A Numerical Approximation On Black-Scholes Equation of Option Pricing

Abstract

This paper considered the notion of European option which is geared towards solving analytical and numerical solutions. In particular, we examined the Black-Scholes closed form solution and modified Black-Scholes (MBS) partial differential equation using Crank-Nicolson finite difference method. These partial differential equations were approximated to obtain Call and Put option prices. The explicit price of both options is found accordingly. The numerical solutions were compared to the closed form prices of Black-Scholes formula. Finally, the graphical solutions and comparisons of other parameters were discussed for the purpose of investment plans.

Keywords: Stock prices , Crank-Nicolson, Option pricing, , MBS PDE , Call and Put Options.

1.Introduction

An option is a tool whose worth is derived from the principal asset which is otherwise known as financial derivative. This type of derivative does have anything in common with mathematical meaning of derivative. In other words, an option on underlying asset is a business between parties who come together to agree on either buying or selling an underlying asset at a determined strike price in the future for a fixed price. The cost of the option lies on the underlying asset, which is usually a stock, commodity, currency or an index. The holder has the right but cannot be compelled to buy, for call option where European put option involves the ability to sell an asset for a certain charge at a prescribed date in the future.

Options are known as "*in- the money*", "*at- the money*", or "*out- of the money*". If S is a stock price and K is the strike price, a call option is in- the money as soon as S >K, at- the money when S=K, and out -of –the- money when S<K. A put option is in the money as soon as S<K, at- the -money once S=K, and out- of- the money once S>K. Obviously, an option is exercised only when it is in- the money. In the nonappearance of transactions costs, an in-the money option is always exercised on the expiration date if it has not been exercised earlier,[1].

However, the relevance of options valuation was first demonstrated by Black-Scholes [2] when option faced difficulties in valuation of option at expiration. They used no-arbitrage argument to explain a partial differential equation which governs the growth of the option price with esteem to the expiration and cost of the fundamental Asset. The Black-Scholes equation has been used widely in many financial applications. The following authors has dealt extensively on this area of study such as [3]- [7]etc.

Option values are obtained by solving partial differential equations with initial and boundary conditions. The finite difference method is one of the famous mathematical tools used in solving partial differential equations. The methods were first explored by Brennam and Schwartz (1978) in valuing financial derivatives. FDM is made up of the following schemes: Explicit method, implicit method and Crank-Nicolson method. All these schemes are used to solve BS PDE's and they are relatively closed to each other but differ in stability and accuracy. In the sequel, we consider CN scheme because it allows us to acquire the option value at different times, including time zero in a single iteration and a combination of Explicit and Implicit methods; it has the best numerical approximations.

In this paper, we shall be interested in Crank-Nicolson(CN) finite difference method for valuation of European call and put options which have gained the interest of researchers for finding approximate solutions to PDEs and this interest is driven by demand of applications of societal problems. Furthermore, this paper describes the analysis of the following: closed form solution of BS formula for call and put options, CN numerical approximation of both option, truncation errors due to the increase of stock volatility and other comparisons such as "in-the-money, at-the-money and out-of-the-money". The above concept motivates the study such that it may serve as a guide to investors, option traders, decision makers, Government and mathematicians alike.

This paper is arranged as follows: Section 2 presents mathematical formulation, Subsection 2.2.1 formulation of the scheme, results and discussion are seen in Section 3, while the paper is concluded in Section 4.

2 Mathematical formulations

Let $S(t)$ be the price of some risky asset at time t ,and μ ,an expected rate of returns on the stock and dt as a relative change during the trading days such that the stock follows a random walk which is govern by a stochastic differential equation. [9].

$$dS(t) = \alpha S(t)dt + \sigma S(t)dW_t \tag{1}$$

Where, α is drift and σ the volatility of the stock, W_t is a Brownian motion or Wiener's process on a probability space $(\Omega, \xi, \wp)$, ξ is a $\sigma -$ algebra generated by $W_t, t \geq 0$.

