Review Article

Research on Abstraction Ability under the Background of Mathematics Key Competencies in Junior High School in China

Abstract: The abstraction ability in junior high school is an extension of the number sense, quantity sense, and symbolic awareness in primary school, and the foundation of mathematical abstraction in senior high school. Abstraction ability is one of the key competencies that junior high school students must have. This study used the China Knowledge Network(CNKI) as a data source to analyze previous research to find out the current situations, deficiencies, and gaps in this direction and found that: (1) Current domestic research on abstraction ability against the background of mathematics key competencies mainly focuses on the following five aspects: strategies for cultivating students' abstraction ability, development status of abstraction ability, evaluation dimension division of abstraction ability, specific manifestations of abstraction ability, influencing factors of students' abstraction ability. (2) The cultivation strategies of students' abstraction ability mainly focus on six aspects, and the researchers have given more comprehensive training suggestions. The previous research mainly investigated the situations of students' abstraction ability and teachers' teaching on the training of abstraction ability. Almost all of the evaluation dimension division of abstraction ability is based on students' mastery of mathematical concepts, principles, rules, and ways of thinking in different situations. There is less research on the specific manifestations and influencing factors of abstraction ability. (3) Based on current research, future research needs to adopt more scientific research tools, a more reasonable analysis framework, expand the research scope, increase validation research, and give more scientific suggestions. So that the research on the abstraction ability of junior high school students is more comprehensive, systematic, and profound.

Keywords: Junior high school; Key competencies; Abstraction ability; Mathematics education

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1. INTRODUCTION

The mathematics key competencies are clearly defined in the Mathematics Curriculum Standards for Compulsory Education of the 2022 edition. In compulsory education, mathematical vision is expressed in terms of abstraction ability (including number sense, quantity sense, and symbolic awareness), geometric intuition, spatial concepts, and innovation consciousness (Ministry of Education of the People's Republic of China, 2022). Moreover, mathematical abstraction has been included as one of the six key competencies in the Mathematics Curriculum Standards for Senior

High School of 2017 edition (Ministry of Education of the People's Republic of China, 2022). Abstraction ability mainly refers to the ability to obtain the object of mathematical research and to form mathematical concepts, properties, rules, and methods through the abstraction of quantitative relations and spatial forms in the real world (Ministry of Education of the People's Republic of China, 2022). The abstraction ability in junior high school is an extension of the number sense, quantity sense, and symbolic awareness in primary school, and the foundation of mathematical abstraction in senior high school, which has a connecting role in content. Therefore, abstraction ability is one of the mathematics key competencies that junior high school students must possess. Since the introduction of mathematics key competencies, mathematics education has gradually shifted from a *knowledge-based* to a *literacy-based* approach (Zhu, 2022). So far, there are more and more studies on abstraction ability against the background of mathematics key competencies, but there is no content to analyze and evaluate them. To find out the current situations, deficiencies, and gaps in this direction, the author analyzes the previous relevant research and tries to provide experiences and references for the subsequent research and improvement of junior high school students' abstraction ability level, to promote the continuous development and deepening of this direction.

The main research questions in this paper are as follows:(1) What are the main aspects of the current research on mathematical abstraction ability in junior high school, and what are the results of each aspect? (2) What are the deficiencies and gaps in the current research on mathematical abstraction ability in junior high school?

2. METHODS

2.1 Data Sources

This paper adopts the method of literary analysis and takes the literature in the database of CNKI as the data source. CNKI provides a multidisciplinary database and is a powerful retrieval tool. It contains a wealth of documents and has a great academic impact. Therefore, it is selected to ensure the reliability and persuasiveness of the research.

2.2 Data Collection

To avoid literature omission, both direct retrieval and reference guidance method were used. Firstly, the author searched with *key competencies*, *abstraction ability*, and *junior high school*, and 96 results were retrieved. After reading through and screening, 34 pieces of literature were selected. Then, 2 articles were selected from the references of literature in the process of data compilation.And the selected articles were written after the mathematical key competencies was proposed in 2014. Finally, a total of 36 results were referenced, read, and analyzed.

2.3 Data Collation

Firstly, the above literature is sorted, numbered, and read in detail, and then the research content, research results, and research methods mentioned therein are

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recorded. Finally, the results are counted and summarized.

