

Sheet-like structure formation inside the core of Massive Neutron Star

Abstract : We investigate the possible effect of ultra-strong magnetic field on the core matter, in particular quark matter inside a massive rotating neutron star. Based on the discovery “Evidence of quark matter cores in massive Neutron Stars” by Annala et al. our main motivation is to understand the type of structure formation appears in the core of a rotating neutron star, magnetar. Taking into account two facts :

i) free u, d, and s quarks can form a composite (i.e. quark matter composite) because of the seed magnetic field located inside the core of a massive neutron star, magnetar in analogy with the observed in the weapon “Bola” , and composite formation in ferromagnetic liquid crystal ;

ii) observation of sheet-like structure in ferromagnetic composites placed in a magnetic field having rotation,

we propose that sheet-like structure might be appeared in the ferromagnetic quark matter composites inside the cores of a massive rotating neutron stars , in particular a magnetar in the presence of its ultra-strong magnetic field which acts as catalyst.

Key Words: quark, quark-matter, neutron star, magnetar, field structured composites.

1. Introduction

Atoms or molecules are the building units of normal matter manifested in the form of solid, liquid or gas in our wonderful world although unknown dark-matter and dark energy dominate the universe. Atoms composed of nuclei and electrons are the building blocks of ordinary matter is termed as “electro-magnetic matter” (or simply electric matter) because of its properties are actually dominated by the electro-magnetic interaction [1]. On the other hand, an atomic nucleus is simply known as “strong matter” since its nature is controlled by the fundamental strong interaction. This strong interaction is of short range and effective at densities ($\sim 10^{15}$ g/cm³) which should be much higher than that of the electric matter (~ 10 g/cm³).

In outer-space the terrestrial atomic nuclei are usually almost spherical. But at sub-nuclear densities i.e. as the density increases to $\sim 10^{14}$ g/cm³ of the uniform nuclear matter [2,3] the nuclei deform from spherical to cylinder, slab, cylindrical hole, spherical hole which have similarity to the known shapes of meat-ball, spaghetti, Lasagna, Macaroni and swiss cheese (so called nuclear pasta) [4]. This type of nuclei with the pasta structures are thought to exist actually inside the core of a supernova or the crust of neutron stars and have important impacts on astrophysical phenomena such as supernova explosions, proto-neutron star cooling, pulsar glitches, etc. In addition to this morphology of nuclear pasta new type of pasta form i.e. gyroid and double diamond morphologies have been proposed [5]. This gyroid and double diamond type nuclear pasta are likely to appear between the cylinder and slab phases. Numerical studies of the whole variety of pasta shapes [6] indicate the slab like is connected the rod like and the gyroid shapes. Of course, most of the pasta phases are liquid crystals [7] . As the nuclear pasta (phase) layer is located between the inner crust and the outer core in a neutron star these special features concerning the shear viscosity can affect the rotation between the crust and the core [8]. Study in [5] estimates a subtle energy difference between gyroid and double diamond morphologies i.e. there is a good chance of the appearance of gyroid morphology near the transition point from cylinder to slab phase the volume fraction of nuclei at this transition point is ~ 0.35 which is very close to the value found for the polymer system [9].

In the core of neutron stars the densities are very larger than 1.5×10^{14} g/cm³ and temperatures are low. The expected density in the cores of massive neutron stars could be even larger $\sim (5 - 10)\rho_s$, where ρ_s being the nuclear saturation density = 2.8×10^{14} g/ cm³. At very high densities zero temperature and zero pressure condition might be exist [12,13] and nuclear matter is believed to experience a phase transition from a neutron liquid to a gas of unconfined quarks (i.e. up (u), down (d)) and gluons [10] along with strange (s)- quarks (produced through weak interaction between electrons and neutrinos). In the highly dense cores it is expected that the

(2)

neutron rich matter can give rise transitioning to a quark matter phase [11,14]. This new form of matter is known as strange quark matter (SQM). As the cold strange quark matter could be absolutely stable [15] a phase transition may occur favouring the creation of a quark matter phase over the entire star and the neutron star will become strange quark star [11, 16]. It is to be noted that at densities much higher than the masses of u, d, and s-quarks one can assume these three quarks as mass less along with their most favored state, so called color-flavor-locked (CFL) phase [17] in this asymptotic region. In fact, the densities at neutron star cores are not asymptotically large rather than intermediate densities at more realistic case (i.e. for neutron star) where other phases (different from CFL) can be realized [16].

Another important element in the cores of neutron stars is the atmosphere of their magnetic fields. The magnitudes of core magnetic fields are stronger than their surface magnetic fields because of conservation of magnetic flux due to high electric conductivity of stellar matter. This ultimately settles a larger magnetic field associated with the core in the denser central region of the star. Typically neutron stars have surface magnetic fields $\sim 10^{12}$ G or even larger ($10^{14} - 10^{15}$) G in the case of magnetars [18] where as core magnetic fields ranging from 10^{18} G for nuclear matter [19] to 10^{20} G for quark matter [20,21]. However, when hydrodynamic equilibrium between gravity and matter pressures has been considered the resultant maximum core field value arises to $\sim 10^{17}$ G for stable configuration [22,23]. Here the significant role of large core magnetic field is that it can produce interesting structural effects inside the core. For example, i) in the case of density-dependent magnetic field model the magnetic field orientation (transverse orientation) in quark stars affects the mass-radius relations for different stages of proto-quark stars [24], (ii) strong magnetic field enhances the possibility of a mixed-phase at high density of moderately dense quark matter with implication for the structure, energetic and vibration spectrum of neutron stars [25], etc.

