Original Research Article

Method of Lines Analysis of Soret and Dufour Effects on an Unsteady Heat and Mass Transfer MHD Natural Convection CouetteFlow

.

ABSTRACT

This study examines the numerical solutions of an unsteady natural convection Couette flow of a viscous, incompressible and electrically conducting fluid between the two vertical parallel plates in the presence of thermal radiation, Soret and Dufour. The fundamental dimensionless governing partial differential equations for the impulsive movement of the plate are solved by method of lines (MOL). The numerical simulations for the effects of Soret and Dufour on the velocity profile, the temperature profile and the concentration profile of the flow are shown graphically. The analysis indicates that the fluid velocity is an increasing function of Soret and Dufour numbers. Also, the concentration profile and the temperature profileincrease with increase in the Soret number andDufour numberrespectively.

Keywords: MHD flow, Method of Lines (MOL), Dufour, Soret*,* Couette Flow

1. INTRODUCTION

In fluid dynamics, Couette flow refers to the laminar, incompressible, steady flow between two infinitely long parallel plates, top plate moving steadily and sustains the flow, with bottom plate being stationary. Such flow is driven by virtue of viscous drag force acting on fluid and the applied pressure gradient parallel to the plates. Magnetohydrodynamic (MHD) flow between two parallel plates in the presence of a transversely applied magnetic field has several applications in many devices such: MHD power generators, MHD pumps, accelerators, aerodynamics, heating, electrostatic precipitation, polymer technology, petroleum industry, purification of molten metals from non-metallic inclusions and fluid droplets-sprays, Srinivasa et al.[1].

Double diffusion or heat and mass transfer hasnumerous applications in engineering processes:exchanger devices, petroleum reservoirs, chemical catalytic reactors and processes, nuclear waste disposal etc.Double diffusive flow is driven by buoyancy due to temperature and concentration gradients. When heat and mass transfer occur together in a fluid flow, the reactions between the energy fluxes induced by the transverseaction of both temperature and composition gradients and the driving potentials are more complicated. While the energy flux induced by a composition gradient is termed Dufouror diffusion-thermal effect, the mass fluxes induced by a temperature gradient is termed Soret or thermal-diffusion effect. In general, Dufour and Soretare of smaller order of magnitude than the effects prescribed by Fick’s laws and are often neglected inheat and mass transfer processes by many researchers. The effects of Soret for instance has been utilized for isotopeseparation, Alao et al. [2]. Hayat et al. [3] had investigated soret and dufour effects on magnetohydrodynamic (MHD) flow of cassonfluid.Raoand Viswanatha [4] studied the effects of soret and dufour on hydromagneticheat and mass transfer over a vertical plate ina porous medium with a convective surface boundary condition and chemical reaction. Bhavana et al. [5] worked onthe soret effect on free convective unsteady MHD flow over a vertical plate with heat source.

Many authors had worked on various aspects of heat transfer of Couette flow under the influence of some pertinent physical parameters. Umavathi et al. [6] studied the analytical solution of generalized plain heat transfer of Couette flow in a composite channel. Rajput and Sahu[7] analyzed the effects of thermal radiation and heat source/sink on the natural convection in unsteady hydromagneticCouette flow of a viscous incompressible electricallyconducting fluid confined between two vertical parallel plates with constant heat flux at one boundary.The system of non-linear differential equations of a Newtonian magnetic lubricant squeeze film flow with magnetic induction effects had been solved and analyzed using the combination of the differential transform method and Padé approximation, by Mohammad et al. [8].Victor and Sreedhara[9] used galerkin's finite element method while studyingunsteady hydromagnetic natural convectionCouette flow through a vertical channel in thepresence of thermal radiation under anexponentially decaying pressure gradient withviscous andjoule dissipation effects.Seth et al. [10] worked on influence of Hall current on unsteady MHD convective Couetteflow of heat absorbing fluid due to accelerated movement of one of the plates of the channel in a porousmedium. InAbderrahim et al. [11], the Gear-Chebyshev-Gauss-Lobatto collocation method was used in analysing the unsteady Couettenanofluid flow with heat transfer for copper-water nanofluidunder the combined effects of the thermal radiation and a uniform transverse magnetic field with variable thermo-physical properties.Durojaye et al. [12] analysed theeffects of some thermo-physical properties offluid on heat and mass transfer flow past semi-infinitemoving vertical plate with viscousdissipation using method of lines.The method of lines analysis ofthe effects of some flow parameters on unsteady mhd fluid flowpast a moving vertical plate embedded in porous medium inthe presence of hall current and rotating system, was studied by Durojaye et al. [13]