On the general note our interest in this paper is the stochastic boundary value problem which is governed with the dynamics of option pricing; hence we have the following:

$$\frac{\partial V}{\partial t} + \frac{1}{2}S^2\sigma^2\frac{\partial^2 V}{\partial S^2} + [\theta - v_t]\mathrm{S}\frac{\partial V}{\partial S} - rV = 0.\ t > 0. \tag{2}$$

Where r represents interest rate, θ represents mean reversion level, k represents mean reversion speed, σ represents union of all volatilities v_t represents volatility of the underlying assets and t represents time of maturity. The details of the above option model can be expressly found in the following books: [10] ,[11] ,[14] and [15]etc.

However, Black-Scholes model is based on seven assumptions:
The asset price follows a Brownian motion with μ and σ as constants, there are no transaction costs or taxes, All securities are perfectly divisible, there is no dividend during the life of the derivatives, there are no riskless arbitrage opportunities, the security trading is continuous.

The analytic formula for the prices of European call option is given as:

$$
\left.\begin{array}{c}
C = SN\left(d_1\right) - Ke^{-rt}N\left(d_2\right) \\
\dfrac{ln\left(\dfrac{S}{K}\right) + \left(\dfrac{r+\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\
d_2 = d_1 - \sigma\sqrt{T}
\end{array}\right\} \quad (3)
$$

where C is Price of a put option, S is price of underlying asset, K is the strike price, r is the riskless rate, T is time to maturity, σ^2 is variance of underlying asset, σ is standard deviation of the(generally referred to as volatility) underlying asset, and N is the cumulative normal distribution.

Similarly , the analytic formula for the prices of European put option is given as:

$$
\left.\begin{array}{c}
P = SN\left(d_1\right) - Ke^{-rt}N\left(d_2\right) \\
\dfrac{ln\left(\dfrac{S}{K}\right) + \left(\dfrac{r+\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\
d_2 = d_1 - \sigma\sqrt{T}
\end{array}\right\} \quad (4)
$$

where P is Price of a put option, any other parameter has the same meaning with that of call option.

2.1 European Options

In the work of [2] they obtained a mathematical structure for finding the reasonable price of European options by the use of no-arbitrage principle to describe a PDE which governs the growth of the option price that evolves time to expiration. The details of this options can be found in the following books: [1] ,[10] and [11] etc.

2.1.1 European Call Option

The BS PDE for European Call and Put Options with value $C(\mathrm{S},t)$ and $P(S,t)$ is given in the following equations:

$$
\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \left[\theta - v_t\right] S \frac{\partial C}{\partial S} - rC = 0 \quad (5)
$$

$$
\frac{\partial P}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 P}{\partial S^2} + \left[\theta - v_t\right] S \frac{\partial P}{\partial S} - rP = 0 \quad (6)
$$

With the following initial and boundary conditions:

$$\left.\begin{array}{c} C(0,t)=0 \\ C(S,t)=0 \;\; when \;\; S\to\infty \\ C(S,T)=\max(S-k,0) \end{array}\right\} \quad (7)$$

$$\left.\begin{array}{c} P(0,t)=Ke^{-rt} \\ P(S,t)=0 \;\; when \;\; S\to\infty \\ P(S,T)=\max(K-S,0) \end{array}\right\} \quad (8)$$

2.2 The Numerical Scheme and Analysis

The Crank-Nicolson finite difference method is to conquer the stability short-comings by applying the stability and convergence restrictions of the explicit finite difference methods. It is essentially an average of the implicit and explicit methods. However, to implement Crank-Nicolson approximation scheme on Black-Scholes partial differential equation, there must be a price time mesh in order to enhance efficiency as solution exits, the vertical axis in the mesh denotes the stock prices, while the horizontal axis denotes time. Therefore, every grid point in the mesh denotes a horizontal index i and a vertical index j such that every point in the mesh is the option price for a distinct time and a distinct stock price. At every time in the mesh $j\Delta s$ is equivalent to the stock price, and $i\Delta t$ is equivalent to the time. There exist boundary conditions which help in the numerical calculations; by means of the pay-off function. The maturity period, $t = T$ and the option are well computed for all the different initial stock prices using a boundary conditions for uniqueness of solution. To get the prices at $t = 0$, the model solves backwards for every time step from $t = T$, [12].