In this study we have proposed the formation of sheet-like structure of quark matter inside the core of a neutron star as well as magnetar as a result of high speed rotation and presence of ultra-strong magnetic field.

2. Neutron Star and compositions of its core

Neutron stars are compact objects formed as an aftermath of supernova explosions, resulting from gravitational collapse of massive stars. Its matter is charge neutral and can be considered as cold ($T=0$) and β -equilibrium, in general. The outer part of a neutron star, called atmosphere, consists of partially ionized atoms and electrons where mass densities below about 10^4 g/cm³. At higher densities (i.e. $> 10^4$ g/cm³) the spatial region consists of inhomogeneous

(3)

Nucleonic matter and electrons is called crust. It can be divided into two : a) an outer crust with a plasma of nuclei and electrons as degree of freedom ; and b) an inner crust where unbound neutrons exist.

The standard picture of the outer crust is that it is composed of completely ionized nuclei in a sea of electrons of almost constant density (because of incompressibility of the highly degenerate electron fluid) and the nuclei for a body centered cubic (bcc) lattice. A crystal of ⁵⁶Fe nuclei is expected to form at densities of $\sim 10^7$ g/cm³ and below [26]. Based on recent progress on theoretical studies [26 – 29] it is generally accepted that heavy cluster of matter with exotic shapes, so called "pasta phases" could arise in the bottom part of the inner crust [30 – 34]. Numerical studies considering the constant proton fraction and β -equilibrium matter

[33,35] in the inner crust suggest that the transition densities between the different geometries and the crust-core transition are affected in a very weak and non-monotonous way but a sizeable effect arises only for very strong magnetic fields, $B = 10^{18}$ G, for which an important decrease of the crust-core transition density was observed.

Core

In 1972 Ruderman [36] first suggested the nuclear matter in the neutron star interior may have anisotropic features at very high densities $\sim 10^{15}$ g/cm³. This anisotropic sources may be of — a mixture of different types fluid, presence of superfluids or magnetic fluids, existence of a solid core, phase transition etc. But the composition of the neutron star in the super-dense state i.e. in the core of the neutron star remains uncertain. It is not yet known what exactly is at the center of the neutron star [37]. Our present understanding indicates that the core is divided into three — the outer core, inner core and its center. This core contains superfluid neutron degenerate matter, mostly composed of neutrons (90%) and a small percentage of protons and neutrons (10%) [38]. Many exotic forms of matter are also possible in the core. It could be quarks, and gluons roaming freely [39]. Even such extreme energies of the core could lead to the creation of hyperons (these particles contain three quarks). Note that neutrons contain the most basic and lowest energy quarks i.e. up (‘u’) and down (‘d’) quarks where as hyperon has at least one of those replaced with an exotic strange quark [40] . Another possibility is that the centre of neutron star is a Bose-Einstein condensate [39, 41,42]. However, the nuclear matter in the core is mainly composed of neutrons, protons, electrons and muons that maintain the system in β -equilibrium [43,44]. For massive neutron star the existence of an exotic inner core is assumed i.e. at high densities extremely neutron rich uniform matter in the outer core and possibly exotic states of matter such as strange baryons, deconfined quarks may appear in the inner core [45 – 47]. This means that due to extreme gravitational pressure the interior neutrons of neutron star will get deformed and turn into deconfined quarks, hyperons, strange

(4)

quarks [37]. As contraction continues the extreme pressures and density push the quarks into their asymptotic freedom phase, strong forces among quarks are zero (almost vanish) [48]. As a result the core compactness reduces, quarks roam freely and finally reduce in core mass as well as stellar mass [49 – 51]. Other investigations revealed the existence of pion condensation [52,53] ; of solid core at densities $10^{14 - 15}$ g/cm³ [54, 55] as well as the presence of a type 3A superfluid [56] which are considered to offer a more realistic view of the structure of the ultra-dense core of compact stellar objects.

The cores of massive neutron stars contain large number of quarks (u....u, d....d, s....s) (i.e. multi-quark droplets) resulting which the core must have a mass larger than ordinary nuclei and be stable [57]. However, the situation is different for droplets of strange quark matter which

would contain approximately the same amount of u-, d- and s-quarks. Considering the characteristics of the deconfined phase Eemeli Annala et al [58] showed the presence of quark-matter core in the interior of maximally massive stable neutron star. According to their result, neutron stars with mass $M > \sim 2M_{\odot}$ and radius around 12 Km are more likely to have a quark core of approximately 6.5 Km.

Another important fact exhibited inside the core is quark clustering. At realistic baryon density (i.e. $\sim 2 - 10 \rho_0$) almost free quarks in dense matter inside the core could be coupled strongly and ultimately grouped into quark cluster [59]. Using Lennard-Jones model of clustered quark matter Lai and Xu [60] estimated the number of quarks inside a single quark cluster to be $N_q < \sim 10^3$, if the state equation of clustering quark matter stiffens to support compact stars with maximum mass $M_{\max} > 2M_{\odot}$.

