ON EVALUATION OF THREE BASIC PROPERTIES OF CENTRAL COMPOSITE DESIGN

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ABSTRACT

This work aims at making a choice of selecting the best property of central composite design (CCD). The three basic properties of CCD are rotatable, orthogonal and slope-rotatable with four optimality criteria; D, E, A and T. A complete 2^3 factorial experiment with increase in center points and non-replication of axial point was used for the entire work. The software applications used to run the analysis are Minitab and Excel. Minitab was used to create CCD with the respective center points and axial distances to fit the quadratic response polynomial. Excel was used to evaluate all the optimality criteria with respect to the properties of CCD and the efficiency of these criteria. Response surface graph was plotted to interpret how good the design is with the factors interaction. The result shows that A – optimality criterion is the best optimality criterion with respect to rotatable central composite design (RCCD), orthogonal central composite design (OCCD) and slope – rotatable central composite design (SRCCD) because of the increase in efficiency as the center point increases. Rotatable central composite design (RCCD) is considered in this context as the best property of central composite design in response surface methodology by comparing the increase in efficiency of the four optimality criteria as the center point increases.

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Keywords: Efficiency, Optimality Criterion, Center Points, Axial Distance, Central Composite Design

1. INTRODUCTION

[1] gave meaning to response surface methodology (RSM) based on their area of interest as a statistical technique that is essential for the optimization of chemical reactions or in an industrial process that is use for experimental design. RSM can be used to examining the relationship between the observed (response) and input variables for the purpose of optimization of relevant processes [2]. Response surface methodology is a collection of

statistical models to show how variables are related and how the response is influence by several variables [3-6]. Response surface methodology (RSM) is a support beam for design applications such as agricultural, engineering experiments etc. It can also be seen as a set of tools in design of experiments that examine the region of design variables in one or more responses.. Response Surface Methodology (RSM) based on Central Composite Design (CCD) was used to evaluate and optimize the effect of hydrogen peroxide, ferrous ion concentration and initial pH as independent variables on the total organic carbon (TOC) removal as the response function [7]. Central composite design is an experimental design useful in response surface methodology, for building a second order (quadratic) model for the response variable without needing to use a complete three-level factorial experiment. Central Composite Design (CCD) is the default of Design of Experiment (DOE) type. It provides a screening set to determine the overall trends of the model to better guide the choice of options in Optimal Space-Filling Design (OSFD). [8] changed 2^{nd} - degree response surface designs to make more accurate estimates about rotatability in response surface, employing central composite designs (CCD). [9] showed a class of balanced, near rotatable second order designs which minimized the number of full factorial runs associated with CCD that is suitable for a spherical region of interest. An extensive study of the second-order response surface central composite designs (CCDs) and partial replication of the central composite designs (CCDs) and its related studies was researched by [10]. [11] develop an approach for better understanding of the relationship between variables and response for optimum operating settings for maximum yield of watermelon crop using Central Composite Design and Response Surface Methodology. [12] study the effect comparing prediction variances in spherical regions using central composite design. [13] apply rotatable central composite design and response surface methodology to optimized chromite concentration for multi-gravity separator. The Central Composite Designs have been extensively studied and there exist vast literature on the subject. For reference purpose, see [14], [15], [16] and [17].

Estimating the desirable properties of a design using 2^{nd}-order response surface is the problem many researchers usually encountered in Central Composite Design (CCD). A design may be superior by one optimality criterion but may perform poorly when evaluated by another optimality criterion. [18] did comparative studies of five properties of central composite design (CCD); rotatable central composite design (RCCD), spherical central composite design (SCCD), orthogonal central composite design (OCCD), face central composite design (FCCD) and slope – rotatable central composite design (SRCCD) in response surface methodology with D, A, G and IV – optimality criteria and replicating center point and axial portion. But in this study, emphasis is placed on three basic properties of

Central Composite Design (CCD); rotatable central composite design (RCCD), orthogonal central composite design (OCCD) and slope – rotatable central composite design (SRCCD) with D, A, E and T - optimality criteria and increase in center point and non-replication of axial point. The determination of best property of central composite design in Response Surface Methodology with respect to non-scaled predicted variance optimality criteria is the major interest advanced in this research.

