

Sheet-like structure formation inside the core of Massive Neutron Star

Abstract : We investigate the possible effect of **an** ultra-strong magnetic field on the core matter, in particular quark matter inside a massive rotating neutron star. Based on the discovery “Evidence of quark matter cores in massive Neutron Stars” by Annala et al. our main motivation is to understand the type of structure formation **that** appears in the core of a rotating neutron star, magnetar. Taking into account two facts :

i) free u, d, and s quarks can form a composite (i.e. quark matter composite) because of the seed magnetic field located inside the core of a massive neutron star, magnetar in analogy with the observed in the weapon “Bola” , and composite formation in **a** ferromagnetic liquid crystal ;

ii) observation of sheet-like structure in ferromagnetic composites placed in a magnetic field having rotation,

we propose that **a** sheet-like structure might be appeared in the ferromagnetic quark matter composites inside the cores of massive rotating neutron stars , in particular a magnetar in the presence of its ultra-strong magnetic field which acts as catalyst.

Key Words: quark, quark-matter, neutron star, magnetar, field structured composites.

1. Introduction

“Atoms or molecules are the building units of normal matter manifested in the form of solid, liquid, **or** gas in our wonderful world although unknown **Dark-matter** and dark energy dominate the universe”. [1] Atoms composed of nuclei and electrons are the building blocks of ordinary matter **and is** termed as “electro-magnetic matter” (or simply electric matter) because **their** properties are dominated by the electro-magnetic interaction [1]. On the other hand, an atomic nucleus is simply known as “strong matter” since its nature is controlled by the fundamental strong interaction. This strong interaction is of **short-range** and effective at densities ($\sim 10^{15}$ g/cm³) which should be much higher than that of the electric matter (~ 10 g/cm³).

“In outer-space the terrestrial atomic nuclei are usually almost spherical. But at sub-nuclear densities i.e. as the density increases to $\sim 10^{14}$ g/cm³ of the uniform nuclear matter [2,3] the nuclei deform from spherical to cylinder, slab, cylindrical hole, spherical hole which have similarity to the known shapes of meat-ball, spaghetti, Lasagna, Macaroni and swiss cheese (so-called nuclear pasta)” [4]. “This type of nuclei with the pasta structures are thought to exist actually inside the core of a supernova or the crust of neutron stars and have important impacts on astrophysical phenomena such as supernova explosions, proto-neutron star cooling, pulsar glitches, etc. In addition to this morphology of nuclear pasta new types of pasta form i.e. gyroid and double diamond morphologies have been proposed” [5]. This gyroid and double diamond type nuclear pasta are likely to appear between the cylinder and slab phases. Numerical studies of the whole variety of pasta shapes [6] indicate the slab-like is connected to the rod-like and the gyroid shapes. Of course, most of the pasta phases are liquid crystals [7] . As the nuclear pasta (phase) layer is located between the inner crust and the outer core in a neutron star these special features concerning the shear viscosity can affect the rotation between the crust and the core [8]. A study in [5] estimates “a subtle energy difference between gyroid and double diamond morphologies i.e. there is a good chance of the appearance of gyroid morphology near the transition point from cylinder to slab phase the volume fraction of nuclei at this transition point is ~ 0.35 which is very close to the value found for the polymer system” [9].

In the core of neutron stars, the densities are very larger than 1.5×10^{14} g/cm³ and temperatures are low. The expected density in the cores of massive neutron stars could be even larger $\sim (5 - 10)\rho_s$, where ρ_s being the nuclear saturation density = 2.8×10^{14} g/ cm³. “At very high densities zero temperature and zero pressure conditions might be exist [12,13] and the nuclear matter is believed to experience a phase transition from a neutron liquid to a gas of unconfined quarks (i.e. up (u), down (d)) and gluons [10] along with strange (s)- quarks (produced through weak interaction between electrons and neutrinos)”. In the highly dense cores, it is expected that the neutron-rich matter can give rise transitioning to a quark matter phase [11,14]. “This new form of matter is known as strange quark matter (SQM). As the cold strange quark matter could be stable [15] a phase transition may occur favoring the creation of a quark matter phase over the entire star and the neutron star will become a strange quark star” [11, 16]. “It is to be noted that at densities much higher than the masses of u, d, and s-quarks one can assume these three quarks as massless along with their most favored state, so-called color-flavor-locked (CFL) phase [17] in this asymptotic region. In fact, the densities at neutron star cores are not asymptotically large rather than intermediate densities in more realistic cases (i.e. for neutron star) where other phases (different from CFL) can be realized” [16].