3. RESULTS

After sorting and analyzing the previous studies, it is found that the current domestic research on abstraction ability against the background of mathematics key competencies mainly focuses on the following five aspects: strategies for cultivating students' abstraction ability, development status of abstraction ability, evaluation dimension division of abstraction ability, specific manifestations of abstraction ability, influencing factors of students' abstraction ability. The results of each aspect are discussed below.

3.1 Strategies for cultivating students' abstraction ability

Strategies for developing students' abstraction ability have been analyzed and discussed by researchers at six aspects: teachers' level, teaching methods, students' level, use of modern educational tools and resources, deepening of mathematical thought, and joint training with other key competencies.

3.1.1 Teachers' level

At the teachers' level, teachers should learn advanced teaching concepts, improve their mathematics abstraction ability, and pay attention to cultivating students' mathematics abstraction literacy.

Qiu and Zhu believe that the cultivation of disciplinary core literacy is gradually valued by society with the progress and development of China's education industry, therefore, the cultivation of students' key competencies in mathematics requires teachers to constantly update their teaching concepts and use more scientific teaching methods to improve students' learning efficiency and learning quality (Qiu & Zhu, 2018).

Chen believes that most junior high school students' learning is still at the stage of passive acceptance, and teachers want to effectively guide students to master and use mathematical knowledge, which requires teachers themselves to master the concepts, basic processes, and teaching theories of mathematical abstract thinking ability and improve their abstraction ability (Chen, 2022).

Huang believes that teachers must recognize the importance of abstraction ability, and the mathematical activities designed by teachers should make students experience the process of mathematical abstraction (Huang, 2020).

3.1.2 Teaching methods

3.1.2.1 Create a reasonable teaching situation

In the teaching process, we should reasonably create teaching situations, guide students to abstract specific mathematical things into general mathematical laws, focus on understanding the abstraction process, and reasonably help students understand the concepts (Gesaletti,2022; Zhong,2022; Lin, 2020; Zeng, 2020; Gao, 2019). Li, Yu, and others believe that living materials are the breeding base for

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mathematical abstraction and that it is necessary to design realistic backgrounds and let students experience the process of knowledge formation (Li, 2022; Yu, 2021; Zhou, 2020; Chang, 2018). The abstraction process should take questions as the carrier. Fan Yang and others believe that it is possible to create problem situations in which students are interested so that students can investigate the knowledge under the guidance of problems and find the essence of the problems (Fan,2022; Yang, 2022; Xie, 2020; Shao, 2020).

3.1.2.2 Guide students to think and learn correct learning methods

Zhong, Qiu, Zhu, and others argue that true competence is developed when students can break away from what the teacher has explained and think according to their way. Therefore, in junior high school mathematics teaching, teachers should take students as the main body, guide them to think independently and actively learn, form the correct methods, and let them gradually establish their mathematical abstract thinking (Qiu & Zhu, 2018; Zhong, 2022; Fan, 2022).

3.1.2.3 Attach importance to the overall teaching

Wang, Zhong, and Huang propose that the cultivation of students' abstraction ability runs through the whole education stage. Teachers should do a good job in the connection between learning stages based on textbooks; grasp the teaching content as a whole and make comparisons in different directions (Huang,2020; Wang & Zhong, 2022). Liu believes that teachers should carry out unit teaching, develop students' summarizing ability, outline chapter key points, and train students' abstract thinking abilities (Liu, 2020).

3.1.2.4 Pay attention to the transition from arithmetic thinking to algebraic thinking

Bao put forward that the transition from arithmetic to algebraic thinking is a key link to developing abstraction ability. In teaching, attention should be paid to symbolic representations (i.e. using symbols or algebraic expressions and equations composed of symbols to represent objects or structures in mathematics (or other disciplines or real life)), symbolic transformations (i.e. the transformations between various representations) and meaning construction(i.e. the interpretation or discovery of the mathematical structures or actual models behind formal symbols or expressions and the meaning and role of various symbolic operations)(Bao, 2022).