3. Nature of Quarks

Quarks are considered as the fundamental building blocks of hadrons as well as the second group of fundamental particles (Leptons are the first group) [61]. Originally, three quark type (or flavors) namely — up ('u'), down ('d') and strange ('s'). all these quarks have half integral spin and are thus fermions. Quarks only form triplets called baryons (such as protons, neutrons) or doublets so called mesons (such as kaons, pi-mesons). It is known at present that quarks exist in six varieties or flavors: u-, d-, s-, charm ('c'), top ('t') and bottom ('b') and each quark has antimatter counterpart known as anti-quark which have opposite charge, baryon number, strangeness, etc. Quarks have electric charge which is a fraction of the standard charge 'e'. The important known facts are:

- i) Out of these six flavors only u- and d- quarks (although they are by far the lightest) appear to play a direct role in normal matter [62].

(5)

- ii) The four forces i.e. strong, electromagnetic, weak and gravitational forces, act between quarks.
- iii) Gluons, i.e. the quantum of strong force, bind quarks or quarks and anti-quarks.
- iv) Due to beta -decay the weak force allows a quark of one type to change into another where as the gravitational force couples quark mass.
- v) Although it is believed that quarks confinement is an unavoidable circumstance but a free quark, which is separated from a nucleon, would be detectable because of its charge (i.e. $-2/3$ or $1/3$ of the charge of an electron).
- vi) The peculiarity of the Omega baryon Ω^- (sss) is that it was found to have a spin of $3/2$ i.e. all three quarks to have spin up.
- vii) When quarks are close together the binding forces (carried by gluons) tend to be weak. For example, in the case of protons or other hadrons, the constituted quarks behave as if they were nearly free at distance of less than 10^{-15} meter.

viii) Quarks are fermions and obey Pauli's Exclusion Principle.

4. Strong Magnetic Fields of Neutron Stars

Compact stars such as neutron stars, in particular magnetars show their possession of very strong magnetic fields. The typical values of the surface magnetic field (inferred from simple magnetic dipole models and spin down rates) are in the range of $10^8 - 10^{13}$ G [63,64]. The inferred periods of anomalous x-ray pulsars (AXPs), soft gamma-ray repeaters (SGRs) suggest that neutron stars have larger surface magnetic field of $10^{14} - 10^{15}$ G [65,66] and even it may be further strong i.e. $\sim 10^{16} - 10^{17}$ G [67,68]. Numerical and theoretical studies argued that at the center of inhomogeneous, ultra-dense and gravitationally bound compact stars such as neutron stars, magnetars may have fields $\sim 10^{19} - 10^{20}$ G [69,70].

It is very difficult to detect the presence of ultra-strong magnetic field inside the neutron stars (compact stars) but theoretical studies, modeling can help us to investigate the approximate effects of such high magnetic fields on the physical parameters of the compact stellar objects i.e. neutron stars, magnetars. The core of a neutron star (form from nuclear matter after supernova explosion) consists of neutrons, protons, electrons (arising from nuclear matter) and other particles such as pions, mesons etc [71]. As the nuclear matter is meta-stable, so it can convert into strange quark matter after releasing a lot of energy to achieve stability [72]. This new form of matter, called strange quark matter, in the cores of neutron stars composed of a large number of deconfined quarks i.e. u-, d- and s-quarks in β -equilibrium with electric and color charge neutrality [73]. Currently, it is argued the possible formation of another new class of compact star which comes from the collapse of neutron star and these are more stable compared to neutron star [74]. This means that collapse of a neutron star may lead to formation

(6)

of a strange quark star or a hybrid star. In the case of strange quark star it is made from strange quarks matter only extending from its center to surface and a layer of nuclear matter may exist on its surface [75]. While in hybrid star its core composed of strange quark matter [76] and crust of hadronic matter.

5. Primitive Weapon Bola

South American Indians used a special type weapon, called 'Bolas' for hunting [77]. "Bola" is a Spanish word meaning 'balls', came from the word "boleadoras". It consists of stone balls, usually in a group of three, attached to long, slenderropes (see fig.1). It is a special type of throwing weapon made of weights on the ends of interconnected cords, used to capture animals by entangling their legs. Depending on the exact design the thrower gives the ball momentum

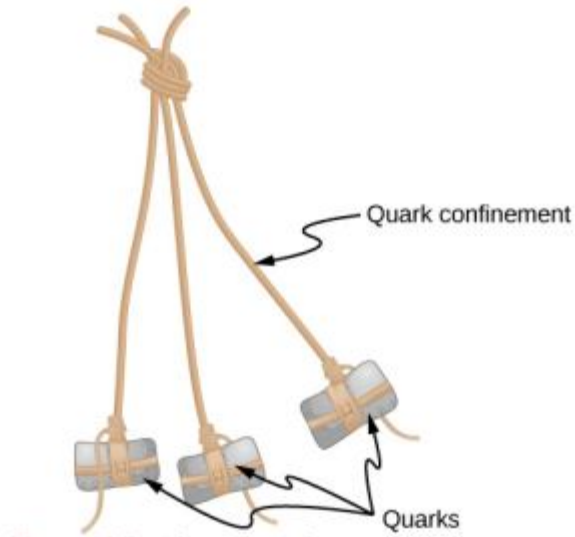


Fig.1 : A baryon is analogous to a bola, a weapon used for hunting. The rocks in this image correspond to the baryon quarks which are free to move about and stray too far from one another but must remain close to the other quarks as it a single entity (adopted from ref [77]). The bola corresponds to a baryon, the stones correspond to quarks, and the string corresponds to the gluons that hold the system together.