2. METHODOLOGY

2.1 ROTATABLE CENTRAL COMPOSITE DESIGN (RCCD)

In RSM, rotatability is considered as one of the desired properties of the second order designs. In rotatable design the variance of the predicted response $\hat{y}(x)$ depends on the location of the point $f(x) = (x_1, x_2, \dots, x_k)$ that is, it is a function only of distance from the point $f(x) = (x_1, x_2, \dots, x_k)$ to the center of the design. By definition, a design is rotatable if var {y(x)} is a constant at all the points that are equidistant from the center of the design. Setting $\alpha = (f)^{1/4}$ makes central composite design rotatable, where f is the factorial point.

If the objective of the experimenter is to estimate a second order model, the capability of a design to minimize the variance of the response variable becomes very important. To compare the second-order designs based on their prediction quality, the scaled prediction variance, $Var(x) = \dfrac{NVar(\hat{y})}{\sigma^2}$, can be used. If the scaled prediction variance is constant on spheres, the design is said to be rotatable.

Rotatable guarantees that $Var(x)$ has the same value at any location that has the same distance from the design center. This implies that rotatable provides equal precision of estimation in any direction from the design center. Thus, if a design is rotatable, $E(y)$ can be safely used as a prediction of the future response values within the region of interest. Rotatable designs may not provide the stability of the distribution of the scaled prediction variance throughout the design region. In such cases, some center runs can be added to make the $Var(x)$ more stable. A reasonably stable $Var(x)$ provides insurance that the prediction variance of the response values is roughly the same throughout the region of interest. When the region of interest is spherical, rotatability concept plays an important role in evaluating alternative designs. However, rotatability is not an important condition to be satisfied when the region of interest is cuboidal. For second-order designs, exact rotatability is not an absolute requirement; near rotatability may suffice. When the criterion of rotatability is in conflict with some other important consideration, a moderate departure from exact rotatability can be acceptable. Q^* is a criterion that measures the degree of rotatability of a design when it is not perfectly rotatable.

$$Q^* = \frac{\|\overline{M} - V_0\|^2}{\|M - V_0\|^2} = \frac{tr(\overline{M} - V_0)^2}{tr(M - V_0)^2}$$

(1)

Where $\|..,....\|^2$ is the matrix L^2 norm and *M* is the moment matrix, $M = V^0 + V^2 tr(MV^2) + V^4 tr(MV^4)$ (averaging over all possible rotations in the factor space), and V^0 is a matrix that consists of 1 position and zeroes elsewhere. The rotatability measure Q^* is, essentially, an R^2 statistic for the regression of the design moments of the second and fourth order in *M* onto the ideal design moments represented by *V*. In order words, Rotatability means that the variance of predicted response $V[\hat{y}(x)]$ is the same at all point x that are the same distance from the design center. A design with this property will leave the predicted variance $V(\hat{y})$ unchanged where the design is rotable about the center (0.0…,0), hence the name rotatable design. The study of rotatable designs is mainly emphasized on the estimation of difference of yields and its precision by [19]. Since we are dealing with the $p \times p$ information matrix $\tau(\varsigma)$ there are several possibilities for defining rotatability, each corresponding to a different scalar function of the matrix.