2.2.1 Formulation of the Scheme

One of the normal ways of approximating the solution of partial differential equations is applying Crank-Nicolson finite difference method which we shall use our proposed model to transform into the scheme. Hence , we have the price time mesh below.

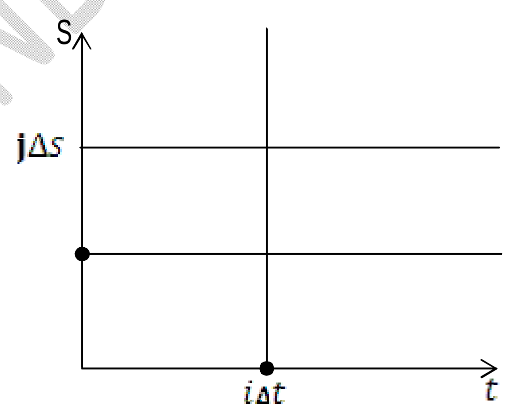

Figure 1: An illustration of Price time mesh.

Recall that the Black-Scholes partial differential equation (2). Let a function $V(\mathrm{S},t)$ in two dimensional grid points, that is to say i and j stands for the index for stock price, S and time, t respectively. The function $V(S,t)=V_i^j$ can be stated as follows in the subsequent difference scheme [1] and [15] etc.

$$Z_i^{\,j} = \frac{1}{2}\sigma^2 S^2 DSS + [\theta - v_t] S_i DS - rV_i^{\,j} \tag{9}$$

where

$S = i\Delta s$, for $0 \le i \le m$, $t = j\Delta t$ for $0 \le j \le i$

$$DSS = \frac{V^{\,j}_{\,i+1} - 2V^{\,j}_{\,i} + V^{\,j}_{\,i-1}}{\Delta^2} \tag{10}$$

$$DS = \frac{V^{\,j}_{\,i+1} - V^{\,j}_{\,i-1}}{2\Delta S} \tag{11}$$

Taking forward difference and backward difference approximations respectively yields implicit and explicit schemes given below.

If we use a forward difference approximation to the time partial derivative we obtain explicit scheme

$$\frac{V_i^{j+1} - V_i^{j}}{\Delta t} + Z_i^{j} = 0 \tag{12}$$

and similarly we obtain the implicit scheme

$$\frac{V_i^{j+1} - V_i^{j}}{\Delta t} + Z_i^{j+1} = 0 \tag{13}$$

The averages of equations (12) and (13) yields Crank-Nicolson method of approximation

$$\frac{V_i^{j+1} - V_i^{j}}{\Delta t} + \frac{1}{2}\left(Z_i^{j} + Z_j^{j+1}\right) = 0 \tag{14}$$

From equation (14)

$$V_i^{j} - \frac{\Delta t}{2} Z_i^{j} = V_i^{j+1} + \frac{\Delta t}{2} Z_i^{j+1} \tag{15}$$

$$\frac{\Delta t}{2} Z_i^{j} = V_i^{j+1} - V_i^{j} + \frac{\Delta t}{2} Z_i^{j+1}$$

$$\therefore Z_i^j = \frac{2}{\Delta t}\left(u_i^{j+1} - u_i^j\right) - Z_i^{j+1} \qquad (16)$$

Substituting (8) in (16) gives in view of (15) and (16) we obtain after collecting like term in V_{i-1} ,

$$\frac{\sigma^2 S^2}{2}\left[\frac{V_{i+1}^j - 2V_i^j + V_{i-1}^j}{(\Delta S)^2}\right] + (\theta - v_t)S_i\left[\frac{V_{i+1}^j - V_{i-1}^j}{2\Delta S}\right] - rV_i^j = \frac{2}{\Delta t}\left(V_i^{j+1} - V_i^j\right) - Z_i^{j+1}$$