By swing them grasping the boleadora by one of the weights or by nexus of the cords, and then releases the bola. Usually, three weights bola are designed such that two shorter cords with heavier weights and the longer cord with a light weight. The heavier weights fly at the front parallel to each other and then hit either side of the legs i.e. by entangling the animal's legs. Practically, once it is set into motion, each ball at the end of the bola can be thought of as

(7)

single object in uniform circular motion [78]. However, some bolas even had up to eight weights in which the longer one cord used as to guide it through the air.

6. Effects of Strong Magnetic fields

i) Observational evidence of sheet-like structure formation in composite

Particle composite means the particles into assemblies. The properties of the composite will depend on the structure of the particle assemblies and thus there is some optimal structure for any given property [79]. The magnetic particle composites can be processed by triaxial magnetic fields or electric fields for optimizing property are known as field structured composites. Thus field structured composites are anisotropic magnetic particle composites.

In practical, magnetic fields are an ideal way of creating structures inside composites. To understand the effects of high magnetic fields on the ferro-magnetic particle composites (i.e. particle distribution and arrangement) Williamson and Martin [80] used liquid crystal sample of anisotropic, filled structured composites (prepared by hosting magnetic particles i.e. Ni, Fe or Co in a liquid monomer and polymerizing the mixture under rotating magnetic field) and then placed it in a magnetic field. Their observational results during the experiment were :

- a) When the sample is placed in a static or uniform magnetic field chains form along the field lines as a result of induced dipolar forces between the magnetized particles.
- b) When an oscillating field, instead of static field, is used the observed result is same as before.
- c) In the case of field direction inverted periodically, not net effect on the process.
- d) If a slowly rotating, uniform magnetic field is applied, the resulting particle chains rotate in the fluid initially but with the increase in the frequency of rotation, the linear chains distort, then begin to break up due hydro-magnetic drag and finally sheets form in the plane of rotating field i.e.” *chain-like structure transformed into sheet-like structure due to the effect of high speed rotation of the field*”.
- e) If uniform, uniaxial or rotating bi-axial field is used in processing the field structured composites, in that case also chain or sheet like structure appears.
- f) The uniaxial field strength is initially zero and then it is slowly increased: under this condition one expects that the uniaxial field begins to interrupt the sheet-like structure initially but later continues the formation.

(8)

A similar but slightly different result is observed by Martin et al in another experiment [81]. They used magnetic field structured composites (prepared by polymerizing magnetic particles in suspending resin) using uniaxial as well as biaxial (i.e. rotating) magnetic fields. They observed : “ *Chain like particle structures arise in uniaxial field case while sheet like particle structure in the case of biaxial field*”. However, in analyzing the chain formation mechanism Wu et al [82] suggested that the efficiency of particle chain formation is affected by the strength the magnetic field, volume fraction and particle sizes also.

ii) *Numerical simulation study of structure formation of magnetic particles*

Ando et al [83] performed numerical simulation of ferromagnetic particles which are randomly dispersed. The influence of gravity was ignored. The particle composites are not magnetic field structured. When a magnetic field is applied to the magnetic particles (i.e. Ni) dispersed in the

medium their observation result was: i) “the particles make chain-like cluster with connection in a parallel direction to the magnetic field” . ii) Using the Non-Dimensional Boundary Area (NBA) method they inferred — “ the structure formed by the magnetic particles does not depend on the particle diameter but depends on the particle volume concentration. In particular,

- a) In the case of particle volume concentration (ϕ) = 5 vol % no bundle of chain-like cluster is made and each chain-like cluster by magnetic particles is almost single chain.
- b) When particle volume concentration is more than (ϕ) = 10 vol % then — “the bundle structure formed by contacts of multiple chain like clusters” and the process continues.

iii) *Anisotropic Ferromagnetic fluid*

As the density of the core of neutron star is beyond $\sim 10^{15}$ g/cm³ the pressure anisotropy is active and affects the physical properties, stability and structure of the matter inside the core [84,85]. Strong magnetic fields present inside the neutron star act as the source of anisotropy in the system [86]. Not only that Tatsumi [69] found theoretically that the core of a magnetar (i.e. neutron star) may be a quark nuggets i.e. composed of approximately equal number of up, down and strange quarks and the internal state of this core may exist as a ferromagnetic-liquid with a surface magnetic fields $B_{\text{surface}} = 10^{12+1}$ T [87].

iv) *Possible Sheet-like structure formation inside the neutron star core*

At supra-nuclear densities inside the neutron star core it is expected that the matter contains nucleons, electrons and other particles like muons, pions, kaons and their condensates, hyperons, strange quark matter [88]. i.e. large number of quarks (i.e. so called multi-quark

