functions $\lambda(t|\boldsymbol{\theta}) = dm(t|\boldsymbol{\theta})/dt$ for the mean functions (3.2) are given by,

$$\begin{aligned} \lambda PLP(t|\boldsymbol{\theta}) &= (\alpha/\sigma)(t/\sigma)^{\alpha-1}, \quad \text{where} \quad \boldsymbol{\theta} = (\alpha, \sigma); \alpha, \sigma > 0, \\ \lambda MOP(t|\boldsymbol{\theta}) &= \beta/(t + \alpha), \quad \text{where} \quad \boldsymbol{\theta} = (\alpha, \beta); \alpha, \beta > 0, \\ \lambda GOP(t|\boldsymbol{\theta}) &= \alpha\beta \exp(-\beta t), \quad \text{where} \quad \boldsymbol{\theta} = (\alpha, \beta); \alpha, \beta > 0, \\ \lambda GGOP(t|\boldsymbol{\theta}) &= \alpha\beta\gamma t^{\gamma-1} \exp(-\beta t^{\gamma}), \quad \text{where} \quad \boldsymbol{\theta} = (\alpha, \beta, \gamma); \alpha, \beta, \gamma > 0, \\ \lambda GPLP(t|\boldsymbol{\theta}) &= G(t)/[1 - F_{EW}(t)], \quad \text{where} \quad \boldsymbol{\theta} = (\alpha, \beta, \sigma); \alpha, \beta, \sigma > 0, \end{aligned} \tag{3.3}$$

$G(t) = \alpha\beta\sigma^{-1}1–\exp[-(t/\sigma)^{\alpha}]^{\beta-1} \exp[-(t/\sigma)^{\alpha}](t/\sigma)^{\alpha-1}$ and $F_{EW}(t)$ is defined in (3.2).

Remark 1. The intensity functions given by (3.3) define the hazard rates of the time between occurrence of events in the respective models.

Remark 2. From (3.3), the intensity function $\lambda PLP(t|\boldsymbol{\theta})$ gives different forms for the PLP depending on the value of α which could be constant, decreasing or increasing depending on whether $\alpha = 1, \alpha < 1$ or $\alpha > 1$, respectively. The intensities $\lambda MOP(t|\boldsymbol{\theta})$ and $\lambda GOP(t|\boldsymbol{\theta})$ presents a decreasing behavior as functions of t and $\lambda GGOP(t|\boldsymbol{\theta})$ describes the situation where the intensity increases slightly at the beginning and then begins to decrease with t.

Remark 3. For the rate $\lambda GPLP(t|\boldsymbol{\theta})$ we observe that: if $\alpha \geq 1$ and $\alpha\beta \geq 1, \lambda(t)$ is a increasing function of t; if $\alpha \leq 1$ and $\alpha\beta \leq 1$, $\lambda(t)$ is a decreasing function of t; if $\lambda > 1$ and $\alpha\beta < 1, \lambda(t)$ has a bathtub form; if $\alpha < 1$ and $\alpha\beta > 1, \lambda(t)$ is unimodel.

In this study, we focus on the use of the PLP process in the analysis of the North America climate data sets.

To have more flexibility of fit considering climate data, we also could assume superposition of NHPP processes [23]. The sum of count NHPP processes $M(t) = \sum_{(j=1)}^{J} M_j(t)$ is also a NHPP where the intensity function is given by $\lambda(t|\boldsymbol{\theta}) = \lambda_1(t|\boldsymbol{\theta}_1) + \lambda_2(t|\boldsymbol{\theta}_2) + \cdots + \lambda_J(t|\boldsymbol{\theta}_J)$, where $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \cdots, \boldsymbol{\theta}_J)$ [49]. Different superpositions of NHPP processes could be assumed as superposition of power law (PLP) processes; superposition of Musa–Okumoto (MOP) processes; superposition of Goel–Okumoto (GOP) processes; superposition of a generalized form of Goel–Okumoto (GGOP) processes, superposition of exponentiated-Weibull (GPLP) processes or even superposition of diferent parameterized forms of the values function.

In this way, we could consider the intensity function or the mean value function assumed as a polynomial, there is, $\lambda(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \cdots + \beta_J t^J$ (a superposition of a homogeneous Poisson process and Weibull processes assuming known shape parameters). As a special case, we assume a polynomial structure for the mean value function, instead of a polynomial structure for the intensity function, given by,

$$mPOLY(t|\boldsymbol{\theta}) = \beta_0 + \beta_1 t + \beta_2 t^2 + \cdots + \beta_J t^J \quad \text{where} \quad \boldsymbol{\theta} = (\beta_0, \beta_1, \beta_2, \cdots, \beta_J) \tag{3.4}$$

which implies in the intensity function given by,

$$\lambda POLY(t|\boldsymbol{\theta}) = \beta_1 + 2\beta_2 t + 3\beta_3 t^2 + \cdots + J\beta_J t^{J-1} \tag{3.5}$$

for a fixed value of J. With an appropriate choice for J, we get accurate estimators for the mean value function implying in accurate estimators for the intensity function.