3.1.2.5 Focus on the teaching of concepts, theorems, and formulas

According to Bao’s viewpoints, concepts are one of the basic forms of logical thinking and are the goal of mathematical abstraction. In teaching, it is important to make students experience the abstraction process of concepts as much as possible and to understand them from different angles (Bao, 2022). Qiu believes that mathematics concepts are the embodiment of the spatial forms and quantitative relationships of the real world in thinking, and are an important result of scientific abstraction. Teachers should strengthen the application of examples, guide students to summarize abstract laws, make symbolic representations, promote the formation of concepts, and then

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return to examples for abstract analysis (Qiu,2019). Zhou believes that it is important to create familiar and interesting problem situations for students, allow them to formulate concepts, and allow them to master concepts in their self-knowledge construction in teaching concepts (Zhou,2021).

3.1.2.6 Focus on the teaching of concepts, theorems, and formulas

Mathematical problem-solving is composed of two important parts: problem-finding and problem-solving, so problem-solving is the heart of mathematics (Qiu,2019). Yang mentions the need to design some novel problems, asking students to read the problems carefully, to see the essence through the phenomena, and to find a breakthrough by relating what they have learned (Yang, 2022). Xie proposes to carefully select exercise questions, and improve students' reading ability so that the students can refine the mathematical information by carefully examining the problems, and building mathematical models after processing to finally solve the problems (Xie, 2020). Shao believes that variation training should be designed to facilitate students to achieve knowledge transfer and maintain the level of knowledge activity (Shao,2020). Wang Haiyan believes that problems are varied, and the steps and methods of solving the problems are also diverse, so students' abstraction ability can be cultivated through the form of multiple solutions to one problem (Wang,2020).

3.1.2.7 Simplify the abstraction process

Fan and Liu point out that mathematics is a subject that combines logic and abstraction, so students must have certain abstraction skills to learn mathematics well. However, students who have just entered junior high school are generally weak in abstraction ability. So, teachers need to optimize the teaching content by cutting out content related to an abstraction of quantitative relations or spatial abstraction, and effectively eliminating information that is not conducive to students' active thinking (Fan, 2022; Liu, 2021).

3.1.2.8 Make a good lateral transfer of abstraction ability from different disciplines.

Huang proposes that abstraction ability is also needed in other disciplines, such as the concepts of mass and vector in physics, which have certain relations and differences with mathematics (Huang, 2020).

3.1.2.9 Strengthen the evaluation of students' mathematical abstract literacy

First, the process evaluation of mathematical abstract literacy level should be carried out, that is, experiencing the process of mathematical abstraction. Attention is paid to the thinking process of obtaining mathematical objects by creating situations. Second, an outcome evaluation of the level of mathematical abstraction should be carried out, that is, the level of mathematical abstraction achieved by students. After a certain period of learning has been completed with abstraction content, teachers can design test papers that examine mathematical abstraction literacy to assess it quantitatively (Huang, 2020).

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3.1.2.10 Teach students in accordance with their aptitude

Zhao proposes to combine the physical and psychological characteristics of junior high school students in teaching. The physical and mental qualities of junior high school students have developed rapidly, and the transformation of neural activity in the brain lays the foundation for the development of abstract thinking and the learning of scientific theories (Zhao, 2021).

Chen supports to use of the connection between knowledge points to design problems in the ZPD (Zone *of Proximal Development)* of students, starting from the teaching design. So that the teaching process revolves around the cultivation of students' abstract thinking (Chen, 2022). Zhou suggests that "class-specific teaching" should be carried out in ZPD. Teachers should take into account the differences in the abstraction abilities of students in ordinary and experimental classes, make reference to the curriculum standards, and formulate reasonable teaching plans according to the curriculum standards and the situation of their classes (Zhou, 2021).

3.1.3 Students' level

Active learning is always more effective than passive learning, and it is important to give full play to students' initiative in the development of their abstraction ability. Chen suggests that, first of all, junior high students should learn to observe graphs and analyze mathematical models, and try to eventually convert these results into practical mathematical knowledge, mathematizing the various practical problems in real life. Secondly, junior secondary students should learn to explore mathematical concepts and theorems through independent practice (Chen, 2022).

3.1.4 Use of modern educational tools and resources

3.1.4.1 Assisting teaching with information technology

At the junior secondary level, students' thinking skills are developed from imaginal thinking to abstract thinking. Abstract knowledge is difficult to understand. Information technology such as making animations and PPT or applying *Geometer's Sketchpad* and electronic whiteboards should be used wisely to concretize abstract knowledge and effectively promote the development of students' abstract thinking ability (Chen, 2022; Zhong, 2022; Liu, 2020; Wang, 2020; Zhao, 2021; Lv, 2021; Fang, 2019; Dai, 2021). Or introduce visualization tools to make abstract thinking visible, such as mind maps and flowcharts (Li, 2022).