That is

$$\frac{\sigma^2 S_i^2}{2(\Delta S)^2}\left[V_{i+1}^j - 2V_i^j + V_{i-1}^j\right] + \frac{(\theta - v_t)S_i}{2\Delta S}\left[V_{i+1}^j - V_{i-1}^j\right] - rV_i^j = \frac{2}{\Delta t}\left(V_i^{j+1} - V_i^j\right) + Z_i^{j+1}$$

Collecting like terms in of V_{i-1}, V_i *and* V_{i+1} and simplifying gives

$$Z_i^j = V_{i-1}^j\left[\frac{\Delta t\sigma^2 S_i^2}{2(\Delta S)^2} - \frac{(\theta - v_t)\Delta tS_i}{2\Delta S}\right] + V_i^j\left[\frac{2\Delta t}{\Delta t} - \frac{\Delta t\sigma^2 S_i^2}{(\Delta S)^2} - r\Delta t\right] + V_{i+1}^j\left[\frac{\Delta t\sigma^2 S_i^2}{2(\Delta S)^2} + \frac{(\theta - v_t)\Delta tS_i}{2\Delta S}\right]$$

and

$$Z_i^{j+1} = V_{i-1}^{j+1}\left[\frac{\Delta t\sigma^2 S_i^2}{2(\Delta S)^2} - \frac{(\theta - v_t)\Delta tS_i}{2\Delta S}\right] + V_i^{j+1}\left[\frac{2\Delta t}{\Delta t} - \frac{\Delta t\sigma^2 S_i^2}{(\Delta S)^2} - r\Delta t\right] + V_{i+1}^{j+1}\left[\frac{\Delta t\sigma^2 S_i^2}{2(\Delta S)^2} + \frac{(\theta - v_t)\Delta tS_i}{2\Delta S}\right]$$

Using (8) in (14) solving simultaneously and taking the average of these two equations we obtain

$$\left.\begin{aligned} &V_{i-1}^j\left[\frac{\Delta t\sigma^2 S_i^2}{4(\Delta S)^2} - \frac{(\theta - v_t)S_i\Delta tS_i}{4\Delta S}\right] + V_i^j\left[1 - \left(\frac{\Delta t\sigma^2 S_i^2}{2(\Delta S)^2} - \frac{r\Delta t}{2}\right)\right] + V_{i+1}^j\left[\frac{\Delta t\sigma^2 S_i^2}{4(\Delta S)^2} + \frac{(\theta - v_t)S_i\Delta tS_i}{4\Delta S}\right] \\ &= V_{i-1}^{j+1}\left[\frac{\Delta t\sigma^2 S_i^2}{4(\Delta S)^2} - \frac{(\theta - v_t)S_i\Delta tS_i}{4\Delta S}\right] + V_i^{j+1}\left[1 - \frac{\Delta t\sigma^2 S_i^2}{2(\Delta S)^2} + \frac{r\Delta t}{2}\right] + V_{i+1}^{j+1}\left[\frac{\Delta t\sigma^2 S_i^2}{4(\Delta S)^2} + \frac{(\theta - v_t)S_i\Delta tS_i}{4\Delta S}\right] \end{aligned}\right\} \qquad (17)$$

The expressions inside the square brackets will be replaced with the coefficients a, b, c. The following equations obtained.

$$aV_{i-1}^{j}+bV_{i}^{j}+cV_{i+1}^{j}=aV_{i-1}^{j+1}+bV_{i}^{j+1}+cV_{i+1}^{j+1} \qquad (18)$$

Where

$a_i = \frac{\Delta t}{4}\left((\sigma^2 S_i^2)-(\theta - v_t)\right)S_i$ $b_i = -\frac{\Delta t}{2}\left(\sigma^2 S_i^2 - r\right)$ and $c_i = -\frac{\Delta t}{2}\left((\sigma^2 S_i^2)-(\theta - v_t)\right)S_i$

a_i, b_i, c_i are random variables; $i = 0,1,...,M$.

