Minireview Article

Some consequences of Bertrand's extended postulate

Abstract

Bertrand's postulate establishes that for all positive integers n > 1 there exists a prime number between n and 2n. We consider a generalization of this theorem as: for integers n ≥ k ≥ 2 is there a prime number between kn and (k + 1)n? We use elementary methods of binomial coefficients and the Chebyshev functions to establish the cases for 2 ≤ k ≤ 8. We then move to an analytic number theory approach to show that there is a prime number in the interval (kn,(k + 1) n) for at least n ≥ k and 2 ≤ k ≤ 519. This is a generalization of Bertrand's postulate extended as proved at link 1706.01009.pdf. And here are the consequences to be deduced from it.

Keywords : Bertrand's extended postulate

Introduction

1) $\left(x^2;(x+1)^2\right)$ has a prime year within the range, even 2 prime numbers

In effect, $(1.1;1.2]$; $[1.2;2.2)$ with k equals 1.

$(2.2;2.3)$; $(2.3;3.3)$ with k equals 2.

…

$(x.x;x(x+1))$; $(x(x+1);(x+1)(x+1))$ with k equals x.

Thus, the Legendre conjecture is true when the other property is true.

2) Oppermann's conjecture.

+ For any integer $x>1$, there is at least one prime number between $x(x-1)$ and x^2.

In effet, $(1.2;2.2]$ with k equals 1.

$(2.3;3.3)$ with k equals 2.

…

$((x-1)x;x.x)$ with k equals x.

+ For any integer $x>1$, there is at least one prime number between $x.x$ and $x(x+1)$.

In effet, $(2.2; 2.3)$ with k equals 2.

$(3.3; 3.4)$ with k equals 3.

…

$(x.x; x(x+1))$ with k equals x.

Thus, the Oppermann conjecture is true when the other property is true.

3) Bertrand's conjecture.

There are at least four prime numbers between P_n^2 and P_{n+1}^2, for all $n>1$, whereø P_n is the nth prime number.

Easy to see $P_{n+1} - P_n \geq 2$.

We consider $P_{n+1} - P_n = 2$.

We must then prove that for n being a positive integer, if exists a prime number between P_n^2 and $(P_n+2)^2$.

Applying the property of element 2, we divide it into 4 intervals

$(P_n^2; P_n(P_n+1))$; $(P_n(P_n+1); (P_n+1)^2)$; $((P_n+1)^2; (P_n+1)(P_n+2))$;
$((P_n+1)(P_n+2); (P_n+2)^2)$

Thus, Bertrand's conjecture is true when the other property is true.

4) $P_{n+1} - P_n < \sqrt{P_n} \Leftrightarrow P_{n+1} < \sqrt{P_n}\left(\sqrt{P_n}+1\right)$

We must then prove that for n being a positive integer, there exists a prime number between P_n andø $\sqrt{P_n}\left(\sqrt{P_n}+1\right)$. The other property is true when property 2 is applied.

5) $KP_n < P_{n+\alpha} < (K+1)P_n$, It means $K < \frac{P_{n+\alpha}}{P_n} < K+1$

6) Assuming that two prime numbers p and q and have a difference of n, then there are at least 2*n* prime numbers between p^2 et q^2.

By applying the property of element 2, we divide it into 2*n* intervals.

(+) $(P^2; P(P+1))$; $(P(P+1); (P+1)^2)$; $((P+1)^2; (P+1)(P+2))$;
$((P+1)(P+2); (P+2)^2)$

$(+)$ $\left((P+2)^2;(P+2)(P+3)\right)$; $\left((P+2)(P+3);(P+3)^2\right)$; $\left((P+3)^2;(P+3)(P+4)\right)$; $\left((P+3)(P+4);(P+4)^2\right)$

$(+)$ $\left((P+n-2)^2;(P+n-2)(P+n-1)\right)$; $\left((P+n-2)(P+n-1);(P+n-1)^2\right)$; $\left((P+n-1)^2;(P+n-1)(P+n)\right)$; $\left((P+n-1)(P+n);(P+n)^2\right)$

Thus, property 6 is correct.