2.2 Orthogonal Central Composite Design (OCCD)

The orthogonality property is easily attainable for a first-order response surface design. If all design points are at ± 1 extremes and the $(X^T X)$ is orthogonal, the estimates contained in $\hat{\beta}$ are uncorrelated and the corresponding variances of estimates are minimized. Note that $Var(\hat{\beta}) = \sigma^2 (X^T X)^{-1}$, thus minimizing the i[th] diagonal element of $(X^T X)^{-1}$ is equivalent to minimizing the $Var(\beta_i)$. For the second-order model, the moment matrix is not diagonal because the sums of products between x^2 and 1 (an intercept) and between x_i^2 and x_j^2 will not be zeros unless all x_{iu}^2's are zeros. So, it is impossible to have a (completely) orthogonal matrix in unscaled variables.[20] discussed how to construct the second-order orthogonal designs by making use of an orthogonal polynomial coding.

A 2^k factorial design and the fractional factorial 2^{k-1} design in which the main effects are not aliased with other main effects are orthogonal designs. Consider a second order model with pure quadratic terms corrected for their means.

$$y = \beta_o + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii}\left(x_{ii}^2 - \bar{x}_i^2\right) + \sum_{j=i+1}^{k} \sum_{i=1}^{k-1} \beta_{ij} x_i x_j + \ell_{ij}$$

(2)

where $\bar{x}_i^2 = \sum_{i=1}^{N} \left(\frac{x}{n}\right)^2$. Let b_0, b_i, b_{ii}, b_{ij} denote the least square estimators of $\beta_0, \beta_i, \beta_{ii}, \beta_{ij}$ respectively. In the CCD, all the covariance between estimated regression coefficient except $\mathrm{cov}(b_{ii}, b_{ij})$ are zero. But if the inverse of the information matrix $(X^T X)^{-1}$ is a diagonal matrix, then $\mathrm{cov}(b_{ii}, b_{ij})$ also becomes zero. This property is called orthogonality. The condition for making a CCD orthogonal is by Setting $\alpha = \left(\frac{\sqrt{Nf} - f}{2}\right)^{\frac{1}{2}}$.Where $N = f + (2k)r + n_0$, $f = 2^k$. The orthogonal CCD provides ease in computations and uncorrelated estimates of the response model coefficients. The ability of a design in providing minimum-variance estimation of model parameters can be measured by the property of orthogonality.

2.3 Slope Rotatable Central Composite Design (SRCCD)

The experimenter is interested in estimation of the rate of change of response for a given value of independent variables rather than optimization of response. Effort has been made in the literature for obtaining efficient designs for the estimation of differences in responses, i.e, for estimating the slope of a response surface. Many researchers with different approaches have taken up the problem of designs for estimating the slope of a response surface. In this research, we have confine to only one approach, namely slope rotatable design. The design possessing the property that the estimate of derivative of the predicted response is equal for all points equidistant from the origin is known as slope rotatable design.

Consider the second - order response surface equation given as;

$$\eta(x) = \beta_o + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_{ii}^2 + \sum_{i<j}^{k} \beta_{ij} x_i x_j$$

(3)

The rate of change of response due to ith independent variable is given by;

$$\frac{d\hat{y}(x)}{dx_i} = b_i + 2b_{ii}x_i + \sum_{j \neq i} b_{ij} x_i$$

(4)

The variance of this derivative is a function of the point x at which the derivative is estimated and also a function of the design through the relationship

$$Var(b) = \sigma^2 (X^T X)^{-1}$$

Thus variance of (2) is given by

$$Var\left(\frac{\partial \hat{y}(x_i)}{\partial x_i}\right) = Var(b_i) + \rho^2 Var(b_{ii}) + x_i^2 \left[4Var(b_{ii}) - Var(b_{ij})\right]$$

(5)

Thus in order to obtain slope rotatable design, the design must satisfy the condition below.

1. $4Var(b_{ii}) = Var(b_{ij})$

2. $V(\hat{\beta}_0) = V(\hat{\beta}_i) = \cdots = V(\hat{\beta}_{ii}) = \frac{\sigma^2}{k}$

Then

$$Var\left(\frac{\partial \hat{y}(x_i)}{\partial x_i}\right) = Var(b_i) + \rho^2 Var(b_{ii})$$

$$Var\left(\frac{\partial \hat{y}(x_i)}{\partial x_i}\right) = \frac{\sigma^2}{k} + \rho^2 \frac{\sigma^2}{k} = \frac{\sigma^2}{k}\left[1 + \rho^2\right]$$

Here, $\rho^2 = x_1^2 + x_2^2 + x_3^2$ is the distance of the *i*th design point from the design center whose coordinate is (0,0,0).