3.1 Likelihood function without the presence of change-points

Without the presence of change-points, let us assume the data set denoted by $D_T = \{n; t_1, \cdots, t_n; T\}$ where n is the number of observed occurrence times such that $0 < t_1 < t_2 < \cdots < t_n < T$. In the application considered here these values are the epochs of occurrence of climate standard violations (above observed average) up to time T. The likelihood function for $\boldsymbol{\theta}$ considering the time truncated model is (see for example [50] given by,

$$L(\boldsymbol{\theta}|D_T) = \left[\prod_{(i=1)}^{n} \lambda(t_i \mid \boldsymbol{\theta}) \exp[-m(T \mid \boldsymbol{\theta})] \right] \tag{3.6}$$

3.2 Likelihood function in the presence of a change-point

In many applications, we could have changes in the counting process over the time range $(0, T)$ linked to some kind of event (e.g., in climate series there is observed precipitation or temperature changes in all world in the last decades). That is, we have a single change-point τ making a transition between two NHPP models of the same type but with different parameters. In this way, the intensity function (see [51], [52]) of the overall process is given by,

$$\lambda(t \mid \boldsymbol{\theta}) = \begin{cases} \lambda(t \mid \boldsymbol{\theta}_1) & \text{if } 0 \leq t \leq \tau \\ \\ \lambda(t \mid \boldsymbol{\theta}_2) & \text{if } t \geq \tau \end{cases} \tag{3.7}$$

where $\lambda(t|\boldsymbol{\theta_j}), j = 1, 2$ are intensity functions related to the mean functions defined in (3.2) and $\boldsymbol{\theta}_j, j = 1, 2$ are the parameters associated to the NHPP before and after the change-point. The corresponding mean value functions $m(t|\boldsymbol{\theta}_j), j = 1, 2$, are given by,

$$m(t \mid \boldsymbol{\theta}) = \begin{cases} m(t \mid \boldsymbol{\theta}_1) & \text{if} \quad 0 \leq t \leq \tau \\ \\ m(\tau \mid \boldsymbol{\theta}_1) + m(t \mid \boldsymbol{\theta}_2) - m(\tau \mid \boldsymbol{\theta}_1) & \text{if} \quad t \geq \tau \end{cases} \tag{3.8}$$

In this way, the data set is given by: $D_T = \{n; t_1, \cdots, t_{(N_\tau)}; t_{N_{(\tau+1)}}, \cdots, t_n; T\}$ where t_k , $k = 1, 2, \cdots, n$ is the time of occurrence of the k^{th} event (in the present case is the k^{th} violation of the climate standard) and τ is the change-point. Therefore, the likelihood function of the model is given by,

$$\begin{aligned} L(\boldsymbol{\theta} \mid D_T) = & \left[\prod_{i=1}^{N_\tau} \lambda(t_i \mid \boldsymbol{\theta}_1)\right] \exp[-m(\tau \mid \boldsymbol{\theta})] \\ & \times \left[\prod_{N_{\tau+1}}^{n} \lambda(t_i \mid \boldsymbol{\theta}_2)\right] \exp[-m(T \mid \boldsymbol{\theta}_2) + m(\tau \mid \boldsymbol{\theta}_2)] \end{aligned} \tag{3.9}$$

As a special case, considering PLP models in the presence of a change-point, the intensity function (3.3) is given by,

$$\lambda(t \mid \boldsymbol{\theta}) = (\alpha_1/\sigma_1)(t/\sigma_1)^{\alpha_1 - 1} \quad \text{if} \quad 0 \leq t \leq \tau \tag{3.10}$$

and

$$\lambda(t \mid \boldsymbol{\theta}) = (\alpha_2/\sigma_2)(t/\sigma_2)^{\alpha_2 - 1} \quad \text{if} \quad t \geq \tau$$

with corresponding mean value function given by,

$$m(t \mid \boldsymbol{\theta}) = (t/\sigma_1)^{\alpha_1} \quad \text{if} \quad 0 \leq t \leq \tau \tag{3.11}$$

and

$$m(t \mid \boldsymbol{\theta}) = (\tau/\sigma_1)^{\alpha_1} + (t/\sigma_2)^{\alpha_2} - (\tau/\sigma_2)^{\alpha_2} \quad \text{if} \quad t \geq \tau$$

In the same way, we get the intensity function and mean value functions for the other NHPP processes in presence of one change-point.

Given the complexity of the likelihood function assuming non-homogeneous Poisson processes especially in presence of change-points, where we usually have difficulties to get maximum likelihood estimators for the parameters of the models assuming the different parameterized forms of the intensity function (or the mean value function), we consider a Bayesian approach using MCMC (Markov Chain Monte Carlo) simulation methods as the Gibbs sampling or the Metropolis-Hastings algorithms [34], [36] to get the estimators for the parameters of the models. Using a Bayesian approach we also could incorporate prior opinions of experts leading to more accurate inferences. Although it is possible to get accurate parameter estimation for the parameters of the assumed NHPP model using Bayesian methods, usually it is difficult to get accurate fit for the mean value function when compared to the empirical accumulated function even considering the NHPP processes defined by (3.2) and (3.3) in the presence of many change-points.

In this work, we consider the use of a polynomial mean value function (3.4) to get very accurate fit for the mean value function and the corresponding intensity function, although the existence of great computational problems considering the likelihood function (3.6) where it is included only the information of the epochs where occurs climate violation $D_T = \{n; t_1, \cdots, t_n; T\}$, n is the number of observed occurrence times such that $0 < t_1 < t_2 < \cdots < t_n < T$. These computational problems are related to non convergence of the MCMC simulation algorithm to get samples of the joint posterior distribution of interest using different prior distributions elicited by the researcher and the convergence of the simulation algorithm is only obtained using very informative prior distributions for the parameters $\beta_0, \beta_1, \beta_2, \cdots, \beta_J$, to get the convergence of the MCMC simulation algorithm.

4 A polynomial regression model for the mean value function (accumulated number of violations)

With the polynomial mean value function (3.4), we usually have great difficulties to get accurate fit for the mean value function assuming the likelihood function (3.6) under the time truncated model even assuming very informative prior distributions for the parameters of the model as mentioned above. In this way, we propose a simple polynomial regression model also under a Bayesian approach and using MCMC estimation methods to get the posterior summaries of interest.