Equation (18) can now be represented in matrix form as follows

$$XV^{j} = YV^{j+1},\ j = 0,1,2,...$$

$$\Rightarrow u^{j} = X^{-1}YV^{j+1}$$

where $V^{j} = (V_{1,i,}V_{2,i,}V_{3,i},...,V_{m,i})^{T}$, ($V^{i}$ *is an* $m\times n$)

$$\begin{bmatrix} b & c & 0 & \cdots & 0 \\ -a & b & c & \cdots & 0 \\ 0 & -a & b & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & c_{M-2} \\ 0 & 0 & \cdots & -a_{M-1} & b_{M-1} \end{bmatrix} \begin{bmatrix} V_1^{j} \\ V_2^{j} \\ V_3^{j} \\ \vdots \\ V_{M-1}^{j} \end{bmatrix} = \begin{bmatrix} X_1^{j} \\ X_2^{j} \\ X_3^{j} \\ \vdots \\ X_{M-1}^{j} \end{bmatrix}$$

$$\begin{bmatrix} b & c & 0 & \cdots & 0 \\ a & b & c & \cdots & 0 \\ 0 & a & b & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & c_{M-2} \\ 0 & 0 & \cdots & a_{M-1} & b_{M-1} \end{bmatrix} \begin{bmatrix} V_1^{j+1} \\ V_2^{j+1} \\ V_3^{j+1} \\ \vdots \\ V_{M-1}^{j+1} \end{bmatrix} = \begin{bmatrix} Y_1^{j+1} \\ Y_2^{j+1} \\ Y_3^{j+1} \\ \vdots \\ Y_{M-1}^{j+1} \end{bmatrix}$$

3 Results and Discussions

Here we present simulation results obtained using equations (3) , (4) and (17) via matlab codes for Black-Scholes exact values and Crank-Nicolson numerical solutions. This is for Call and Put options.

Table 1: Comparison the between Black-Scholes exact values and Crank-Nicolson numerical approximations for Call Option when the initial stock prices are 40 and 50 with K = 25, r = 0.2 and T = 1.

Sigma	**S_o = 40, K = 25, $\theta = 0.1, v_t = 0.035$ r=0.2**			**S_o = 50, K = 25, r=0.2, $\theta = 0.1, v_t = 0.035$**		
	BS Exact values	**CN**	**Relative Error**	**BS Exact Values**	**CN**	**Relative Error**
0.25	19.5398	19.5378	1.0236E-04	29.5321	29.4815	1.7134E-03
0.3	19.5695	19.5564	6.6941E-04	29.5357	29.3841	5.1328E-03
0.35	19.6371	19.5926	2.266E-04	29.5506	29.2378	0.0106
0.4	19.7508	19.6468	5.2656E-03	29.5877	29.0650	0.0177
0.45	19.9117	19.7185	9.7028E-03	29.6565	28.8896	0.0259
0.5	20.1167	19.8070	0.0154	29.7625	28.7302	0.0347
0.55	20.3607	19.9121	0.0220	29.9075	28.5990	0.0438
0.6	20.6383	20.0334	0.0293	30.0906	28.5022	0.0528
0.65	20.9441	20.1704	0.0369	30.3094	28.4420	0.0616
0.7	21.2733	20.3221	0.0447	30.5604	28.4178	0.0701
0.75	21.6219	20.4873	0.0525	30.8401	28.4269	0.0782
0.8	21.9861	20.6641	0.06013	31.1446	28.4655	0.0860

The Tables presented in this Section are basically describing process and procedures' of convergence rates as well as the behaviors of financial market variables or quantities. However an increase in the volatility increases Black-Scholes close form prices. This is quite realistic because volatility causes significant changes in the price history of stock market over time; in the initial stock price of 40 for a call option.

Secondly, increasing the volatility and initial stock price of 50 increases Black-Scholes close form prices; while reducing the Crank-Nicolson numerical approximate prices. By implication, it means that an investor stands to minimize profit to large extent because someone who wants to buy will not be able to buy when prices are high. While reduction of option prices in this situation will only favor an investor who wants to buy for the purpose of call option. See Table 1.