(9)

droplets). This absolutely stable strange quark matter may exist even without gravity [74, 89]. As the density inside the neutron star’s core is high enough, the degrees of freedom i.e. the freely three flavors of quark — u, d, and s-quarks, might appear, showing ferromagnetic phase transition, as ferromagnetic liquid []. The typical magnetic field strength on the surface of rotating neutron stars (i.e. pulsars) could be of the order of 10^{12} G and for magnetars it could be $\sim 10^{13} - 10^{15}$ G as inferred from the observation of AXPs [90]. In the high dense cores of massive neutron stars and in the interior of magnetars the magnetic field strength can reach a values of about $10^{16} - 10^{18}$ G [73, 91, 92] and even large values of about $10^{18} - 10^{20}$ G [93,94]. Dvornikov [95] predicts amplification of the seed magnetic field 10^{12} G in the core of a neutron star (in particular a hybrid star or a quark star) , driven by the electroweak interaction of quarks, generates such strong magnetic fields in dense quark matter. This strong magnetic fields can affect the shape, mass and radius of neutron stars and magnetars (i.e. compact objects). This means the geometry of the interior magnetic field is at least as important as the

field strength itself on the structure formation inside the core. Study [96] indicates a tangled, isotropic magnetic field has a relatively smaller impact on mass, radius of the core of a neutron star . As the core contains ferromagnetic liquid, the strange quark matter i.e. magnetized ferromagnetic quark particles form clusters in the presence of strong magnetic field after magnetically attracted into aggregating.

Neutron star’s dipole magnetic field axis is not aligned with its rotational axis and its rotation / spin period is only a fraction of a second. Thus, high frequency of rotation and ultra-strong magnetic field both might be active on its core and affect the core materials into structure formation. In the presence of ultra-strong magnetic field large number of quark clusters thus turn into chains that ultimately deformed and finally re-shape into sheet-like structure due to high frequency of rotation of the core system. This newly formed sheets extend inside the whole core i.e. from center to outer core of the neutron star. In Table 1 we compare the structure formation in composites and neutron star’s core materials.

7. Conclusion

Comparing the experiment on composites with the neutron star case it can be stated as:

The core of a neutron star means the ferromagnetic liquid , multi-quark droplets are present as if a composite form available in strange quark matter. The seed magnetic field of strength $\sim 10^{12}$ G is located inside the core and amplification of this seed magnetic field generates strong and ultra-strong magnetic fields which are associated with the neutron stars , magnetars at their surfaces and inside the stars, respectively. This means initially the ferromagnetic quark particles turn into a composite cluster form (i.e. field structure type) due to the effect of seed magnetic field and later this quark composite clusters are situated under the amplified

(10)

Table 1: Comparison between Experimental / observational results using composites and the requirements /availability in the core material of neutron star

Particulars	Composites	NS core material	Requirements or availability in the Core of NS, Magnetar
Used Material	Anisotropic Ferromagnetic Particles (Ni, Fe or Co)	Anisotropic ferromagnetic Quark particles (u-, d- and s- quarks)	Satisfied
Type	Liquid Crystal Composites	Fermi Liquid Strange Quark Matter	Satisfied
Processing for Field Structured composite	Uniaxial or biaxial or Triaxial Magnetic Fields	Uniaxial seed Magnetic Field	Satisfied
Chain Formation	Processed i.e. field structured Composites are placed under Magnetic Field	Quark Clustered are under Amplified super-strong or Ultra-strong magnetic field	Satisfied. Inference—Chains might be formed in

			quark cluster.
Sheet-like Structure formation	Magnetic Field is rotated at desired frequency	Whole Core system i.e. core Material and magnetic field rotates with the same frequency of stellar rotation	Satisfied. Inference—Sheet-like structure might be appeared in core strange quark matter.

magnetic fields. This has a similarity with the fact that field structured liquid crystal composites are placed in a magnetic field during the experiment and as a result particle chains appear in composites. In the case of neutron star quark particle chains might appear due to the effect of strong / ultra-strong magnetic fields produced after the amplification of the seed magnetic field in the interior of the star. In the final phase of the experiment magnetic field is rotated at frequency 50 – 100 Hz so that the field structured composites deform and turn into sheet-like structure. In the case of rotating neutron star (and magnetar also) the whole interior system rotates with the same frequency of stellar rotation. As an effect of the star's rotation on its internal core material the already appeared chains are deformed and ultimately re-shaped into sheet-like structure. This means that sheet-like structure formation appears in the interior strange quark matter of the core of the rotating massive neutron star (in the case of magnetar also). Finally we conclude that appearance of sheet-like structure might be possible inside the cores of rotating neutron stars, magnetars.

Data availability statement : No data has been used in this paper.

Ethical Interest : None

References:

1. Lai X, Xia C, Xu R. Bulk strong matter: the Trinity. *Adv. in Phys.*2022; **8**: 2137433
2. Schuetrumpf B, Klatt M A, Iida K., Maruhn J A, Mecke K, Reinhard PG. Time dependent Hartee Fock approach to Nuclear 'Pasta' at finite temperature. *Phys. Rev.C* 2013 ; 87: 055805
3. Bethe H A. Supernova Mechanism. *Rev. Mod. Phys.* 1990 ; 62 : 801
4. Nakazato K, Oyamatsu K, Yamada S. Pasta phase with Gyroid Morphology at subnuclear densities. *AIP Conf. Proc.* 2010 ;1238 : 218
5. Nakazato K, Oyamatsu K, Yamada S. Gyroid Phases in Nuclear Pasta. *Phys. Rev. Lett.* 2009 ; 103 : 132501
6. Schuetrumpf B, Klatt MA, Iida K, Schroder-Turk GE,Maruhn JA, Mecke K, Reinhard PG. Minimal surfaces in Nuclear Pasta with the time-dependent Hartee Fock approach. *Proc. Int. Winter Meeting on Nuclear Physics* (2014)
7. Pethick CJ, Potekhin A. Liquid Crystals in the mantle of neutron stars. *Phys. Lett. B* 1998 ; 427 : 7

