Original Research Article

A resilient adaptive event-triggered H_∞ tracking control of T-S fuzzy systems with Markovian jump parameters under DOS attacks

Abstract

In this paper, the problem of H_∞ output-tracking control for T-S fuzzy markov jump systems(MJSs) with imperfect premise matching is investigated. Firstly, due to the influence of network-induce delay, the system mode information can not be transmitted to the controller synchronously, thus, a discrete-time hidden markov model(HMM) is established to describe the asynchronous phenomenon between the system and the controller; Secondly, form the prospectively of practical applications, packet loss in two channels is taken into consideration, which is caused by denial-of-service(DOS) attacks; Thirdly, in order to improve the data transmission efficiency and save network bandwidth resources, a novel resilient adaptive event-triggered(AET) mechanism is proposed, in which the event-based threshold parameter is dynamically regulated over the processing. Additionally, resorted the compensation principle and the lyapunov-krasovski(L-K) functions methods, some sufficient conditions in form of liner matrix inequality(LMI) are achieved to co-guarantee the stability and H_∞ tracking property of the closed-loop system; Finally, two examples are given to verify the effectiveness of our design mentality.

Keywords:
Fuzzy markov jump systems; Asynchronous H_∞ control; Denial-of-service attacks; Resilient adaptive event triggered mechanism; Output tracking control.

1. Introduction

In practice, the analysis and synthesis of the systems become more difficult since most of complex network systems are nonlinear. The T-S fuzzy system technology is one of the most effective methods to deal with the problems of nonlinear systems. T-S fuzzy model is a nonlinear system described by a group of "if-then" fuzzy rules. Each rule represents a subsystem, and the whole fuzzy system is a linear combination of all subsystems. It was first proposed in [1]. Based on this system, many stability and synchronization analysis methods of linear systems can be extended to the study of nonlinear systems. In [2], the safety control problem of fuzzy interval type-2 has been proposed. Using a random communication scheduling protocol to control the transmission from the sensor to the controller, a mathematical model based on the compensation scheme is also constructed. The T-S fuzzy methods have been widely researched in the complex nonlinear systems, such as bioengineer, medicine, and many other results have been obtained in the relevant literature, see [3],[4],[5],and [6].

A MJS is a stochastic system with diversified modes. Based on a set of markov chains, the transition of the system switches among the various modes. As a statistical analysis model, HMM was founded in the 1970s and developed in the 1980s which has become an important direction of signal procession. Its functions are used through a conditional probability matrix between the system and the controller. The conditional probability reflects the degree of asynchrony between the system and the controller. In[7], the stability and H_∞ performance of nonlinear MJSs with general transition probabilities (TPs) are investigated. Its TPs consist of three situations which are known completely, known partially, and unknown completely,respectively. [8] studies an asynchronous control problem of nonlinear MJSs and, the sampling data controller based on the fuzzy quantization is designed. The stochastic stability of fuzzy nonlinear MJSs is guaranteed effectively. There are also many other results on HMM [9]. For another example, [10] considered a finite-time asynchronous H_∞ fault tolerant control for a nonlinear HMM. In [11], the stability problems of both continuous-time and discrete-time linear MJSs with partly unknown transition probabilities are investigated. [12] studies the finite time robust stability of MJSs with partially known transition rate. The robust stability conditions are derived on account of the lyapunov stability theory and LMI technique.

Generally, communication network systems may suffer from malicious attacks inevitably , which can damage the stability and the transmission performance of the network system. Thus, the scholars have conducted more in-depth research on the security problems of network from the perspective of control theory. In general, common attacks in network control systems(NCSs) include the DOS attacks and deception attacks. The destructiveness of the deception attacks is mainly reflected in changing the data reliability, while DOS attacks mainly hinder the data transmission. According to the past experience, DOS attacks are more possible to occur in NCSs. Generally speaking, the description of the DOS attacks process in a NCS can be described by two types of random process: markov chain and Bernoulli distribution variable. [13] studies an asynchronous AET control for IT-2 fuzzy NCSs subject to the aperiodic DOS attacks. In [13], two resilient AETMs are applied to the output of the sensor and the controller independently to resist the affect of aperiodic DOS attacks. [14] investigates the design of resilient dispersion sampling data filters for linear interconnected systems under DOS attacks. A method based on the piecewise L-K functions is proposed to determine filter parameters and sample data communication schemes.

As an emerging communication mechanism, event triggered strategy (ETS) has attracted wide attention because of its excellence in decreasing the network communication bandwidths [15], [16], [17]. The main idea of ETSs is that the signals are sampled or transmitted only when the pre-specified event-triggered conditions are violated. This applies that no data samples occupy the network channel during the event triggered instants. Thus, this can save the network bandwidths resources effectively. [18] studies the problem of network control for T-S fuzzy MJSs with communication latency. Based on aperiodic sampling data, a mode-dependent ETS can improve the efficiency of data transmission greatly. [18] also designs an asynchronous controller based on HMM to stabilize a fuzzy MJS with general transition probabilities. However, few researchers investigates the problem of ETS in the correlated fuzzy tracking control under DOS attacks, which is one of the initial motivations of this paper.

In [19], based on the H_∞ tracking control method, an adaptive dynamic programming (ADP) is proposed for uncertain continuous time nonlinear MJSs. A neural network (NN) observer with the input and output of the system is constructed. Thus, the equations of completely uncertain system and an augmented system consist of dynamic tracking error and reference trajectory can be formulated. [20] studies the tracking control problem of nonlinear MJSs based on T-S fuzzy model with incomplete modal information. Furthermore, the designed controller can not fully control the mode which the system maintains at runtime. In this imperfect modal information scenario, a mechanism based on HMM is improved to simulate the defect of modal asynchrony. The main work of [21] is to solve the tracking control problem of the fuzzy MJSs. Additionally, the impact of both disturbance and uncertainty in fuzzy MJSs is also considered. In [22],[23],[24], they all assume that the designed tracking controllers keep the same mode as the system. But actually, the mode information of the controller and the system are not always synchronized, which is also one of the initial motivations of this paper.

This paper presents an asynchronous H_∞ fuzzy tracking control method for the fuzzy MJSs with partially known modal information. The L-K function with fuzzy basis and mode-dependent are constructed to obtain some sufficient criteria. These criteria can ensure that the state and the tracking error of the closed-loop system achieve a stable stochastically for a specified H_∞ tracking performance. The main contributions of this paper are summarized as follows:

1) Due to the influence of network-induce delay, the system mode information can not be transmitted to the controller synchronously. Therefore, a discrete-time HMM is established to describe the asynchronous phenomenon. Moreover, the conditional probability between the system and the controller is known incompletely. Specially, the method proposed in this paper also covers the case where the condition probability is completely known. During the process of analysing the system stability, two inequality conditions are introduced to deal with the problem of incomplete mode information;

2) This paper considers the DOS attacks occurring in both sensor-to-controller(S-C) and controller-to-actuator(C-A) channels. This can make the proposed method more general;

3) A novel resilient AET mechanism is proposed to relieve the network communication burden and decrease the affect caused by DOS attacks. When DOS attacks are not activated, the triggered detectors will be updated to save more communication resource. Moreover, the triggered threshold is both related to

the membership functions(MFs) of the system and the MFs of the controller. Moreover, the average value of the latest transmitted data also has influence on the change of the threshold.

4) Based on the latest transmitted data and the system mode signals, an asynchronous fuzzy controller is designed to ensure the stochastic stability of the system. Compared with [25], the advantage of this paper is that the designed controller combines the resilient AET mechanism with the attack compensation principle. Furthermore, in order to solve the mismatch issue between the fuzzy MJSs and the controller, the inequality relationship of MFs is constructed to deal with this problem in this paper.

The remainder of this paper is arranged as follows: In Section 2, an asynchronous fuzzy controller is designed for the fuzzy MJS with AET mechanism under DOS attacks; In Section 3, some sufficient principles which can guarantee the stochastic stability and a prescribed H_∞ tracking performance index for the nonlinear system; Two examples are listed in Section 4; Finally, some conclusions can be obtained from this paper in Section 5.

Notations: $l_2[0,\infty)$ represents the square summable sequences space; $\|\mathcal{X}\|$ denotes the Euclidean norm of $\mathcal{X}$; $\mathbb{R}^{\hat{n}}$ represents the $\hat{n}$ dimensional Euclidean space; $\mathbb{R}^{\hat{m}\times\hat{n}}$ denotes the $\hat{m}\times\hat{n}$ real matrices; $\mathcal{P}>0(\mathcal{P}<0)$ applies that the matrix $\mathcal{P}$ is symmetric and positive(negative)-definite. $\mathcal{Y}^T$ denotes the transposition of $\mathcal{Y}$; $*$ in the symmetric matrices represents the symmetric term which are omitted. $Prob\{\hat{a}\}$ and $\mathcal{E}\{\hat{a}\}$ represent the probability and the mathematical expectation of $\hat{a}$ in probability statistics; $diag\{\mathcal{Y}\}$ means that $\mathcal{Y}$ is a block-diagonal matrix; $\tilde{\lambda}_{min}(\cdot)$ and $\tilde{\lambda}_{max}(\cdot)$ denote the minimum eigenvalue and maximum eigenvalue of a real matrix, respectively.

2. Preliminaries

2.1 System Dynamics

Let s be the number of the fuzzy rules. A nonlinear fuzzy MJS is described by the following IF-THEN rules:

Plant Rule i : IF $\kappa_1(k)$ is ι_{i1},$\kappa_2(k)$ is ι_{i2} ,$\cdots$,and $\kappa_\nu(k)$ is $\iota_{i\nu}$, THEN

$$\begin{cases}\eta(k+1)=\mathcal{A}_{\tau_k,i}\eta(k)+\mathcal{B}_{\tau_k,i}u(k)+\mathcal{E}_{\tau_k,i}\omega(k)\\ z(k)=\mathcal{C}_{\tau_k,i}\eta(k)+\mathcal{D}_{\tau_k,i}u(k)+\mathcal{F}_{\tau_k,i}\omega(k)\end{cases}\tag{1}$$

where $i\in\mathcal{I}=\{1,2,\cdots,s\}$. $\kappa(k)=(\kappa_1(k),\kappa_2(k),\cdots,\kappa_\nu(k))^T$ denotes the vector consisting of the premise variables; $\{\iota_{ij}\}$ $(j=1,2,\cdots,\nu)$ denotes a fuzzy set; $\eta(k)\in\mathbb{R}^{n_1}$, $u(k)\in\mathbb{R}^{n_2}$, and $z(k)\in\mathbb{R}^{n_3}$ represent the state of the system, control input, and the output of the system, respectively. $\omega(k)\in\mathbb{R}^{o}$ denotes the external interference belongs to $l_2[0,\infty)$. $\mathcal{A}_{\tau_k,i}$, $\mathcal{B}_{\tau_k,i}$, $\mathcal{C}_{\tau_k,i}$, $\mathcal{D}_{\tau_k,i}$, $\mathcal{E}_{\tau_k,i}$, $\mathcal{F}_{\tau_k,i}$ are all real matrices which have been known. $\tau_k\in\mathbb{L}=\{1,2,\cdots,L\}$ represents the mode number of the system, which is used to described the markov jump. The transition probability matrix $\Pi = \{\pi_{mn}\}$ can be described as:

$$Prob\{\tau_k + 1 = n|\tau_k = m\} = \pi_{mn}, \quad m \in \mathbb{L}, n \in \mathbb{L}$$

where π_{mn} is the transition probability of the system mode τ_k , Thus, $\pi_{mn} \in [0, 1]$,and $\sum_{n=1}^{L} \pi_{mn} = 1$. Through the fuzzy approach, let $\tau_k = m$, the system (1) can be modified as:

$$\begin{cases} \eta(k+1) = \mathcal{A}_{mh}\eta(k) + \mathcal{B}_{mh}u(k) + \mathcal{E}_{mh}\omega(k) \\ z(k) = \mathcal{C}_{mh}\eta(k) + \mathcal{D}_{mh}u(k) + \mathcal{F}_{mh}\omega(k) \end{cases} \tag{2}$$

where

$$\begin{cases} h_i(\kappa(k)) = \frac{\iota_i(\kappa(k))}{\sum_{i=1}^{s}\iota_i(\kappa(k)} \\ \iota_i(\kappa(k) = \prod_{j=1}^{\nu}\iota_{ij}(\kappa_j(k)) \end{cases}$$

and

$$\mathcal{A}_{mh} = \sum_{i=1}^{s} h_i(\kappa(k))\mathcal{A}_{mi},\ \mathcal{B}_{mh} = \sum_{i=1}^{s} h_i(\kappa(k))\mathcal{B}_{mi},\ \mathcal{E}_{mh} = \sum_{i=1}^{s} h_i(\kappa(k))\mathcal{E}_{mi}$$
$$\mathcal{C}_{mh} = \sum_{i=1}^{s} h_i(\kappa(k))\mathcal{C}_{mi},\ \mathcal{D}_{mh} = \sum_{i=1}^{s} h_i(\kappa(k))\mathcal{D}_{mi},\ \mathcal{F}_{mh} = \sum_{i=1}^{s} h_i(\kappa(k))\mathcal{F}_{mi}$$