The objective of this paper is to examine the effects of thermal diffusion (Soret) and diffusion thermo (Dufour)on an unsteady two-dimensional heat and mass transfer radiativemagnetohydrodynamic natural convective Couette flow of a viscous, incompressible, electrically conducting fluid between two vertical parallel plates with suction, in a porous medium, under the influence of a uniform transverse magnetic field with appropriate boundary conditions in the case of impulsive movement in the plate. The coupled nonlinear partial differential equations whichgovern the flow is solved using the method of lines (MOL).

2. MATHEMATICAL MODEL AND ANALYSIS

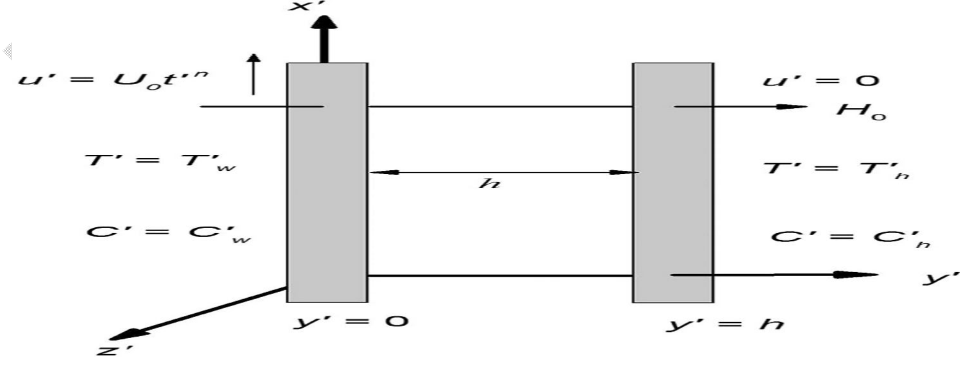

Figure 1: Physical Model of the Problem (Source:Srinivasa et al. [1])

We consider the two-dimensional unsteady natural convective Couette flow of a viscous, electrically conducting fluid past a vertical porous plate with suction, under the influence of a uniform transverse magnetic field, thermal radiation, heat and mass transfer. The x'-axis and y'-axis are taken along the plate in the vertical upward and normal direction to the plate respectively. Let the plates be separated by a distance h. At time $t' \leq 0$, the fluid and the plates of the channel are assumed to be at rest and at same temperature T_h' and concentration C_h' when time $t' > 0$, the plate at $y' = 0$starts moving with time dependent velocity $U_o t'^m$where U_o is a constant and n is a non-negative integer, in its own plane and at the same time the plate temperature and concentration is raised to T_W' and C_W' respectively while the plate at $y' = h$is kept fixed. At the same time $t' > 0$, the wall at $y' = h$ is stationary and maintained at a constant temperature T_h' and constant concentrationC_h'. It is also assumed that the transverse magnetic field of the uniform strength H_o is to be applied normal to the plate while the hall effect, viscous dissipation and induced magnetic field are negligible due to the very small magnetic Reynolds number being considered. Furthermore, it is assumed that the voltage is not applied which implies the absence of an electric field while the homogeneous chemical reaction of first order with rate constant $\bar{K}$ between the diffusion species and the fluid is neglected. In addition, the fluid has constant thermal conductivity and kinematic viscosity.In view of the assumptions above and considering the Boussinesq's approximation, the governing equations for the flow is given by the following partial differential equations Srinivasa et al.[1]

