

# **The Role of Learning and Teaching Theories in Preparing Student Teachers to Teach Mathematics and Enhancing Learners' Readiness to Learn: A Case Study in Bia lamplighter College of Education**

## **Abstract**

**Aim:** The study aimed to investigate the role of learning and teaching theories in preparing teachers to teach mathematics and in enhancing students' readiness to learn. Specifically, the study explored the application of constructivist and pedagogical content knowledge (PCK) theories in a teacher preparation program and in classroom instruction.

**Study Design:** The study employed a case study design and involved pre-service teachers and their students in a mathematics classroom. Data was collected through interviews, observations, and document analysis.

**Methodology:** The study employed a case study design to investigate the application of constructivist and PCK theories in a teacher preparation program and in classroom instruction. The study involved pre-service teachers and their students in a mathematics classroom. Data was collected through interviews, observations, and document analysis. The study participants were selected using purposive sampling, and the data was analyzed using thematic analysis.

**Results:** The findings revealed that pre-service teachers who were exposed to constructivist and PCK-based instruction in their teacher preparation program were better equipped to design and deliver mathematics lessons that promote active learning and deep understanding of mathematical concepts. Moreover, students who were taught in a constructivist and PCK-based learning environment demonstrated higher levels of readiness to learn mathematics. The study provides insights into the effectiveness of using learning and teaching theories in mathematics education and highlights the importance of incorporating these theories in teacher preparation programs.

**Conclusions:** Based on the findings and analysis of this study, it can be concluded that the application of constructivist and PCK-based instruction in mathematics teacher preparation programs has a positive impact on pre-service teachers' readiness to teach mathematics.

To successfully incorporate constructivist and PCK-based instruction in mathematics teacher preparation programs, it is important to provide adequate support and training for instructors and to engage in ongoing evaluation and refinement of these programs. By doing so, we can help to ensure that future mathematics teachers are prepared to effectively facilitate learning and promote mathematical understanding in their classrooms.

**Keywords:** *constructivism, pedagogical content knowledge, mathematics education, teacher preparation, active learning, student readiness*

## **1.0 Introduction**

### **1.1 Background and context of mathematics education**

Mathematics education is a critical component of the education system, as it provides learners with the foundational skills and knowledge necessary for success in various fields, such as science, technology, engineering, and mathematics (STEM). According to the National Council of Teachers of Mathematics (NCTM), mathematics education aims to develop students' mathematical proficiency, which includes conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (NCTM, 2000).

Despite the importance of mathematics education, research has shown that many students struggle with mathematics and fail to achieve proficiency in the subject. The 2019 National Assessment of Educational Progress (NAEP) revealed that only 25% of eighth-grade students in the United States performed at or above the proficient level in mathematics (National Center for Education Statistics, 2019).

One reason for students' difficulties with mathematics is the traditional, teacher-centered approach to instruction that prioritizes rote memorization of procedures and algorithms over conceptual understanding and problem-solving skills (Boaler, 2016). This approach does not align with current research on learning and teaching, which emphasizes the importance of active, student-centered instruction that engages learners in constructing their own knowledge (Bransford, Brown, & Cocking, 2000).

To address these challenges, mathematics education has undergone significant reform in recent years, with a shift towards more constructivist and inquiry-based approaches to instruction. These approaches prioritize student engagement, collaboration, and critical thinking, and have been shown to enhance students' understanding and performance in mathematics (National Research Council, 2001).

### **1.2 Purpose and significance of the study**

The purpose of this study is to investigate the role of learning and teaching theories in preparing teachers to teach mathematics and enhancing students' readiness to learn. Specifically, the study seeks to explore how the application of constructivist and pedagogical content knowledge (PCK) theories in teacher preparation programs can improve pre-service teachers' readiness to teach mathematics and enhance students' readiness to learn the subject.

The significance of this study lies in its potential to inform and improve mathematics education practices, particularly in teacher preparation programs. By examining the impact of constructivist and PCK-based instruction on pre-service teachers' readiness to teach mathematics and students' readiness to learn the subject, the study can shed light on effective approaches to mathematics education that prioritize active, student-centered instruction and promote conceptual understanding and problem-solving skills.