$\iota_{ij}(\kappa_j(k))$ denotes the grade of MFs $\kappa_j(k)$ in ι_{ij}. $h_i(\kappa(k))$ is the normalized MF of rule i. For $\forall i \in \mathcal{I}, h_i(\kappa(k)) \geq 0$ and $\sum_{n=1}^{L} h_i(\kappa(k)) = 1$. For simplicity, h_i will replace $h_i(\kappa(k))$ in the subsequent sections.

Remark 2.1 Both the state vector $\eta(k)$ and the output vector $z(k)$ in the system (2) consider the external disturbance $\omega(k)$ due that the network systems will inevitably be disturbed by various external factors at any time.

2.2 Reference model

The tracking control problem implies that the asymptotic tracking of the system should be achieved while suppressing the interference. So as to track the expected output signal well, the following steady reference system is selected by this paper:

$$\begin{cases} \eta_r(k+1) = \mathcal{A}_r\eta_r(k) + \mathcal{B}_r r(k) \\ z_r(k) = \mathcal{C}_r\eta_r(k) \end{cases} \tag{3}$$

where $\eta_r(k), z_r(k), r(k)$ are the state vector, output vector, and the bounded reference input,respectively. $\mathcal{A}_r, \mathcal{B}_r, \mathcal{C}_r$ are known real matrices with appropriate dimensions.

Remark 2.2 According to [25] and [26], the system (3) is selected as the reference model of H_∞ tracking control. What's more, the reference system is known and stable.

2.3 Resilient AET Mechanism

The main principle of AET mechanism is that the measurement output or control signal will be released through the network if the given event-triggered conditions are violated. This paper considers a resilient AET mechanism to regulate data transmission.

According to [26], let $\xi(k) = [\eta^T(k), \eta_r^T(k)]^T$ and $T_n = [t_n, t_n + l_n)$ as the separate packet transmission in the system and no attack time intervals,respectively. When the invader does not launch attacks, the first trigger moment is $k = 0$, and the next event-triggered moment needs to be analyzed by the following two situations:

Given $0 \leq \varepsilon_1 \leq \varepsilon_2, \mathcal{W} = \mathcal{W}^T > 0$. There are two possible cases which we should consider:

(a) If the sampling data $\xi(k)$ satisfies :

$$\|\mathcal{W}^{\frac{1}{2}}\varepsilon(k, k_{n^*})\|^2 \geq \epsilon(k)\|\mathcal{W}^{\frac{1}{2}}\xi_(k_n)\|^2 \tag{4}$$

where $\varepsilon(k) = \xi(k_{n^*}) - \xi(k)$, and $\epsilon(k)$ is the threshold function described by:

$$\epsilon(k) = \epsilon_1 + (\epsilon_2 - \epsilon_1)\psi(k)e^{-\alpha_1\|\xi(k) - \frac{1}{T}\sum_{t=0}^{T-1}\xi(k_{n^*-t})\|^{\alpha_2}} \tag{5}$$

(b) The sampling data $\xi(k)$ is abandoned purposely.

When the invaders launch attacks, the above AET mechanism is unable to decrease the adverse effects of the attacks timely. Therefore, This paper construct a new resilient AETM:

$$k_{n^*+1} \in \{k_{n^*} \ \ satisfying \ \ (4)|k_{n^*} \in T_n\} \bigcup \{t_n\} \tag{6}$$

Remark 2.3 For the threshold function $\epsilon(k)$, $\psi(k) = \frac{r - \sum_{i=1}^{r}[h_i(\eta(k)) - \hat{h}_i(\varrho(k_s))]^2}{r}, \alpha_1 > 0, \alpha_2 > 0, T > 0$, these parameters can adjust the threshold resiliently through $\epsilon(k)\xi(k_{n^*-t}) = \xi(0), n^* - t < 0$ in (5). AET generator can determine the time of transmitting data in a dynamic way. In general, it can be summarized as two situation: (i)$\xi(k_{n^*+1}) = \xi(k)$; (ii) the sampling data $\xi(k_{n^*+1})$ will be transmitted to the controller through the system.

2.4 HMM-Based Asynchronous Fuzzy Controller

In consideration of the environment of exist communication networks, it is unjustified to believe that the system has the same premise variables with the controller. As a result, the IPM method of imperfect premise variables matching is adopted. The fuzzy controller which can describe the asynchronous phenomenon is constructed as:

Controller rule j : IF $\varrho_1(k)$ is ψ_{j1}, $\varrho_2(k)$ is ψ_{j2}, $\cdots$, $\varrho_g(k)$ is ψ_{jg}, Then

$$u_1(k) = K_{\phi_k,j}u_2(k) \tag{7}$$

where $j \in \mathcal{I} = \{1, 2, \cdots, s\}$, s denotes the number of the fuzzy rules; $\varrho_k = \{\varrho_1(k), \varrho_2(k), \cdots, \varrho_g(k)\}$ represents the premise variable. $\{\psi_{j\iota}\}$ is a fuzzy set; $K_{\phi_k,j} \in \mathbb{R}^{m\times n}$ denotes the controller gain matrix.

$u_1(k)$ is the controller input. $u_2(k)$ is the controller output. $\phi_k \in \mathcal{T} = \{1, 2, \cdots, T\}$ denotes the controller mode. Figs.1 depicts that the structure of the communication network with the AET mechanism and DOS attacks.

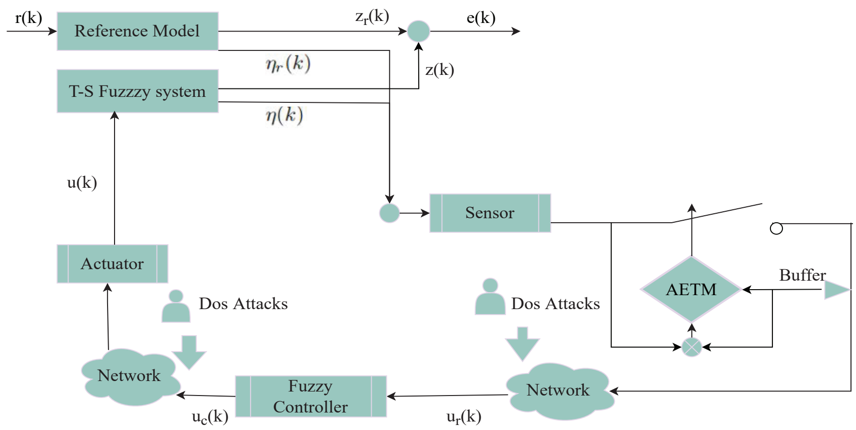

Figure 1: The configuration of T-S fuzzy tracking control system with the AET mechanism and DOS attacks.

The conditional probability between the controller and the system can be expressed by the matrix $\Omega = \delta_{mt}$.

The probability of ϕ_k can replace τ_k with:

$$Prob\{\phi_k = t | \tau_k = m\} = \delta_{mt}, \quad t \in \mathcal{T}, \ m \in \mathbb{L}$$

where $0 \leq \delta_{mt} \leq 1$, $\sum_{t=1}^{M} \delta_{mt} = 1$.

This paper considers the problem that the information of matrix Ω is known incompletely. For an example,when $L = 2$ and $T = 3$, matrix Ω may be as:

$$\begin{bmatrix} \delta_{11} & ? & ? \\ \delta_{21} & ? & \delta_{23} \end{bmatrix}$$

where "?" represents the unknown elements. For the convenience of analyzing, divided the set $\mathcal{T}$ into two parts, i.e., $\mathcal{T} = \mathcal{T}_{\mathcal{K}}^{(m)} + \mathcal{T}_{\mathcal{UK}}^{(m)}$ with:

$$\begin{cases} \mathcal{T}_{\mathcal{K}}^{(m)} \triangleq \{t : \delta_{mt} \ is \ known\}, \\ \mathcal{T}_{\mathcal{UK}}^{(m)} \triangleq \{t : \delta_{mt} \ is \ unknown\}. \end{cases}$$

If $\mathcal{T}_{\mathcal{K}}^{(m)} \neq \emptyset$ and $\mathcal{T}_{\mathcal{UK}}^{(m)} \neq \emptyset$, it can be rewritten as:

$$\begin{cases} \mathcal{T}_{\mathcal{K}}^{(m)} = \{\mathcal{K}_1^{(m)}, \cdots, \mathcal{K}_{s_1}^{(m)}\}, \ 1 \leq s_1 \leq T \\ \mathcal{T}_{\mathcal{UK}}^{(m)} = \{\mathcal{UK}_1^{(m)}, \cdots, \mathcal{UK}_{s_2}^{(m)}\}, \ 1 \leq s_2 \leq T \end{cases}$$

where $\mathcal{K}_{s_1}^{(m)}$ are the s_1th known elements with the index $\mathcal{K}_{s_1}^{(m)}$ in the mth row of matrix Ω, and correspondingly, $\mathcal{UK}_{s_2}^{(m)}$ is the s_2th unknown element with the index $\mathcal{UK}_{s_2}^{(m)}$. Then, through the fuzzy approach, let $\phi_(k) = t$, the controller (7) can be modified as:

$$u_1(k) = K_{th} u_2(k) \tag{8}$$

where

$$K_{th} = \sum_{j=1}^{s} \hat{h}_j(\varrho(k)) K_{tj}, \;\; \bar{h}_j(\varrho(k)) = \frac{\psi_j(\varrho(k))}{\sum_{j=1}^{s} \psi_j(\varrho(k))}, \;\; \psi_j(\varrho(k)) = \Pi_{t=1}^{g} \psi_{jt}(\varrho(k))$$

Similar to the previous, $\psi_{jt}(\varrho(k))$ is the grade of membership $\varrho(k)$ in ψ_{jt}. $\bar{h}_j(\varrho(k))$ denotes the normalized MFs of rule j. For $\forall j \in \mathcal{I}$, $\bar{h}_j(\varrho(k)) \geq 0$ and $\sum_{j=1}^{s} \bar{h}_j(\varrho(k)) = 1$. For notation simplicity, $\bar{h}_j$ stands for $\bar{h}_j(\varrho(k))$ in the subsequent sections.

Remark 2.4 Note that the fuzzy controller in (8) is different from the controller in the system (2). The HMM based fuzzy controller emphasizes the mismatch of the premise variables between the controller and the system. The controller in (8) also emphasizes the asynchronous phenomenon between the controller and the system. Unlike [27] emphasizes that the system transition probability matrix is incompletely known, this paper is focus on the partially unknown conditional probability matrix between the controller and the system.