Momentum equation:

$$\frac{\partial u'}{\partial t'} = v\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_h') + g\beta^*(C' - C_h') - \frac{\sigma\mu_e^2 H_0^2}{\rho}(u' - U_o t'^m) \qquad (1)$$

Energy equation

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{D_m k_T}{c_s c_p}\frac{\partial^2 c'}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} \qquad (2)$$

Species diffusion equation:

$$\frac{\partial c'}{\partial t'} = D\frac{\partial^2 c'}{\partial y'^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T'}{\partial y'^2} \qquad (3)$$

The corresponding boundary conditions are:

$$t' \leq 0: u' = 0,\ T' = T_h', C' = C_h'\ for\ 0 \leq y' \leq h$$
$$t' > 0: u' = U_0 t'^m,\ T' = T_W',\ C' = C_W'\ at\ y' = 0 \qquad (4)$$
$$u' = 0, T' = T_h', \qquad C' = C_h'\ at\ y' = h$$

where u'- velocity component in x'- direction(ms^{-1}), T'- temperature of fluid (K), C'- concentration of fluid ($mol\,m^{-3}$), C_h'- concentration of the fluid near to the stationary plate ($mol\,m^{-3}$), T_h'- Temperature of the fluid near to the stationary plate (K), β- Coefficient of volume expansion for heat transfer (K^{-1}), β^* - Coefficient of volume expansion for mass transfer (m^3kg^{-1}), g- acceleration due to gravity, σ- electrical conductivity of the fluid (Sm^{-1}), U_0- Reference velocity (ms^{-1}), ρ- density of the fluid (kgm^{-3}), H_0- magnetic field component along y'-axis (Am^{-1}), μ- viscosity (m^2s^{-1}), t'- time in x', y'- coordinate systems (s), k-Thermal conductivity of the fluid ($Wm^{-1}K^{-1}$), C_p- Specific heat at constant pressure ($Jkg^{-1}K$), C_s- Concentration susceptibility ($m\,mol^{-1}$), D_m- Mass diffusivity (m^2s^{-1}), k_T - Thermal diffusion ratio, q_r – radiative heat flux, D - Chemical molecular diffusivity (m^2s^{-1}), T_m - Mean fluid temperature (K). $g\beta(T' - T_h')$ - thermal buoyancy effect, $g\beta^*(C' - C_h')$- concentration buoyancy effect, $\frac{\sigma\mu_e^2 H_0^2}{\rho}(u' - U_o t'^m)$- magnetohydrodynamic effect due to the Lorentz force, $\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'}$ - radiation term, $\frac{D_m k_T}{c_s c_p}\frac{\partial^2 c'}{\partial y'^2}$ – Dufour effect, $\frac{D_m k_T}{T_m}\frac{\partial^2 T'}{\partial y'^2}$ – Soret effect, $D\frac{\partial^2 c'}{\partial y'^2}$ – molecular diffusivity term.The radiative heat flux q_r in radiation term in equation (2) is simplified by making use of the Rosseland approximation,Adegbie and Fagbade [14]:

$$q_r = -\frac{4\bar{\sigma}}{3k^*}\frac{\partial T'^4}{\partial y'} \qquad (5)$$

where $\bar{\sigma}$ - Stefan–Boltzmann constant (W$m^{-2}K^{-4}$), k^*- mean absorption coefficient (m^{-1}).