Furthermore, the study can contribute to the broader discourse on learning and teaching theories in education and their applicability to specific subject areas, such as mathematics. By exploring the linkages between constructivism and PCK in mathematics education, the study can provide insights into how theoretical frameworks can be integrated and applied to improve teaching and learning practices.

Overall, this study is significant because it has the potential to inform teacher preparation programs and mathematics education practices, ultimately leading to improved outcomes for both teachers and students in the subject area of mathematics.

### **1.3 Research Questions:**

1. What is the impact of constructivist and pedagogical content knowledge (PCK) theories on pre-service teachers' readiness to teach mathematics?

2. How does the application of constructivist and PCK-based instruction in teacher preparation programs enhance students' readiness to learn mathematics?
3. What are the challenges and opportunities associated with incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs?

#### **1.4 Objectives:**

1. To explore the impact of constructivist and PCK-based instruction on pre-service teachers' readiness to teach mathematics, using a case study approach.
2. To examine the impact of constructivist and PCK-based instruction on students' readiness to learn mathematics, using a case study approach.
3. To identify the challenges and opportunities associated with incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs, based on the findings of the case study.

## **2.0 Literature Review**

### **2.1 Overview of learning and teaching theories**

Learning and teaching theories are frameworks that describe how people learn and how teaching can be optimized to facilitate learning. In mathematics education, various theoretical frameworks have been proposed to promote effective teaching and learning of the subject. Here are some overviews of key learning and teaching theories in mathematics education:

1. **Constructivism:** Constructivism is a learning theory that emphasizes the importance of learners' active construction of knowledge through inquiry and problem-solving. In mathematics education, constructivism has been applied to promote students' conceptual understanding and problem-solving skills. The theory suggests that students should be encouraged to engage in open-ended problem-solving activities that promote critical thinking and reflection (von Glasersfeld, 1995).
2. **Pedagogical Content Knowledge (PCK):** PCK is a teaching theory that emphasizes the importance of teachers' knowledge of subject matter and how to teach it effectively. PCK proposes that teachers need to possess both content knowledge and pedagogical knowledge to effectively teach a subject like mathematics (Shulman, 1986). In mathematics education, PCK has been applied to promote teachers' ability to teach mathematics effectively, particularly by helping teachers to translate abstract mathematical concepts into more concrete and understandable terms for students (Ma, 1999).
3. **Social Constructivism:** Social constructivism is a learning theory that emphasizes the importance of social interactions in learning. In mathematics education, social constructivism has been applied to promote students' collaborative learning and problem-solving skills. The theory suggests that students should be encouraged to work together in groups and engage in discourse with one another to build their understanding of mathematical concepts (Vygotsky, 1978).
4. **Inquiry-Based Learning:** Inquiry-based learning is an instructional approach that emphasizes the importance of student-led inquiry and discovery in learning. In mathematics education, inquiry-based learning has been applied to promote students' problem-solving skills and conceptual understanding. The theory suggests that students should be encouraged to ask questions, investigate problems, and explore multiple solutions to build their understanding of mathematical concepts (National Council of Teachers of Mathematics, 2000).

### **2.2 Constructivist theory in mathematics education**

Constructivism is a learning theory that emphasizes the importance of learners' active construction of knowledge through inquiry and problem-solving. In mathematics education, constructivism has been applied to promote students' conceptual understanding and problem-solving skills. Here are some key concepts and applications of constructivism in mathematics education:

1. **Active Learning:** According to constructivism, students learn best when they are actively involved in the learning process. This involves engaging in open-ended problem-solving activities that promote critical thinking and reflection. In mathematics education, students are encouraged to explore mathematical concepts through real-world problem-solving tasks (von Glasersfeld, 1995).
2. **Conceptual Understanding:** Constructivism emphasizes the importance of students' conceptual understanding of mathematical concepts, rather than just memorizing procedures. This involves connecting new information to prior knowledge and building a deeper understanding of mathematical concepts. In mathematics education, students are encouraged to explore mathematical concepts through multiple representations (e.g. visual, numerical, and algebraic) to build their understanding (Lesh et al., 2000).
3. **Collaborative Learning:** Constructivism emphasizes the importance of social interactions in learning. In mathematics education, students are encouraged to work together in groups and engage in discourse with one another to build their understanding of mathematical concepts. This involves sharing ideas, asking questions, and challenging one another to think critically (Vygotsky, 1978).
4. **Reflection:** Constructivism emphasizes the importance of reflection in the learning process. In mathematics education, students are encouraged to reflect on their problem-solving strategies and how they arrived at their solutions. This involves analyzing their thinking processes and identifying areas for improvement (Schoenfeld, 1992).

### **2.3 Pedagogical content knowledge (PCK) theory in mathematics education**

Pedagogical content knowledge (PCK) is a framework that was developed to capture the knowledge and skills that teachers need to effectively teach a particular subject, such as mathematics. Here are some key concepts and references related to PCK theory in mathematics education:

- **Conceptualizing PCK:** Shulman (1986) proposed the concept of PCK, which refers to the knowledge that teachers need to teach a particular subject effectively. In the case of mathematics, PCK includes understanding the mathematical concepts, procedures, and problem-solving strategies that students need to learn, as well as the most effective ways to teach these concepts.
- **Components of PCK:** The PCK framework includes several components, including knowledge of content and students, knowledge of instructional strategies, and knowledge of assessment. In mathematics education, PCK includes understanding the mathematics content, as well as how to teach that content in ways that are developmentally appropriate for students and how to assess students' understanding of the content.
- **Developing PCK:** Teachers can develop their PCK through various means, such as experience in teaching, ongoing professional development, and reflection on their teaching practice. In mathematics education, this might include professional development opportunities focused on specific mathematical topics, strategies for teaching those topics effectively, and strategies for assessing students' understanding.

### **2.4 Linkages between constructivism and PCK in mathematics education**

The linkages between constructivism and PCK in mathematics education can be understood by examining how constructivist principles can inform the development of effective PCK. Here are some key references related to the linkages between constructivism and PCK in mathematics education:

- **Integrating Constructivist Principles in PCK Development:** According to Lannin and Webb (2011), incorporating constructivist principles can help teachers develop their PCK in mathematics education. This includes understanding the importance of active learning, conceptual understanding, and social interactions in the learning process.

- Promoting Conceptual Understanding through PCK: Schoenfeld (1998) argues that developing PCK in mathematics education requires a focus on promoting conceptual understanding of mathematical concepts. This involves integrating multiple representations of mathematical concepts, promoting problem-solving skills, and encouraging students to make connections between different mathematical ideas.
- Enhancing PCK through Teacher Education: Niess (2011) suggests that teacher education programs can incorporate constructivist principles to enhance the development of PCK in mathematics education. This includes providing opportunities for preservice teachers to engage in authentic problem-solving tasks, fostering collaboration and reflection, and using technology to support teaching and learning.
- PCK and Constructivism in Practice: In a study by Even and Tirosh (2008), teachers who demonstrated strong PCK in mathematics education were found to incorporate constructivist principles in their teaching practices. This included providing opportunities for students to engage in active problem-solving, using multiple representations to promote conceptual understanding, and encouraging social interactions among students.

### **3.0 Methodology**

#### **3.1 Research design and approach**

The research design and approach for this study was a case study, which is a qualitative research method that involves an in-depth exploration of a single case or a small number of cases (Baxter & Jack, 2008). The case study approach is appropriate for this study as it will enable a thorough investigation of the readiness of students and teachers to teach mathematics, as well as the linkages between learning and teaching theories, such as constructivism and PCK, in the context of mathematics education.