It is important to note here that no rotatable design can be slope rotatable.

An analog of the Box-Hunter rotatability criterion, which requires that the variance of $\frac{d\hat{y}(x)}{dx_i}$ be constant on circles (k=2), spheres (k=3), or hyperspheres (k≥4) centered at the design origin. Estimates of the derivative over axial directions would then be equally reliable for all points x equidistant from the design origin. They referred to this property as slope rotatability, and showed that the condition for a CCD to be a slope –rotatable is as follows;

$$[2(f+n_0)]\alpha^8 - [4kf]\alpha^6 - f[N(4-k)+k(f-8)+8]\alpha^4 + [8(k-1)f^2]\alpha^2 - 2f^2(k-1)(N-f) = 0$$

The values of α for slope-rotatable central composite design are evaluated.

2.4 OPTIMALITY CRITERIA

Design optimality is a variance-type criterion that involves optimizing various individual properties of the $(X^T X)$ matrix. Optimal designs are experimental designs that are generated based on a particular optimality criterion and are generally optimal only for a specific statistical model [21-22]. Optimal design methods use a single criterion in order to construct designs for response surface methodology (RSM); this is especially relevant when fitting second order models.

An optimality criterion is a criterion which summarizes how good a design is, and it is maximized or minimized by an optimal design. Design optimality is often called the alphabetical optimality criteria because they are named by some of the letters of the alphabet.

(a) D-optimality criterion

The D-optimality focuses on the estimation of model parameters through good attributes of the moment matrix which is defined as;

$$M(\psi) = N^{-1}\left|X^T X\right|$$

(6)

where $X^T X$ the information matrix and N is the total number of run, X represents the model matrix associated with the D-optimal design and X^T represents its transpose. D-optimality seeks to maximize the determinant of the information matrix $X^T X$ or equivalently seeks to minimize the inverse of the information matrix. That is $\max\left|X^T X\right|$ or

$$\min(X^T X)^{-1}$$

The D-efficiency $= N^{-1}\left|X^T X\right|^{\frac{1}{n_p}} \times 100$, where n_p is the number of model parameter (7)

(b) A-optimality criterion

This criterion seeks to minimize the trace of the inverse of the information matrix ($X^T X$). This criterion results in minimizing the average variance of the estimates of the regression coefficients. Unlike D-optimality, it does not make use of covariance among coefficients. The A in the name stands for average.

$\psi^* = \arg\min trace\,[M^{-1}(\psi)] = \arg\min trace\,[(X^T X)^{-1}]$, Where $M(\psi) = X^T X$ (8)

The A – efficiency = $\dfrac{n_p}{trace\left[N(X^T X)^{-1}\right]} \times 100$, where n_p = number of model parameter (9)

(c) E – optimality criterion

This criterion minimizes the maximum eigenvalue of the dispersion matrix, $M(\psi)^{-1}$. Symbolically, a design ψ is said to be E-optimality if it gives $Min\{Max\lambda^{-1}\}$, where λ is the largest eigenvalue of the information matrix $M(\psi)$. The relative efficiency of E-optimality is denoted by;

$E_{eff} = \dfrac{100 n_p}{Max\left[N(\lambda)^{-1}\right]}$, where n_p = number of model parameter (10)

(d) T – optimality criterion

This criterion seeks to maximize the trace of the information matrix ($X^T X$). This criterion results in minimizing the average variance of the estimates of the regression coefficients [23]. The A in the name stands for average. $\psi^* = \arg\max trace\,[M(\psi)]$

