Some consequences of Bertrand's extended postulate

Abstract

Bertrand's postulate establishes that for all positive integers n > 1 there exists a prime number between n and 2n. We consider a generalization of this theorem as: for integers n ≥ k ≥ 2 is there a prime number between kn and (k + 1)n? This is a generalization of Bertrand's postulate extended as proved at link 1706.01009.pdf. The example is deduced that there are at least k -1 prime numbers between n and kn where n, k is a positive integers greater than 1. Then we can prove a number of hypotheses and some properties below. And here are the consequences to be deduced from it.

Key words : Bertrand's extended postulate, prime number, integer

Introduction

" In 1850, P. L. Chebyshev proved the famous Bertrand postulate (1845) that every interval [n, 2n] contains a prime (for a very elegant version of his proof, see Theorem 9.2 in " [10]). " Other nice proofs were given by S. Ramamujan in 1919 [8] and P. Erd˝os in 1932 (reproduced in [4], pp.171-173) ". " In 2006, M. El. Bachraoui [1] proved that every interval [2n, 3n] contains a prime, while A. Loo [6] proved the same statement for every interval [3n, 4n] ". Moreover, A. Loo found a lower estimate for the number of primes in the interval [3n, 4n]. Note also that already in 1952 J. Nagura [7] proved that, " for n ≥ 25, there is always a prime between n and 65n. From his result it follows that the interval [5n, 6n] always contains a prime. In this paper we prove the following. From here we can generalize that (kn, (k+1)n) always has a prime number where n, k are positive integers greater than 1 ".

1) $\left(x^2; (x+1)^2\right)$ has at least 1 prime, even 2 prime numbers.

In effect, $\left(1.1; 1.2\right]$; $\left[1.2; 2.2\right)$ with k equals 1.

$\left(2.2; 2.3\right)$; $\left(2.3; 3.3\right)$ with k equals 2.

…

$\left(x.x; x(x+1)\right)$; $\left(x(x+1); (x+1)(x+1)\right)$ with k equals x.

Thus, the Legendre conjecture is true when the other property is true.

2) Oppermann's conjecture.

+ For any integer $x > 1$, there is at least one prime number between $x(x-1)$ and x^2.

In effet, $\left(1.2; 2.2\right]$ with k equals 1.

$\left(2.3; 3.3\right)$ with k equals 2.

…

$\left((x-1)x; x.x\right)$ with k equals x-1.

+ For any integer $x > 1$, there is at least one prime number between $x.x$ and $x(x+1)$.

In effet, $\left(2.2; 2.3\right)$ with k equals 2.

$\left(3.3; 3.4\right)$ with k equals 3.

…

$\left(x.x; x(x+1)\right)$ with k equals x.

Thus, the Oppermann conjecture is true when the other property is true.

3) Brocard's conjecture.

There are at least four prime numbers between P_n^2 and P_{n+1}^2, for all $n > 1$, where P_n is the nth prime number.

Easy to see $P_{n+1} - P_n \geq 2$.

We consider $P_{n+1} - P_n = 2$.

We must then prove that for n being a positive integer, if exists a prime number between P_n^2 and $(P_n + 2)^2$.

Applying the property of element 2, we divide it into 4 intervals

$\left(P_n^2; P_n(P_n+1)\right)$; $\left(P_n(P_n+1); (P_n+1)^2\right)$; $\left((P_n+1)^2; (P_n+1)(P_n+2)\right)$; $\left((P_n+1)(P_n+2); (P_n+2)^2\right)$

Thus, Bertrand's conjecture is true when the other property is true.

4) $P_{n+1} - P_n < \sqrt{P_n} \Leftrightarrow P_{n+1} < \sqrt{P_n}\left(\sqrt{P_n} + 1\right)$

We must then prove that for n being a positive integer, there exists a prime number between P_n and $\sqrt{P_n}\left(\sqrt{P_n}+1\right)$. The other property is true when property 2 is applied.

5) $KP_n < P_{n+\alpha} < (K+1)P_n$, It means $K < \frac{P_{n+\alpha}}{P_n} < K+1$

6) Assuming that two prime numbers p and q and have a difference of n, then there are at least 2*n* prime numbers between p^2 et q^2.

By applying the property of element 2, we divide it into 2*n* intervals.

$(+)$ $(P^2; P(P+1))$; $(P(P+1); (P+1)^2)$; $((P+1)^2; (P+1)(P+2))$;
$((P+1)(P+2); (P+2)^2)$

$(+)$ $((P+2)^2; (P+2)(P+3))$; $((P+2)(P+3); (P+3)^2)$; $((P+3)^2; (P+3)(P+4))$;
$((P+3)(P+4); (P+4)^2)$

$(+)$ $((P+n-2)^2; (P+n-2)(P+n-1))$; $((P+n-2)(P+n-1); (P+n-1)^2)$;
$((P+n-1)^2; (P+n-1)(P+n))$; $((P+n-1)(P+n); (P+n)^2)$

Thus, property 6 is correct.

7) Andrica's conjecture

$$\sqrt{P_{n+1}} - \sqrt{P_n} < 1 \Leftrightarrow P_{n+1} < P_n + 2\sqrt{P_n} + 1$$

$But\ P_{n+1} < P_n + \sqrt{P_n}$ (according to the property 4)

8) Assuming that two prime numbers and have a difference of n, then there are at least mn prime numbers between p^m and q^m where m is a positive entry greater than 1.

By applying property 6 and the induction method, we obtain property 8 correctly.

9) If q is a prime number, there is less $q-1$ prime numbers between q and q^2

By applying the property of element 2, we divide it into q-1 intelvalles.
$(q; 2q); (2q; 3q); \ldots; ((q-1)q; q^2)$.

So property 9 is correcte.

10) Where q is prime and m and k are natural numbers greater than 1 such that $m<k$ there is at least $(q-1)(k-m)$ prime numbers between q^m and q^k.

Applying the element property 2, we divide it into $(q-1)(k-m)$ intervalles.

$$(q^m; 2q^m); (2q^m; 3q^m); \ldots; ((q-1)q^m; q^{m+1})$$

$\left(q^{m+1}; 2q^{m+1}\right); \left(2q^{m+1}; 3q^{m+1}\right); \left(\left(q-1\right)q^{m+1}; q^{m+2}\right)$

...

$\left(q^{k-1}; 2q^{k-1}\right); \left(2q^{k-1}; 3q^{k-1}\right); \dots; \left(\left(q-1\right)q^{k-1}; q^{k}\right)$

So property 9 is correct.

11) Weak form Redmond–Sun conjecture.

With x, y, m, n having positive integers such that $x < y$ and $m<n$ there is at least *am* + $\left(y-1\right)\left(n-m\right)$ prime numbers between x^m and y^n with y – x = a.

By applying properties 9 and 10, we get the correct property 11.

Conclusions

From the fact that (n, 2n), (2n, 3n), ..., (kn, (k+1)n) in turn, there is always 1 prime number in the ranges above where n is a positive integer, we get get that (n, kn) always has at least k - 1 primes where n, k are positive integers greater than 1. For example, (n, 4n) has at least 3 primes. Besides k positive integers greater than 1, we can easily see that Andrica's conjecture is also true because k is always greater than 1.

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