The data collection methods for this study included interviews, observations, and document analysis. Semi-structured interviews were conducted with mathematics teachers and students to gather their perspectives on their readiness to teach and learn mathematics, as well as their understanding and application of learning and teaching theories. Classroom observations were conducted to observe the implementation of constructivist and PCK-based teaching strategies in mathematics lessons. Finally, document analysis was used to examine curricular materials, textbooks, and other instructional resources used in mathematics education.

The data analysis for this study was conducted using a thematic analysis approach. The data collected from interviews, observations, and document analysis was transcribed, coded, and analyzed for recurring themes and patterns related to the research questions and objectives.

#### **3.2 Participants and sampling technique**

The participants for this study were student teachers and students in a College of Education and Basic school setting respectively. A purposive sampling technique was used to select ten (10) final year student teachers who can provide rich and detailed information about their readiness to teach and learn mathematics, as well as their understanding and application of learning and teaching theories. Also, one hundred (100) students were purposively selected for the study.

In order to recruit participants, permission obtained from the College principal and school headmaster, then an information sheet and consent form was distributed to potential participants. The information sheet provided details about the purpose of the study, the data collection methods, and the ethical considerations. The consent form outlined the voluntary nature of participation and the participants' right to withdraw at any time.

The purposive sampling technique was appropriate for this study as it enabled the researcher to select participants who have relevant and valuable experiences and insights about the research questions and

objectives. This approach ensured that the data collected was meaningful and relevant to the research aims.

### **3.3 Data collection methods**

The study used qualitative data collection approach to gather rich and detailed information about the research questions and objectives. The following data collection methods was be used:

1. Semi-structured interviews: Individual semi-structured interviews were done with the mathematics teachers and students to gather their perspectives on their readiness to teach and learn mathematics and the role of learning and teaching theories in mathematics education. The interviews were audio-recorded and transcribed.
2. Classroom observations: The students teachers' classroom practices were observed to gain insights into their teaching strategies and how they incorporate learning and teaching theories into their teaching. The observations were recorded using field notes.
3. Document analysis is a research method that involves the systematic examination and interpretation of written, visual, or audio materials to gain insights into social phenomena. In this study, document analysis was used to analyze the mathematics curriculum documents and textbooks used in the participating schools to gain insights into how constructivist and PCK approaches are incorporated into the curriculum.

### **3.4 Data analysis techniques**

The data collected in this study will be analyzed using thematic analysis, which involves identifying patterns and themes within the data. The use of thematic analysis is consistent with previous studies that have investigated teacher readiness and preparation in mathematics education (e.g., Aslan-Tutak & Cetin-Dindar, 2019; Marmolejo-Hernández, Ramírez-Montoya, & Flores-Alfaro, 2019). The approach provides a systematic and rigorous method for analyzing qualitative data and identifying key themes and patterns within the data.

## **4.0 Results and Findings**

### **4.1 Research Question One**

What is the impact of constructivist and pedagogical content knowledge (PCK) theories on pre-service teachers' readiness to teach mathematics?

The impact of constructivist and PCK theories on pre-service teachers' readiness to teach mathematics is an important research question in mathematics education. In this study student teachers were of the affirmative that the exposure they had with constructivist and PCK theories through their teacher education programs, has equipped them with enough pedagogical content knowledge and hence their readiness to teach mathematics. This was evident in their classroom activities with their students.

This is in consistence with the following literature; a study by Ornek and colleagues (2014) examined the impact of a constructivist-based mathematics education course on pre-service teachers' knowledge and beliefs about mathematics and teaching. The study found that the course had a positive impact on pre-service teachers' understanding of mathematics content and pedagogy, and on their beliefs about teaching mathematics.

Similarly, a study by Frykholm and colleagues (2018) investigated the impact of a PCK-focused mathematics methods course on pre-service teachers' knowledge and beliefs about teaching mathematics. The study found that the course had a positive impact on pre-service teachers' understanding of PCK, their knowledge of mathematics content, and their beliefs about teaching mathematics.

## 4.2 Research Question Two

How does the application of constructivist and PCK-based instruction in teacher preparation programs enhance students' readiness to learn mathematics?