Another important element in the cores of neutron stars is the atmosphere of their magnetic fields. The magnitudes of core magnetic fields are stronger than their surface magnetic fields because of the conservation of magnetic flux due to the high electric conductivity of stellar matter. “This ultimately settles a larger magnetic field associated with the core in the denser

central region of the star. Typically neutron stars have surface magnetic fields $\sim 10^{12}$ G or even larger ($10^{14} - 10^{15}$) G in the case of magnetars [18] where as core magnetic fields ranging from 10^{18} G for nuclear matter [19] to 10^{20} G for quark matter” [20,21]. “However, when the hydrodynamic equilibrium between gravity and matter pressures has been considered the resultant maximum core field value arises to $\sim 10^{17}$ G for stable configuration” [22,23]. Here the significant role of a large core magnetic field is that it can produce interesting structural effects inside the core. For example, i) in the case of density-dependent magnetic field model the magnetic field orientation (transverse orientation) in quark stars affects the mass-radius relations for different stages of proto-quark stars [24], (ii) strong magnetic field enhances the possibility of a mixed-phase at high density of moderately dense quark matter with implication for the structure, energetic and vibration spectrum of neutron stars [25], etc.

In this study, we have proposed the formation of the sheet-like structure of quark matter inside the core of a neutron star as well as magnetar as a result of high-speed rotation and the presence of an ultra-strong magnetic field.

2. Neutron Star and compositions of its core

Neutron stars are compact objects formed as an aftermath of supernova explosions, resulting from the gravitational collapse of massive stars. Its matter is charge neutral and can be considered as cold ($T=0$) and β -equilibrium, in general. The outer part of a neutron star, called atmosphere, consists of partially ionized atoms and electrons where mass densities are below about 10^4 g/cm³. At higher densities (i.e., $> 10^4$ g/cm³) the spatial region consists of inhomogeneous nucleonic matter and electrons are called the crust. It can be divided into two : a) an outer crust with a plasma of nuclei and electrons as degree of freedom ; and b) an inner crust where unbound neutrons exist.

The standard picture of the outer crust is that it is composed of completely ionized nuclei in a sea of electrons of almost constant density (because of incompressibility of the highly degenerate electron fluid) and the nuclei for a body-centered cubic (bcc) lattice. A crystal of ⁵⁶Fe nuclei is expected to form at densities of $\sim 10^7$ g/cm³ and below[26]. Based on recent progress in theoretical studies [26 – 29] it is generally accepted that heavy clusters of matter with exotic shapes, so-called ”pasta phases” could arise in the bottom part of the inner crust [30 – 34]. Numerical studies considering the constant proton fraction and β -equilibrium matter [33,35] in the inner crust suggest that the transition densities between the different geometries and the crust-core transition are affected in a very weak and non-monotonous way but a sizeable effect arises only for very strong magnetic fields, $B = 10^{18}$ G, for which an important decrease of the crust-core transition density was observed.

Core

In 1972 Ruderman [36] first suggested the nuclear matter in the neutron star interior may have anisotropic features at very high densities $\sim 10^{15}$ g/cm³. These features may be — a mixture of