2.5 DOS Attacks

Considering the random data packets missing caused by DOS attacks. Two variables which obey the Bernoulli distribution $\theta_1(k), \theta_2(k) \in \{0, 1\}$ are selected to describe the phenomenon of data missing in the two channels: S-C and C-A. If either of $\theta_1(k) = 0$ and $\theta_2(k) = 0$ holds, the packet is lost. As a result, the controller input and the actuator output can be described as follows:

$$\begin{cases} u_2(k) = \theta_1(k)(\eta(k) - \eta_r(k)) \\ u_e(k) = \theta_2(k) u_1(k) \end{cases} \tag{9}$$

where $u_e(k)$ denotes the output of the controller under DOS attacks. Then we define $\upsilon(k) = \theta_1(k)\theta_2(k)$. It is obvious that $\upsilon(k) = 1$ only when $\theta_1(k) = \theta_2(k) = 1$, which implies that the data transmit successfully. There are the following probability expressions for $\upsilon(k)$:

$$\begin{cases} Prob\{\upsilon(k) = 1\} = \mathcal{E}\{\upsilon(k)\} = \mathcal{E}\{\upsilon^2(k)\} = \gamma \\ Prob\{\upsilon(k) = 0\} = 1 - \gamma \end{cases}$$

where γ represents the packet arriving rate.

From (8) and (9), we can obtain:

$$u_e(k) = \upsilon(k) K_{th}(\eta(k) - \eta_r(k)) \tag{10}$$

In order to compensate the lost data packets due to the DOS attack, the actuator will always be the current output if the controller is not renovated successfully. Hence,the ultimate controller can be designed as follows:

$$u(k) = \upsilon(k)u_e(k_{n^*}) + (1 - \upsilon(k))u(k-1) \tag{11}$$

Remark 2.5 The cyber attacks are inevitable in network control systems. Among many attacks, DOS attacks are considered to be more possible to occur. In the course of engineering practice, attacks launched by invader are not always be successful because of the existence of safety protection from the protection agency. Thus, under these circumstances, attacks occur in a random fashion. Therefore, two Bernoulli variables are used to describe the packet loss phenomenon caused by DOS attacks in the two channels: S-C and C-A. The product of the two variables is selected as the arrival rate of the actual packet. It is important to note that: when $\theta_i(k)(i = 1, 2) = 0$, DOS attacks is active, data packets are lost. When $\theta_i(k)(i = 1, 2) \neq 0$, DOS attacks is silent, data packets can be transmitted successfully.

Remark 2.6 Refers to [26], this paper designs the controllers (11) which adopts the compensation principle. Specifically, if DOS attacks are active, that is $\upsilon(k) = 0$, the event triggers will no longer act on. At this time, the output of the present controller will be replaced by the output of the last moment. If DOS attacks are silent, that is $\upsilon(k) = 1$, the output of the controller will be related to the start time of the event triggers.

2.6 Tracking Error Dynamic System

Combining (2),(3),(11),and $e(k) = z(k) - z_r(k)$.The following augmented tracking error dynamic system can be obtained:

$$\begin{cases} \zeta(k+1) = \bar{A}_{mth}\zeta(k) + \bar{B}_{mth}\varepsilon(k) + \bar{E}_{mh}\bar{\omega}(k) \\ e(k) = \bar{C}_{mth}\zeta(k) + \bar{D}_{mth}\varepsilon(k) + \bar{F}_{mh}\omega(k) \end{cases} \tag{12}$$

where

$$\zeta(k) = [\eta^T(k) \;\; \eta_r^T(k) \;\; u^T(k-1)]^T, \;\; \bar{\omega}(k) = [\omega^T(k) \;\; r^T(k)]^T,$$
$$\bar{\upsilon}(k) = \upsilon(k) - \gamma, \;\; \varepsilon(k) = \xi(k_{n^*}) - \xi(k), \;\; \xi = [\eta^T(k) \;\; \eta_r^T(k)]^T$$

$$\bar{G}_{mth} = \bar{G}_{1mth} + \upsilon(k)\bar{G}_{2mth}, \;\; \bar{E}_{mh} = \sum_{i=1}^{s} h_i\bar{E}_{mi}, \;\; \bar{F}_{mh} = \sum_{i=1}^{s} h_i\bar{F}_{mi},$$
$$\bar{G}_{1mth} = \sum_{i=1}^{s}\sum_{j=1}^{s} h_i\bar{h}_j\bar{G}_{1mtij}, \;\; \bar{G}_{2mth} = \sum_{i=1}^{s}\sum_{j=1}^{s} h_i\bar{h}_j\bar{G}_{2mtij}$$

with $G = A$, B , C , D, where

$$\bar{A}_{1mtij} = \begin{bmatrix} \mathcal{A}_{mi} + \gamma\mathcal{B}_{mi}K_{tj} & -\gamma\mathcal{B}_{mi}K_{tj} & K_{tj} - \gamma\mathcal{B}_{mi}K_{tj} \\ 0 & \mathcal{A}_r & 0 \\ \gamma K_{tj} & -\gamma K_{tj} & K_{tj} - \gamma K_{tj} \end{bmatrix}, \;\; \bar{A}_{2mtij} = \begin{bmatrix} \mathcal{B}_{mi}K_{tj} & -\mathcal{B}_{mi}K_{tj} & -\mathcal{B}_{mi}K_{tj} \\ 0 & 0 & 0 \\ K_{tj} & -K_{tj} & -K_{tj} \end{bmatrix},$$

$$\bar{B}_{1mtij} = \begin{bmatrix} \gamma\mathcal{B}_{mi}K_{tj} & -\gamma\mathcal{B}_{mi}K_{tj} \\ 0 & 0 \\ \gamma K_{tj} & -\gamma K_{tj} \end{bmatrix}, \ \bar{B}_{2mtij} = \begin{bmatrix} \mathcal{B}_{mi}K_{tj} & -\mathcal{B}_{mi}K_{tj} \\ 0 & 0 \\ K_{tj} & -K_{tj} \end{bmatrix},$$

$$\bar{C}_{1mtij} = \begin{bmatrix} \mathcal{C}_{mi} + \mathcal{D}_{mi}K_{tj} & -\mathcal{C}_r - \gamma\mathcal{D}_{mi}K_{tj} & \mathcal{D}_{mi}K_{tj} - \gamma\mathcal{D}_{mi}K_{tj} \end{bmatrix}, \ \bar{C}_{2mtij} = \begin{bmatrix} 0 & -\mathcal{D}_{mi}K_{tj} & -\mathcal{D}_{mi}K_{tj} \end{bmatrix},$$

$$\bar{D}_{1mtij} = \begin{bmatrix} \gamma\mathcal{D}_{mi}K_{tj} & -\gamma\mathcal{D}_{mi}K_{tj} \end{bmatrix}, \ \bar{D}_{2mtij} = \begin{bmatrix} \mathcal{D}_{mi}K_{tj} & -\mathcal{D}_{mi}K_{tj} \end{bmatrix},$$

$$\bar{E}_{mi} = \begin{bmatrix} \mathcal{E}_{mi} & 0 \\ 0 & \mathcal{B}_r \\ 0 & 0 \end{bmatrix}, \ \bar{F}_{mi} = \begin{bmatrix} \mathcal{F}_{mi} & 0 \end{bmatrix}.$$

In this paper, a fuzzy tracking controller (11) which can ensure the stochastic stability of the system(12) and satisfy the given H_∞ tracking performance level need us to design. The following definitions and some related lemmas are necessary to explain.

Lemma 2.1. *(see [28]) Given a matrix $\mathcal{R} > 0$, then the following inequality holds:*

$$-\mathcal{X}^T\mathcal{R}^{-1}\mathcal{X} \leq -\mathcal{X}^T - \mathcal{X} + \mathcal{R}$$

where $\mathcal{X}$ is a matrix with appropriate dimensions.

Definition 2.1. *(see [29])The system (12) is considered to achieve a stochastic stability when $\bar{\omega} = 0$, if for an arbitrary zero initial condition $\zeta(0)$,$\tau_0 \in \mathbb{L}$,$\phi_0 \in \mathcal{T}$, there has a inequality holds:*

$$\mathcal{E}\{\sum_{k=0}^{\infty} \|\zeta(k)\|^2 | \zeta(0), \tau_0, \phi_0\} < \infty$$

The aim of this work is to construct an HMM-based asynchronous fuzzy controller in the form of (7) such that the system (12) is stochastically stable and meets an H_∞ tracking performance index ρ, i.e. for all nonzero $\bar{\omega}(k) \in l_2[0,\infty)$ and under zero initial state, the following inequality holds:

$$\mathcal{E}\{\sum_{k=0}^{\infty} e^T(k)e(k)\} \leq \rho^2 \sum_{k=0}^{\infty} \bar{\omega}^T(k)\omega(k).$$

3. Main Results

This section will firstly analyze the stochastic stability and H_∞ tracking performance for the augment system(12) with AETM and DOS attacks.

Theorem 3.1. *For the given scalars ℓ_j, $\rho > 0$, $\epsilon_2(\epsilon_2 < 1)$, the controller gain matrices $K_{tj}(j \in \mathbb{I})$, and $\gamma \in [0,1]$, the system (12) can achieve a stochastic stability and satisfy the prescribed H_∞ tracking performance index ρ, if $\bar{h}_j - \ell_j h_j \geq 0$, and some real matrices $\mathcal{P}_{mi} = \mathcal{P}_{mi}^T > 0$, $\mathcal{Q}_{mti} = \mathcal{Q}_{mti}^T > 0$,$\mathcal{W} = \mathcal{W}^T > 0$,$\Upsilon_i = \Upsilon_i^T$ with appropriate dimensions such that for $\forall m \in \mathbb{L}$, $\forall t \in \mathbb{M}$,and $\forall f, i \in \mathcal{I}$, the inequalities hold:*

$$\sum_{t \in \mathcal{T}_{\mathcal{K}}^{(m)}} \delta_{mt} \mathcal{Q}_{mti} < \mathcal{P}_{mi}, \quad \sum_{t \in \mathcal{T}_{\mathcal{UK}}^{(m)}} \delta_{mt} \mathcal{Q}_{mti} < \mathcal{P}_{mi} \tag{13}$$

$$\Theta_{mtfij} - \Upsilon_i < 0 \tag{14}$$

$$\ell_i \Theta_{mtfij} + (1 - \ell_i)\Upsilon_i < 0 \tag{15}$$

$$\ell_j \Theta_{mtfij} + \ell_i \Theta_{mtfji} + (1 - \ell_j)\Upsilon_i + (1 - \ell_i)\Upsilon_j < 0, \quad i < j \tag{16}$$

where

$$\Theta_{mtfij} = \begin{bmatrix} -\mathcal{Q}_{mti} + \epsilon_2 I_1^T \mathcal{W} I_1 & \epsilon_2 I_1^T \mathcal{W} & 0 & \tilde{C}_{mtij}^T & \Phi_{15}^T \\ 0 & (\epsilon_2 - 1)\mathcal{W} & 0 & \tilde{D}_{mtij}^T & \Phi_{25}^T \\ 0 & 0 & -\rho^2 I & \tilde{F}_{mi}^T & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\Phi_{55} \end{bmatrix}$$

$$\Phi_{15} = \begin{bmatrix} \sqrt{\pi_{m1}} \tilde{P}_{1f} \tilde{A}_{mtij} \\ \sqrt{\pi_{m2}} \tilde{P}_{2f} \tilde{A}_{mtij} \\ \cdots \\ \sqrt{\pi_{mL}} \tilde{P}_{Lf} \tilde{A}_{mtij} \end{bmatrix}, \quad \Phi_{25} = \begin{bmatrix} \sqrt{\pi_{m1}} \tilde{P}_{1f} \tilde{B}_{mtij} \\ \sqrt{\pi_{m2}} \tilde{P}_{2f} \tilde{B}_{mtij} \\ \cdots \\ \sqrt{\pi_{mL}} \tilde{P}_{Lf} \tilde{B}_{mtij} \end{bmatrix}, \quad \Phi_{55} = \begin{bmatrix} \tilde{P}_{1f} & & & \\ & \tilde{P}_{2f} & & \\ & & \cdots & \\ & & & \tilde{P}_{Lf} \end{bmatrix},$$

$$\tilde{C}_{mtij} = \begin{bmatrix} \bar{C}_{1mtij} \\ \bar{\gamma} \bar{C}_{2mtij} \end{bmatrix}, \quad \tilde{D}_{mtij} = \begin{bmatrix} \bar{D}_{1mtij} \\ \bar{\gamma} \bar{D}_{2mtij} \end{bmatrix}, \quad \tilde{F}_{mi} = \begin{bmatrix} \bar{F}_{mi} \\ 0 \end{bmatrix}, \quad \tilde{A}_{mtij} = \begin{bmatrix} \bar{A}_{1mtij} \\ \bar{\gamma} \bar{A}_{2mtij} \end{bmatrix},$$

$$\tilde{B}_{mtij} = \begin{bmatrix} \bar{B}_{1mtij} \\ \bar{\gamma} \bar{B}_{2mtij} \end{bmatrix}.$$

with $\bar{\gamma} = \sqrt{(\gamma(1-\gamma))}$, $I_1 = [I \quad 0]$.