8. Pons JA, Viganò D, Rea N. A highly resistive layer within the crust of X-ray pulsars limits their spin periods. *Nature Physics*. 2013 ; 9 : 431
9. Laurer JH, Hajduk DA, Fung JC, Sedat JW, Smith SW, Gruner SM, Agard DA, Spontak RJ. Micro-structure analysis of a Cubic Bi-continuous Morphology via near SIS Triblock Copolymer. *Macromolecules*. 1997; 30: 3938
10. Lopez-Fune E. Magnetized strange quark matter under stellar equilibrium and finite temperature. Arxiv: **1902.02717** [HE] (2019)
11. Weber F. Strange Quark Matter and Compact Stars. *Prog. Parti. Nucl. Phys.* 2005 ; 54, 193
12. Ivanenko DD, Kurdgelaidze DF. *Lett. Nuovo Cim.* 1969 ; 2 : 13
13. Itoh, N. Hydrostatic Equilibrium of Hypothetical Quark Stars. *Prog. Theor. Phys.* 1970 ; 44 : 291
14. J.M. Lattimer. *New Astron. Rev.* 2010 ; 54 : 101
15. Witten E. Cosmic separation of phases. *Phys. Rev. D.* 1984 ; 30 : 272
16. Ferrer EJ, de la Incera V, Sanson P. Quark matter contribution to the heat capacity of magnetized neutron stars. *Phys. Rev. D* 2021 ; 103 : 123013
17. Alford M, Rajagopal K, Wilczek F. QCD at finite baryon density: nucleon droplets and color superconductivity. *Phys. Lett. B* 1998 ; 422 : 247
18. Olausen SA, Kaspi VM. The MCGILL MAGNETAR Catalog. *Astrophys. J. Suppl. Ser.* 2014 ; 212 : 6
19. Dong L, Shapiro SL. Cold Equation of State in a Strong Magnetic Field: Effects of Inverse beta –Decay. *Astrophys. J.* 1991 ; 383 : 745
20. Ferrer EJ, de la Incera V, Keith JP, Portillo I, Springsteen PL. Equation of state of a dense and magnetized Fermion systems. *Phys. Rev. C* 2010 ; 82: 065802
21. Paulucci L, Ferrer EJ, de la Incera V, Horvath JE. Equation of state for the magnetic-color-flavor-locked phase and its implications for compact star models. *Phys. Rev.D* 2011 ; 83 : 043009
22. Cardall CY, Prakash M, Lattimer JM. Effects of Strong Magnetic Fields on Neutron Star Structure. *Astrophys. J.* 2001 ; 554 : 322
23. Ferrer EJ, Hackebill A. Thermodynamics of neutrons in a magnetic field and its implications for neutron stars. *Phys. Rev. C* 2019 ; 99 : 065803
24. Chu PC, Li X-H, Ma H-Y, Wang B, Dong Y-M, Zhang X-M. Quark matter and quark stars in strong magnetic fields at finite temperature within the confined-isospin-density-dependent mass model. *Phys. Lett. B* 2018 ; 778 : 447
25. Mondal T, Jaikumar P. Effect of strong magnetic field on competing order parameters in Two-flavor Dense Quark matter. *Adv. High. Energy Phys.* 2017 ; 2017: 6272909 (2017)
26. Ortel M, Hampel T, Klahn, S. Type I, Equation of States for Supernovae and Compact stars. *Rev. Mod. Phys.* 2017 ; 89 : 015007

27. Oyamatsu K, Iida K. Symmetry energy at subnuclear densities and nuclei in neutron star crusts. *Phys. Rev. C* 2007 ; 75 : 015801
28. Kubis S, Porębskab J, Alvarez-Castillo DE. Low Density Symmetry Energy Effects and the Neutron Star Crust Properties . *Acta Phys. Polonica B* 2010 ; 41: 2449
29. Roca-Maza X, Brenna M., Agrawal BK, Bortignon P, Colo G, Cao LG, Paar N, Vretenar D. Giant quadrupole resonances in ^{208}Pb , the nuclear symmetry energy, and the neutron skin thickness. *Phys. Rev. C* 2013 ; 87, 034301
- 30 Pethick CJ, Potekhin AY. Liquid Crystals in the Mantles of Neutron Stars. *Phys. Lett. B* 1998 ; 427 : 7
- 31 Ravenhall DG, Pethick CJ, Wilson JR, Structure of Matter below Nuclear Saturation Density. *Phys. Rev. Lett.* 1983 ; 50 : 2066
- 32 Avancini SS, Chiacchiera S, Menezes DP, Providencia C. Warm pasta phase in the Thomas-Fermi approximation. *Phys. Rev. C* 2010 ; 82 : 055807
- 33 Bao SS, Hu JN, Zhang ZW, Shen H. Effects of the symmetry energy on properties of neutron star crusts near the neutron drip density. *Phys. Rev. C* 2014 ; 90 : 045802
- 34 Scurto L, Pais H, Gulminelli F. Strong Magnetic fields and Pasta phases revisited, *Phys. Rev. D* . eprint arXiv:**2212.09355** (2023)
- 35 Nandi R, Bandyopadhyay D, Mushustin I, Greiner W. Inner crusts of neutron stars in strongly quantizing magnetic fields. *Astrophys. J.* 2011 ; 736 : 156
- 36 Ruderman R. Pulsars: Structure and Dynamics. *Ann. Rev. Astron. Astrophys.* 1972 ; 10 : 427
- 37 Baym G, Hatsuda T, Kojo T, Powel PD, Song Y, Takatsuka T. From hadrons to quarks in neutron stars: a review *Rep. Prog. Phys.* 2018 ; 81 : 056902
- 38 Khanna KM, Kandie DK, Tonui JK, Cherop HK. Incommensurate Crystallization of Neutron Matter in Neutron Stars. *East Europe J. Phys.* 2020 ; 2 : 57
39. Mann A. The golden age of neutron-star physics has arrived. *Nature* 2020 ; 579:20
40. Grassi F. Quark core stars, quark stars and strange stars. *Zeitsch. Fur Phys. C* 1989 ; 44 : 129
41. Stein AW. Frontiers the Physics of Dense Matter for Neutron Stars. *J. Phys. Conf. Series* 2016 ; 706 : 022001
42. Pethick CJ, Schäfer T, Schwenk A. in Proc. “Universal Themes of B-E condensation”. eds: D.W. Snoke, W.P. Littlewood (Cambridge Univ. Press. UK) 2017 p. 573
43. Helström S. Neutron Star structure and Equation of State.
Web: theory.uchicago.edu/teaching >final paper >helstrom
44. Anntic S, Stone JR, Thomas AW. Neutron stars from crust to core within the Quark-meson coupling model. in Proc.HIAS 2019, EPJ web of Conf. 2020 ; 232: 03001
45. Hebeler K, Lattimer JM, Pethick CJ, Schwenk A. Equation of state and neutron star properties constrained by nuclear physics and observation. *Astrophys. J.* 2013 ; 713 : 11