different types of fluid, presence of superfluids or magnetic fluids, existence of a solid core, phase transition, etc. But the composition of the neutron star in the super-dense state i.e. in the core of the neutron star remains uncertain. It is not yet known what exactly is at the center of the neutron star [37]. Our present understanding indicates that the core is divided into three — the outer core, inner core and its center. This core contains superfluid neutron degenerate matter, mostly composed of neutrons (90%) and a small percentage of protons and neutrons (10%) [38]. Many exotic forms of matter are also possible in the core. It could be quarks, and gluons roaming freely [39]. Even such extreme energies of the core could lead to the creation of hyperons (these particles contain three quarks). Note that neutrons contain the most basic and lowest energy quarks i.e. up (‘u’) and down (‘d’) quarks where as hyperon has at least one of those replaced with an exotic strange quark [40] . Another possibility is that the centre of neutron star is a Bose-Einstein condensate [39, 41,42]. However, the nuclear matter in the core is mainly composed of neutrons, protons, electrons, and muons that maintain the system in β -equilibrium [43,44]. For massive neutron star the existence of an exotic inner core is assumed i.e. at high densities extremely neutron-rich uniform matter in the outer core and possibly exotic states of matter such as strange baryons, deconfined quarks may appear in the inner core [45 – 47]. This means that due to extreme gravitational pressure the interior neutrons of neutron star will get deformed and turn into deconfined quarks, hyperons, strange quarks [37]. As contraction continues the extreme pressures and density push the quarks into their asymptotic freedom phase, and strong forces among quarks are zero (almost vanish) [48]. As a result, the core compactness reduces, quarks roam freely and finally reduce in core mass as well as stellar mass [49 – 51]. Other investigations revealed the existence of pion condensation [52,53] ; the solid core at densities 10^{14-15} g/cm³ [54, 55] as well as the presence of a type 3A superfluid [56] which are considered to offer a more realistic view of the structure of the ultra-dense core of compact stellar objects.

“The cores of massive neutron stars contain a large number of quarks (u....u, d....d, s....s) (i.e. multi-quark droplets) resulting which the core must have a mass larger than ordinary nuclei and be stable” [57]. However, the situation is different for droplets of strange quark matter which would contain approximately the same amount of u-, d- and s-quarks. Considering the characteristics of the deconfined phase Eemeli Annala et al [58] showed the presence of quark-matter core in the interior of the maximally massive stable neutron star. According to their result, neutron stars with mass $M > \sim 2M_{\odot}$ and radius around 12 Km are more likely to have a quark core of approximately 6.5 Km.

Another important fact exhibited inside the core is quark clustering. At realistic baryon density (i.e. $\sim 2 - 10 \rho_0$) almost free quarks in dense matter inside the core could be coupled strongly and ultimately grouped into quark clusters [59]. Using Lennard-Jones model of clustered quark matter Lai and Xu [60] estimated “the number of quarks inside a single quark cluster to be $N_q < \sim 10^3$, if the state equation of clustering quark matter stiffens to support compact stars with maximum mass $M_{\max} > 2M_{\odot}$ ”.

3. Nature of Quarks

Quarks are considered as the fundamental building blocks of hadrons as well as the second group of fundamental particles (Leptons are the first group) [61]. Originally, three quark type (or flavors) namely — up ('u'), down ('d'), and strange ('s'). all these quarks have half integral spin and are thus fermions. Quarks only form triplets called baryons (such as protons, neutrons) or doublets so called mesons (such as pions, pi-mesons). It is known at present that quarks exist in six varieties or flavors : u-, d-, s-, charm ('c'), top ('t') and bottom ('b') and each quark has antimatter counterpart known as anti-quark which have opposite charge, baryon number, strangeness, etc. Quarks have electric charge which is a fraction of the standard charge 'e'. The important known facts are :

- i) Out of these six flavors only u- and d- quarks (although they are by far the lightest) appear to play a direct role in the normal matter [62].
- ii) The four forces i.e. strong, electromagnetic, weak and gravitational forces, act between quarks.
- iii) Gluons, i.e. the quantum of strong force, bind quarks or quarks and anti-quarks.
- iv) Due to beta –decay the weak force allows a quark of one type to change into another where as the gravitational force couples quark mass.
- v) Although it is believed that quarks confinement is an unavoidable circumstance but a free quark, which is separated from a nucleon, would be detectable because of its charge (i.e. $-2/3$ or $1/3$ of the charge of an electron).
- vi) The peculiarity of the Omega baryon Ω^- (sss) is that it was found to have a spin of $3/2$ i.e. all three quarks have spin up.
- vii) When quarks are close together the binding forces (carried by gluons) tend to be weak. For example, in the case of protons or other hadrons, the constituted quarks behave as if they were nearly free at a distance of less than 10^{-15} meter.
- viii) Quarks are fermions and obey Pauli's Exclusion Principle.