3.1.4.2 Realize graphic transformation with the help of teaching aids

Liu, Fang, and others argue that abstract concepts can be visualized through the use of objects and examples. For example, in the teaching of triangle interior angle summation theorem, experiments are conducted using triangular pieces of paper (Chen, 2022; Li, 2022; Liu, 2020; Fang, 2019).

3.1.4.3 Organize exploration and practice activities

Practical exploration can turn abstract knowledge into practical problems. Li,

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Guan, and others believe that classroom verification activities or comprehensive practical activities should be carried out to help students internalize abstract knowledge, transform abstract knowledge into practical operation ability, let students experience the formation process of knowledge, and exercise their ability to abstract the essence of mathematics (Li, 2022; Shao, 2020; Liu, 2020; Fang, 2019; Guan, 2021).

3.1.4.4 Make full use of various teaching materials

Sun believes that the reading and thinking content of the textbook is good teaching material. On the one hand, students are encouraged to read and explore independently, on the other hand, teachers guide students to summarize and reveal the essence of abstract problems (Sun, 2022). Zhou mentions the need to tell classical mathematical stories through the use of modern equipment such as audio, video, pictures, materials, and other forms, to reproduce the mathematical abstraction so that everyone has a new understanding and interpretation of mathematics and abstraction. For example, the Anglo-German dispute over the invention of calculus, the issue of the seven bridges in Konisburg, and so on (Zhou, 2020).

3.1.5 Deepening of mathematical thought

According to Bao, the most challenging aspect of the development of abstraction skills is the ability to abstract mathematical methods and thoughts (Bao, 2022). The following are the main mathematical thinking methods that researchers pay attention to in the training of abstract ability.

Regarding the idea of combining numbers and shapes, Guan, Li, and others argue that mathematics knowledge in junior high school is more abstract, and the flexible transformation between intuition and abstraction can be achieved through combining numbers and shapes, which builds a platform for students to think abstractly and gradually enhances their abstraction ability (Qiu & Zhu, 2018; Xie, 2020; Wang & Zhong, 2022; Liu, 2020; Guan, 2021; Li, 2018).

As for analogical thinking, analogies can connect seemingly unrelated knowledge. Wang, Zhong, and others believe that when learning abstract new concepts or meeting new situations, they can associate and distinguish the relationship and difference between new and old knowledge by analogy with the knowledge structure, learning methods, or exploration paths of old knowledge, which is also an important process in developing mathematical abstraction ability (Yu, 2021; Wang & Zhong, 2022; Li, 2018).

3.1.6 Joint training with other key competencies

Wu and Fang argue that the cultivation of abstraction ability is not an isolated process, but a process that interacts with other learning links. The establishment of mathematical models is a process of mathematical abstraction. Focusing on students' thinking in the process of mathematical modeling can effectively grasp students' thinking processes and characteristics, and can also effectively develop students' ability to abstract thinking (Fang, 2019; Wu, 2020).

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Jiang believes that a high level of arithmetic is basically reflected in three levels: accurate calculation, understanding of calculation, and reasonable methods. The computing ability has a certain hierarchy and development and gradually develops as the theory increases and the level of abstraction rises. Therefore, the teaching of arithmetic and students' abstract thinking skills are inextricably linked (Jiang, 2019).

Qiu and Zhu believe that the cultivation of students' abstraction ability in junior high school mathematics teaching must focus on the cultivation of students' thinking and logical ability and cultivating the learning habits of active thinking and observation so that students' logical ability and generalization ability can be enhanced (Qiu & Zhu, 2018).

Previous researchers have put forward comprehensive training strategies on how to cultivate students' abstraction ability, including six aspects: teachers' level, teaching methods, students' level, use of modern educational tools and resources, deepening of mathematical thought, and joint training with other key competencies. At the teacher level, teachers should have a correct understanding of abstraction ability, update teaching concepts in time, and constantly improve their abstraction ability.

The teaching methods are the dimension that has received the most attention, with the vast majority of researchers referring to the reasonable construction of teaching situations. Students must play the initiative and learn to observe, analyze and practice. In terms of the use of modern educational tools and resources, most researchers have referred to information technology-assisted teaching in addition to the appropriate use of teaching aids, practical activities, in-class reading materials, and mathematical stories.

