Alternative Method (Penalty method) for Solving Assignment and Comparative Study with Existing Methods.

Abstract

Assignment problem is an important area in Operation Research and is also discussed in real physical world. In this paper an attempt has been made to solve the assignment problem using a new Method called the Penalty method. We discuss a numerical example by using the new Method and compare it with standard existing method which is the Hungarian method. We compare the optimal solution of the new Method and the Hungarian method. The new method is a simple procedure, easy to apply for solving assignment problem.

Keywords: Assignment problem, Hungarian method, Alternative method, Operation Research.

I. Introduction

The assignment problem is one of the most important areas in the area of allocation of resource to activity on one to one basis. It is one of the most studied area in Combinatorial Optimization problem or Operation research in Mathematics. It is a special case of transportation problems. The assignment problem finds numerous applications in various diverse business situations, such as assigning machines to factory orders, assigning sales people to sales territories, assigning contract to contract bidders, assigning teachers to classes and assigning accountants to accounts of the clients. In this paper we developed another method for solving assignment problem. Finally we compare the Optimal Solution of the new method and the Hungarian method.

II. Mathematical Formulation of Assignment Problem

Like any transportation problem, the assignment problem has a tableau which is matrix associated with it. Consider the problem of assigning n resources to m activity on one to one basis in such a way that cost or time is minimized. The cost matrix (C_{ij}) is given below:

Table 1.1 Representation of the Assignment Problem

ACTIVITY

	A_1	A_2	$\dots$		A_n	
R_1	$C_{1,1}$	$C_{1,2}$	$\dots$	C_{1n-1}	C_{1n}	
R_2	$C_{2,1}$	$C_{2,2}$	$\dots$	C_{2n-1}	C_{2n}	
Resource	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$	
	$C_{n-1,1}$	$C_{2,n-1}$	$\dots$	$C_{n-1,n-1}$	$C_{n-1,n-1}$	
R_n	$C_{n,1}$	C_{2n}	$\dots$	$C_{n,n-1}$	$C_{n,n}$	1
Required	1	1	$\dots$		1	

The cost matrix is the same as that of a transportation problem except that the availability at each of the resource and the requirement at each of the destinations is unity. Let X_{ij} denote the assignment of i^{th} resource to j^{th} activity, then

$$X_{ij} = \begin{cases} 1; if\ resource\ i\ is\ assigned\ to\ activity\ j \\ 0; \quad otherwise \end{cases}$$

Then the mathematical formulation of the problem is Minimize $Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} X_{ij}$

Subject to the constraints $\sum_{i=1}^{n} X_{ij} = 1$ and $\sum_{j=1}^{n} X_{ij} = 1$ $X_{ij} = 0\ or\ 1$

For all $i = 1,2, \dots n$ and $j = 1,2, \dots n$

III. Alternative Method For Solving Assignment Problem

In this section we introduce a new method for solving assignment problem the algorithm for the alternative method is as follows:

Step 1: Compute the row and column penalties by subtracting the least cost from the next least cost

Step 2: Identify the highest penalty from the row and column penalties and assign the least cost. Cross out all entries in the row and column penalties of the assigned cost.

Step 3: When there is a tie in the penalties, assign the penalty with the least with the least cost in the cost matrix.

Step 4: When there is a tie in the assigned cost, search through the row or column of each of the tied entry and assign the least cost on it row or column.

Step 5: Stopping Criteria: Stop when all rows and columns of the cost matrix have been crossed out.

IV. Numerical Examples Using the Algorithms of the Alternative Method

1. Solving the following assignment problem using the Alternative Method.

Consider the problem of assigning six jobs to six persons. The assignment table is given below

Persons	Jobs 1	2	3	4	5	6
A	73	21	10	9	86	95
B	73	55	72	91	14	82
C	80	18	39	11	73	27
D	44	99	90	36	100	14
E	71	33	73	42	37	86
F	34	11	39	35	52	9

Determine the optimum assignment schedule and minimum assignment cost.

Step 1: Compute the row and column Penalties

Persons	Jobs 1	2	3	4	5	6	
A	73	21	10	9	86	95	1
B	73	55	72	91	4	82	51
C	80	18	39	11	73	27	7
D	44	99	90	36	100	14	22
E	71	33	73	42	37	86	9
F	34	11	39	35	52	9	2
	10	7	29	2	33	5	

The highest Penalty is 51, we assign 4 which is the least on it row.

Step 2: Compute the row and column penalties on the reduced matrix in Step 1.

Persons	Jobs 1	2	3	4	5	6		
A	73	21	10	9	86	95	1	1
B	73	55	72	91	4	82	51	--
C	80	18	39	11	73	27	7	7
D	44	99	90	36	100	14	22	22
E	71	33	73	42	37	86	9	9
F	34	11	39	35	52	9	2	2
	10	7	29	2	33	5		
	10	7	29	2	--	5		

29 is the highest penalty, so we assign 10 which is the smallest on it column.