4. Strong Magnetic Fields of Neutron Stars

“Compact stars such as neutron stars, in particular magnetars show their possession of very strong magnetic fields. The typical values of the surface magnetic field (inferred from simple magnetic dipole models and spin-down rates) are in the range of $10^8 - 10^{13}$ G” [63,64]. “The inferred periods of anomalous x-ray pulsars (AXPs), and soft gamma-ray repeaters (SGRs) suggest that neutron stars have larger surface magnetic field of $10^{14} - 10^{15}$ G [65,66] and even it may be further strong i.e. $\sim 10^{16} - 10^{17}$ G” [67,68]. “Numerical and theoretical studies argued that at the center of inhomogeneous, ultra-dense and gravitationally bound compact stars such as neutron stars, magnetars may have fields $\sim 10^{19} - 10^{20}$ G” [69,70].

It is very difficult to detect the presence of ultra-strong magnetic field inside the neutron stars (compact stars) but theoretical studies, modeling can help us to investigate the approximate

effects of such high magnetic fields on the physical parameters of the compact stellar objects i.e. neutron stars, magnetars. The core of a neutron star (formed from nuclear matter after supernova explosion) consists of neutrons, protons, electrons (arising from nuclear matter) and other particles such as pions, mesons, etc [71]. “As the nuclear matter is meta-stable, so it can convert into strange quark matter after releasing a lot of energy to achieve stability” [72]. “This new form of matter, called strange quark matter, in the cores of neutron stars composed of a large number of deconfined quarks i.e. u-, d- and s-quarks in β -equilibrium with electric and color charge neutrality” [73]. “Currently, it is argued the possible formation of another new class of compact **star comes** from the collapse of neutron star and these are more stable compared to neutron star” [74]. This means that **the** collapse of a neutron star may lead to formation

(6)

of a strange quark star or a hybrid star. In the case of strange quark star it is made from strange quarks matter only extending from its center to surface and a layer of nuclear matter may exist on its surface [75]. While in hybrid star **it's** core **is composed** of strange quark matter [76] and crust of hadronic matter.

5. Primitive Weapon Bola

South American Indians used a special type weapon, called ‘Bolas’ for hunting [77]. “Bola” is a Spanish word meaning ‘balls’, came from the word “boleadoras”. It consists of stone balls, usually in a group of three, attached to long, slender ropes (see fig.1). It is a special type of throwing weapon made of weights on the ends of interconnected cords, used to capture animals by entangling their legs. Depending on the exact design the thrower gives the ball momentum

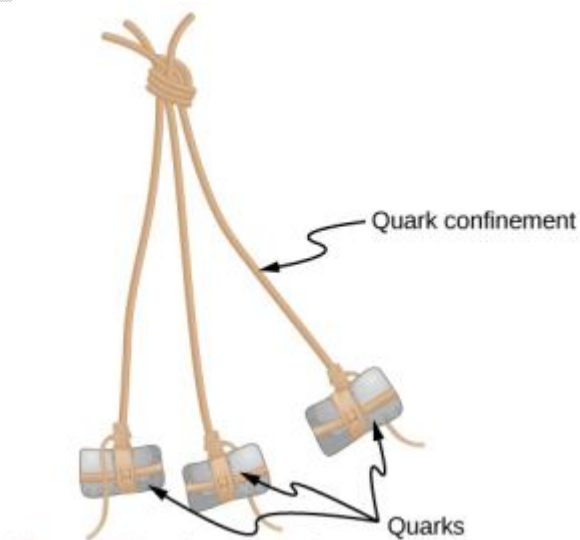


Fig.1 : A baryon is analogous to a bola, a weapon used for hunting. The rocks in this image correspond to the baryon quarks which are free to move about and stray too far from one another but must remain close to the other quarks as it a single entity (adapted from ref [77]). The bola corresponds to a baryon, the stones correspond to quarks, and the string corresponds to the gluons that hold the system together.

By swinging them grasping the boleadora by one of the weights or by nexus of the cords, and then releasing the bola. Usually, three weights bola are designed such that two shorter cords with heavier weights and the longer cord with a lightweight. The heavier weights fly at the front parallel to each other and then hit either side of the legs i.e. by entangling the animal's legs. Practically, once it is set into motion, each ball at the end of the bola can be thought of as a single object in uniform circular motion [78]. However, some bolas even had up to eight weights in which the longer one cord used as to guide them through the air.

6. Effects of Strong Magnetic Fields

i) Observational evidence of sheet-like structure formation in composite

Particle composite means the particles into assemblies. The properties of the composite will depend on the structure of the particle assemblies and thus there is some optimal structure for any given property [79]. The magnetic particle composites can be processed by triaxial magnetic fields or electric fields for optimizing property known as field structured composites. Thus field structured composites are anisotropic magnetic particle composites.