The development of abstraction ability is closely related to the thought of combining numbers and figures and analogical thinking. Mathematical abstraction skills are also closely related to the performance of other key competencies, such as modeling concepts and calculation ability.

3.2 Development situations of abstraction ability

Concerning the development status of abstraction ability in junior high school, previous researchers mainly collected data through questionnaire surveys, tests, and interviews, and then analyzed the data to draw conclusions. The previous research conclusions mainly focus on two levels: the current situation of students' abstraction ability and the current situation of the cultivation of abstraction ability in teachers' teaching.

3.2.1 The current situation of students' abstraction ability

The research on the current situation of junior high school students' abstraction ability mainly focuses on the following four aspects: cognitive attitude, overall levels, gender differences, and class and regional differences.

Students have a positive attitude towards the abstraction ability, but there is an inconsistency between cognition and action. Zhou concluded from the analysis of the questionnaire that most of the study participants considered mathematical abstraction

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skills important for learning, life, and work, but they rarely applied them to solve practical problems in their lives, that is, there was inconsistency between cognition and action (Zhou, 2021).

There is still much room for development at the overall level. Researchers such as Wang, Zhong, and Zhou analyzed the test papers and get that the overall level of mathematical abstraction ability of the study participants was mostly in the middle or even low state, and there was still much room for improvement (Huang, 2020; Shao, 2020; Wang & Zhong, 2022; Zhou, 2021; Dai, 2021). Only Yin and Zhao analyzed the data on mathematical abstraction problems from the Mathematics Academic Quality Monitoring Test for second-year junior secondary school students in Jiangsu Province and found that the development of mathematical abstraction ability of second-year junior secondary school students in Jiangsu Province was generally good, with the majority of students the middle or above level (Yin & Zhao, 2017).

Fan found that 7th-grade students were better at representational abstraction than principle-based abstraction and constructive abstraction through a survey (Fang, 2019). Chen found that junior secondary students were weak in representational abstraction and principle-based abstraction, so he believed that students' abstraction ability in mathematical concepts and laws, abstraction of special mathematical symbols, and generalization of mathematical laws should be improved (Chen, 2022).

There is no significant difference in the level of abstraction ability between boys and girls. Almost all researchers who have analyzed gender differences on this question have concluded that there is no significant difference in the abstraction ability of boys and girls in junior high school (Chen, 2022; Huang, 2020; Zhou, 2021; Fang, 2019; Dai, 2021; Yin & Zhao, 2017). Therefore, the viewpoint that boys' logical thinking and abstraction ability are better than girls' is a stereotype of the public, and math teachers need to develop students' mathematical abstraction abilities equally.

There are class and regional differences in students' abstraction abilities. Zhou and Dai analyzed the data and concluded that there was a significant difference in the mathematical abstraction ability between students in the experimental and ordinary classes (Zhou, 2021; Dai, 2021). Fang went further and analyzed the gaps in each sub-dimension, concluding that students in the experimental class had better principle-based abstraction ability and constructive abstraction ability than those in the ordinary class and that there was no significant difference in representational abstraction ability (Fang, 2019). Yin and Zhao analyzed the scores and level distribution of mathematical abstraction-related problems in the Mathematics Academic Quality Monitoring for second-year junior high school students in Jiangsu Province, and the results reflected that students in urban schools were better than students in their mathematical abstraction ability, students in schools in southern and central Jiangsu were better than students in schools in northern Jiangsu, and students in private schools were better than students in public schools (Yin & Zhao, 2017).

3.2.2 Current situations of teachers' teaching on the training of abstraction ability

First of all, teachers' teaching concepts are backward. Shao's analysis showed that quite a number of math teachers still remain in the mechanical teaching stage of

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knowledge points, and lack the awareness of cultivating students' independent thinking (Shao, 2020). Wu believes that traditional mathematics teaching tends to focus more on the learning and application of knowledge and lacks the awareness of developing students' abstraction ability in the design and implementation of teaching (Wu, 2020).

Secondly, teachers themselves do not have a deep understanding of the concept of abstraction competence. Chen and Huang believe that teachers have a single view of f the elements of mathematical expression ability and do not have a deep enough understanding of mathematical abstraction ability (Chen, 2022; Huang, 2020).