Proof. Select the Lyapunov functional candidate for the system (12) as follows:

$$V(k) = \zeta^T(k) \mathcal{P}_{mh} \zeta(k) \tag{17}$$

where $\mathcal{P}_{mh}=\sum_{i=1}^{s}h_i\mathcal{P}_{mi}$,$\mathcal{P}_{mi}=diag\{\mathcal{P}_{1mi},\mathcal{P}_{2mi},\mathcal{P}_{3mi}\}$.

When $\bar{\omega}(k)\equiv 0$.From (17), we can calculate the difference and mathematical expectation of $V(k)$:

$$\begin{aligned}\mathcal{E}\{\triangle V(k)\}&=\mathcal{E}\{\zeta^T(k+1)\mathcal{P}_{nh^+}\zeta(k+1)-\zeta^T(k)\mathcal{P}_{mh}\zeta(k)\}\\&=\mathcal{E}\{\zeta_1^T(k)\sum_{t=1}^{M}\delta_{mt}\Omega_{mth}^T\tilde{P}_{mh^+}\Omega_{mth}\zeta_1(k)-\zeta^T(k)\mathcal{P}_{mh}\zeta(k)\}\end{aligned}\tag{18}$$

where

$$\Omega_{mth}=\begin{bmatrix}\bar{A}_{mth} & \bar{B}_{mth}\end{bmatrix},\ \ \zeta_1(k)=\begin{bmatrix}\zeta(k)\\ \varepsilon(k)\end{bmatrix},\ \ \tilde{P}_{mh^+}=diag\{\mathcal{P}_{mh^+},\mathcal{P}_{mh^+}\}.$$

with $\mathcal{P}_{mh^+}=\sum_{n=1}^{L}\pi_{mn}\mathcal{P}_{nh^+}$, $\mathcal{P}_{nh^+}=\sum_{f=1}^{r}h_f^+\mathcal{P}_{nf}$, $\ h^+=h_{k+1}$, $\ \Omega_{mth}=\sum_{i=1}^{s}\sum_{j=1}^{s}h_i\bar{h}_j\Omega_{mtij}$.

From the event-triggered condition (6), we can obtain :

$$\varepsilon^T(k)\mathcal{W}\varepsilon(k)<\varepsilon_2[\xi(k)+\varepsilon(k))^T\mathcal{W}(\xi(k)+\varepsilon(k)]\tag{19}$$

Then, we can get:

$$\begin{aligned}\mathcal{E}\{\triangle V(k)\}<&\mathcal{E}\{\zeta_1^T(k)\sum_{t=1}^{M}\delta_{mt}\Omega_{mth}^T\tilde{P}_{mh^+}\Omega_{mth}\zeta_1(k)\}-\zeta^T(k)\mathcal{P}_{mh}\zeta(k)\\&+\varepsilon_2[\xi(k)+\varepsilon(k))^T\mathcal{W}(\xi(k)+\varepsilon(k)]-\varepsilon^T(k)\mathcal{W}\varepsilon(k)\\&=\mathcal{E}\{\zeta_1^T(k)\sum_{t=1}^{M}\delta_{mt}\Omega_{mth}^T\tilde{P}_{mh^+}\Omega_{mth}\zeta_1(k)\}+\zeta_1^T(k)W_{mh}\zeta_1(k)\end{aligned}\tag{20}$$

where

$$W_{mh}=\begin{bmatrix}-\mathcal{P}_{mh}+\varepsilon_2I_1^T\mathcal{W}I_1 & \varepsilon_2I_1^T\mathcal{W}\\ * & (\varepsilon_2-1)\mathcal{W}\end{bmatrix}$$

By calculating the mathematical expectation of the first term of (20), we can get:

$$\mathcal{E}\{\triangle V(k)(\zeta(k),\tau(k))|\zeta(k),\tau(k),\phi(k)\}=\zeta_1^T(k)\Theta_{1mth}\zeta_1(k)\tag{21}$$

where

$$\Theta_{1mth}=\begin{bmatrix}\tilde{A}_{mth}^T\tilde{P}_{mh^+}\tilde{A}_{mth}-\mathcal{P}_{mh}+\varepsilon_2I_1^T\mathcal{W}I_1 & \tilde{A}_{mth}^T\tilde{P}_{mh^+}\tilde{B}_{mth}+\varepsilon_2I_1^T\mathcal{W}\\ * & \tilde{B}_{mth}^T\tilde{P}_{mh^+}\tilde{B}_{mth}+(\varepsilon_2-1)\mathcal{W}\end{bmatrix}$$

with $\tilde{A}_{mth}=\sum_{i=1}^{s}\sum_{j=1}^{s}h_i\hat{h}_j\tilde{A}_{mtij}$, $\ \tilde{B}_{mth}=\sum_{i=1}^{s}\sum_{j=1}^{s}h_i\bar{h}_j\tilde{B}_{mtij}$.

In order to make full use of MFs information and decrease the conservativeness, the matrices Υ_i are introduced. Based on the property of MFs, i.e., $\sum_{i=1}^{s}h_i-\sum_{j=1}^{s}\bar{h}_j=0$. Then, according to [30], [31], [32], [33], combining (14),(15),(16) with the system (12) and considering $\bar{h}_j-\ell_jh_j\geq 0$,we obtain:

$$\sum_{i=1}^{s}\sum_{j=1}^{s}h_i(h_j-\hat{h}_j)\Upsilon_i=\sum_{i=1}^{s}h_i(\sum_{j=1}^{s}h_j-\sum_{j=1}^{s}\bar{h}_j)\Upsilon_i=0\tag{22}$$

Then , the following relationship is obtained:

$$\begin{aligned}
\Theta_{mth} =& \sum_{f=1}^{s}\sum_{i=1}^{s}\sum_{j=1}^{s} h_f^+ h_i \bar{h}_j \Theta_{mtfij} \\
=& \sum_{f=1}^{s}\sum_{i=1}^{s}\sum_{j=1}^{s} h_f^+ h_i \bar{h}_j \Theta_{mtfij} + \sum_{f=1}^{s}\sum_{i=1}^{s}\sum_{j=1}^{s} h_f^+ h_i (h_j - \bar{h}_j + \ell_j h_j - \ell_j \bar{h}_j)\Upsilon_i \\
=& \sum_{f=1}^{s} h_f^+ [\sum_{i=1}^{s} h_i^2 (\ell_j \Theta_{mtfij} + (1-\ell_j)\Upsilon_i) + \sum_{i=1}^{s}\sum_{j=1}^{s} h_i(\bar{h}_j - \ell_j h_j)(\Theta_{mtfij} - \Upsilon_i) \\
&+ \sum_{i=1}^{s-1}\sum_{j=1}^{s} h_i h_j (\ell_j \Theta_{mtfij} + \ell_i \Theta_{mtfij} + (1-\ell_j)\Upsilon_i + (1-\ell_i)\Upsilon_j)] < 0
\end{aligned}$$

where

$$\Theta_{mth} = \begin{bmatrix} -\mathcal{Q}_{mth} + \varepsilon_2 I_1^T \mathcal{W} I_1 & \varepsilon_2 I_1^T \mathcal{W} & 0 & \tilde{C}_{mth}^T & \tilde{\Phi}_{15}^T \\ 0 & (\varepsilon_2 - 1)\mathcal{W} & 0 & \tilde{D}_{mth}^T & \tilde{\Phi}_{25}^T \\ 0 & 0 & -\rho^2 I & \tilde{F}_{mh}^T & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\tilde{\Phi}_{55} \end{bmatrix}$$

$$\tilde{\Phi}_{15} = \begin{bmatrix} \sqrt{\pi_{m1}}\tilde{P}_{1h^+}\tilde{A}_{mth} \\ \sqrt{\pi_{m2}}\tilde{P}_{2h^+}\tilde{A}_{mth} \\ \cdots \\ \sqrt{\pi_{mL}}\tilde{P}_{Lh^+}\tilde{A}_{mth} \end{bmatrix}, \quad \Phi_{25} = \begin{bmatrix} \sqrt{\pi_{m1}}\tilde{P}_{1h^+}\tilde{B}_{mth} \\ \sqrt{\pi_{m2}}\tilde{P}_{2h^+}\tilde{B}_{mth} \\ \cdots \\ \sqrt{\pi_{mL}}\tilde{P}_{Lh^+}\tilde{B}_{mth} \end{bmatrix}, \quad \Phi_{55} = \begin{bmatrix} \tilde{P}_{1h^+} & & & \\ & \tilde{P}_{2h^+} & & \\ & & \cdots & \\ & & & \tilde{P}_{Lh^+} \end{bmatrix}.$$

Base on the complement of Schur and $\Theta_{mth} < 0$, we can obtain that:

$$\Theta_{2mth} = \tilde{W}_{mh} + \tilde{\Omega}_{mth} < 0 \tag{23}$$

where

$$\tilde{W}_{mh} = \begin{bmatrix} -\mathcal{Q}_{mh} + \varepsilon_2 I_1^T \mathcal{W} I_1 & \varepsilon_2 I_1^T \mathcal{W} \\ * & (\varepsilon_2 - 1)\mathcal{W} \end{bmatrix},$$

$$\tilde{\Omega}_{mth} = \begin{bmatrix} \tilde{A}_{mth}^T \tilde{P}_{mh^+}\tilde{A}_{mth} & \tilde{A}_{mth}^T \tilde{P}_{mh^+}\tilde{B}_{mth} \\ * & \tilde{B}_{mth}^T \tilde{P}_{mh^+}\tilde{B}_{mth} \end{bmatrix}.$$

and

$$\Pi_{1mth}^T \tilde{P}_{mh^+}\Pi_{1mth} + \Pi_{2mth}^T \Pi_{2mth} + \Pi_{3mth} < 0 \tag{24}$$

where

$$\Pi_{1mth} = \begin{bmatrix} \tilde{A}_{mth} & \tilde{B}_{mth} & 0 \end{bmatrix}, \quad \Pi_{2mth} = \begin{bmatrix} \tilde{C}_{mth} & \tilde{D}_{mth} & \tilde{F}_{mh} \end{bmatrix},$$

$$\Pi_{3mth} = \begin{bmatrix} -\mathcal{Q}_{mh} + \varepsilon_2 I_1^T \mathcal{W} I_1 & \varepsilon_2 I_1^T \mathcal{W} & 0 \\ * & (\varepsilon_2 - 1)\mathcal{W} & 0 \\ 0 & 0 & -\rho^2 I \end{bmatrix}$$

with

$$\tilde{C}_{mth} = \sum_{i=1}^{s}\sum_{j=1}^{s} h_i \bar{h}_j \tilde{C}_{mtij}, \quad \tilde{D}_{mth} = \sum_{i=1}^{s}\sum_{j=1}^{s} h_i \bar{h}_j \tilde{D}_{mtij}, \quad \tilde{F}_{mh} = \sum_{i=1}^{s} h_i \tilde{F}_{mi}.$$