Following Rajput and Sahu [7], we assume that the differences in temperature within the flow are sufficiently small such that q_r may be expressed as a linear function of T'. Hence on expanding T'^4 in a Taylor Series about T'_h up to first order approximation, we have:

$$T'^4 \cong T_h'^4 + 4(T' - T_h')T_h'^3 = 4T'T'^3 - 3T_h'^4 \qquad (6)$$

Using equations (5) and (6) in the last term of equation (2), we obtain:

$$\frac{\partial q_r}{\partial y'} = -\frac{16\bar{\sigma}T_h'^3}{3k^*}\frac{\partial^2 T'}{\partial y'^2} \qquad (7)$$

Substituting equation (7) in the equation (2), the energy equation becomes:

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{16\bar{\sigma}T_h'^3}{3k^*}\frac{\partial^2 T'}{\partial y'^2} + \frac{D_m k_T}{c_s c_p}\frac{\partial^2 c'}{\partial y'^2} \qquad (8)$$

To transform the governing equations and boundary conditions into dimensionless form, the following non-dimensional quantities are introduced. Srinivasaet al. [1]

$$u = \frac{u'}{h}, \quad y = \frac{y'}{h}, \quad t = \frac{t'v}{h^2}, \theta = \frac{T' - T_h'}{T_w' - T_h'}, \quad \phi = \frac{c' - C_h'}{C_w' - C_h'}, \quad G_r = \frac{g\beta h(T_w' - T_h')}{v} P_r = \frac{\rho v C_p}{K},$$

$$S_c = \frac{v}{D}$$

$$G_c = \frac{g\beta^* h(C_w' - C_h')}{v}, F = \frac{U_* h^{2n-1}}{v^n}, \quad M^2 = \frac{\sigma\mu_e^2 H_0^2 h^2}{\rho v}, \quad R = \frac{3kk^*}{4\bar{\sigma}T_h'^3}, \quad D_r = \frac{D_m k_T h^2 (C_w' - C_h')}{v c_s C_p (T_w' - T_h')}, \qquad (9)$$

$$S_r = \frac{D_m k_T (T_w' - T_h')}{v T_m (C_w' - C_h')}, \qquad Re_x^{-1} = \frac{U_o x'}{v}$$

In view of non-dimensional quantities in equation (9), the equations (1), (3) and (8) reduce to the following non-dimensional form:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r\theta + G_c\phi - M^2(u - Ft^n) \qquad (10)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\left(\frac{3R+4}{3R}\right)\frac{\partial^2 \theta}{\partial y^2} + D_r\frac{\partial^2 \phi}{\partial y^2} (11)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} + S_r\frac{\partial^2 \theta}{\partial y^2} \qquad (12)$$

Similarly, using equation (9), the initial and boundary conditions in equation (4) reduce to:

$t \leq 0 \;: u = 0, \quad \theta = 0, \quad \phi = 0 \; for \; 0 \leq y \leq 1$

$t > 0 : u = Ft^n, \quad \theta = 1, \quad \phi = 1, \quad at \; y = 0$(13)

$u = 0, \quad \theta = 0, \quad \phi = 0 \; at \; y = 1$

Considering the impulsive movement of the plate at $y' = 0 \; i.e n = 0,$ equation (10) becomes:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r\theta + G_c\phi - M^2(u - F) \qquad (14)$$

and the corresponding initial and boundary conditions (13) become:

$t \leq 0 \;: u = 0, \quad \theta = 0, \quad \phi = 0 \; for \; 0 \leq y \leq 1$

$t > 0 : u = F, \quad \theta = 1, \quad \phi = 1, \; at \; y = 0$ (15)

$u = 0, \quad \theta = 0, \quad \phi = 0 \; at \; y = 1$

where G_r-Grashof number for heat transfer, G_c-Grashof number for mass transfer, M-Hartmann number P_r-Prandtl number, S_c-Schmidt number, R-Thermal radiation parameter, F-Accelerating parameter, S_r- Soret number, D_r-Dufour number, u-velocity of the fluid, θ-temperature of the fluid, ϕ-concentration of the fluid, t-time in dimensionless forms, Srinivasa et al. [1].