In this way, using the information of the epochs where occurs climate violation denoted by $D_T = \{n; t_1, \cdots, t_n; T\}$, with n denoting the number of observed occurrence times such that $0 < t_1 < t_2 < \cdots < t_n < T$, we assume a standard multiple linear polynomial regression model given by,

$$y_i = \beta_0 + \beta_1 t_i + \beta_2 t_i^2 + \; + \beta_J t_i^J + \epsilon_i \tag{4.1}$$

where y_i is the accumulated number of violations at time t_i of each violation occurrence and ϵ_i is a error term (non-observed random variable) assumed to be independent with a normal distribution $N(0, \sigma^2)$.

In this situation, the likelihood function is given by,

$$L(\boldsymbol{\theta} \mid D_T, y) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{\sigma^2}\left(y_i - \beta_0 - \beta_1 t_i - \cdots - \beta_J t_i^J\right)^2\right] \tag{4.2}$$

where $\boldsymbol{\theta} = (\beta_0, \beta_1, \beta_2, \cdots, \beta_J, \sigma^2)$; $D_T = \{n; t_1, \cdots, t_n; T\}$ and $y = (y_1, y_2, \cdots, y_n)$. We assume normal $N(a, b^2)$ prior distributions for the parameters $\beta_0, \beta_1, \beta_2, \cdots, \beta_J$ with known hyperparameters a and b and a $Gamma(c, d)$ prior distribution with mean c/d and variance c/d^2 for the parameter $\tau = 1/\sigma^2$. We further assume prior independence.

5 Results

In this section, we present the Bayesian results assuming the two model formulations presented in sections 3 and 4 for the climate data of the five climate stations in North America (USA) introduced in section 2.

5.1 Use of the PLP-NHPP introduced in section 3

The first model denoted as “model 1” to be used in the data analysis of the climate data of the five climate stations in North America (USA) introduced in section 2 is the PLP process defined in (3.2) and (3.3) in presence of one change-point considering uniform U(1,141) prior distribution for the change-point τ; uniform $U(0, 10)$ prior for the scale parameters $\sigma_j, j = 1, 2$; uniform $U(0, 3)$ prior distributions for the shape parameters $\alpha_j, j = 1, 2$ for all climate series (annual mean temperature and annual mean rain precipitation). Simulation of samples for the joint posterior distributions were obtained using MCMC methods and the OpenBugs software (Spiegelhalter et al., 2003) from where it was generated 1,000 Gibbs samples (taking every 100^{th} sample from 100,000 simulates sample) after a burn-in-sample of 11,000 samples discarded to eliminate the effect of the initial values. Convergence of the simulation algorithm was verified from trace plots of the simulated sample for each parameter.

Table 2 shows the posterior summaries of interest (posterior means, posterior standard-deviations and 95% credible intervals for the parameters of the model) assuming the PLP-NHPP process in presence of a change-point for the accumulated numbers of violations (accumulated number of years in each time where it is observed a climate violation, that is, the annual mean of temperature or rain precipitation is above a threshould considering as threshould the averages of the observed annual temperatures and annual rain precipitations).

5.2 Use of the polynomial regression for the accumulated number of violations (mean value function) introduced in section 4

A second model used in the data analysis denoted as "model 2" assumed in this work is the polynomial regression model defined in (3.4) and (3.5) for the accumulated number of climate violations considering normal $N(0,1)$ prior distribution for β_0; normal $N(1,1)$ prior for β_1; normal $N(0,0.1)$ prior for β_2; normal $N(0,0.01)$ prior for $\beta_j, j \geq 3$; and a Gamma $G(1,1)$ prior distributions for the parameter $\tau = 1/\sigma^2$ for all climate series (annual mean temperature and annual mean rain precipitation). Simulation of samples for the joint posterior distributions were obtained using MCMC methods and the OpenBugs software [37] from where it was generated 1,000 Gibbs samples (taking every 100^{th} sample from 100,000 simulates sample) after a burn-in-sample of 111,000 samples discarded to eliminate the effect of the initial values. Convergenge of the simulation algorithm was verified from trace plots of the simulated sample for each parameter.

Table 3 shows the posterior summaries of interest (posterior means, posterior standard-deviations and 95% credible intervals for the parameters of the model) assuming the polynomial regression model for the accumulated numbers of violations (accumulated number of years in each time where it is observed a climate violation, that is, the annual mean of temperature or rain precipitation is above a threshold considering as threshold, the averages of annual temperatures and annual rain precipitations). The choice of the polynomial order J in each case was decided using the Bayesian discrimination method DIC (Deviance Information Criterian), [53] available in the OpenBugs software and parcimony.

Figure 2 shows the plots of the empirical accumulated numbers of climate violations (above the annual averages) and the fitted Bayesian mean value function $m(t)$ considering the PLP process in presence of a change-point and the polynomial NHPP process (superposition of non-homogeneius Poisson processes) for the annual mean temperatures of the five climate stations in USA.

From the plots of Figure 2 we observe a very good fit of "model 2" assuming a polynomial mean value function for all cases considering the annual mean temperatures. Figure 3 shows the plots of the empirical accumulated numbers of climate violations (above the annual averages) and the fitted Bayesian mean value function $m(t)$ considering the PLP process in presence of a change-point and the polynomial NHPP process (superposition of non-homogeneous Poisson processes) for the annual mean rain precipitations of the five climate stations in USA.