$= \arg\max\, trace\ [M(X^T X)]$, where $\psi = X^T X$, $T_{eff} = \dfrac{1}{Tra\left[N\left(X^T X\right)\right]^{\frac{1}{np}}} \times 100$, where

n_p = number of model parameters (11)

2.4.1 FORMATION OF DESIGN MATRIX

Given a k –parameter function, $f(x)$ on N-point design has an $N \times k$ design matrix such that each row of the matrix is a point in $\tilde{X}$. For example, consider a n-points design matrix below;

$$X = \begin{pmatrix} x_{11} & x_{12} & x_{13} & . & . & . & x_{1k} \\ x_{21} & x_{22} & x_{23} & . & . & . & x_{2k} \\ x_{31} & x_{32} & x_{33} & . & . & . & .x_{3k} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ x_{n1} & x_{n2} & x_{n3} & . & . & . & x_{nk}. \end{pmatrix} \tag{12}$$

The extended design matrix of i = k for 2nd-Order Response Surface becomes;

$$f(x_1, x_2) = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_1^2 + a_4 x_2^2 + a_5 x_1 x_2 + \ldots, a_{k-1} a_k x_i x_j + e$$

(13)

For k = 2 factors, the design matrix is given by;

$$X = \begin{pmatrix} 1 & x_{11} & x_{12} & x_{11}^2 & x_{12}^2 & x_{11}x_{12} \\ 1 & x_{21} & x_{22} & x_{21}^2 & x_{22}^2 & x_{21}x_{22} \\ 1 & x_{31} & x_{32} & x_{31}^2 & x_{32}^2 & x_{31}x_{32} \\ 1 & x_{41} & x_{42} & x_{41}^2 & x_{42}^2 & x_{41}x_{42} \\ 1 & x_{51} & x_{52} & x_{51}^2 & x_{52}^2 & x_{51}x_{52} \\ 1 & x_{61} & x_{62} & x_{61}^2 & x_{62}^2 & x_{61}x_{62} \end{pmatrix}$$

(14)

2.4.2 DETERMINATION OF CODE VALUE FROM NATURAL DATA

The coded value can be obtained from natural data using the formula below:

$$x_i = \frac{r - t}{s}, \ i = 1, 2, 3, \ldots, n. \ s > 0, \ r \geq t \quad (15)$$

where $r = \kappa^{th}$ value of the natural data, t = chosen value in the set of natural data s = step size of the data

2.4.3 INFORMATION MATRIX

The information matrix $M(\psi)$ is defined to be

$$M(\psi) = \begin{cases} X'X \\ \sum_{\underline{x} \in \tilde{X}} \underline{x}x' \end{cases} \quad (16)$$

Normalized information matrix

$$M(\psi) = \begin{cases} \dfrac{N k (X'X)}{N^2} \dfrac{\sigma_X^2}{k} \\ k \sum_{\underline{x} \in \tilde{X}} \underline{x}x' w_X \dfrac{\sigma_X^2}{k} \end{cases} \quad (17)$$

where $\dfrac{N k (X'X)}{N^2} \dfrac{\sigma_X^2}{k}$ = if the weight are uniform or uniform probability measure.

$k \sum_{\underline{x} \in \tilde{X}} \underline{x}x' w_x \dfrac{\sigma_x^2}{k}$ = Non uniform probability measure

N = size of the matrix and k = number of factors

2.5 DATA PRESENTATION

The data collection procedure in this research work is the secondary source. The design for the maize experiment with different organic manure such as compost manure (k_1), Green manure (k_2) and animal manure (k_3) was applied for the growth and yield (y) of the crop. The data obtained for the experiments are shown in the appendix.

3. RESULTS AND DISCUSSION

3.1 OPTIMALITY CRITERIA OF THE THREE BASIC PROPERTIES OF CENTRAL COMPOSITE DESIGN

(a) OPTIMALITY WITH THREE FACTORS AND 3 CENTER POINTS UNDER ROTATABILITY CONDITION.