46. Weber F. Strangeness in Neutron Stars. *Acta Phys. Polonica B* 1999 ; 30: 3149
47. Lattimer JM. Neutron star masses and radii. in Xiamen CUSTIPEN workshop on the EoS of dense neutron rich matter in the Era of Gravitational Wave Astronomy, AIP Con. Proc. 2019 ; 2127 : 020001
48. Gross D, Wilczek F. Ultraviolet Behavior of Non-Abelian Gauge Theories. *Phys. Rev. Lett.* 1973 ; 30 : 1343; Asymptotically Free Gauge Theories-I. *Phys. Rev. D* 1973 ; 9 : 980
49. Wilczek F. Asymptotic Freedom: From Paradox to Paradigm. Lecture given in acceptance of the Nobel Prize, Dec'2004, PNAS. 2005 ; 102 : 8403
50. Huwang JK. Asymptotic freedom, quark confinement, proton crisis, neutron structure, dark matter and relative force strength.
Doi: 10.20944/preprints202102.0395vi
51. Shuryak E. Physics of Strongly coupled Quark-Gluon Plasma. *Prog. Parti. Nucl. Phys.* 2009 ; 62 :, 48
52. Sawyer RF. Condensed π^- Phase in Neutron-Star Matter. *Phys. Rev. Lett.* 1972 ; 29 : 382
53. Dev K, Gleiser M. Anisotropic Stars: Exact Solutions. *Gen. Rel. Grav.* 2000 ; 34 :1793
54. Cameron A G W, Canuto V. Neutron stars: general review. in Proc. 16th Solvay Conf. on Astrophysics and Gravitation, Neutron Stars (Brussels: Editions de l'Universite de Bruxelles, 1973) p. 221
55. Canuto V. 8th Texas Symp. on Relativistic Astrophys. 1977 ; 302, 514
56. Kippenhahn R, Weigert A. *Stellar Structure and Evolution* (Springer Verlag, NY, 1990) p. 192
57. Greiner C. Physics of Strange Star. In Proc. IV Int. Conf. on Strangeness in Quark Matter, *J. Phys. G* 1999 ; 25 : 389
58. Annala E, Gorda T, Kurkela A, Nättilä S, Vuorinen A. Evidence for Quark Matter Cores in massive Neutron Star. *Nature Phys.* 2020 ; 16 : 907
59. Dai S, Xu R. Thermal and non-thermal Radiation from Pulsars: Hints of Physics. In ASP Conf. Series : *Electromagnetic Radiation from Pulsars and Magnetars.* 2012 ; 466 : 129
60. Lai, X Y, Xu RX . A note on the discovery of a 2Mo Pulsar. *Phys. Rev. C* 2021 ; 104 : 045805
61. Ling S J, Sanny J, Maebis W, University Physics. Vol-3, Chapter 11.3 (Openstax, . Texas, USA, 2018)
62. Poudel PR. Quarks and their discovery. www.nepjol.info/index.RNP
63. Taylor JH, Manchester RN, Lyne AG. Catalog of 558 Pulsars. *Astrophys. J. Suppl. Series* 1993 ; 88 : 529
64. Mclaughlin M, Stairs IH, Kaspi VM. et al. *Astrophys. J. Lett.* 2003 ; 591 : L185
65. Duncan RC, Thompson C. Formation of Very Strongly Magnetized Neutron Stars *Astrophys. J. Lett.* 1992 ; 392, L9
66. Thompson C, Duncan RC. The Soft Gamma Repeaters as Very Strongly Magnetized Neutron Stars. II. Quiescent Neutrino, X-Ray, and Alfvén Wave Emission. *Astrophys. J.* 1996 ; 473, 322
67. Senco SB, Froul DA, Harrison FA et al. *Astrophys. J.* 2010 ; 711 : 641