In practical, magnetic fields are an ideal way of creating structures inside composites. To understand the effects of high magnetic fields on the ferro-magnetic particle composites (i.e. particle distribution and arrangement) Williamson and Martin [80] used liquid crystal samples of anisotropic, filed structured composites (prepared by hosting magnetic particles i.e., Ni, Fe ,or Co in a liquid monomer and polymerizing the mixture under rotating magnetic field) and then placed it in a magnetic field. Their observational results during the experiment were :

- a) When the sample is placed in a static or uniform magnetic field chains form along the field lines as a result of induced dipolar forces between the magnetized particles.
- b) When an oscillating field, instead of a static field, is used the observed result is same as before.
- c) In the case of field direction inverted periodically, not net effect on the process.
- d) If a slowly rotating, uniform magnetic field is applied, the resulting particle chains rotate in the fluid initially but with the increase in the frequency of rotation, the linear chains distort, then begin to break up due to hydro-magnetic

drag and finally sheets form in the plane of rotating field i.e.” *chain-like structure transformed into sheet-like structure due to the effect of high-speed rotation of the field*”.

- e) **If a uniform**, uniaxial or rotating bi-axial field is used in processing the field structured composites, in that case also chain or sheet like structure appears.
- f) The uniaxial field strength is initially zero and then it is slowly increased: under this condition one expects that the uniaxial field begins to interrupt the sheet-like structure initially but later continues the formation.

A similar but slightly different result is observed by Martin et al in another experiment [81]. They used magnetic field structured composites (prepared by polymerizing magnetic particles in suspending resin) using uniaxial as well as biaxial (i.e. rotating) magnetic fields. They observed : “ *Chain like particle structures arise in uniaxial field case while sheet like particle structure in the case of biaxial field*”. However, in analyzing the chain formation mechanism Wu et al [82] suggested that the efficiency of particle chain formation is affected by the strength the magnetic field, volume fraction and particle sizes also.

ii) *Numerical simulation study of structure formation of magnetic particles*

Ando et al [83] performed a numerical simulation of ferromagnetic particles which are randomly dispersed. The influence of gravity was ignored. The particle composites are not magnetic field structured. When a magnetic field is applied to the magnetic particles (i.e. Ni) dispersed in the medium their observation result was: i) “the particles make chain-like cluster with connection in a parallel direction to the magnetic field” . ii) Using the Non-Dimensional Boundary Area (NBA) method they inferred — “ the structure formed by the magnetic particles does not depend on the particle diameter but depends on the particle volume concentration. In particular,

- a) In the case of particle volume concentration (ϕ) = 5 vol % no bundle of chain-like cluster is made and each chain-like cluster by magnetic particles is almost single chain.
- b) When particle volume concentration is more than (ϕ) = 10 vol % then — “the bundle structure formed by contacts of multiple chain like clusters” and the process continues.

iii) *Anisotropic Ferromagnetic fluid*

As the density of the core of neutron star is beyond $\sim 10^{15}$ g/cm³ the pressure anisotropy is active and affects the physical properties, stability and structure of the matter inside the core [84,85]. Strong magnetic fields present inside the neutron star act as the source of anisotropy in the system [86]. Not only that Tatsumi [69] found theoretically that the core of a magnetar (i.e. neutron star) may be a quark nuggets i.e. composed of approximately equal number of up,

down and strange quarks and the internal state of this core may exist as a ferromagnetic-liquid with a surface magnetic fields $B_{\text{surface}} = 10^{12+1}$ T [87].