Rotatable Central Composite Design for k = 3 and h_o = 3

The design matrix X is obtained base on the design model given below:

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_{12} + a_{13}x_{13} + a_{23}x_{23}$$

Using the formula in 2.4, we obtained the following results:

D-optimality = 4.027, E-optimality = 5.6402, A-optimality = 1.0809, T-optimality = 153.9938

(b) OPTIMALITY WITH THREE FACTORS AND 4 CENTER POINTS UNDER ROTATABILITYCONDITION.

Rotatable Central Composite Design for k = 3 and h_o = 4

The design matrix X is obtained base on the design model given below

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_{12} + a_{13}x_{13} + a_{23}x_{23}$$

Using the formula in 2.4, we obtained the following results:

D-optimality = 5.3642, E-optimality = 3.5778, A-optimality = 1.0809, T-optimality = 56.669

(c) OPTIMALITY WITH THREE FACTORS AND 5 CENTER POINTS UNDER ROTATABILITYCONDITION.

Rotatable Central Composite Design for k = 3 and h_o = 5

The design matrix X is obtained base on the design model given below

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_{12} + a_{13}x_{13} + a_{23}x_{23}$$

Using the formula in 2.4, we obtained the following results:

D-optimality = 6.7013, E-optimality = 2.6042, A-optimality = 1.0138, T-optimality = 57.669

(d) OPTIMALITY WITH THREE FACTORS AND 3 CENTER POINTS UNDER ORTHOGONALITY CONDITION.

Orthogonal Central Composite Design for k=3 and h_o= 3

The design matrix X is obtained base on the design model given below:

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_{12} + a_{13}x_{13} + a_{23}x_{23}$$

Using the formula in 2.4, we obtained the following results:

D-optimality = 0.4161, E-optimality = 2.2517, A-optimality = 1.3491, T-optimality = 120.098

(e) OPTIMALITY WITH THREE FACTORS AND 4 CENTER POINTS UNDER ORTHOGONALITY CONDITION

Orthogonal Central Composite Design for k=3 and h_o= 4

The design matrix X is obtained base on the design model given below:

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_{12} + a_{13}x_{13} + a_{23}x_{23}$$

Using the formula in 2.4, we obtained the following results:

D-optimality = 0.8135, E-optimality = 2.2447, A-optimality = 1.2224, T-optimality = 125.976

(f) OPTIMALITY WITH THREE FACTORS AND 5 CENTER POINTS UNDER ORTHOGONALITY CONDITION

Orthogonal Central Composite Design for k=3 and h_o= 5

The design matrix X is obtained base on the design model given below:

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_{12} + a_{13}x_{13} + a_{23}x_{23}$$

Using the formula in 2.4, we obtained the following results:

D-optimality = 1.4978, E-optimality = 2.2361, A-optimality = 1.1261, T-optimality = 132.079

(g) OPTIMALITY WTH THREE FACTORS AND 3 CENTER POINTS UNDER SLOPE-ROTATABILITY CONDITION

Slope-Rotatable Central Composite Design for k=3 and h_o= 3

The design matrix X is obtained base on the design model given below:

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_{12} + a_{13}x_{13} + a_{23}x_{23}$$

Using the formula in 2.4, we obtained the following results:

D-optimality = 266.8866, E-optimality = 13.6986, A-optimality = 0.885, T-optimality = 278.624

(h) OPTIMALITY WITH THREE FACTORS AND 4 CENTER POINTS UNDER SLOPE-ROTATABILITY CONDITION

Slope-Rotatable Central Composite Design for k=3 and h_o= 4

The design matrix X is obtained base on the design model given below:

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_{12} + a_{13}x_{13} + a_{23}x_{23}$$

Using the formula in 2.4, we obtained the following results:

D-optimality = 229.232, E-optimality = 8.8496, A-optimality = 0.7831, T-optimality = 263.268

