Minireview Article

Some consequences of Bertrand's extended postulate

Abstract(Copy in full from [2])

Bertrand's postulate establishes that for all positive integers n > 1 there exists a prime number between n and 2n. We consider a generalization of this theorem as: for integers n ≥ k ≥ 2 is there a prime number between kn and (k + 1)n? We use elementary methods of binomial coefficients and the Chebyshev functions to establish the cases for 2 ≤ k ≤ 8. We then move to an analytic number theory approach to show that there is a prime number in the interval (kn,(k + 1) n) for at least n ≥ k and 2 ≤ k ≤ 519. This is a generalization of Bertrand's postulate extended as proved at link 1706.01009.pdf. And here are the consequences to be deduced from it.

Keywords :Bertrand's extended postulate

Introduction

1) $\left(x^2;\left(x+1\right)^2\right)$ has a prime year within the range, even 2 prime numbers(Title?)

In effect, $\left(1.1;1.2\right]$; $\left[1.2;2.2\right)$ with k equals 1.

$\left(2.2;2.3\right)$; $\left(2.3;3.3\right)$ with k equals 2.

…

$\left(x.x;x\left(x+1\right)\right)$; $\left(x\left(x+1\right);\left(x+1\right)\left(x+1\right)\right)$ with k equals x.

Thus, the Legendre conjecture is true when the other property is true.

2) Oppermann's conjecture.

+ For any integer $x>1$, there is at least one prime number between $x\left(x-1\right)$ and x^2.

In effet, $\left(1.2;2.2\right]$ with k equals 1.

$\left(2.3;3.3\right)$ with k equals 2.

…

$\left(\left(x-1\right)x;x.x\right)$ with k equals x.

+ For any integer $x>1$, there is at least one prime number between $x.x$ and $x\left(x+1\right)$.

In effet, $(2.2; 2.3)$ with k equals 2.

$(3.3; 3.4)$ with k equals 3.

…

$(x.x; x(x+1))$ with k equals x.

Thus, the Oppermann conjecture is true when the other property is true.

3) Bertrand's conjecture.(Bertrand or Brocard?)

There are at least four prime numbers between P_n^2 and P_{n+1}^2 ,for all $n > 1$, whereø P_n is the nth prime number.

Easy to see $P_{n+1} - P_n \geq 2$.

We consider $P_{n+1} - P_n = 2$.

We must then prove that for n being a positive integer, if exists a prime number between P_n^2 and $(P_n + 2)^2$.

Applying the property of element 2, we divide it into 4 intervals

$$\left(P_n^2; P_n(P_n+1)\right); \left(P_n(P_n+1); (P_n+1)^2\right); \left((P_n+1)^2; (P_n+1)(P_n+2)\right);$$
$$\left((P_n+1)(P_n+2); (P_n+2)^2\right)$$

Thus, Bertrand's conjecture is true when the other property is true.

4) $P_{n+1} - P_n < \sqrt{P_n} \Leftrightarrow P_{n+1} < \sqrt{P_n}\left(\sqrt{P_n} + 1\right)$

We must then prove that for n being a positive integer, there exists a prime number between P_n andø $\sqrt{P_n}\left(\sqrt{P_n} + 1\right)$.The other property is true when property 2 is applied.

5) $KP_n < P_{n+\alpha} < (K+1)P_n$, It means $K < \frac{P_{n+\alpha}}{P_n} < K + 1$

(4and 5, propertiespresentedwithoutexplanation)

6) Assuming that two prime numbers p and q and have a difference of n, then there are at least 2$n$prime numbers between p^2 et q^2.

By applying the property of element 2, we divide it into 2$n$intervals.

$(+)\ \left(P^2; P(P+1)\right);\ \left(P(P+1); (P+1)^2\right);\ \left((P+1)^2; (P+1)(P+2)\right);$

$\left((P+1)(P+2); (P+2)^2\right)$

$(+)\ \left((P+2)^2; (P+2)(P+3)\right);\ \left((P+2)(P+3); (P+3)^2\right);\ \left((P+3)^2; (P+3)(P+4)\right);$

$\left((P+3)(P+4); (P+4)^2\right)$

$(+)\ \left((P+n-2)^2; (P+n-2)(P+n-1)\right);\ \left((P+n-2)(P+n-1); (P+n-1)^2\right);$

$\left((P+n-1)^2; (P+n-1)(P+n)\right);\ \left((P+n-1)(P+n); (P+n)^2\right)$

Thus, property 6 is correct.

7) Andrica's conjecture

$\sqrt{P_{n+1}} - \sqrt{P_n} < 1 \Leftrightarrow P_{n+1} < P_n + 2\sqrt{P_n} + 1$

$But\ P_{n+1} < P_n + \sqrt{P_n}$ (according to the property4)

8) Assuming that two prime numbers and have a difference of n, then there are at least mn prime numbers between p^m and q^m where m is a positive entry greater than 1.

By applying property 6 and the induction method, we obtain property 8 correctly.

9) If q is a prime number, there is less $q-1$ prime numbers between q and q^2

By applying the property of element 2, we divide it into q-1 intelvalles.
$(q; 2q); (2q; 3q); \ldots; \left((q-1)q; q^2\right).$

So property 9 is correcte.

10) Where q is prime and m and k are natural numbers greater than 1 such that $m<k$ there is at least $(q-1)(k-m)$ prime numbers between q^m and q^k.

Applying the element property 2, we divide it into $(q-1)(k-m)$ intervals.

$(q^m; 2q^m); (2q^m; 3q^m); \ldots; \left((q-1)q^m; q^{m+1}\right)$

$(q^{m+1}; 2q^{m+1}); (2q^{m+1}; 3q^{m+1}); \left((q-1)q^{m+1}; q^{m+2}\right)$

...

$(q^{k-1}; 2q^{k-1}); (2q^{k-1}; 3q^{k-1}); \ldots; \left((q-1)q^{k-1}; q^k\right)$

So property 9 is correct.

11) Weak form Redmond–Sun conjecture.

With x, y, m, n having positive integers such that $x < y$ and $m < n$ there is at least $am + (y-1)(n-m)$ prime numbers between x^m and y^n with y–x=a.

By applying properties 9 and 10, we get the correct property 11.

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[12] https://en.wikipedia.org/wiki/Oppermann%27s_conjecture

[13] https://en.wikipedia.org/wiki/Andrica%27s_conjecture