From the condition(13),we can get:

$$\sum_{t\in\mathcal{T}_{\mathcal{K}}^{(m)}} \delta_{mt}\mathcal{Q}_{mth} < \mathcal{P}_{mh}, \quad \sum_{t\in\mathcal{T}_{\mathcal{UK}}^{(m)}} \delta_{mt}\mathcal{Q}_{mth} < \mathcal{P}_{mh} \tag{25}$$

Combining (23),(24) and (25),it can be derived that:

$$\sum_{t\in\mathbb{M}} \delta_{mt}\tilde{\Omega}_{mth} + W_{mh} < \tilde{\Omega}_{mth} + \tilde{W}_{mh} = \Theta_{2mth} < 0 \tag{26}$$

and

$$\sum_{t\in\mathbb{M}} \delta_{mt}[\Pi_{1mth}^T \tilde{P}_{mh^+} \Pi_{1mth} + \Pi_{2mth}^T \Pi_{2mth} + \tilde{\Pi}_{3mth}] < 0 \tag{27}$$

where

$$\tilde{\Pi}_{3mth} = \begin{bmatrix} -\mathcal{P}_{mh} + \varepsilon_2 I_1^T \mathcal{W} I_1 & \varepsilon_2 I_1^T \mathcal{W} & 0 \\ * & (\varepsilon_2 - 1)\mathcal{W} & 0 \\ 0 & 0 & -\rho^2 I \end{bmatrix}$$

According to (26),it yields

$$\begin{aligned}\mathcal{E}\{\triangle V(k)(\zeta(k),\tau(k))|\zeta(k),\tau(k),\phi(k)\} =& \mathcal{E}\{\zeta^T(k+1)P_{nh^+}\zeta(k+1) - \zeta^T(k)P_{mh}\zeta(k)\} \\ <& -\tilde{\lambda}_{min}[-\sum_{t\in\mathbb{M}} \delta_{mt}\tilde{\Omega}_{mth} - W_{mh}]\zeta_1^T(k)\zeta_1(k) \\ <& -\sigma\zeta_1^T(k)\zeta_1(k) < 0\end{aligned}$$

where $\sigma = \inf\{\tilde{\lambda}_{min}(-\sum_{t\in\mathbb{M}} \delta_{mt}\tilde{\Omega}_{mth} - W_{mh})\}$. Then, it can be concluded that:

$$\mathcal{E}\{\sum_{k=0}^{\infty} \|\zeta(k)\|^2 |\zeta(0),\tau_0,\phi_0\} < \frac{1}{\sigma}\mathcal{E}\{V(0) - V(\infty)\} < \frac{1}{\sigma}\mathcal{E}\{V(0)\} < \infty$$

Up to now, we can obtain that the system (12) has achieved a stochastic stability. The next work is that the H_∞ performance for the tracking control system (27) should be considered:

$$\mathcal{E}\{\sum_{k=0}^{\infty} e^T(k)e(k) - \rho^2\bar{\omega}^T(k)\bar{\omega}(k)\}$$

$$
\begin{aligned}
&< \mathcal{E}\{\sum_{k=0}^{\infty} \triangle V(k) + e^T(k)e(k) - \rho^2 \bar{\omega}^T(k)\bar{\omega}(k)\} \\
&< \sum_{k=0}^{\infty} \zeta_2^T(k) \sum_{t\in\mathbb{M}} \delta_{mt}(\Pi_{1mth}^T \tilde{P}_{mh+}\Pi_{1mth} + \Pi_{2mth}^T\Pi_{2mth} + \tilde{\Pi}_{3mth})\zeta_2(k) \\
&< 0 \qquad (28)
\end{aligned}
$$

where $\zeta_2^T(k) = [\zeta_1^T(k)\ \ \bar{\omega}^T(k)]^T$. Thus, i.e.

$$
\sum_{k=0}^{\infty} \mathcal{E}\{e^T(k)e(k)\} < \rho^2 \bar{\omega}^T(k)\bar{\omega}(k)
$$

The above formula explains that for $\forall \bar{\omega}(t) \in l_2[0,\infty)$, the system (12) satisfies the prescribed H_∞ tracking performance ρ. Thus the proof is completed.

Remark 3.1 In system (12), $\bar{h}_j(\varrho_(k)) \neq h_i(\kappa(k))$. Thus, we can not use the conventional parallel distribute compensation(PDC) strategy. Therefore, this paper adopted an IPM approach to solve the mismatch problems. Due to $\sum_{i=1}^{s}\sum_{j=1}^{s} h_i h_j = 1$ and $\sum_{i=1}^{s}\sum_{j=1}^{s} h_i \bar{h}_j = 1$, those properties of the membership function can bring less conservative stability.

In **Theorem 3.1**, some sufficient criteria which can guarantee the stochastic stability and achieve the given H_∞ performance for the tracking control system (12) are derived. However, based on the theory analysis results in Theorem 3.1, the controller gains K_{tj} can not be parameterized directly because of the presence of nonlinear terms such as $\tilde{P}_{Lf}\tilde{A}_{mtij}$. For the purpose of using the method of LMIs to solve the problem, the results of the controller gains are calculated in the following theorem.

Theorem 3.2. *For some given scalars ℓ_j, $\rho > 0$, $\epsilon_2(\epsilon_2 < 1)$, and $\gamma \in [0,1]$, the controller gain matrices $K_{t,j}$, the system (12) can achieve a stochastic stability and satisfy the prescribed H_∞ tracking performance ρ, if there exist the inequality $\bar{h}_j - \ell_j h_j \geq 0$, and matrices $\hat{P}_{mi} = \hat{P}_{mi}^T > 0$,$\hat{Q}_{mti} = \hat{Q}_{mti}^T > 0$,$\mathcal{W} = \mathcal{W}^T > 0$,$\hat{\Upsilon}_i = \hat{\Upsilon}_i^T$,Λ_t,$\hat{K}_{t,j}$ with appropriate dimensions such that for $p,q = 1,2$, $\forall m \in \mathbb{L}$, $t \in \mathbb{M}$, and $\forall f \in \mathcal{I}$, $i,j \in \mathcal{I}$,the inequalities hold:*

$$
\begin{bmatrix} -\hat{P}_{mi} & \mathbb{P}_{mi}^{(p)} \\ * & -\mathbb{Q}_{mi}^{(q)} \end{bmatrix} < 0 \qquad (29)
$$

$$
\hat{\Theta}_{mtfij} - \hat{\Upsilon}_i < 0 \qquad (30)
$$

$$
\ell_i\hat{\Theta}_{mtfii} + (1-\ell_i)\hat{\Upsilon}_i < 0 \qquad (31)
$$

$$
\ell_j\hat{\Theta}_{mtfij} + \ell_i\hat{\Theta}_{mtfji} + (1-\ell_j)\hat{\Upsilon}_i + (1-\ell_i)\hat{\Upsilon}_j < 0,\ \ i<j \qquad (32)
$$

$$
Q_{mti} - \varepsilon_2 I_1^T \mathcal{W} I_1 > 0 \qquad (33)
$$

where

$$\begin{cases} \mathbb{P}_{mi}^{(1)} \triangleq \hat{P}_{mi}[\sqrt{\delta_{m\mathcal{K}_1^{(m)}}} \ \ \sqrt{\delta_{m\mathcal{K}_2^{(m)}}} \ \cdots \ \sqrt{\delta_{m\mathcal{K}_{s_1}^{(m)}}}] \\ \mathbb{Q}_{mi}^{(1)} \triangleq diag\{\hat{Q}_{m\mathcal{K}_1^{(m)}i}, \ \hat{Q}_{m\mathcal{K}_2^{(m)}i}, \cdots, \hat{Q}_{m\mathcal{K}_{s_1}^{(m)}i}\} \\ \mathbb{P}_{mi}^{(2)} \triangleq \hat{P}_{mi}[1_{m\mathcal{U}\mathcal{K}_1^{(m)}} \ \ 1_{m\mathcal{U}\mathcal{K}_2^{(m)}} \ \cdots \ 1_{m\mathcal{U}\mathcal{K}_{s_2}^{(m)}}] \\ \mathbb{Q}_{mi}^{(2)} \triangleq diag\{\hat{Q}_{m\mathcal{U}\mathcal{K}_1^{(m)}i}, \ \hat{Q}_{m\mathcal{U}\mathcal{K}_2^{(m)}i}, \cdots, \ \hat{Q}_{m\mathcal{U}\mathcal{K}_{s_2}^{(m)}i}\} \end{cases}$$

$$\hat{\Theta}_{mtfij} = \begin{bmatrix} \hat{\Phi}_{11} & \hat{\Phi}_{12} & 0 & \hat{C}_{mtij}^T & \hat{\Phi}_{15}^T \\ 0 & \hat{\Phi}_{22} & 0 & \hat{D}_{mtij}^T & \hat{\Phi}_{25}^T \\ 0 & 0 & -\rho^2 I & \tilde{F}_{mi}^T & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\hat{\Phi}_{55} \end{bmatrix}$$

$$\hat{\Phi}_{11} = \tilde{Q}_{mti} - \Lambda_{1t}^T - \Lambda_{1t}, \ \ \tilde{Q}_{mti} = (\mathcal{Q}_{mti} - \varepsilon_2 I_1^T \mathcal{W} I_1)^{-1},$$

$$\hat{\Phi}_{12} = \Lambda_{1t}^T + \Lambda_{2t} - \hat{W}, \ \ \hat{W} = (\varepsilon_2 I_1^T \mathcal{W})^{-1},$$

$$\hat{\Phi}_{22} = (\tilde{W} - \Lambda_{2t}^T - \Lambda_{2t}), \ \ \Lambda_{1t} = diag\{\Lambda_t \ \ \Lambda_t \ \ \Lambda_t\},$$

$$\Lambda_{2t} = diag\{\Lambda_t \ \ \Lambda_t\}, \ \ \tilde{W} = [(1-\varepsilon_2)\mathcal{W}]^{-1}$$

$$\hat{\Phi}_{15} = \begin{bmatrix} \sqrt{\pi_{m1}}\hat{A}_{mtij} \\ \sqrt{\pi_{m2}}\hat{A}_{mtij} \\ \cdots \\ \sqrt{\pi_{mL}}\hat{A}_{mtij} \end{bmatrix}, \ \ \hat{\Phi}_{25} = \begin{bmatrix} \sqrt{\pi_{m1}}\hat{B}_{mtij} \\ \sqrt{\pi_{m2}}\hat{B}_{mtij} \\ \cdots \\ \sqrt{\pi_{mL}}\hat{B}_{mtij} \end{bmatrix}, \ \ \hat{\Phi}_{55} = \begin{bmatrix} \hat{P}_{1f} & & & \\ & \hat{P}_{2f} & & \\ & & \cdots & \\ & & & \hat{P}_{Lf} \end{bmatrix}$$

$$\hat{A}_{mtij} = \begin{bmatrix} \hat{A}_{1mtij} \\ \bar{\gamma}\hat{A}_{2mtij} \end{bmatrix}, \ \ \hat{B}_{mtij} = \begin{bmatrix} \hat{B}_{1mtij} \\ \bar{\gamma}\hat{B}_{2mtij} \end{bmatrix}, \ \ \hat{C}_{mtij} = \begin{bmatrix} \hat{C}_{1mtij} \\ \bar{\gamma}\hat{C}_{2mtij} \end{bmatrix}, \ \ \hat{D}_{mtij} = \begin{bmatrix} \hat{D}_{1mtij} \\ \bar{\gamma}\hat{D}_{2mtij} \end{bmatrix},$$