Table 2: Posterior summaries assuming a PLP process in presence of a change-point ("model 1")

Charleston n	Parameter	Means	Sd	95% Credible Interval
Temperature	α_1	1.023	0.2783	(0.6508;1.819)
(annual	α_2	1.362	0.1921	(0.9231;1.672)

average = 18.714);	σ_1	2.474	1.359	(0.5104;5.885)
n=68 violations;	σ_2	6.093	2.627	(0.8873;9.831)
T=141 years)	τ	82.21 (years 1961)	41.66	(3.969;131.40)
Precipitation	α_1	1.306	0.5544	(0.5932;2.710)
(annual	α_2	1.400	0.1544	(0.9919;1.599)
average =102.24);	σ_1	3.682	1.882	(0.5823;8.129)
n=67 violations;	σ_2	7.243	2.266	(1.635;9.908)
T=141 years	τ	36.25 (year 1915)	27.31	(4.458;105.60)
New Orleans	Parameter	Means	SD	95% Credible Interval
Temperature	α_1	1.492	0.2517	(0.8046;1.840)
(annual	α_2	1.222	0.1593	(0.8747;1.468)
average =20.881)	σ_1	6.562	2.021	(2.249;9.710)
n=65 violations;	σ_2	6.433	2.519	(1.269;9.865)
T=141 years	τ	74.52 (year 1954)	18.97	(10.540;117.80)
Precipitation	α_1	1.089	0.4528	(0.4309;2.327)
annual	α_2	1.286	0.2183	(0.6671;1.572)
average =130.47);	σ_1	3.330	1.963	(0.5784;8.046)
n=68 violations;	σ_2	5.927	2.610	(0.5294;9.785)
T=141 years	τ	34.64 (year 1914)	37.88	(1.739;139.10)
Washington	Parameter	Means	SD	95% Credible Interval
Temperature	α_1	0.6102	0.2919	(0.0928;1.227)
(annual	α_2	1.486	0.1584	(1.101;1.709)
average =13.819);	σ_1	6.822	2.340	(1.682;9.893)
n=72 violations;	σ_2	7.109	2.192	(1.947;9.881)
T=141 years	τ	46.33 (year 1925)	7.642	(27.93;58.39)
Precipitation	α_1	1.263	0.5012	(0.7124;2.611)
(annual	α_2	1.189	0.2070	(0.7509;1.533)
average =87.27);	σ_1	2.433	1.356	(0.5017;5.355)
n=63 violations;	σ_2	5.110	2.670	(0.4638;9.718)
T=141 years	τ	53.12 (year 1932)	46.58	(2.401;137.70)
Chicago	Parameter	Means	SD	95% Credible Interval
Temperature	α_1	1.148	0.3643	0.5117;2.259)
(annual	α_2	1.164	0.2084	(0.7495;1.524)
average =10.153);	σ_1	3.047	1.706	(0.8046;7.559)
n=70 violations;	σ_2	4.672	2.726	(0.4763;9.656)
T=141 years	τ	62.32 (year 1925)	45.04	(1.432;139.60)
Precipitation	α_1	1.211	0.5382	(0.5343;2.587)
(annual	α_2	1.384	0.1585	(0.9939;1.606)
average =73.55);	σ_1	2.750	1.630	(0.4251;6.926)
n=67 violations	σ_2	6.977	2.242	(1.581;9.901)

T=141 years	τ	31.68 (year 1911)	35.04	(2.156;130.70)
Portland	Parameter	mean	SD	95% Credible Interval
Temperature	α_1	1.111	0.5236	0.2973;2.579)
(annual	α_2	1.473	0.1589	(1.056;1.679)
average =12.081);	σ_1	6.975	2.260	(1.701;9.927)
n=75 violations;	σ_2	7.245	2.223	(1.674;9.928)
T=141 years	τ	38.67 (year 1918)	27.75	(2.486;126.60)
Precipitation	α_1	1.115	0.2386	(0.7442;1.693)
(annual	α_2	1.072	0.2763	(0.3098;1.468)
average =84.68);	σ_1	2.871	1.506	(0.7152;6.682)
n=69 violations;	σ_2	4.824	2.797	(0.3045;9.705)
T=141 years	τ	80.68 (year 1960)	44.23	(2.318;140.10)

Table 3: Posterior summaries assuming a polynomial process of order J ("model 2")

Charleston	Parameter	Means	Sd	95% Credible Interval
Temperature	β_0	-0.2066	0.5827	(-1.313;0.9396)
(annual	β_1	0.6408	0.0355	(0.5726;0.7074)
average =18.714); J=3;	β_2	-0.00553	0.00061	(-0.0067;-0.0043)
n=68 violations;	β_3	0.0000309	< 0.0001	(0.000025;0.000036)
T=141 years	$1/\sigma^2$	0.3689	0.05977	(0.2595;0.4915)
Precipitation	β_0	1.044	0.6644	(-0.1976;2.629)
(annual	β_1	0.5656	0.0804	(0.3714;0.7202)
average =102.24); J=4;	β_2	-0.01233	0.0022	(-0.0165;-0.0068)
n=67 violations;	β_3	0.000166	< 0.0001	(0.00011;0.00021)
T=141 years	β_4	< 0.0001	< 0.0001	(-0.0000007;-0.0000004)
	$1/\sigma^2$	0.6915	0.1209	(0.4735;0.9453)
New Orleans	Parameter	Means	Sd	95% Credible Interval
Temperature	β_0	2.189	0.8476	(0.4977;3.847)
(annual	β_1	-0.6786	0.0806	(-0.8513;-0.5009)
average =20.881); J=4;	β_2	0.03793	0.0023	(0.03232;0.04257)
n=65 violations;	β_3	-0.0004	< 0.0001	(-0.00045;-0.00034)
T=141 years	β_4	< 0.0001	< 0.0001	(0.0000011;-0.0000014)
	$1/\sigma^2$	0.2517	0.04665	(0.1722;0.3597)
Precipitation	β_0	1.976	0.3506	(1.292;2.649)
(annual	β_1	0.1515	0.0192	(0.1152;0.1875)
average =130.47); J=3;	β_2	0.00443	0.00031	(0.00382;0.00506)