7) Andrica's conjecture

$$\sqrt{P_{n+1}}-\sqrt{P_n}<1 \Leftrightarrow P_{n+1}<P_n+2\sqrt{P_n}+1$$

$But\ P_{n+1} < P_n + \sqrt{P_n}$ (according to the property 4)

8) Assuming that two prime numbers and have a difference of n, then there are at least mn prime numbers between p^m and q^m where m is a positive entry greater than 1.

By applying property 6 and the induction method, we obtain property 8 correctly.

9) If q is a prime number, there is less $q-1$ prime numbers between q and q^2

By applying the property of element 2, we divide it into q-1 intelvalles.
$(q;2q);(2q;3q);\ldots;\left((q-1)q;q^2\right)$.

So property 9 is correcte.

10) Where q is prime and m and k are natural numbers greater than 1 such that $m<k$ there is at least $(q-1)(k-m)$ prime numbers between q^m and q^k.

Applying the element property 2, we divide it into $(q-1)(k-m)$ intervals.

$(q^m;2q^m);(2q^m;3q^m);\ldots;\left((q-1)q^m;q^{m+1}\right)$

$(q^{m+1};2q^{m+1});(2q^{m+1};3q^{m+1});\left((q-1)q^{m+1};q^{m+2}\right)$

...

$(q^{k-1};2q^{k-1});(2q^{k-1};3q^{k-1});\ldots;\left((q-1)q^{k-1};q^k\right)$

So property 9 is correct.

11) Weak form Redmond–Sun conjecture.

With x, y, m, n having positive integers such that $x<y$ and $m<n$ there is at least $am + (y-1)(n-m)$ prime numbers between x^m and y^n with y – x = a.

By applying properties 9 and 10, we get the correct property 11.

Références

[1] M. El Bachraoui. Primes in the Interval [2n, 3n]. Int. J. Contemp. Math. Sci., 1:617–621, 2006.

[2] Kyle D. Balliet. On the prime numbers in intervals https://arxiv.org/pdf/1706.01009.pdf?fbclid=IwAR1dHSZTR-hi6XDT1hb1IBpDJ8lLIFNk-rftxhT4aJDREQzqXd5lre5BQiY

[3] P. Chebyshev. M´emoire sur les nombres premiers. M´em. Acad. Sci. St. Ptersbourg, 7:17–33, 1850.

[4] P. Erd˝os. Beweis eines satzes von tschebyschef. Acta Litt. Univ. Sci., Szeged, Sect. Math., 5:194–198, 1932.

[5] P. Erd˝os. On a new method in elementary number theory which leads to an elementary proof of the prime number theorem. Proc. Nat. Acad. Scis., 35:374– 384, 1949.

[6] P. Erd˝os and J. Sur´anyi. Topics in the Theory of Numbers. Springer Verlag, 2003.

[7] A. Loo. On the Primes in the Interval [3n, 4n]. Int. J. Contemp. Math. Sci., 6:1871–1882, 2011.

[8] https://fr.wikipedia.org/wiki/Postulat_de_Bertrand?fbclid=IwAR0thE-AQBmzT0jUvykPYXJBBOPayyG4pGO0VUOB3Q_d40Xl3-dinlDeIyY

[9] L. Schoenfeld. Sharper bounds for the Chebyshev functions θ(x) and ψ(x). II. Math. Comp., 30:337–360, 1976.

[10] https://en.wikipedia.org/wiki/Legendre%27s_conjecture

[11] https://en.wikipedia.org/wiki/Brocard%27s_conjecture

[12] https://en.wikipedia.org/wiki/Oppermann%27s_conjecture

[13] https://en.wikipedia.org/wiki/Andrica%27s_conjecture