$$\hat{A}_{1mtij} = \begin{bmatrix} \mathcal{A}_{mi}\Lambda_t + \gamma\mathcal{B}_{mi}\hat{K}_{tj} & -\gamma\mathcal{B}_{mi}\hat{K}_{tj} & \hat{K}_{tj} - \gamma\mathcal{B}_{mi}\hat{K}_{tj} \\ 0 & \mathcal{A}_r\Lambda_t & 0 \\ \gamma\hat{K}_{tj} & -\gamma\hat{K}_{tj} & \hat{K}_{tj} - \gamma K_{tj} \end{bmatrix},$$

$$\hat{A}_{2mtij} = \begin{bmatrix} \mathcal{B}_{mi}\hat{K}_{tj} & -\mathcal{B}_{mi}\hat{K}_{tj} & -\mathcal{B}_{mi}\hat{K}_{tj} \\ 0 & 0 & 0 \\ \hat{K}_{tj} & -\hat{K}_{tj} & -\hat{K}_{tj} \end{bmatrix},$$

$$\hat{B}_{1mtij} = \begin{bmatrix} \gamma\mathcal{B}_{mi}\hat{K}_{tj} & -\gamma\mathcal{B}_{mi}\hat{K}_{tj} \\ 0 & 0 \\ \gamma\hat{K}_{tj} & -\gamma\hat{K}_{tj} \end{bmatrix}, \ \ \hat{B}_{2mtij} = \begin{bmatrix} \mathcal{B}_{mi}\hat{K}_{tj} & -\mathcal{B}_{mi}\hat{K}_{tj} \\ 0 & 0 \\ \hat{K}_{tj} & -\hat{K}_{tj} \end{bmatrix},$$

$$\hat{C}_{1mtij} = \begin{bmatrix} \mathcal{C}_{mi}\Lambda_t + \mathcal{D}_{mi}\hat{K}_{tj} & -\mathcal{C}_r\Lambda_t - \gamma\mathcal{D}_{mi}\hat{K}_{tj} & \mathcal{D}_{mi}\hat{K}_{tj} - \gamma\mathcal{D}_{mi}\hat{K}_{tj} \end{bmatrix},$$

$$\hat{C}_{2mtij} = \begin{bmatrix} 0 & -\mathcal{D}_{mi}\hat{K}_{tj} & -\mathcal{D}_{mi}\hat{K}_{tj} \end{bmatrix}, \ \ \hat{D}_{1mtij} = \begin{bmatrix} \gamma\mathcal{D}_{mi}\hat{K}_{tj} & -\gamma\mathcal{D}_{mi}\hat{K}_{tj} \end{bmatrix},$$
$$\hat{D}_{2mtij} = \begin{bmatrix} \mathcal{D}_{mi}\hat{K}_{tj} & -\mathcal{D}_{mi}\hat{K}_{tj} \end{bmatrix}, \ \ \hat{P}_{Lf} = \tilde{P}_{Lf}^{-1}, \ \ \hat{\Upsilon}_i = diag\{\Upsilon_{1i}, \Upsilon_{2i}, 0\}.$$

Moreover, the H_∞ fuzzy controller gains in(8)can be obtained by:

$$K_{tj} = \hat{K}_{tj}\Lambda_t^{-1}$$

.

Proof. Due to $\hat{P}_{mi} = \hat{P}_{mi}^T > 0$, $\hat{Q}_{mti} = \hat{Q}_{mti}^T > 0$, let $\hat{P}_{mi} = \mathcal{P}_{mi}^{-1}$, $\hat{Q}_{mti} = \mathcal{Q}_{mti}^{-1}$. Considering the Schur complement and (29), it can be inferred that :

$$\begin{cases} \sum_{t\in\mathcal{T}_{\mathcal{K}}^{(m)}} \delta_{mt}\hat{P}_{mi}\hat{Q}_{mti}^{-1}\hat{P}_{mi}^T < \hat{P}_{mi} \\ \sum_{t\in\mathcal{T}_{\mathcal{UK}}^{(m)}} \delta_{mt}\hat{P}_{mi}\hat{Q}_{mti}^{-1}\hat{P}_{mi}^T < \hat{P}_{mi} \end{cases} \tag{34}$$

Pre and post-multiplying (34) by $\hat{P}_{mi}^{-1}$, one can obtain (13). This means that (29) is equivalent to (13). Then, pre and post-multiplying (30) by $diag\{I, I, I, I, \hat{\Phi}_{55}^{-1}\}$ and its transpose, respectively. It leads to:

$$\hat{\Theta}_{mtfij}^{(1)} - \hat{\Upsilon}_i < 0 \tag{35}$$

where

$$\hat{\Theta}_{mtfij}^{(1)} = \begin{bmatrix} \hat{\Phi}_{11} & \hat{\Phi}_{12} & 0 & \hat{C}_{mtij}^T & \bar{\Phi}_{15}^T \\ 0 & \hat{\Phi}_{22} & 0 & \hat{D}_{mtij}^T & \bar{\Phi}_{25}^T \\ 0 & 0 & -\rho^2 I & \tilde{F}_{mi}^T & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\Phi_{55} \end{bmatrix}$$

with

$$\bar{\Phi}_{15} = \begin{bmatrix} \sqrt{\pi_{m1}}\tilde{P}_{1f}\hat{A}_{mtij} \\ \sqrt{\pi_{m2}}\tilde{P}_{2f}\hat{A}_{mtij} \\ \cdots \\ \sqrt{\pi_{mL}}\tilde{P}_{Lf}\hat{A}_{mtij} \end{bmatrix}, \ \ \bar{\Phi}_{25} = \begin{bmatrix} \sqrt{\pi_{m1}}\tilde{P}_{1f}\hat{B}_{mtij} \\ \sqrt{\pi_{m2}}\tilde{P}_{2f}\hat{B}_{mtij} \\ \cdots \\ \sqrt{\pi_{mL}}\tilde{P}_{Lf}\hat{B}_{mtij} \end{bmatrix}.$$

Notice that $(\mathcal{Q}_{mti} - \varepsilon_2 I_1^T \mathcal{W} I_1) > 0$, hence the following inequality holds:

$$[(\mathcal{Q}_{mti} - \varepsilon_2 I_1^T \mathcal{W} I_1)^{-1} - \Lambda_{1t}^T](\mathcal{Q}_{mti} - \varepsilon_2 I_1^T \mathcal{W} I_1)[(\mathcal{Q}_{mti} - \varepsilon_2 I_1^T \mathcal{W} I_1)^{-1} - \Lambda_{1t}] > 0$$

According to the **Lemma 2.1**, it can lead to:

$$-\Lambda_{1t}^T(\mathcal{Q}_{mti} - \varepsilon_2 I_1^T \mathcal{W} I_1)\Lambda_{1t} < (\mathcal{Q}_{mti} - \varepsilon_2 I_1^T \mathcal{W} I_1)^{-1} - \Lambda_{1t}^T - \Lambda_{1t} \tag{36}$$

Similarly, we can get:

$$\begin{aligned} & -\Lambda_{2t}^T[(1-\varepsilon_2)\mathcal{W}]\Lambda_{2t} < [(1-\varepsilon_2)\mathcal{W}]^{-1} - \Lambda_{2t}^T - \Lambda_{2t} \ , \\ & \Lambda_{1t}^T \varepsilon_2 I_1^T \mathcal{W} \Lambda_{2t} > \Lambda_{1t} + \Lambda_{2t} - (\varepsilon_2 I_1^T \mathcal{W})^{-1} \end{aligned} \tag{37}$$

From (35),(36) ,(37), one can obtain:

$$\hat{\Theta}_{mtfij}^{(2)} - \hat{\Upsilon}_i < 0 \tag{38}$$

where

$$\hat{\Theta}_{mtfij}^{(2)} = \begin{bmatrix} -\Lambda_{1t}^T(\mathcal{Q}_{mti} - \varepsilon_2 I_1^T \mathcal{W} I_1)\Lambda_{1t} & \Lambda_{1t}^T \varepsilon_2 I_1^T \mathcal{W} \Lambda_{2t} & 0 & \hat{C}_{mtij}^T & \bar{\Phi}_{15}^T \\ 0 & \Lambda_{2t}^T[(\varepsilon_2 - 1)\mathcal{W}]\Lambda_{2t} & 0 & \hat{D}_{mtij}^T & \bar{\Phi}_{25}^T \\ 0 & 0 & -\rho^2 I & \tilde{F}_{mi}^T & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\Phi_{55} \end{bmatrix}$$

Multiplying both side of (38) with the matrix $diag\{\Lambda_{1t}^{-T}, \Lambda_{2t}^{-T}, I, I, I\}$ and its transpose, respectively. (14) can be obtained. This indicates that (14) can be deduced from (30).

Using the similar method, it can be obtained that (15),(16) is equivalent to (31) ,(32). Up to now, the proof is completed.

Remark 3.2 It is worth emphasizing that the method adopted in this paper can calculate the matrix $\hat{K}_{tj}$ through the technique of LMI. Then, we can obtain the controller gains quickly, which greatly decrease the complexity of the computation. In this paper, the information of the conditional probability between the system and the controller is partially unknown. However, the method proposed in Section 3 is also applicable to the situation where the conditional probability is completely known. The specific description is summarized in **Corollary 3.1**.

Corollary 3.1. *For some given scalars ℓ_j, $\rho > 0$, $\epsilon_2(\epsilon_2 < 1)$, and $\gamma \in [0,1]$, the controller gain matrices $K_{t,j}$, the system (12) can achieve a stochastic stability and satisfy the prescribed H_∞ tracking performance ρ, if there exist the inequality $\bar{h}_j - \ell_j h_j \geq 0$, and matrices $\hat{P}_{mi} = \hat{P}_{mi}^T > 0$,$\hat{Q}_{mti} = \hat{Q}_{mti}^T > 0$,$\mathcal{W} = \mathcal{W}^T > 0$,$\hat{\Upsilon}_i = \hat{\Upsilon}_i^T$,Λ_t,$\hat{K}_{t,j}$ with appropriate dimensions such that for $p, q = 1, 2$, $\forall m \in \mathbb{L}$, $t \in \mathbb{M}$, and $\forall f \in \mathcal{I}$, $i, j \in \mathcal{I}$, the inequalities hold:*

$$\begin{bmatrix} -\hat{P}_{mi} & \mathbb{P}_{mi} \\ * & -\mathbb{Q}_{mi} \end{bmatrix} < 0$$

$$\hat{\Theta}_{mtfij} - \hat{\Upsilon}_i < 0$$

$$\ell_i \hat{\Theta}_{mtfii} + (1-\ell_i)\hat{\Upsilon}_i < 0$$

$$\ell_j \hat{\Theta}_{mtfij} + \ell_i \hat{\Theta}_{mtfji} + (1-\ell_j)\hat{\Upsilon}_i + (1-\ell_i)\hat{\Upsilon}_j < 0, \ \ i < j$$

$$Q_{mti} - \varepsilon_2 I_1^T \mathcal{W} I_1 > 0$$

where

$$\begin{cases} \mathbb{P}_{mi} \triangleq \hat{P}_{mi}[\sqrt{\delta_{m1}} \ \ \sqrt{\delta_{m2}} \ \ \cdots \ \ \sqrt{\delta_{mT}}] \\ \mathbb{Q}_{mi} \triangleq diag\{\hat{Q}_{m1i}, \ \hat{Q}_{m2i}, \cdots, \hat{Q}_{mTi}\} \end{cases}$$

Moreover, the H_∞ *fuzzy controller gains in(8)can be obtained by:*

$$K_{tj} = \hat{K}_{tj}\Lambda_t^{-1}$$

and the definition of other parameters are the same as those of ***Theorem 3.2****.*

4. Examples

To illustrate the effectiveness of the proposed methods, this paper cites two simulation examples in this section.