Finally, the level of teachers' teaching in developing students' abstraction ability needs to be improved. The survey results of Wang and Zhong showed that teachers do not give students enough time for independent thinking in teaching, the deep integration of information technology and disciplines needs further study, and teachers do not study the overall teaching of the unit enough (Wang & Zhong, 2022). Huang's survey concluded that a few teachers had never combed the mathematical knowledge system in the classroom, and the overall level of teachers needed to be improved (Huang, 2020).

The previous research mainly investigated the current situations of students' abstraction ability and teachers' teaching on the training of abstraction ability. The situations of students' abstraction ability are mainly about students' cognitive attitudes, overall situation, gender differences, and class and regional differences. The main conclusions about the situation teachers' teaching on the training of abstraction ability mainly focus on the problems of teachers themselves and the problems in their teaching.

3.3 Evaluation dimension division of abstraction ability

A correct evaluation of students' abstraction ability is an important basis for adjusting teaching directions and strategies. In previous studies, researchers have divided different evaluation dimensions on the evaluation t of students' abstraction ability and specified the specific performance of each dimension. Their main views are as follows.

Wang and Zhong divide the levels of abstraction ability into three levels according to situations. Level 1 is the ability to abstract mathematical problems from familiar situations, obtain mathematical concepts and rules, understand the propositions and conclusions of propositions, and imitate the mathematical methods learned to solve problems. Level 2 is the ability to abstract mathematical concepts and rules from related situations, put forward mathematical propositions and models, select and apply mathematical methods to solve problems in new situations, and understand the mathematical ideas therein. Level 3 is the ability to abstract mathematical concepts and rules in comprehensive situations, formulate mathematical propositions and models, put forward mathematical methods and ideas, and recognize mathematical structures and systems (Wang & Zhong, 2022).

Zhou, referring to the previous research and the cognitive level of junior high school students, also divides abstraction ability into three levels. Level 1 manifests

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itself in the representation of numbers, quantitative relationships, and laws of change in symbols. Level 2 manifests itself in abstracting mathematical concepts from quantities and quantitative relationships, and graphs and graphical relationships. Level 3 manifests itself in the abstraction of general laws and structures through calculation and reasoning with the help of symbols (Zhou, 2021).

Chen and Lin divide the abstraction ability in junior high school into two dimensions: abstract mathematical concepts and abstract general laws and structures and set up four levels respectively. Level 1 is demonstrated by the ability to give examples in new situations under both dimensions. Level 2 is demonstrated by being able to explain the concepts in new situations, discover some components of mathematical concepts, explain the general rules and structures in new situations, and correctly summarize some general laws and structures in a non-symbolic way. Level 3 is demonstrated by the ability to discover all the elements that make up mathematical concepts in new contexts, to popularize general laws and structures, to correctly generalize general laws and structures in a non-symbolic way, and to correctly use mathematical symbols to represent some general laws and structures. Level 4 is expressed by the correct use of symbolic language in new situations (Chen & Lin, 2020).

Referring to Bloom's classification of cognitive goals, PISA, and the level division methods in curriculum standards, Cheng takes into account the complexity of contexts, cognitive processes, and creativity of thinking to classify each key competency into three levels: Reproduction (Level 1), Connection (Level 2) and Reflection (Level 3).The level of representation is to be able to abstract mathematical concepts, rules, and propositions in familiar situations, understand the meaning of mathematical expressions, and solve some simple problems by imitating the solutions of similar problems. The level of connection is to be able to abstract mathematical concepts, rules, and propositions in situations of relevance, to express them in mathematical language, and to explain abstract mathematical concepts, rules, and propositions with appropriate examples. The level of reflection is the ability to abstract more general situations from known mathematical propositions in order to distill mathematical methods to solve a class of problems. Be able to abstract mathematical concepts, rules, and propositions in complex situations and express them in mathematical language. Be able to form new propositions and create new methods based on their knowledge. Be able to establish the connection between the real world and mathematics, grasp mathematical principles in real problems, and use them to explain real-world phenomena (Cheng, 2018).