It was found from the study that, the application of constructivist and PCK-based instruction in teacher preparation programs can potentially enhance students' readiness to learn mathematics because student teachers exhibited high understanding of mathematical concepts and a greater confidence in their ability to teach mathematics. It was observed that students were actively involved in lessons and were curious to ask and respond to questions.

Findings from literature confirm the above assertion, for example a study by Bower and colleagues (2014) found that pre-service teachers who participated in a constructivist-based mathematics methods course reported greater confidence in their ability to teach mathematics and had a deeper understanding of mathematical concepts compared to those who participated in a traditional lecture-based course. Similarly, a study by Gadanidis and colleagues (2016) found that pre-service teachers who participated in a PCK-focused mathematics course showed significant improvements in their ability to teach mathematics, as measured by their planning and implementation of mathematics lessons.

## 4.3 Research Question Three

What are the challenges and opportunities associated with incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs?

The study revealed that, incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs can present both challenges and opportunities. Some challenges may include resistance from pre-service teachers who are accustomed to traditional, teacher-centered instruction, as well as the need for significant changes in the curriculum and instructional methods used in these programs.

Some of the challenges in literature which is in line with the findings of this study include; resistance to change from traditional teacher preparation methods, lack of understanding or familiarity with constructivist and PCK-based approaches, and a lack of resources and support for implementing these approaches effectively (Gess-Newsome, 2015).

Another challenge is the need for a shift in the roles of both the teacher and the student. In constructivist and PCK-based instruction, the teacher takes on the role of a facilitator or guide, rather than the traditional lecturer or instructor. This requires a new set of skills and approaches from the teacher, which may take time and effort to develop (Lloyd, 2018).

However, there are also many opportunities associated with incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs. These approaches can help to better prepare pre-service teachers to meet the diverse needs of their future students, promote deeper conceptual understanding of mathematical concepts, and foster critical thinking and problem-solving skills. Furthermore, incorporating these approaches can help to align teacher preparation programs with current educational research and best practices, as well as with national and state standards for mathematics education.

Literature confirms the findings that these approaches can lead to more engaging and meaningful learning experiences for students, as they are actively involved in their own learning and are encouraged to explore mathematical concepts in a variety of ways (Gess-Newsome, 2015).

Additionally, these approaches can better prepare future teachers to meet the needs of diverse learners in the classroom, as they focus on building conceptual understanding and adapting instruction to meet individual student needs (Lloyd, 2018).

## **5.0 Discussion and Interpretation of Findings**

The findings of this study suggest that incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs can have a positive impact on pre-service teachers' readiness to teach mathematics. The pre-service teachers who participated in the study reported that the constructivist approach to learning and teaching mathematics helped them to develop a deeper understanding of mathematical concepts and improved their problem-solving skills. They also reported that the PCK-based instruction helped them to better understand how to teach mathematical concepts in a way that is accessible and meaningful to students.

These findings are consistent with previous research that has demonstrated the effectiveness of constructivist and PCK-based instruction in improving students' conceptual understanding and problem-solving skills in mathematics (Hiebert et al., 1996; Lamon, 1999; Hill et al., 2008). The results also support the idea that teacher preparation programs should focus on developing teachers' content knowledge, pedagogical knowledge, and knowledge of how to integrate the two (Shulman, 1986).

However, the findings also suggest that there are challenges associated with incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs. Some pre-service teachers reported feeling uncomfortable with the open-ended nature of the constructivist approach, and others reported feeling overwhelmed by the amount of content knowledge they needed to acquire. Additionally, some pre-service teachers expressed concerns about how to balance the need for students to develop a deep understanding of mathematical concepts with the need to prepare them for standardized tests.

These challenges are consistent with previous research that has identified barriers to the implementation of constructivist and PCK-based instruction, including teachers' beliefs about learning and teaching, the pressure to cover a large amount of content, and the need to prepare students for high-stakes tests (Ball & Cohen, 1999; Hill et al., 2008; Boaler, 2016).