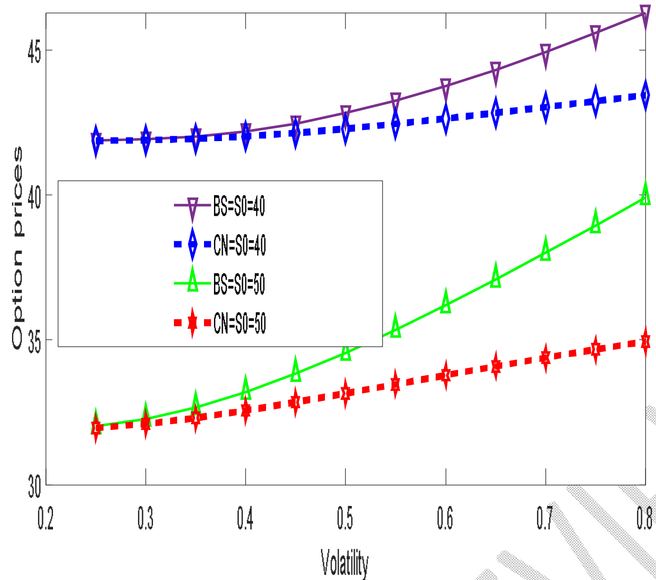

Figure 2: Comparison between Black-Scholes exact values and Crank-Nicolson Numerical approximations when initial stock price is 40 and 50 for call option.

In Figure 2, the plots describes an upward tends over the trading activities. It implies the investments started at the same level of profit making before they begin to grow in their separate margins of financial increase.

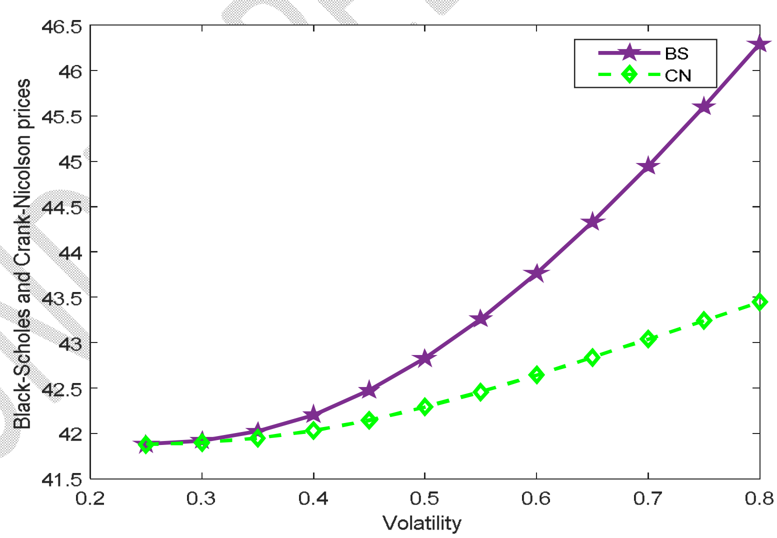

Figure 3: The error differences between Black-Scholes exact values and Crank-Nicolson Numerical approximations when initial stock prices is 40 and 50 for Call option.

Figure 3, shows clear error differences in BS and CN. which shows an increase in the initial stock prices; increase level of errors for call option All plots tend towards semi linear growth function with its low convergence rates. The plots also read that the human errors in the investments plans were not much and cannot affect the future trading activities

Table 2: Comparing the performance of the Black-Scholes exact values and Crank-Nicolson finite difference method for European Put Option when initial stock prices are 40 and 50 with K = 100, r = 0.2 and T = 1