n=68 violations;	β_3	-0.000016	< 0.0001	(-0.000019;-0.000013)
T=141 years	$1/\sigma^2$	1.629	0.2651	(1.140;2.166)
Washington	Parameter	Means	Sd	95% Credible Interval
Temperature	β_0	1.753	0.8797	(-0.0283;3.500)
(annual	β_1	-0.4123	0.03175	(-0.4710;-0.3458)
average =13.819); J=3;	β_2	0.00994	0.00041	(0.00914;0.01071)
n=72 violations;	β_3	-0.000025	< 0.0001	(-0.000028;-0.000021)
T=141 years	$1/\sigma^2$	1.406	0.2424	(0.9624;1.928)
Precipitation	β_0	0.7493	0.4603	-0.1265;1.640)
(annual	β_1	0.7043	0.0308	(0.6458;0.7646)
average = 87.27); J=3;	β_2	-0.00451	0.00054	(-0.0056;-0.0035)
n=63 violations;	β_3	0.000018	< 0.0001	(0.000013;0.000023)
T=141 years	$1/\sigma^2$	0.6157	0.1106	(0.4290;0.8627)
Chicago	Parameter	Means	Sd	95% Credible Interval
Temperature	β_0	1.591	0.5149	(0.6030;2.647)
(annual	β_1	0.2684	0.0442	(0.1739;0.3555)
average =10.153); J=4;	β_2	0.00867	0.0012	(0.0062;0.0110)
n=70 violations;	β_3	-0.000094	0.00001	(-0.00019;-0.000066)
T=141 years	β_4	< 0.0001	< 0.0001	(0.0000002;0.0000004)
	$1/\sigma^2$	0.8932	0.1554	(0.6199;1.2130)
Precipitation	β_0	1.949	0.5818	(0.8306;3.293)
(annual	β_1	0.2851	0.0751	(0.0971;0.4233)
average =73.55); J=4;	β_2	-0.0008	0.0023	(-0.0050;0.0049)
n=67 violations;	β_3	0.000028	< 0.0001	(-0.000034;0.000073)
T=141 years	β_4	< 0.0001	< 0.0001	(-0.0000002;0.0000001)
	$1/\sigma^2$	0.7511	0.1354	(0.5068; 1.039)
Portland	Parameter	Means	Sd	95% Credible Interval
Temperature	β_0	0.5842	0.7437	(-0.8659;2.001)
(annual	β_1	-0.1092	0.0347	(-0.1789;-0.0412)
average =12.081); J=3;	β_2	0.00706	0.00053	(0.00914;0.01071)
n=75 violations;	β_3	-0.000018	¡0.0001	(-0.000023;-0.000013)
T=141 years	$1/\sigma^2$	0.5525	0.09361	(0.3872;0.7520)
Precipitation	β_0	0.7100	0.5612	(-0.3427;1.931)
(annual	β_1	0.5500	0.0509	(0.4401;0.6472)
verage =84.68); J=4;	β_2	-0.00399	0.0014	(-0.0069;-0.00079)
n=69 violations;	β_3	0.000072	0.00001	(0.000036;0.000106)
T=141 years	β_4	¡0.0001	¡0.0001	(-0.0000005;-0.0000002)
	$1/\sigma^2$	0.6761	0.1177	(0.4608; 0.9259)

Figure 4 shows the plots of the Bayesian fitted intensity functions $\lambda(t)$ for the numbers of climate violations (above the annual averages) considering the polynomial NHPP process (don't repeat) for the annual mean temperatures and the annual rain precipitations for the five climate stations in USA. The intensity functions where obtained from (3.5).

6 Interpretation of the obtained results

Table 4 shows the annual averages assumed as thresholds in the NHPP processes, the number (n) of violations in each time, that is, the annual mean is greater than the annual average in the follow-up period of $T = 141$ years (1880 to 2020) and the Bayesian estimate of the year change-point assuming a PLP process in presence of a chang-point

Table 4: Estimated Bayesian years of change-points using PLP processes in presence of a change-point

Cimate station	Annual average	Number of violations (n)	Estimated change-point (year)
Annual mean temperatures(C^0)			
Charleston	18.714	68	1961
New Orleans	20.881	65	1954
Washington	13.889	72	1925
Chicago	10.153	70	1941
Portland	12.081	75	1918
Annual mean precipitations(ml)			
Charleston	102.24	67	1915
New Orleans	130.47	68	1914
Washington	87.27	63	1932
Chicago	73.55	67	1911
Portland	84.68	69	1960