Fang and Chen divide mathematical abstraction into three dimensions according to the nature of abstract objects: representational abstraction ability, principle-based abstraction ability, and constructive abstraction ability. Representational abstraction ability refers to the initial abstraction ability that allows direct observation of phenomena and some surface characteristics of things. There are two levels under this dimension: abstraction of mathematical concepts and rules and abstraction of special mathematical symbols. Principle-based abstraction ability refers to the abstraction of the intrinsic causality, regularity, and relationships of things, the

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result of which is mathematical laws, theorems, etc. This dimension is subdivided into two levels: summary of mathematical methods and induction of laws. Constructive abstraction ability refers to the mathematical constructive activities that build on these abstractions. This dimension is divided into two levels: the building of mathematical models and the proof of practical problems (Chen, 2022; Fang, 2019).

Believing that junior secondary students have a limited level of understanding of mathematics, Dai categorized the abstraction ability of junior secondary students into two dimensions: representational abstraction and principled abstraction, which are almost consistent in connotation to the eponymous dimension proposed by Fang. On this basis, representational abstraction was subdivided into the abstraction of mathematical concepts, the abstraction of quantitative relations, and the acquisition of abstract symbols. The principled abstraction is subdivided into the refinement of problem strategies and the establishment of mathematical models (Dai, 2021).

In previous research, the evaluation dimensions of abstract ability are mainly divided into two types. One is to divide the abstraction ability into several levels and to specify the specific standards to be achieved at each level, so as to judge the level of students. The other is to divide the mathematical abstraction ability into different sub-dimensions and to evaluate the level of each different dimension. Although the evaluation frameworks and grading criteria are different, almost all of them are based on students' mastery of mathematical concepts, principles, rules, and ways of thinking in different situations.

3.4 Specific manifestations of abstraction ability

Bao believes that the abstraction ability of junior high school students is mainly reflected in the abstraction of mathematical concepts, relationships, and methods. The details are as follows. Firstly, it is demonstrated by the further development of number sense and the ability to abstract the concepts of rational numbers and real numbers r according to the actual situations or mathematical problem situations. Secondly, it is expressed in the further development of a sense of quantity, an understanding of the r significance of measurement in the study of geometry, and the cultivation of initial geometric intuition. Thirdly, it is expressed in the further development of symbolic awareness, an understanding of algebra as a generalization of arithmetic, and the ability to use algebraic methods to solve problems. Fourthly, it is expressed in the ability to abstract mathematical concepts, propositions, methods, and systems in contexts and to accumulate experience from the concrete to the abstract (Bao, 2022).

Yin and Zhao summarize the concrete manifestations of mathematical abstraction in the following four aspects: understanding the meaning of numbers in real life and estimating the results of calculations, using symbols to express numbers, quantity relations and laws of change, abstracting mathematical concepts from quantity and quantity relations and graph and graph relations and abstracting general rules and structures by means of operations and reasoning with the help of symbols (Yin & Zhao, 2017).

3.5 Influencing factors of students' abstraction ability

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Zhao conducted questionnaire surveys on students to know their learning situation, tested their level of abstraction ability through test papers, and then analyzed them through SPSS to find that: students' interest in learning mathematics, whether they are good at discovering mathematical problems in life, whether they are good at solving mathematical problems with their life experience, whether they are good at summarizing laws, whether they will take the initiative to construct a framework of knowledge, their attitude when they encounter abstract mathematical problems, teachers' teaching skills, other teaching aids, and other factors are significantly correlated with students' mathematical abstraction ability (Zhao, 2021).

Factors affecting students' levels of abstraction ability are important for the cultivation of students' abstraction skills, but only one researcher has carried out quantitative research on this aspect and put forward strategic suggestions to effectively cultivate students' abstraction ability.

4 DISCUSSION

current domestic research on abstraction ability against the background of mathematics key competencies mainly focuses on the following five aspects: strategies for cultivating students' abstraction ability, development status of abstraction ability, evaluation dimension division of abstraction ability, specific manifestations of abstraction ability, influencing factors of students' abstraction ability. These aspects explore the various perspectives of abstraction ability comprehensively, but there are still great differences in the breadth and depth of research in each aspect, as follows.