Example 1(numerical example): For the system (2), three operational modes are selected. The corresponding three-dimensional transition probability matrix is :

$$\Pi_{33} = \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0.1 & 0.6 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$$

The condition probability matrices are given except that some elements is partially unknown. Here lists two cases: partially known (**Theorem 3.2**) and completely known (**Corollary 3.1**) , respectively ($T = 3$). For the following simulation, the two cases are named as Case 1 and Case 2 whose corresponding condition probability matrices are as follows:

$$\Omega^{(1)} = \begin{bmatrix} 0.3 & 0.3 & 0.4 \\ 0.2 & 0.1 & 0.7 \\ 0.5 & 0.4 & 0.1 \end{bmatrix}, \ \Omega^{(2)} = \begin{bmatrix} 0.3 & ? & ? \\ ? & ? & 0.7 \\ 0.5 & ? & ? \end{bmatrix}$$

Consider a 2-rules fuzzy MJS (1):

Fuzzy Rule 1 :IF $\eta_1(k)$ is $-m_{\tau_k}$, then

$$\begin{cases} \eta(k+1) = \mathcal{A}_{\tau_k,1}\eta(k) + \mathcal{B}_{\tau_k,1}u(k) + \mathcal{E}_{\tau_k,1}\omega(k) \\ z(k) = \mathcal{C}_{\tau_k,1}\eta(k) + \mathcal{D}_{\tau_k,1}u(k) + \mathcal{F}_{\tau_k,1}\omega(k) \end{cases} \tag{39}$$

Fuzzy Rule 2 :IF $\eta_2(k)$ is m_{τ_k}, then

$$\begin{cases} \eta(k+1) = \mathcal{A}_{\tau_k,2}\eta(k) + \mathcal{B}_{\tau_k,2}u(k) + \mathcal{E}_{\tau_k,2}\omega(k) \\ z(k) = \mathcal{C}_{\tau_k,2}\eta(k) + \mathcal{D}_{\tau_k,2}u(k) + \mathcal{F}_{\tau_k,2}\omega(k) \end{cases} \tag{40}$$

Let $m_1 = m_2 = 0.5$ and the values of the coefficient matrix are as follows:

$$\mathcal{A}_{11} = \mathcal{A}_{12} = \begin{bmatrix} 0.15 & 0.01 \\ 0.01 & 0 \end{bmatrix}, \ \mathcal{A}_{21} = \mathcal{A}_{22} = \begin{bmatrix} 0.25 & 0.02 \\ 0.03 & 0 \end{bmatrix}, \ \mathcal{A}_{31} = \mathcal{A}_{32} = \begin{bmatrix} 0.45 & 0.03 \\ 0.05 & 0 \end{bmatrix},$$

$$\mathcal{B}_{11} = \mathcal{B}_{12} = \begin{bmatrix} 0.11 \\ 0.05 \end{bmatrix}, \ \mathcal{B}_{21} = \mathcal{B}_{22} = \begin{bmatrix} 0.21 \\ 0.06 \end{bmatrix}, \ \mathcal{B}_{31} = \mathcal{B}_{32} = \begin{bmatrix} 0.41 \\ 0.08 \end{bmatrix},$$

$$\mathcal{E}_{11} = \mathcal{E}_{12} = \begin{bmatrix} 0.15 \\ 0.05 \end{bmatrix}, \ \mathcal{E}_{21} = \mathcal{E}_{22} = \begin{bmatrix} 0.25 \\ 0.06 \end{bmatrix}, \ \mathcal{E}_{31} = \mathcal{E}_{32} = \begin{bmatrix} 0.45 \\ 0.08 \end{bmatrix},$$

$$\mathcal{C}_{11} = \mathcal{C}_{12} = \begin{bmatrix} 0 & 0.05 \end{bmatrix}, \ \mathcal{C}_{21} = \mathcal{C}_{22} = \begin{bmatrix} 0 & 0.01 \end{bmatrix}, \ \mathcal{C}_{31} = \mathcal{C}_{32} = \begin{bmatrix} 0 & 0.08 \end{bmatrix},$$

$$\mathcal{D}_{11} = \mathcal{D}_{12} = 0 \ , \ \mathcal{D}_{21} = \mathcal{D}_{22} = 0.1 \ , \ \mathcal{D}_{31} = \mathcal{D}_{32} = 0.5 \ ,$$

$$\mathcal{F}_{11} = \mathcal{F}_{12} = 0 \ , \ \mathcal{F}_{21} = \mathcal{F}_{22} = 0.2 \ , \ \mathcal{F}_{31} = \mathcal{F}_{32} = 0.5.$$

The reference model is given by:

$$\mathcal{A}_r = \begin{bmatrix} -0.1 & 0.1 \\ -0.1 & 0.005 \end{bmatrix}, \ \mathcal{B}_r = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \ \mathcal{C}_r = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}.$$

What's more, the MFs of the system and the controller are selected as:

$$\begin{cases} h_1(\eta_1(k)) = 0.2(1 - \frac{\eta_1(k)}{m_{\tau_k}}), \ \ h_2(\eta_1(k)) = 1 - h_1(\eta_1(k)) \\ \tilde{h}_1(\eta_{r1}(k)) = 0.2(1 - \frac{\eta_{r1}(k)}{m_{\tau_k}}), \ \ \tilde{h}_2(\eta_{r1}(k)) = 1 - \hat{h}_1(\eta_{r1}(k)) \end{cases}$$

The external disturbance is selected as $\omega = 0.3sin(0.9k)$, and the reference input are assumed as $r(k) = 0.1cos(0.8k)$. The weight coefficients are given as $\ell_1 = \ell_2 = 0.5$. The initial value of the state vector are selected as $\eta(0) = [0.05 \quad -0.01]^T$, $\eta_r(0) = [0.02 \ \ 0.01]^T$. Both the system mode and the controller mode evolution is shown in Fig.2. Fig.3 shows the output trajectories of system (2) and system (3) with above parameters but without a controller. It can be seen that the system is not stable. When the designed controllers are added to the system, from Fig.5, we can see that the state of the two fuzzy rules achieves a stochastic stability. Fig.6 depicts the state trajectories of $\zeta(k)$ in system (12). It can be obtained that the system (12) achieves the stochastic stability with the controllers in **Theorem 3.2** around $15th$ second. For the above two cases, Figs.7-10 depict both the trajectories of the system output and the tracking errors, respectively. From these figures, it can be obtained that the system output $z(k)$ can track the reference output $z_r(k)$ perfectly with the constructed controllers both in two cases. What's more, when the condition probability information is unknown, the overall tracking effect isn't inferior.Fig.11 depicts the time instants of successful DOS attacks. Fig.12 shows the release instants and release intervals of AET generator. In Fig.12, the triggered times are 105. Through computation, there are 11.3% of the sampled signals be transmitted to the controller. Additionally, it can be obtained that the maximal release interval is 1.100 and the average release interval is 0.432.

Table 1: Optimal Performance ρ^* For Different β under two cases

β	$\rho^*(Case1)$	$\rho^*(Case2)$
0.1	0.0015	0.00039
0.2	0.00023	0.0028
0.3	0.00098	0.0039
0.5	0.0038	0.0056
0.8	0.0049	0.0099
1	0.0098	0.0103

For the above two cases, let γ be the packet arriving rate and assign it a value 0.75. The controller gains can be obtained from **Corollary 3.1** and **Theorem 3.2**, respectively. Firstly, according to the solution of LMIs (30)-(32) and **Theorem 3.2**. Meanwhile, the desired fuzzy controller gains with partially known condition probability can be derived as follows:

$$K_{11} = \begin{bmatrix} 0.1809 & -9.6007 \end{bmatrix}, \; K_{12} = \begin{bmatrix} -0.0267 & 3.6227 \end{bmatrix}, \; K_{21} = \begin{bmatrix} -0.0827 & 3.6876 \end{bmatrix},$$
$$K_{22} = \begin{bmatrix} -0.1161 & 2.0200 \end{bmatrix}, \; K_{31} = \begin{bmatrix} -0.0105 & 3.8608 \end{bmatrix}, \; K_{32} = \begin{bmatrix} -0.0098 & 4.0270 \end{bmatrix}.$$

Then, according to the solution to LMIs in **Corollary 3.1**, the corresponding fuzzy controller gains when the condition probability matrix Ω is completely known can be derived as follows:

$$K_{11} = \begin{bmatrix} 0.0908 & -8.7006 \end{bmatrix}, \; K_{12} = \begin{bmatrix} -0.0168 & 1.8227 \end{bmatrix}, \; K_{21} = \begin{bmatrix} -0.0426 & 1.8976 \end{bmatrix},$$
$$K_{22} = \begin{bmatrix} -0.0171 & 1.0101 \end{bmatrix}, \; K_{31} = \begin{bmatrix} -0.0095 & 1.9304 \end{bmatrix}, \; K_{32} = \begin{bmatrix} -0.0049 & 2.0398 \end{bmatrix}.$$

Then, when DOS attacks occurs, we will investigate that the value of the packet arriving rate γ how affects the optimal H_∞ tracking performance ρ^*.

Let β be the incidence rate of DOS attacks, we can obtain that β and γ have counter proportional relationships. Hence, Let β takes different values, the optimal tracking performance ρ^* obtained by **Theorem 3.2** in two cases are listed in table 1.

From the table 1, Under the two cases, it can be obtained that when β takes the smaller value, ρ^* will also be smaller. What's more, when β takes the same value, ρ^* in Case 2 is relatively larger than ρ^* of Case 1. Additionally, it can be summarized that the stability of the network system will be destroyed by DOS attacks, and the H_∞ tracking performance will decrease when the probability of DOS attacks increases.

Example 2(practical example): Refer to [26], an example of a tunnel diode circuits model is applied in this section to illustrate our effectiveness of proposed methods. The tunnel diode can be described as $T_D(k) = 0.01\varsigma_{D(k)} + \chi\varsigma_D^3(k)$, where $\chi \in [0.03, 0.05]$ is an unknown parameter. The state vectors are $\eta_1(k) = \varsigma_C(t)$ and $\eta_2(k) = T_L(k)$. Let $\Gamma(k) = 0.001 + \chi\varsigma_D^3(k)$, then the tunnel diode circuits system can be modeled as :

$$\begin{cases} \mathcal{M}\eta_1(k+1) = \Gamma(k)\eta_1(k) + \eta_2(k) \\ \mathcal{N}\eta_2(k+1) = \eta_1(k) - \mathcal{S}\eta_2(k) + \omega(k) \end{cases} \tag{41}$$

where the parameters of the system can be selected as $\mathcal{M} = 15mF$,$\mathcal{N} = 1.5H$, $\mathcal{S} = 18\Omega$, $\eta_1(k) \in [-5, 5]$. Then , through the methods of the T-S fuzzy, the tunnel diode circuits system can be described as follows:

$$\eta(k+1) = \sum_{i=1}^{2} \lambda_i(\eta_1(k))[\mathcal{A}_{mi}\eta(k) + \mathcal{B}_{mi}\omega(k)]$$

where

$$\mathcal{A}_{m1} = \begin{bmatrix} \frac{\Gamma_{min}}{C} & 45 \\ 2 & 15 \end{bmatrix}, \ \mathcal{A}_{m2} = \begin{bmatrix} \frac{\Gamma_{max}}{C} & 45 \\ 2 & 15 \end{bmatrix}, \ \mathcal{B}_{m1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ \mathcal{B}_{m2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ \Gamma_{min} = 0.001, \ \Gamma_{max} = 0.322.$$

and the values of other parameters are the same as in example 1, let $\gamma = 0.65$,$\epsilon_1 = 0.4$,$\epsilon_2 = 0.6$, the external disturbance be :

$$\omega(k) = \begin{cases} 2.5, 0 \leq k \leq 5 \\ 2.5, 5 \leq k \leq 20 \\ 0, else \end{cases}$$

The initial states vector $\eta(0) = [1, \ 0]^T$ and $\eta_r(0) = [0, \ 1]^T$. The state trajectories of $\eta_1(k)$ and $\eta_2(k)$ are depicted in Fig.13. The AET instants and intervals are depicted in Fig.14. The simulated results explain that the designed asynchronous fuzzy controller can estimate the signals of the reference output effectively under the existence of DOS attacks.