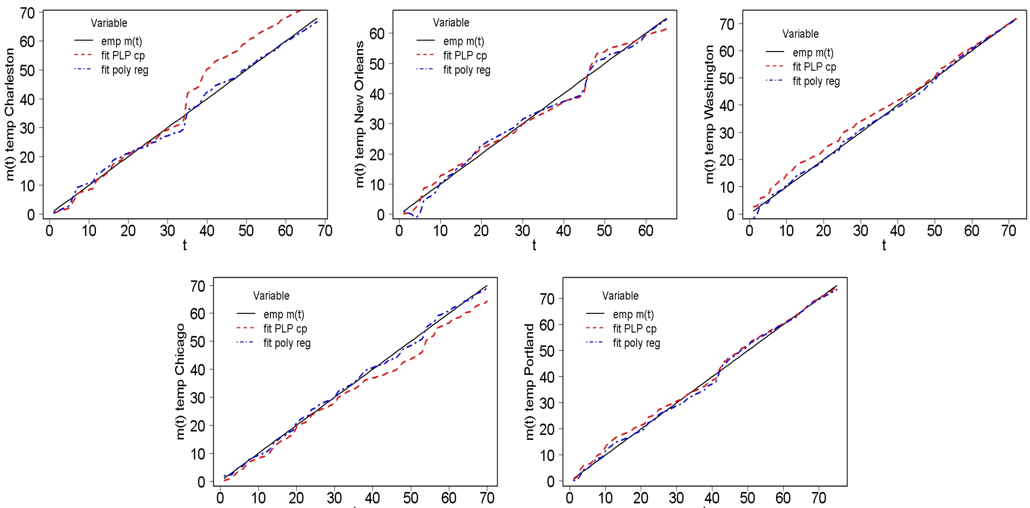


Figure 2: Fitted mean value functions for the annual mean temperatures

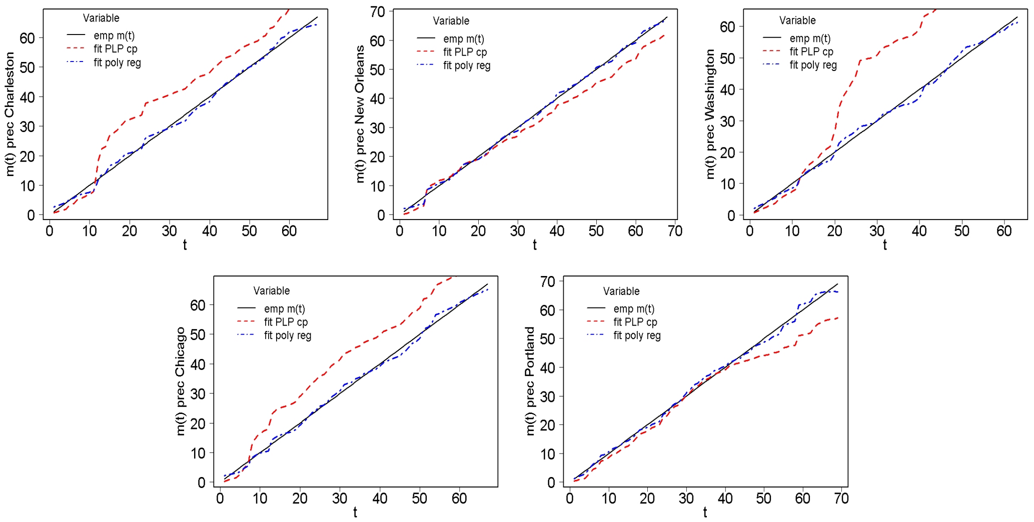
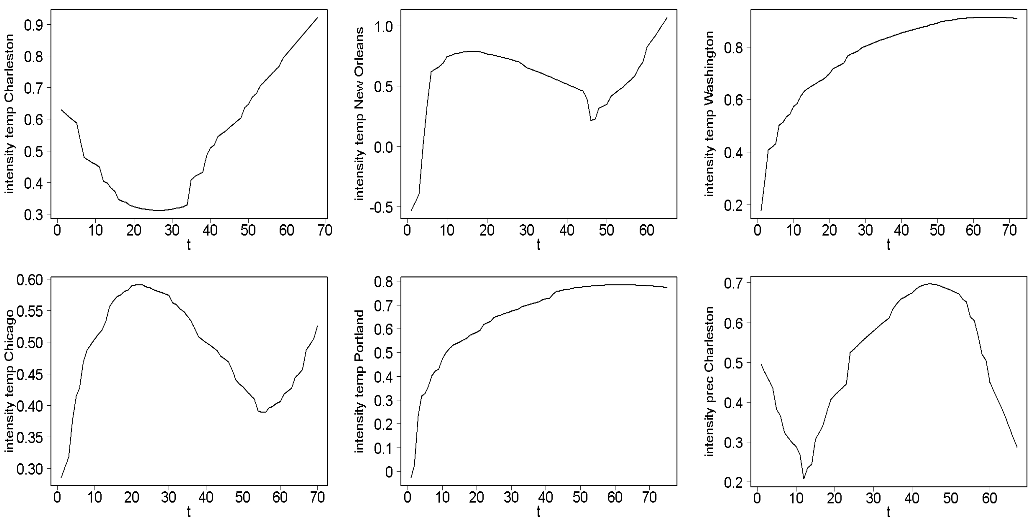


Figure 3: Fitted mean value functions for the annual mean precipitations

Assuming the polynomial NHPP (superposition of NHPP processes) we get very accurate estimation for the mean value functions and the intensity function from where we could get complete information on the climate behavior, as for example, the numbers of violations in each period of 10 years.

Table 5 shows some estimated mean value functions (accumulated numbers of violations) considering the annual mean temperatures fitted by a superposition of NHPP (polynomial NHPP process) from where it is possible to see the periods of 10 years with more violations (annual mean temperatures above the average annual temperature in the follow-up period of T=141 years).