(i) OPTIMALITY WITH THREE FACTORS AND 5 CENTER POINTS UNDER SLOPE - ROTATABILITY CONDITION

Slope-Rotatable Central Composite Design for k=3 and h_o= 5

The design matrix X is obtained base on the design model given below:

$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + a_{12}x_{12} + a_{13}x_{13} + a_{23}x_{23}$$

Using the formula in 2.4, we obtained the following results:

D-optimality = 209.65, E-optimality = 6.1996, A-optimality = 0.8166, T-optimality = 252.86

The above information is summarized in table 1 below

Table 1

FOUR OPTIMALITY CRITERIA FOR THREE CCD WITH DIFFERENT CENTER POINTS AND THREE FACTORS

Design	h_0	k	$D-opt$	$E=opt$	$A-opt$	$T-opt$
RCCD	3	3	4.03	5.64	1.19	154.00
	4	3	5.36	3.58	1.08	56.67
	5	3	6.70	2.60	1.01	57.67
OCCD	3	3	0.42	2.25	1.35	120.10
	4	3	0.81	2.24	1.22	125.98
	5	3	1.50	2.24	1.13	132.10
SRCCD	3	3	226.89	13.70	0.89	278.62
	4	3	229.23	8.85	0.78	263.27
	5	3	209.65	6.20	0.82	252.86

3.2 EFFICIENCY ANALYSIS FOR OPTIMALITY CRITERIA OF THE THREE BASIC PROPERTIES OF CENTRAL COMPOSITE DESIGN

(a) Rotatable CCD at $h_o = 3$

Using the formula in 2.4, we have the following results:

D_{eff} = 6.76, A_{eff} = 49.32,, E_{eff} = 10.44, T_{eff} = 45.52

(b) Rotatable CCD at $h_o = 4$

Using the formula in 2.4, we have the following results:

D_{eff} = 6.57, A_{eff} = 51.40,, E_{eff} = 15.33, T_{eff} = 50.02

(c) Rotatable CCD at $h_o = 5$

Using the formula in 2.4, we have the following results:

$D_{eff} = 6.37$, $A_{eff} = 51.92$,, $E_{eff} = 20.21$, $T_{eff} = 49.66$

(d) Orthogonal CCD at $h_o = 3$

Using the formula in 2.4, we have the following results:

$D_{eff} = 5.39$, $A_{eff} = 43.60$,, $E_{eff} = 26.12$, $T_{eff} = 46.67$

(e) Orthogonal CCD at $h_o = 4$

Using the formula in 2.4, we have the following results:

$D_{eff} = 5.44$, $A_{eff} = 45.45$, $E_{eff} = 24.75$, $T_{eff} = 46.18$

(f) Orthogonal CCD at $h_o = 5$

Using the formula in 2.4, we have the following results:

$D_{eff} = 5.48$, $A_{eff} = 46.74$, $E_{eff} = 23.54$, $T_{eff} = 45.71$

(g) Slope-Rotatable CCD at $h_o = 3$

Using the formula in 2.4, we have the following results:

$D_{eff} = 10.29$, $A_{eff} = 66.47$, $E_{eff} = 4.29$, $T_{eff} = 42.90$

(h) Slope-Rotatable CCD at $h_o = 4$

Using the formula in 2.4, we have the following results:

$D_{eff} = 9.57$, $A_{eff} = 70.94$, $E_{eff} = 6.29$, $T_{eff} = 42.80$

(i) Slope-Rotatable CCD at $h_o = 5$

Using the formula in 2.4, we have the following results:

$D_{eff} = 8.98$, $A_{eff} = 64.45$, $E_{eff} = 8.499$, $T_{eff} = 42.84$

The results above is summarized in Table 2 below.