iv) *Possible Sheet-like structure formation inside the neutron star core*

At supra-nuclear densities inside the neutron star **core**, it is expected that the matter contains nucleons, electrons and other particles like muons, pions, kaons and their condensates, hyperons, strange quark matter [88]. i.e., large number of quarks (i.e. so called multi-quark droplets). **This stable** strange quark matter may exist even without gravity [74, 89]. As the density inside the neutron star's core is high enough, the degrees of freedom i.e. the freely three flavors of quark — u, d, and s-quarks, might appear, showing ferromagnetic phase transition, as ferromagnetic liquid. The typical magnetic field strength on the surface of rotating neutron stars (i.e. pulsars) could be of the order of 10^{12} G and for magnetars it could be $\sim 10^{13} - 10^{15}$ G as inferred from the observation of AXPs [90]. In the high dense cores of massive neutron stars and in the interior of magnetars the magnetic field strength can reach a values of about $10^{16} - 10^{18}$ G [73, 91, 92] and even large values of about $10^{18} - 10^{20}$ G [93,94]. Dvornikov [95] predicts amplification of the seed magnetic field 10^{12} G in the core of a neutron star (in particular a hybrid star or a quark star), driven by the electroweak interaction of quarks, generates such strong magnetic fields in dense quark matter. **These** strong magnetic fields can affect the shape, mass and radius of neutron stars and magnetars (i.e. compact objects). This means the geometry of the interior magnetic field is at least as important as the field strength itself on the structure formation inside the core. Study indicates a tangled, isotropic magnetic field has a relatively smaller impact on mass, radius of the core of a neutron star [96] . As the core contains ferromagnetic liquid, the strange quark matter i.e. magnetized ferromagnetic quark particles form clusters in the presence of strong magnetic field after magnetically attracted into aggregating.

Neutron star's dipole magnetic field axis is not aligned with its rotational axis and its rotation / spin period is only a fraction of a second. Thus, high frequency of rotation and ultra-strong magnetic field both might be active on its core and affect the core materials into structure formation. In the presence of **an** ultra-strong magnetic field large number of quark clusters thus turn into chains that ultimately deformed and finally re-shape into sheet-like structure due to high frequency of rotation of the core system. **These** newly formed sheets extend inside the whole core i.e. from center to outer core of the neutron star. In Table 1 we compare the structure formation in composites and neutron star's core materials.

7. Conclusion

Comparing the experiment on composites with the neutron star case it can be stated as:

The core of a neutron star means the ferromagnetic liquid , multi-quark droplets are present as if a composite form available in strange quark matter. The seed magnetic field of strength $\sim 10^{12}$ G is located inside the core and amplification of this seed magnetic field generates strong and ultra-strong magnetic fields which are associated with the neutron stars , magnetars at their surfaces and inside the stars, respectively. This means initially the ferromagnetic quark particles turn into a composite cluster form (i.e. field structure type) due to the effect of seed magnetic field and later **these** quark composite clusters are situated under the amplified

Table 1: Comparison between Experimental / observational results using composites and the requirements /availability in the core material of neutron star

Particulars	Composites	NS core material	Requirements or availability in the Core of NS, Magnetar
Used Material	Anisotropic Ferromagnetic Particles (Ni, Fe or Co)	Anisotropic ferromagnetic Quark particles (u-, d- and s- quarks	Satisfied
Type	Liquid Crystal Composites	Fermi Liquid Strange Quark Matter	Satisfied
Processing for Field Structured composite	Uniaxial or biaxial or Triaxial Magnetic Fields	Uniaxial seed Magnetic Field	Satisfied
Chain Formation	Processed i.e. field structured Composites are placed under Magnetic Field	Quark Clustered are under Amplified super-strong or Ultra-strong magnetic field	Satisfied. Inference—Chains might be formed in quark cluster.
Sheet-like Structure formation	Magnetic Field is rotated at desired frequency	Whole Core system i.e. core Material and magnetic field rotates with the same frequency of stellar rotation	Satisfied. Inference—Sheet-like structure might be appeared in core strange quark matter.

magnetic fields. This has a similarity with the fact that field structured liquid crystal composites are placed in a magnetic field during the experiment and as a **result**, particle chains appear in composites. In the case of neutron star quark particle chains might appear due to the effect of strong / ultra-strong magnetic fields produced after the amplification of the seed magnetic field in the interior of the star. In the final phase of the experiment magnetic field is rotated at frequency 50 – 100 Hz so that the field structured composites deform and turn into sheet-like structure. In the case **of a rotating** neutron star (and magnetar also) the whole interior system rotates with the same frequency of stellar rotation. As an effect of the star's rotation on its internal core material the already appeared chains are deformed and ultimately re-shaped into sheet-like structure. This means that sheet-like structure formation appears in the interior strange quark matter of the core of the rotating massive neutron star (in the case of magnetar also). **Finally**, we conclude that **the** appearance of sheet-like structure might be possible inside the cores of rotating neutron stars, **and** magnetars.

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