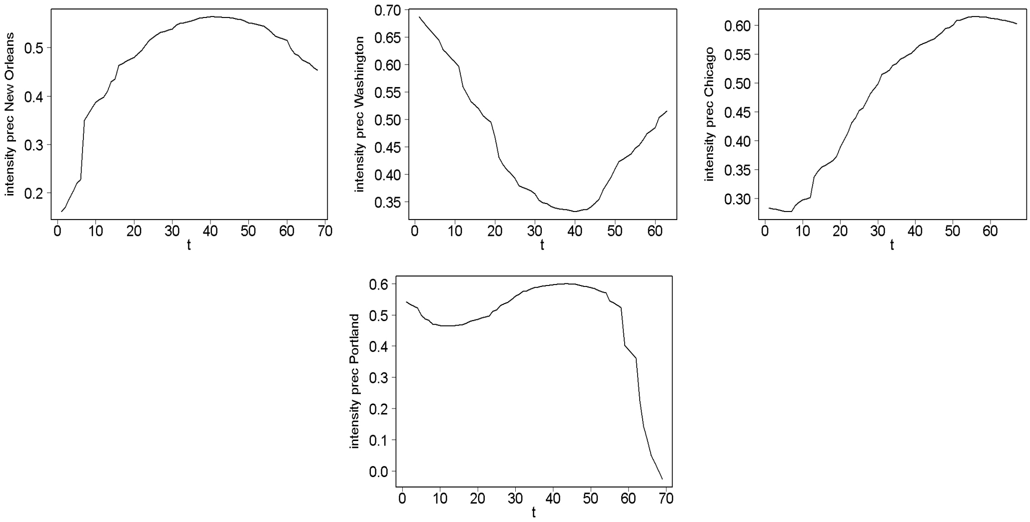


Figure 4: Bayesian fitted intensity functions for the annual mean temperatures and mean rain precipitations assuming polynomial mean value functions

Table 6 shows some estimated mean value functions (accumulated numbers of violations) considering the annual mean rain precipitationss fitted by a superposition of NHPP (polynomial NHPP process) from where it is possible to see the periods of 10 years with more violations (annual mean rain precipitations above the average annual temperature in the follow-up period of T=141 years).

Table 5: . Some estimated mean value functions for the accumulated numbers of violation considering the annual mean temperatures

Climate Station	year - estimated accumulated number of violations		Number of violations in periods of 10 years
Charleston	40(1919)	18.5600	1919-1929= 3.31 (≈ 3)
	50(1929)	21.8700	1929-1939= 3.14 (≈ 3)
	60(1939)	25.0100	1939-1949= 3.14 (≈ 3)
	70(1949)	28.1500	1989-1999= 5.94 (≈ 6)
	110(1989)	44.4700	1999-2009= 7.06 (≈ 7)
	120(1999)	50.4100	2009-2019= 8.37 (≈ 8)
	130(2009)	57.4700	
	140(2019)	65.8400	
New Orleans	30(1909)	6.2420	1909-1919= 7.288 (≈ 7)
	40(1919)	13.5300	1919-1929= 7.85 (≈ 8)
	50(1929)	21.3800	1929-1939= 7.44 (≈ 7)
	60(1939)	28.8200	1939-1949= 6.4 (≈ 6)
	70(1949)	35.2200	1999-2009= 4.47 (≈ 4)
	120(1999)	51.3000	2009-2019= 7.86 (≈ 8)

	130(2009)	55.7700	
	140(2019)	63.6300	
Washington	60(1939)	7.4300	1939-1949= 5.64 (≈ 6)
	70(1949)	13.0700	1949-1959= 6.59 (≈ 6)
	80(1959)	19.6600	1959-1969= 7.38 (≈ 8)
	90(1969)	27.0400	1969-1979= 8.03 (≈ 8)
	100(1979)	35.0700	1979-1989= 8.52 (≈ 9)
	110(1989)	43.5900	1989-1999= 8.87 (≈ 9)
	120(1999)	52.4600	1999-2019= 9.07 (≈ 9)
	130(2009)	61.5300	2009-2009= 9.12 (≈ 9)
	140(2019)	70.6500	
Chicago	21(1900)	10.240	1900-1910= 5.49 (≈ 5)
	31(1910)	15.730	1911-1921= 5.86 (≈ 6)
	32(1911)	16.300	1921-1931= 5.87 (≈ 6)
	42(1921)	22.160	1931-1941= 5.68 (≈ 6)
	52(1931)	28.030	1949-1959= 4.91 (≈ 5)
	62(1941)	33.710	1977-1987= 3.99 (≈ 4)
	70(1949)	38.010	1991-2001= 3.93 (≈ 4)
	80(1959)	43.010	2010-2020= 4.81 (≈ 5)
	98(1977)	50.960	
	108(1987)	54.950	
	112(1991)	56.510	
	122(2001)	60.440	
	131(2010)	64.230	
	141(2020)	69.040	
Portland	36(1915)	4.9370	1915-1925= 3.76 (≈ 4)
	46(1925)	8.6930	1926-1936= 4.74 (≈ 5)
	47(1926)	9.1200	1941-1951= 5.86 (≈ 6)
	57(1936)	13.8600	1959-1969= 6.88 (≈ 7)
	62(1941)	16.5300	1969-1979= 7.29 (≈ 7)
	72(1951)	22.3900	1988-1998= 7.76 (≈ 8)
	80(1959)	27.5200	1999-2009= 7.84 (≈ 8)
	90(1969)	34.4000	2009-2019= 7.81 (≈ 8)
	100(1979)	41.6900	
	109(1988)	48.5100	
	119(1998)	56.2700	
	120(1999)	57.0500	
	130(2009)	64.8900	
	140(2019)	72.7000	

Table 6: Some estimated mean value functions for the accumulated numbers of violation considering the annual mean rain precipitations