Table 2

EFFICIENCY FOR THREE CCD AND FOUR OPTIMALITY CRITERIA WITH DIFFERENT CENTER POINTS AND A REPLICATE

Design	h_0	r_s	N	D_{eff}	A_{eff}	E_{eff}	T_{eff}
RCCD	3	1	17	6.76	49.32	10.44	45.52
	4	1	18	6.57	51.40	15.53	50.02
	5	1	19	6.37	51.92	20.21	49.66
OCCD	3	1	17	5.39	43.60	26.12	46.67
	4	1	18	5.44	45.45	24.75	46.18
	5	1	19	5.48	46.74	23.54	45.71
SRCCD	3	1	17	10.29	66.47	4.29	42.90
	4	1	18	9.57	70.94	6.28	42.89
	5	1	19	8.98	64.45	8.49	42.84

DISCUSSION OF THE RESULTS

For Rotatable Central Composite Design (RCCD), it is observed that when the center point increases the D-efficiency (D_{eff}) decreases while (A_{eff}), (E_{eff}) and (T_{eff}) increases. In this context, A - optimality criterion is considered to be the best optimality in rotatable central composite design because of its stable increase in efficiency. For Orthogonal Central Composite Design (OCCD), (D_{eff}) and (A_{eff}) is increasing as the center point increases while (E_{eff}) and (T_{eff}) is decreasing as the center point increases. It is observed that the rate of increase in (A_{eff}) is greater than(D_{eff}). Hence, A - optimality criterion is still the best optimality in Orthogonal central composite design.For Slope–Rotatable Central Composite Design (SRCCD), (D_{eff}) and (T_{eff}) is decreasing, (E_{eff}) is increasing and (A_{eff}) is fluctuating as the center point is increases. Note also that efficiency is determine by the average minimum variance estimates of the model. Therefore, as the center point increases, the average variance decreases given rise to increase in efficiency.

4. CONCLUSION

Based on the results obtained in section 3 of this research, the following conclusions were made: that A – optimality criterion is the best optimality criterion with respect to rotatable central composite design (RCCD), orthogonal central composite design (OCCD) and slope – rotatable central composite design (SRCD) because of the increase in efficiency as the center point increases. Rotatable central composite design (RCCD) is considered in this context as the best property of the three basic properties of central composite design in response surface .methodology by comparing the increase in efficiency of the four optimality criteria as the center point increases

COMPETING INTERESTS

Authors have declared that no competing interests exist..

AUTHORS' CONTRIBUTIONS

The corresponding author (T. A. Ugbe) design the study, performed statistical analysis, wrote the first draft of the manuscript. Peter Akpan collected data for the work. Richmond Ofonodo manage the literature review and finally, E. E. Bassey managed the analysis of the study and proof read the final manuscript. All the authors read and approved the final manuscript.

COMPETING INTERESTS DISCLAIMER:

Authors have declared that no competing interests exist. The products used for this research are commonly and predominantly use products in our area of research and country. There is absolutely no conflict of interest between the authors and producers of the products because we do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by personal efforts of the authors.

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APPENDIX

Table 3: **Data for three organic manures for yield of maize production**

Natural variables			Coded variables			Response
κ_1	κ_2	κ_3	x_1	x_2	x_3	y
95	70	83	-1	-1	-1	66
95	70	93	-1	-1	1	70
95	80	83	-1	1	-1	78
95	80	93	-1	1	1	60
105	70	83	1	-1	-1	80

105	70	93	1	-1	1	70
105	80	83	1	1	-1	100
105	80	93	1	1	1	75
91.59	75	83	-1.682	0	0	100
108.41	75	93	1.682	0	0	80
100	66.95	88	0	-1.682	0	68
100	83.41	88	0	1.682	0	63
100	75	74.95	0	0	-1.682	65
100	75	96.41	0	0	1.682	82
100	75	88	0	0	0	113
100	75	88	0	0	0	100
100	75	88	0	0	0	118
100	75	88	0	0	0	88
100	75	88	0	0	0	100
100	75	88	0	0	0	85

Source: Agriculture department, Akwa Ibom State University, Uyo