The cultivation strategies of students' abstraction ability are the most concerned aspect by previous researchers, and they have given more comprehensive training suggestions, including six aspects: teachers' level, teaching methods, students' level, use of modern educational tools and resources, deepening of mathematical thought, and joint training with other key competencies. There are the most suggestions on teaching methods. At the teacher level, teachers should have a correct understanding of abstraction ability, update teaching concepts in time, and constantly improve their abstraction ability. These are requirements for teachers, but there is a lack of practical suggestions for them. The main target audience for the development of abstraction ability is students, but most of the strategies are about how teachers teach and very few about how students should learn. while some researchers infer from the survey results, lacking empirical research to verify the effectiveness of a certain method.

The previous research mainly investigated the current situations of students' abstraction ability and teachers' teaching on the training of abstraction ability. The situations of students' abstraction ability are mainly about students' cognitive attitudes, overall situation, gender differences, and class and regional differences. The main conclusions about the situation teachers' teaching on the training of abstraction ability mainly focus on the problems of teachers themselves and the problems in their teaching. Given that the references are all within the last few years, despite the regional school differences, almost all the results show that there is still more room for improvement in the overall level, and the level of abstraction ability is

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independent of gender. However, the test papers, questionnaires, or interview outlines used in the researcher's survey process were all self-made, with different standards.

As for the division of the evaluation dimensions of abstraction ability, researchers have given different standards. Although the evaluation frameworks and grading criteria are different, almost all of them are based on students' mastery of mathematical concepts, principles, rules, and ways of thinking in different situations.

There is less research on the specific manifestations and influencing factors of abstraction ability. However, the factors affecting the level of students' abstraction ability are important to the development of students' abstraction ability and should not be ignored.

There are still some deficiencies in previous studies. First of all, there is less research on the specific manifestations and influencing factors of abstraction ability.Secondly, Most the strategies for developing abstraction ability are about how teachers teach, and very few are about how students learn. Finally, although most researchers used test papers, questionnaires, or interviews to investigate and research, there are still researchers who draw conclusions based on theoretical speculation or teaching experience, lacking strict experimental verification. It can be seen that the current research is not very strict and scientific.

There are some gaps in previous studies. Firstly, in terms of strategies for cultivating students' abstraction ability, the research lacks requirements for other aspects such as families, society, education departments, etc. Secondly, the strategies for cultivating students' abstraction ability put forward by researchers lack practical verification to scientifically judge whether these suggestions and strategies are effective.

Finally, the test papers, questionnaires, and interview outlines used in previous studies were designed by the researchers themselves, and there was no uniform standard research procedure and measurement questionnaire. Also, due to different geographical conditions, educational environments, and other factors, the foreign analytical framework does not apply to China. Therefore, domestic research lacks localized research tools and a systematic analysis framework.

5 CONCLUSIONS AND RECOMMENDATIONS

By analyzing the previous research on abstraction ability, the following conclusions are drawn:

(1) Current domestic research on abstraction ability against the background of mathematics key competencies mainly focuses on the following five aspects: strategies for cultivating students' abstraction ability, development status of abstraction ability, evaluation dimension division of abstraction ability, specific manifestations of abstraction ability, influencing factors of students' abstraction ability.

The cultivation strategies of students' abstraction ability are the most concerned aspect by previous researchers, and they have given more comprehensive training suggestions, including six aspects: teachers' level, teaching methods, students’ level, use of modern educational tools and resources, deepening of mathematical thought,

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and joint training with other key competencies. The previous research mainly investigated the current situations of students' abstraction ability and teachers' teaching on the training of abstraction ability. Almost all of the evaluation dimension division of abstraction ability are based on students' mastery of mathematical concepts, principles, rules, and ways of thinking in different situations. There is less research on the specific manifestations and influencing factors of abstraction ability. However, the factors affecting the level of students' abstraction ability are important to the development of students' abstraction ability and should not be ignored.

(2) Strategies for developing abstraction ability are mostly about how teachers teach, lacking in demands on other levels such as family, school, or society, and lacking in empirical verification. There is less research on the manifestations and influencing factors of abstraction ability. Therefore, these aspects should be expanded. Researchers use self-made test papers, questionnaires or interview outlines to investigate and research, and some researchers base their research on theoretical thinking or teaching experience, lacking domestic and local research tools and systematic analysis framework, which should be improved.

Therefore, based on current research, future research needs to adopt more scientific research tools, a more reasonable analysis framework, expand the research scope, increase validation research, and give more scientific suggestions. So that the research on the abstraction ability of junior high school students is more comprehensive, systematic, and profound.

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