Climate Station	year - estimated accumulated number of violations		Number of violations in periods of 10 years
Charleston	3(1882)	2.635	1882-1892= 4.025 (≈ 4)
	13(1892)	6.660	1912-1922= 2.17 (≈ 2
	33(1912)	11.530	1935-1945= 3.14 (≈ 3)
	43(1922)	13.700	1935-1945= 3.69 (≈ 4)
	56(1935)	17.280	1946-1956= 4.76 (≈ 5)
	66(1945)	20.970	1959-1969= 5.95 (≈ 6)
	67(1946)	21.400	1979-1989= 6.93 (≈ 7)
	77(1956)	26.160	1992-2002= 6.56 (≈ 7)
	80(1959)	27.770	2006-2016= 4.84 (≈ 5)
	90(1969)	33.720	
	100(1979)	40.350	
	110(1989)	47.280	
	113(1992)	49.350	
	123(2002)	55.910	
	127(2006)	58.280	
	137(2016)	63.120	
New Orleans	26(1905)	8.634	1905-1915= 3.81 (≈ 4)
	36(1915)	12.440	1919-1929= 4.53 (≈ 4)
	40(1919)	14.120	1930-1940= 4.98 (≈ 5)
	50(1929)	18.650	1949-1959= 5.48 (≈ 5)
	51(1930)	19.130	1964-1974= 5.63 (≈ 6)
	61(1940)	24.110	1980-1990= 5.56 (≈ 6)
	70(1949)	28.860	1991-2001= 5.36 (≈ 5)
	80(1959)	34.340	2002-2012= 5.05 (≈ 5)
	85(1964)	37.130	
	95(1974)	42.760	
	101(1980)	46.130	
	111(1990)	51.690	
	112(1991)	52.230	
	122(2001)	57.590	
	123(2002)	58.110	
	133(2012)	63.160	
Washington	10(1889)	7.360	1889-1899= 5.82 (≈ 6)
	20(1899)	13.180	1902-1912= 4.95 (≈ 5)
	23(1902)	14.780	1922-1932= 3.97 (≈ 4)
	33(1912)	19.730	1934-1944= 3.60 (≈ 4)
	43(1922)	24.140	1945-1955= 3.39 (≈ 3)
	53(1932)	28.110	1969-1979= 3.40 (≈ 3)
	55(1934)	28.860	1979-1989= 3.58 (≈ 4)

	65(1944)	32.460	1993-2003= 4.03 (≈ 4)
	66(1945)	32.810	2004-2014 = 4.53 (≈ 5)
	76(1955)	36.200	
	90(1969)	40.840	
	100(1979)	44.240	
	110(1989)	47.820	
	114(1993)	49.330	
	124(2003)	53.360	
	125(2004)	53.780	
	135(2014)	58.310	
Chicago	3(1882)	2.798	1882-1892= 2.78 (≈ 3)
	13(1892)	5.578	1892-1902= 2.82 (≈ 3)
	23(1902)	8.396	1921-1931= 3.55 (≈ 4)
	42(1921)	14.280	1935-1945= 4.10 (≈ 4)
	52(1931)	17.830	1951-1961= 4.78 (≈ 5)
	56(1935)	19.350	1965-1975= 5.33 (≈ 5)
	66(1945)	23.450	1977-1987= 5.74 (≈ 6)
	72(1951)	26.110	1991-2001= 6.04 (≈ 6)
	82(1961)	30.890	2009-2019= 6.11 (≈ 6)
	86(1965)	32.920	
	96(1975)	38.250	
	98(1977)	39.360	
	108(1987)	45.100	
	112(1991)	47.470	
	122(2001)	53.510	
	130(2009)	58.420	
	140(2019)	64.530	
Portland	1(1880)	1.256	1880-1890= 5.11 (≈ 5)
	11(1890)	6.368	1891-1901= 4.72 (≈ 5)
	12(1891)	6.853	1902-1912= 4.68 (≈ 5)
	22(1901)	11.570	1916-1926= 4.96 (≈ 5)
	23(1902)	12.030	1927-1937= 5.32 (≈ 5)
	33(1912)	16.710	1940-1950= 5.74 (≈ 6)
	37(1916)	18.620	1959-1969= 5.96 (≈ 6)
	47(1926)	23.580	1970-1980= 5.69 (≈ 6)
	48(1927)	24.100	1996-2006= 3.12 (≈ 3)
	58(1937)	29.420	
	61(1940)	31.090	
	71(1950)	36.830	
	80(1959)	42.180	
	90(1969)	48.140	
	91(1970)	48.730	
	101(1980)	54.420	
	117(1996)	62.040	

127(2006)	65.160

7 Concluding remarks

In this study, we have considered the use of non-homogeneous Poisson processes (NHPP) under a Bayesian approach to analyze annual average temperatures and rain precipitations of climate data for some regions of North America reported for a long period. In the data analysis, we assumed a special parametrical form of the NHPP given by the PLP process which is very flexible for the fit of the climate data (temperature or rain precipitation). The unknown parameters of the intensity function or rate $\lambda(t), t \geq 0$ of the PLP-NHPP process in which the Poisson events occur considering data (accumulated number of years in a given time interval $[0, t)$ with the climate index is above a threshold given by the overal average in the assumed period) in presence or not of a change-point were obtained using MCMC (Markov Chain Monte Carlo) methods and the free OpenBugs software.

Assuming "model 1" (PLP-NHPP process) It is not clear what the author want to say.

The obtained Bayesian inferences using the PLP-HHPP processes ("model 1") were satisfatory, especially in the detection of one change-point, but the use of a polynomial structure for the mean value function ("model 2") improved the estimation of the mean value function (or the intensity function).

Assuming the polynomial NHPP (superposition of NHPP processes) or "model 2", we obtained very accurate estimation for the mean value functions and the intensity functions from where we can get complete information on the climate behavior, as for example, the detection of more than one change-point and the numbers of violations in each period of 10 years.

The obtained results are of great interest for understanding the climate changes occurring across the world. It is important to point out that the statistical techniques proposed in this study can be also used for other regions of the world.

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