Sigma Sigma	$S_o = 40,\ K = 100$ $\theta = 0.1,\ v_t = 0.035$			$S_o = 50, K = 100,\ \theta = 0.1,\ v_t = 0.035$		
	BS Exact values	CN	Relative Error	BS Exact Values	CN	Relative Error
0.25	41.8817	41.8778	9.3119E-05	32.0183	31.9605	1.8052E-03
0.3	41.9211	41.8977	5.5819E-04	32.2717	32.1011	5.2864E-03
0.35	42.0213	41.9458	1.7967E-03	32.6694	32.3076	0.01107
0.4	42.2025	42.0283	4.1277E-03	33.1966	32.5633	0.01908
0.45	42.4721	42.1446	7.7109E-03	33.8322	32.8510	0.029002
0.5	42.8277	42.2898	0.01256	34.5553	33.1562	0.04049
0.55	43.2622	42.4578	0.01859	35.3479	33.4685	0.05317
0.6	43.7659	42.6422	0.02568	36.1951	33.7805	0.06671
0.65	44.3292	42.8374	0.03365	37.0850	34.0870	0.08084
0.7	44.9429	43.0389	0.04236	38.0079	34.3849	0.09532
0.75	45.5988	43.2432	0.05166	38.9559	34.6722	0.10996
0.8	46.2893	43.4474	0.06139	39.9226	34.9476	0.1246

In Table 2 , a little increase in the volatility of stock also increases the close form prices of BS and CN through an initial stock prices of 40 and 50 respectively. This remark is reasonable in the aspect of an investor whose primary aim is to maximize profit; because the investor is only obliged to sell.

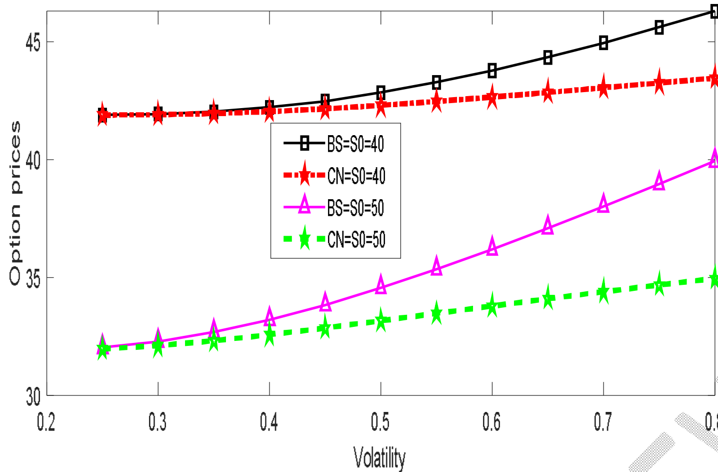

Figure 4: Comparison between Black-Scholes exact values and Crank-Nicolson Numerical approximations when initial stock prices is 40 and 50 for Put option.

Figure 4 describes the nature of option sales between BS and CN. In Black-Scholes, the stock sales in the investment business grew more rapidly and well profiting than the CN. The CN sales grew with the shortest optimum level. That is to say that original BS model is better than the modified Black-Scholes partial differential equations via Crank-Nicolson numerical approximations.

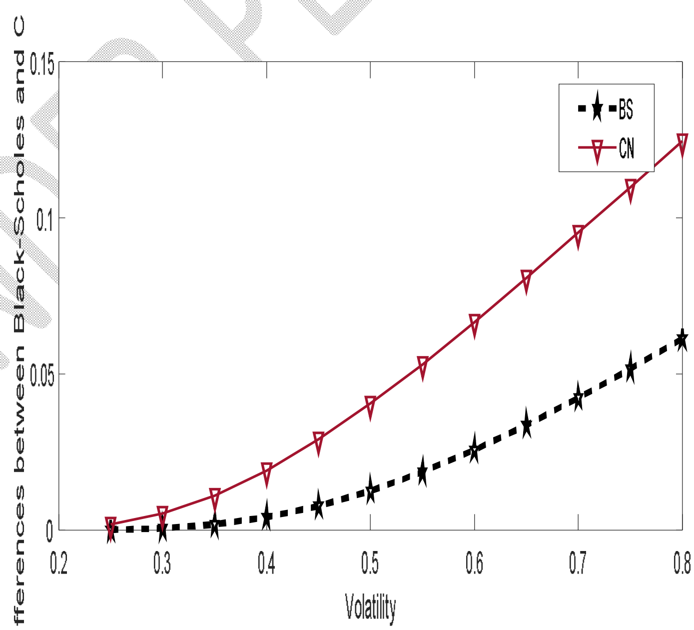

Figure 5: The Error Differences between Black-Scholes exact values and Crank-Nicolson Numerical approximations when initial stock prices is 40,and 50 for Put option.