Step 3: Compute the row and column penalties on the reduced matrix in Step 2.

Persons	Jobs 1	2	3	4	5	6			
A	73	21	10	9	86	95	1	1	--
B	73	55	72	91	4	82	51	--	--
C	80	18	39	11	73	27	7	7	7
D	44	99	90	36	100	14	22	22	22
E	71	33	73	42	37	86	9	9	9
F	34	11	39	35	52	9	2	2	2
	10	7	29	2	33	5			
	10	7	29	2	--	5			
	10	7	--	24	--	5			

The highest penalty is 24, so we assign 11, which is the smallest on it column.

Step 4: Compute the row and column penalties on the reduced matrix in Step 3.

Persons	Jobs 1	2	3	4	5	6				
A	73	21	10	9	86	95	1	1	--	--
B	73	55	72	91	4	82	51	--	--	--
C	80	18	39	11	73	27	7	7	7	--
D	44	99	90	36	100	14	22	22	22	30
E	71	33	73	42	37	86	9	9	9	38
F	34	11	39	35	52	9	2	2	2	2
	10	7	29	2	33	5				
	10	7	29	2	--	5				
	10	7	--	24	--	5				
	10	22	--	--	--	5				

The penalty with the highest is 38, we assign 14 which is the smallest on it row.

Step 5: Compute the row and column penalties on the reduced matrix in Step 4.

Persons	Jobs 1	2	3	4	5	6					
A	73	21	10	9	86	95	1	1	--	--	--
B	73	55	72	91	4	82	51	--	--	--	--
C	80	18	39	11	73	27	7	7	7	--	--
D	44	99	90	36	100	14	22	22	22	30	30
E	71	33	73	42	37	86	9	9	9	38	--
F	34	11	39	35	52	9	2	2	2	2	25
	10	7	29	2	33	5					
	10	7	29	2	--	5					
	10	7	--	24	--	5					
	10	22	--	--	--	5					
	10	--	--	--	--	5					

Total assignment cost $= A3 + B5 + C4 + D6 + E2 + F1$

$= 10 + 4 + 11 + 14 + 33 + 34$

$= 106$

2. Solve the following Assignment Problem using Hungarian Method

Consider the problem of assigning six jobs to six persons. The assignment costs are given below.

Persons	Jobs 1	2	3	4	5	6
A	73	21	10	9	86	95
B	73	55	72	91	14	82
C	80	18	39	11	73	27
D	44	99	90	36	100	14
E	71	33	73	42	37	86
F	34	11	39	35	52	9

Step 1: Subtract the least entry from each row from the entry of each row.

Persons	Jobs 1	2	3	4	5	6
A	64	12	1	0	77	86
B	69	51	68	87	0	78
C	69	7	28	0	62	16
D	30	85	76	22	86	0
E	38	0	40	9	4	53
F	25	2	30	26	43	0

Step 2: Subtract the least entry in each column from the entries of each column.

Persons	Jobs 1	2	3	4	5	6
A	39	12	0	0	77	86
B	44	51	67	87	0	78
C	44	7	27	0	62	16
D	5	85	75	22	86	0
E	13	0	39	9	4	53
F	0	2	29	26	43	0

Step 3: Cover the zeros of the cost matrix with a minimum of lines

Persons	Jobs 1	2	3	4	5	6
A	39	12	0	0	77	86
B	44	51	67	87	0	78
C	44	7	27	0	62	16
D	5	85	75	22	86	0
E	13	0	39	9	4	53
F	0	2	29	26	43	0

Since the number of lines equals the order of the matrix, we assign zero on each row or column.

Step 4:

Persons	Jobs 1	2	3	4	5	6
A	39	12	△0	~~0~~	77	86
B	44	51	67	87	△0	78
C	44	7	27	△0	62	16
D	5	85	75	22	86	△0
E	13	△0	39	9	4	53
F	△0	2	29	26	43	~~0~~

Step 5: Allocate 1 for each assignment on a row or column

	1	2	3	4	5	6
A			1			
B					1	
C				1		
D						1
E		1				
F	1					

Step 6: Total assignment cost $= A3 + B5 + C4 + D6 + E2 + F1$

$$= 10 + 4 + 11 + 14 + 33 + 34$$

$$= 106$$

The total cost by the Alternative method is also 106

Since the Alternative method has the same Optimal Solution as the Hungarian method, we therefore conclude that the new alternative method is effective for solving assignment problem.

V. Conclusion

In this paper, we proposed a new method called the Alternative method for solving Assignment problem. We explained the algorithm for the alternative method and showed it computational efficiency by using numerical example. The Optimal Solution is the same as that of the Optimal Solution of the Hungarian method. We have therefore introduced a different algorithm for solving the Assignment problem.

VI. Reference

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