68. Starling R LC, Starling, E. Rol, A.J. Vander Horst et al. MNRAS 2001 ; 400 : 90
69. Tatsumi T. Ferromagnetism of quark liquid. Phys. Lett. B 200 ; 489 : 280
70. Ferrer EJ, de la Incera V, Kaith JP et al. Equation of state of a dense and magnetized fermion system. Phys. Rev. C **82**, 065802 (2010)
71. Lattimer JM, Prakash M. The Physics of Neutron Stars. Science 2004 ; 304: 536
72. Bordbar G H, Bahri H, Kayanikhoo F. Calculation of the structural properties of a strange quarks star in the presence of a strong magnetic field using a density dependent bag constant. Res. Astron. Astrophys. 2012 ; 12: 1280
73. Hou J-X, PengG-X, Xia C-J, Xu J-F. Magnetized strange quark matter in a mass Density dependent model. Chinese Phys. C 2015 ; 39 : 015101
74. Farhi E, Jaffe R L. Strange matter. Phys. Rev. D 1984 ; 30 : 2379
75. Glendenning N K, Weber F. From Strange Star to Strange Dwarf Astrophys. J. 1993 ; 400 : 647
76. Bhattacharyya A, Ghosh S K, Joarder P S, Mallick R, Raha S. Conversion of a neutron star to a strange star: A two-step process. Phys. Rev. C 2006 ; 74 : 065804
77. <http://www.britannica.com/technology/bola>
78. B.R. Pressley, Primitive, Survival and Wildness Living Skills. <http://wayoftheraven.net/2018/09/19/bola/BolasTheUnsung>
79. Martin JE. Using triaxial magnetic fields to create optimal particle composites. Composites Part A. 2005 ; 36 : 545
80. Williamson R L, Martin J E. Field structured Composite studies. SANDIA Report **No-SAND2004-1291** (2004)
81. Martin J E, Venturini E, Odinek J, Anderson R A. Anisotropic Magnetism in field structured composites. Phys. Rev. E 2000 ; 61 : 2818
82. Wu H, Xu Z, Wang J, Bo X, Tong Z, Jiang S, Zhang G. Chain formation mechanism of magnetic particles in magnetorheological elastomers during prestructure. J. Magnetism. Magnetic Materials. 2021 ; 527 : 167693
83. Ando T, Katayama D, Hirota N, Koike O, Tatsumi R, Yamato M. Structure formation of Magnetic particles under magnetic fields towards anisotropic material. in Proc. 9th Int. Symp. on Electromagnetic Processing of Materials (EPM2018), IOP Conf. Ser. 2018 ; 424 : 012076
84. Dev K, Gleiser M. Anisotropic Stars: Exact Solutions. Gen. Rel. Grav. 2002 ; 34: 1793
85. Banerjee S. Mathematical Model of Relativistic anisotropic compact stellar model admitting linear equation of state. Comm. Theor. Phys. 2018 ; 70 : 585
86. Paret DM, Perez Martinez A, Ayala A, Piccinelli G, Sanchez A. Neutron Star velocities and Magnetic Fields. EPJ Web Conf. (ISMD 2017) 2018 ; 172: 07002
87. Pace van Devender J, Shoemaker I M, Sloan T, van Devender A P, Ulmen B A. Mass distribution of Magnetized Quark Nugget Dark Matter and comparison with requirement and observations. Nature Scientific Reports. 2020 ; 10 : 17903

88. Ortel M, Hampel M, Klähn T. Equation of State for Supernovae and Compact stars. *Rev. Mod. Phys.* 2017 ; 89 : 015007
89. Bhattacharyya A, Mishustin I M, Greiner W. Deconfined phase transition in compact stars : Maxwell vs Gibbs Construction of the mixed phase. *J. Phys. G* 2010 ; 37: 025201
90. Rizaldy R, Sulakson A . Deformation of quark stars under internal strong magnetic fields. *J. Phys. Conf. series.* 2019 ; 1354 : 012006
91. Sinha M, Huang X-G, Sedrakian A. Strange Quark matter in strong magnetic fields Within a confining model. *Phys. Rev. D* 2013 ; 88 : 025008
92. Kharzeev D E, McLerran LD, Warringa H J. The effects of topological charge change in heavy ion collisions. *Nucl. Phys. A* 2008 ; 803 : 227
93. Yang L, Wen X-J. Magnetic effect vs Thermal effect on quark matter with a running coupling at finite densities. *Comm. Theor. Phys.* 2017 ; 67 : 535
94. Kayanikhoo F, Providencia C. The Effect of Magnetic Field on the structure of Strange Quark Star. in *Proceedings of RAGtime 20–22, 15–19 Oct., 16–20 Sept., 19–23 Oct., 2018/2019/2020*, eds. G. Torok and V. Karas, (Silesian University in Opava, 2020) pp. 121
95. Dvornikov M. Generation of strong Magnetic Fields in dense quark matter driven by the Elelctroweak interaction of quarks. *Nucl. Phys. B* 2016 ; 913 : 79
96. Wei W, Liu X-M, Zheng X-P. Quark Stars with strong Magnetic Fields: Considering different magnetic field generation. *Res. Astron. Astrophys.* 2017 ; 17 : 102