In Figure 5, it is clear on the error differences in BS and CN; which shows an increase in the initial stock prices; increase level of errors for put option. The plots tend towards linear growth function whose convergence rates are fast. The plots indicate that the human errors in the investments plans and decisions were not much and cannot affect the future of any investment plans.

. Table 3 : Option values when K=S0 for Call and Put options

K=S0	Call option	Put option
10	1.0190	0.5313
20	2.0776	1.1022
30	3.1269	1.6638
40	4.1741	2.2232
50	5.2207	2.7818
60	6.2641	3.3400
65	6.7698	3.6190
70	7.2329	3.8974
75	7.5984	4.1696
80	7.7597	4.3981

In Table 3, it can be noticed that, K=S0 signifies that the option contract is at- the money because the stock price is equal to the strike price and has zero intrinsic value. Therefore the put option also expires without being exercised before it does not monetize any value. Secondly an increase in strike and stock prices increases the value of Call and Put option prices; which implies that the strike and stock prices has significant effects in pricing of an option.

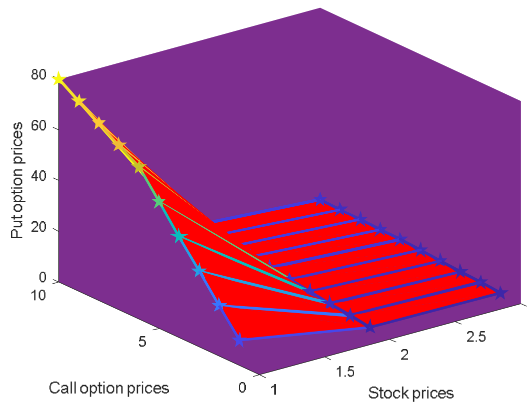

Figure 6: mesh presentation of stock and option prices

Figure 6 considers the value of European call and put options when K=S0.Carefull looking at the diagram indicates the Call option has higher value than the Put option which informs an investor on proper decision to take in terms of buying or selling of assets in time varying investments.

Table 4 : Option values when K<S0 and K>S0 for Call and Put options

K<S0			K>S0		
K	S0	Call option	K	S0	Put option
2	4	2.0976	4	2	1.8067
5	7	2.2538	7	5	1.7011
8	11	3.4116	11	8	2.5403
10	20	10.4878	20	10	9.0254
20	30	10.9940	30	20	8.6101
30	40	11.5887	40	30	8.4355
40	50	12.2931	50	40	8.4894
50	60	13.0832	60	50	8.6952
70	70	14.3404	80	70	9.4485
75	85	14.2368	85	75	8.8442

As seen in Table 4 K>S0 it shows that the option will have intrinsic value ; that is to say that put option with strike price higher than the current price will be in-the money since one can sell the stock higher than the market price and then buy it back for a guaranteed profit. This is realistic because the higher the strike price of a put option the higher the price it can sell the underlying asset; so the put options becomes K=S0. Clearly that this type of option is suitable and profitable to only investors who want to sell its asset which may be indexed in millions of naira throughout the trading days.

On the contrary, when K<S0 indicates that the option has some intrinsic value which is beneficial to exercise the options. It is also known as in-the-money because the stock price is above the strike price at expiration. The call option owner can exercise the option, putting up cash or buy the stock at the strike price or the owner can simply sell the option at its fair market value to another buyer before it expires. This remark is quite profit maximizing in time varying investments; see Table 4.

4.Conclusion

The pricing of European options is considered of closed form prices and numerical approximation of option prices. The simulations of analytical and numerical were effectively carried out using matlab programming software. The results showed as follows: there were no much differences between BS and CN for Call and Put options; little error is due to the variations in stock volatility, a little increase in stock volatility increases option prices for both options, furthermore, results also showed the significant effects of " in –the-money ,"at -of -the money", and "out-of-the-money" respectively.

However, we shall be looking at the controllability studies of both options in the next study.

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