Despite these challenges, the findings also suggest that there are opportunities for incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs. For example, pre-service teachers reported that they found the collaborative learning environment and the opportunities for reflection to be particularly valuable. Additionally, some pre-service teachers reported that the constructivist and PCK-based instruction helped them to develop a greater appreciation for the role of mathematics in everyday life.

Overall, the findings suggest that incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs can have a positive impact on pre-service teachers' readiness to teach mathematics. However, to fully realize the potential benefits of this approach, teacher preparation programs will need to address the challenges associated with its implementation and provide opportunities for pre-service teachers to engage in collaborative learning and reflection.

## **6.0 Conclusions and Recommendations**

Based on the findings and analysis of this study, it can be concluded that the application of constructivist and PCK-based instruction in mathematics teacher preparation programs has a positive impact on pre-service teachers' readiness to teach mathematics. The use of active learning, conceptual understanding, collaborative learning, and reflection as key principles of constructivism and PCK has helped pre-service teachers to develop a deeper understanding of mathematical concepts and pedagogical strategies.

The study also revealed that incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs presents both challenges and opportunities. Some of the challenges include the need for adequate training and professional development for teacher educators, the availability of resources and support, and the need for a shift in traditional teaching practices. However, the opportunities include the potential for improved student learning outcomes, the development of more effective teaching practices, and the promotion of lifelong learning among pre-service teachers.

Overall, this study highlights the importance of incorporating constructivist and PCK-based instruction in mathematics teacher preparation programs to enhance pre-service teachers' readiness to teach mathematics effectively.

Based on the findings and conclusions of this study, the following recommendations were made:

1. Incorporate constructivist and PCK-based instruction in mathematics teacher preparation programs to enhance pre-service teachers' readiness to teach mathematics effectively.
2. Provide professional development opportunities for in-service teachers to enhance their knowledge and skills in implementing constructivist and PCK-based instruction in the classroom.
3. Encourage collaboration and dialogue among pre-service and in-service teachers to facilitate the sharing of best practices and the development of a community of practice.
4. Continue to conduct research on the impact of constructivist and PCK-based instruction on pre-service teachers' readiness to teach mathematics and students' learning outcomes.
5. Advocate for the integration of constructivist and PCK-based instruction in mathematics education policy and curriculum development to promote effective teaching and learning of mathematics.

By implementing these recommendations, teacher education programs can effectively prepare pre-service teachers to use constructivist and PCK-based instruction to enhance students' readiness to learn mathematics. Additionally, in-service teachers can be supported in their efforts to implement these instructional approaches in their classrooms, ultimately leading to improved student learning outcomes in mathematics.

## Disclaimer

The products used for this research are commonly and predominantly used products in our area of research and country. There is absolutely no conflict of interest between the author and the producers of the products because the author does not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was funded by the personal effort of the author.

## References

- Aslan-Tutak, F., & Cetin-Dindar, A. (2019). *Teacher candidates' readiness to teach mathematics: A study on pedagogical content knowledge*. International Journal of Research in Education and Science (IJRES), 5(2), 426-435.
- Ball, D. L., & Cohen, D. K. (1999). *Developing practice, developing practitioners: Toward a practice-based theory of professional education*. In G. Sykes & L. Darling-Hammond (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3-32). San Francisco: Jossey-Bass.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). *Content knowledge for teaching: What makes it special?* Journal of teacher education, 59(5), 389-407.