5. Conclusion

This paper investigates an asynchronous H_∞ tracking control problem for discrete-time FMJSs with a resilient AET mechanism under the DOS attacks. Through the lyapunov functions based on the fuzzy basis and mode-depend, some sufficient conditions which can guarantee the stochastic stability of the system are derived. Moreover , under the condition of event-triggered, the stability of the final closed-loop system with a given H_∞ tracking performance can also have been ensured. This paper also designs an asynchronous controller which covers two cases: partially known and fully known conditional probabilities. Finally, the effectiveness of the proposed method is illustrated with two examples. In the end, it is hoped that the results can be extended to other T-S FMJSs with time-varying delay, deception attacks, and DOS attacks described by the markov chains. More AET mechanisms also encourage us to explore in the future works.

6. Compliance with ethical standards conflict of interest

The authors declare that they have no conflict of interest concerning the publication of this manuscript.

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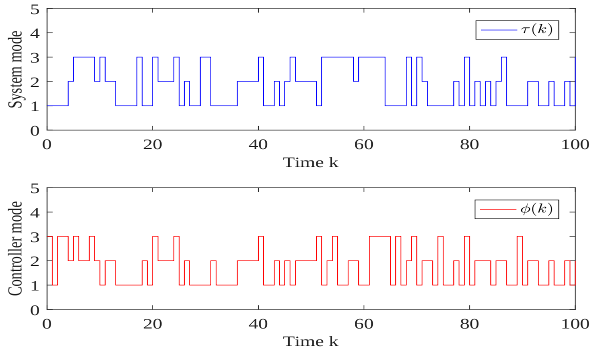

Figure 2: Mode evolution of the system (2) and the controller (11)

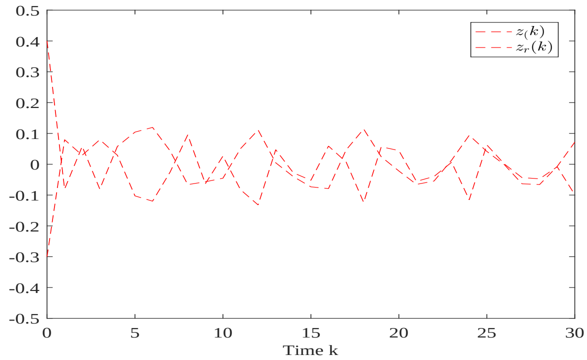

Figure 3: The output trajectories of the system (2) and system (3) without a controller

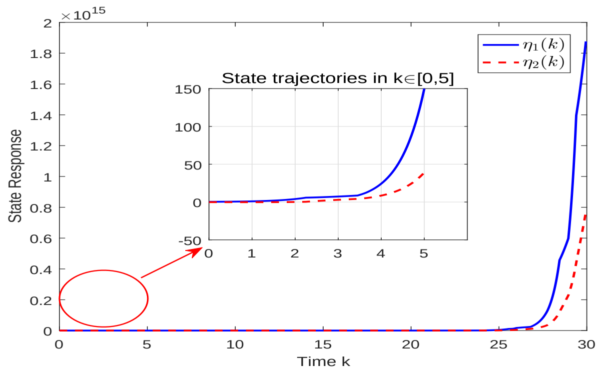

Figure 4: The state trajectories of the system (2) without a controller

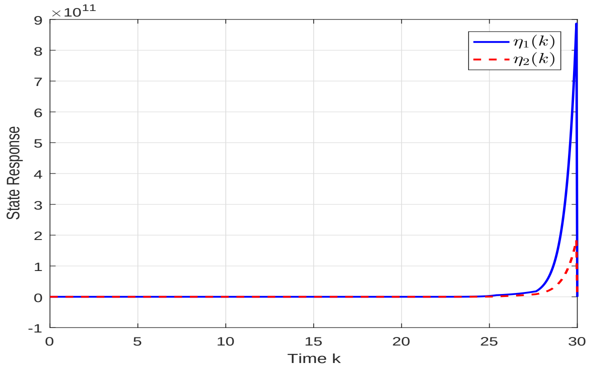

Figure 5: The state trajectories of the system (2) with controllers in **Theorem 3.2**

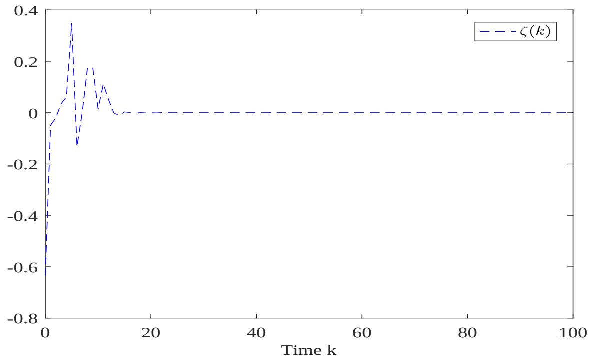

Figure 6: The state trajectories of the system (12) with controllers in **Theorem 3.2**

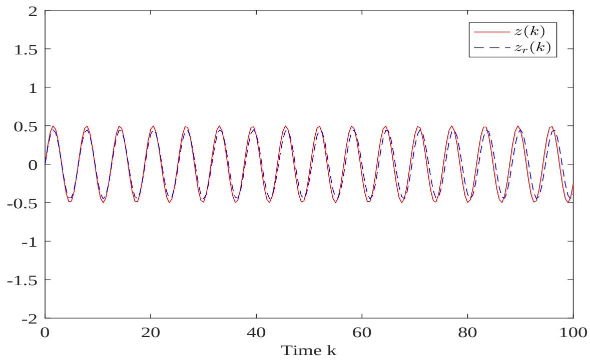

Figure 7: (Case1) The output trajectories of the system (2) and system (3)

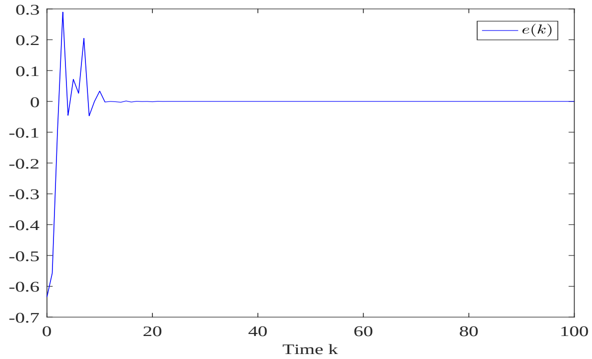

Figure 8: (Case1) The tracking error between the system (2) and system (3)

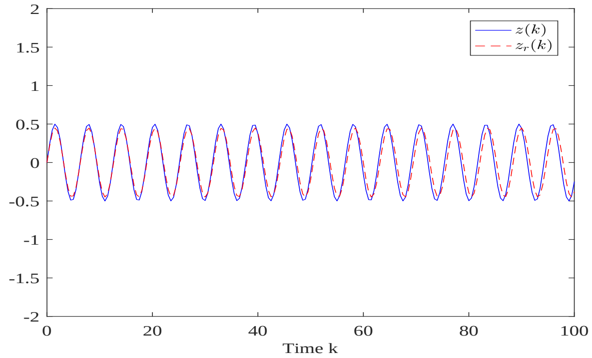

Figure 9: (Case2) The output trajectories of the system (2) and system (3)

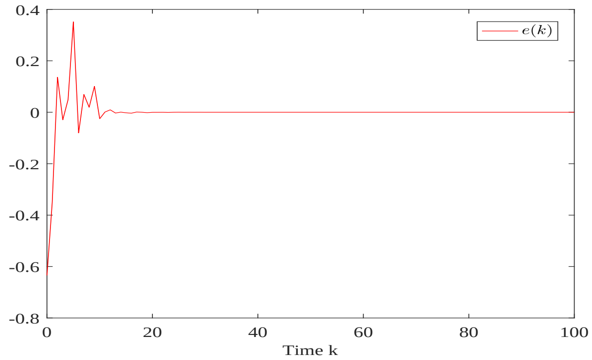

Figure 10: (Case2) The tracking error between the system (2) and system (3)

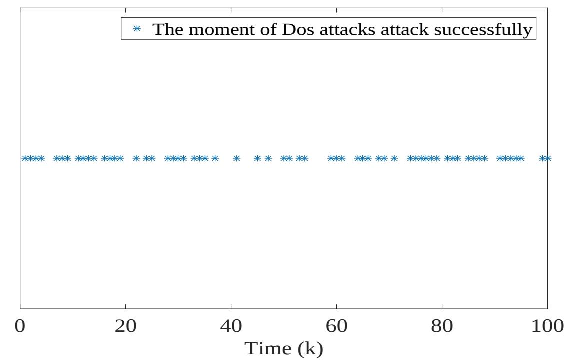

Figure 11: The time instants of successful Dos attacks with $\gamma = 0.75$

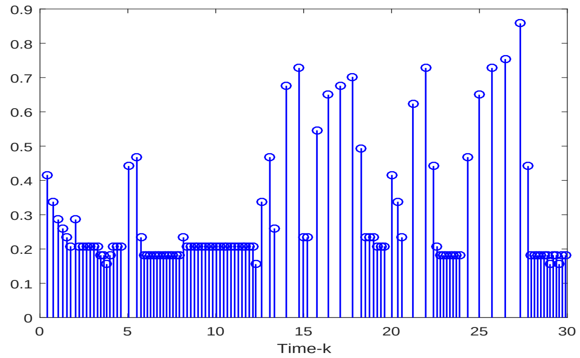

Figure 12: The AET release instants and release intervals in example 1

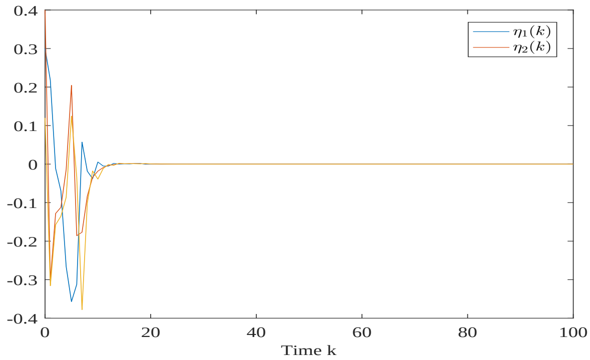

Figure 13: The state trajectories of $\eta_1(k)$ and $\eta_2(k)$ in example 2

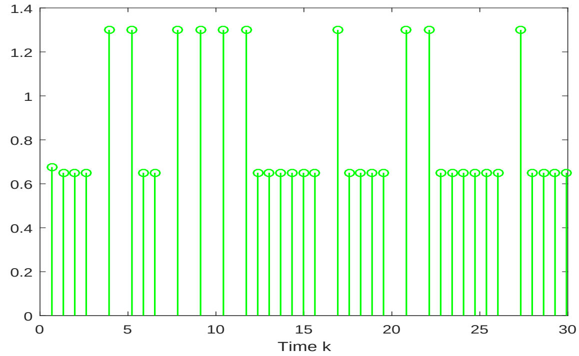

Figure 14: The AET release instants and release intervals in example 2