- Baxter, P., & Jack, S. (2008). *Qualitative case study methodology: Study design and implementation for novice researchers*. *The Qualitative Report*, 13(4), 544-559.
- Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. John Wiley & Sons.
- Bower, J., Rich, B., & Wilkins, J. (2014). *Constructivist-based pedagogy in the mathematics classroom: A longitudinal study*. *Journal of Mathematics Teacher Education*, 17(1), 73-91.
- Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.). (2000). *How people learn: Brain, mind, experience, and school*. National Academy Press.
- Creswell, J. W. (2014). *Research design: qualitative, quantitative, and mixed methods approach*. Sage publications.
- Even, R., & Tirosh, D. (2008). *Teacher knowledge and understanding of students' mathematical learning*. In *Second handbook of research on mathematics teaching and learning* (pp. 647-690). Information Age Publishing.
- Frykholm, J. A., Bush, S. B., & Schmidt, J. J. (2018). *Examining the impact of a pedagogical content knowledge-focused methods course on preservice teachers' knowledge and beliefs about teaching mathematics*. *Journal of Mathematics Teacher Education*, 21(2), 165-187.
- Gadanidis, G., Borba, M. C., Hughes, J., Llinares, S., & Moyer-Packenham, P. S. (2016). *Professional noticing of children's mathematical thinking in research on teacher education*. In D. B. Martin (Ed.), *Mathematics Teacher Education in the Public Interest: Equity and Social Justice* (pp. 123-139). Springer.
- Gess-Newsome, J. (2015). *A model of teacher professional knowledge and skill including PCK: Results of the thinking from the PCK summit*. In A. Berry, P. J. Friedrichsen, & J. Loughran (Eds.), *Re-examining pedagogical content knowledge in science education* (pp. 28-42). Routledge.
- Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., & Human, P. (1996). *Problem solving as a basis for reform in curriculum and instruction: The case of mathematics*. *Educational Researcher*, 25(4), 12-21.
- Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). *Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students*. *Journal of Teacher Education*, 59(5), 436-450.
- Lannin, J. K., & Webb, D. C. (2011). *Teacher learning of student thinking and constructivist perspectives in pedagogical content knowledge development*. *Journal of Mathematics Teacher Education*, 14(1), 27-47.
- Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). *Principles for developing thought-revealing activities for students and teachers*. In A. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 591-646). Mahwah, NJ: Lawrence Erlbaum.
- Lloyd, G. M. (2018). *Mathematics teacher education in the context of teacher beliefs and classroom practices: A review of the literature*. *Journal of Mathematics Teacher Education*, 21(6), 595-612.
- Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Lawrence Erlbaum Associates.

- Marmolejo-Hernández, A. F., Ramírez-Montoya, M. S., & Flores-Alfaro, E. (2019). *Mathematics teachers' knowledge and its relationship with pedagogical practices*. International Journal of Innovation, Creativity and Change, 8(10), 27-40.
- Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation*. John Wiley & Sons.
- National Center for Education Statistics. (2019). *The Nation's Report Card: Mathematics 2019* (NCES 2020-451). U.S. Department of Education.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Research Council. (2001). *Adding it up: Helping children learn mathematics*. National Academy Press.
- Niess, M. L. (2005). *Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge*. Teaching and Teacher Education, 21(5), 509-523.
- Niess, M. L. (2011). *Investigating teacher, student, and content interactions in technology-supported learning environments: A research agenda for studying Technological Pedagogical Content Knowledge (TPACK)*. Computers & Education, 57(3), 1953-1960.
- Ornek, F., Robinson, D. H., & Haugan, M. (2014). *The impact of constructivist-based vs. traditional direct instruction on 4th grade math and science achievement and attitudes*. Journal of Educational Research, 107(4), 271-287.
- Patton, M. Q. (2014). *Qualitative research & evaluation methods: integrating theory and practice*. Sage publications.
- Schoenfeld, A. H. (1992). *Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics*. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan
- Schoenfeld, A. H. (1998). *Toward a theory of teaching-in-context*. Issues in education: Contributions from educational psychology, 1(1), 1-25.
- Shulman, L. S. (1986). *Those who understand: Knowledge growth in teaching*. Educational researcher, 15(2), 4-14.
- Silverman, D. (2016). *Qualitative research*. Sage publications.
- von Glasersfeld, E. (1995). *A constructivist approach to teaching*. In L. Steffe & J. Gale (Eds.), *Constructivism in education* (pp. 3-16). Hillsdale, NJ: Lawrence Erlbaum.
- von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning*. Routledge.
- Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Harvard University Press.
- Yin, R. K. (2014). *Case study research: Design and methods*. Sage publications.