The skin-friction or the shear stress at the moving plate of the channel in non-dimensional form is given by:

$$\tau = -\left(\frac{\tau_w'}{\rho u_o v}\right)_{y'=0} = -\left(\frac{\partial u}{\partial y}\right)_{y=0} (16)$$

The rate of heat transfer at the moving hot plate of the channel in non-dimensional form is given by:

$$Nu_o = -x' \frac{\left(\frac{\partial T'}{\partial y'}\right)_{y'=0}}{T'_w - T'_\infty} \;,\quad Nu_0 Re_x^{-1} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} \qquad (17)$$

The rate of heat transfer on the stationary plate is given by:

$$Nu_1 = -x' \frac{\left(\frac{\partial T'}{\partial y'}\right)_{y'=h}}{T'_w - T'_\infty} \;,\quad Nu_1 Re_x^{-1} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=1} \qquad (18)$$

The Sherwood number at the moving plate of the channel in non-dimensional form is given by:

$$Sh = -x' \frac{\left(\frac{\partial C'}{\partial y'}\right)_{y'=0}}{C'_w - C'_\infty} \;,\quad Sh Re_x^{-1} = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0} (19)$$

where Re_x is the Reynold's number. Srinivasa et al. [1]

3. METHOD OF LINES (MOL)

Method of Lines as a numerical procedure for solving partial differential equations (PDE's) had been reported to give accurate approximate solutions only that, its successful application to new PDE's problem depends on the experience and cleverness of the analyst. Instead of being a unique, direct and clearly defined approach, MOL is a general conceptthat requires specific details of each new PDE problem. The main idea of the MOL is to replace the spatial (boundary value) derivatives in the partial differential equations (PDE's) with algebraic approximations. With this, only the initial value variable, typically time in a physical problem, remains and thus we have a system of ordinary differential equations (ODE's) that approximates the given partial differential equations. Putting stiffness of the system of ODE's generated into consideration, an appropriate integration algorithm for initial value ODE's is chosen for numerical computations. As a result, we obtain approximate solutions to the given PDE's.Griffiths and Schiesser [15,16,17], Schiesser [18], Knapp [19], Biazar and Nomidi [20].

Linearizing and explicitly decoupling equations (11), (12) and (14), the following approximations are adopted: $\frac{\partial^2 \phi}{\partial y^2} \cong 1$ in equation (11), $\frac{\partial^2 \theta}{\partial y^2} \cong 1$ in equation (12), $\theta \cong 1$, $\phi \cong 1$in equation (14). Chung [21]. Considering the adopted approximations, equations (11), (12), (14), are rewritten as:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r + G_c - M^2(U - F) \qquad (20)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\left(\frac{3R+4}{3R}\right)\frac{\partial^2 \theta}{\partial y^2} + D_r (21)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} + S_r \qquad (22)$$

Then, we solve equations (20) – (22) subject to the transformed boundary conditions (15) by Method of Lines (MOL).Discretizing equation (20) in space variable y while leaving time variable t continuous, we have the system of ODEs:

$\left(\frac{du}{dt}\right)_i = \frac{u_{i+1}-2u_i+u_{i-1}}{h^2} + G_r + G_c - M^2 u_i + M^2 F$(23)

$= \frac{1}{h^2}u_{i-1} - \left(\frac{2}{h^2} + M^2\right)u_i + \frac{1}{h^2}u_{i+1} + G_r + G_c + M^2F$

$= \alpha_1 u_{i-1} - \alpha_2 u_i + \alpha_3 u_{i+1} + \alpha_4$(24)

where$\alpha_1 = \alpha_3 = \frac{1}{h^2}$, $\alpha_2 = \frac{2}{h^2} + M^2$, $\alpha_4 = G_r + G_c + M^2F$ (25) Now, equations (24) – (25) with boundary conditions $u(0,t) = u_0(y,t) = F$ and $u(N+1,t) \approx u(1,t) = 0$, can be solved iteratively.
For $i = 1, 2, \dots N$, $u(0,t) = u_0(y,t) = F$ and $u(N+1,t) \approx u(1,t) = 0$, equation (24) can be written in matrix form:

$$\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \vdots \\ \dot{u}_{N-1} \\ \dot{u}_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & \alpha_1 & \alpha_2 & \alpha_3 & 0 & \dots & 0 & 0 \\ 0 & 0 & \alpha_1 & \alpha_2 & \alpha_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} F \\ u_1 \\ u_2 \\ \vdots \\ u_{N-1} \\ u_N \end{bmatrix} + \begin{bmatrix} \alpha_4 \\ \alpha_4 \\ \alpha_4 \\ \alpha_4 \\ \vdots \\ \alpha_4 \end{bmatrix}$$

(26) where the coefficients α_1, α_2, α_3 and α_4 are given by equation (25) and $\dot{u}_i = \left(\frac{du}{dt}\right)_i$
In the same way, equation (21) becomes:

$\left(\frac{d\theta}{dt}\right)_i = \frac{3R+4}{3RP_r}\left(\frac{\theta_{i+1}-2\theta_i+\theta_{i-1}}{h^2}\right) + D_r$(27)

$= \frac{3R+4}{3Rh^2P_r}\theta_{i-1} - 2\left(\frac{3R+4}{3h^2RP_r}\right)\theta_i + \frac{3R+4}{3Rh^2P_r}\theta_{i+1} + D_r$

$= \beta_1\theta_{i-1} - \beta_2\theta_i + \beta_3\theta_{i+1} + \beta_4$(28)

where$\beta_1 = \frac{3R+4}{3Rh^2P_r}$, $\beta_2 = 2\left(\frac{3R+4}{3Rh^2P_r}\right)$, $\beta_3 = \frac{3R+4}{3Rh^2P_r}$, $\beta_4 = D_r$ (29)Now, equations (28) – (29) with conditions $\theta(0,t) = \theta_0, (y,t) = 1$ and $\theta(N+1,t) \approx \theta(1,t) = 0$, can be solved iteratively.
For $i = 1, 2, \dots N$, $\theta(0,t) = \theta_0(y,t) = 1$ and $\theta(N+1,t) \approx \theta(1,t) = 0$, equation (28) can be written in matrix form:

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_{N-1} \\ \dot{\theta}_N \end{bmatrix} = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & \beta_1 & \beta_2 & \beta_3 & 0 & \dots & 0 & 0 \\ 0 & 0 & \beta_1 & \beta_2 & \beta_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} 1 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{N-1} \\ \theta_N \end{bmatrix} + \begin{bmatrix} \beta_4 \\ \beta_4 \\ \beta_4 \\ \beta_4 \\ \vdots \\ \beta_4 \end{bmatrix}$$

(30) where the coefficients $\beta_1, \beta_2, \beta_3$ and β_4 are given by equation (29) and $\dot{\theta}_i = \left(\frac{d\theta}{dt}\right)_i$
Similarly, equation (22) becomes:

$\left(\frac{d\phi}{dt}\right)_i = \frac{1}{S_c}\left(\frac{\phi_{i+1}-2\phi_i+\phi_{i-1}}{h^2}\right) + S_r$(31)

$= \frac{1}{h^2S_c}\phi_{i-1} - \frac{2}{h^2S_c}\phi_i + \frac{1}{h^2S_c}\phi_{i+1} + S_r$

$= \gamma_1\phi_{i-1} - \gamma_2\phi_i + \gamma_3\phi_{i+1} + \gamma_4$(32)

where $\gamma_1 = \frac{1}{h^2 S_c},\ \gamma_2 = \frac{-2}{h^2 S_c},\ \gamma_3 = \frac{1}{h^2 S_c}, \gamma_4 = S_r$ (33)Now, equations (32) – (33) with conditions $\phi(0,t) = \phi_0(y,t) = 1$ and $\phi(N+1,t) \approx \phi(1,t) = 0$, can be solved iteratively.For $i = 1,2,\ldots N,\ \phi(0,t) = \phi_0(y,t) = 1$ and $\phi(N+1,t) \approx \phi(1,t) = 0,$ equation (32) can be written in matrix form:

$$\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \vdots \\ \dot{\phi}_{N-1} \\ \dot{\phi}_N \end{bmatrix} = \begin{bmatrix} \gamma_1 \gamma_2 \gamma_3 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \gamma_1 \gamma_2 \gamma_3 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \gamma_1 \gamma_2 \gamma_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0\ 0 & \cdots \gamma_1 & \gamma_2 \end{bmatrix} \begin{bmatrix} 1 \\ \phi_1 \\ \phi_2 \\ \vdots \\ \phi_{N-1} \\ \phi_N \end{bmatrix} + \begin{bmatrix} \gamma_4 \\ \gamma_4 \\ \gamma_4 \\ \gamma_4 \\ \vdots \\ \gamma_4 \end{bmatrix} \quad (34)$$

where the coefficients $\gamma_1,\ \gamma_2, \gamma_3$ and $\gamma_4$are given by equation (33) and $\dot{\phi}_i = \left(\frac{d\phi}{dt}\right)_i$.

4.**RESULTS AND DISCUSSION**

This paper examinesthe effects ofSoret and Dufour on an unsteady two-dimensional heat and mass transfer radiative MHD natural convective Couette flow of a viscous, incompressible, electrically conducting fluid between the two vertical parallel plates with suction, in a porous medium, and subject to a uniform transverse magnetic field. In the analysis, method of lines (MOL) is used to solve the dimensionless forms of the governing equations of the flow.Unless otherwise stated, the values, $G_r = 5.0, G_c = 5.0, M = 2.0, h = 0.1, P_r = 0.71, D_r = 1, S_c = 0.22, S_r = 1, R = 2.0, F = 0.5,$ for the flow parameters are used in MATLAB codes for the computations and graphical simulations. Figure 2 and Figure 4, respectively, show the effects of variations inDufour number on velocity distribution and temperature distribution of the flow. The Dufour number establishes the contribution of concentration gradients to the thermal energy flux of the flow. AsDufour number increases, both the velocity and the temperature profiles of the flow increase. Also, Figure 3 andFigure 5, respectively, show the effects of variations in Soret number on velocitydistribution and concentrationdistribution of the flow. The Soret number signifies the effect of the temperature gradients causing significant mass diffusion effects. As Soret number increases, the velocity profile as well as the concentration profile of the flow increases.

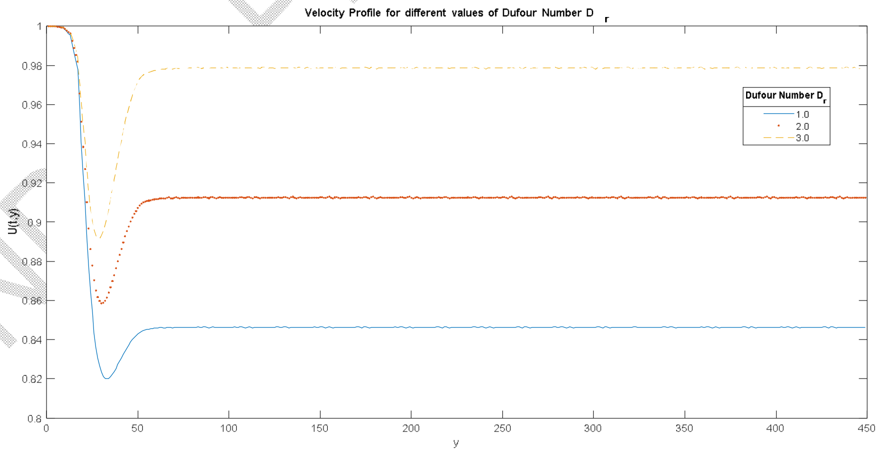

Figure 2: Velocity profile with variations in Dufour Number (D_r)

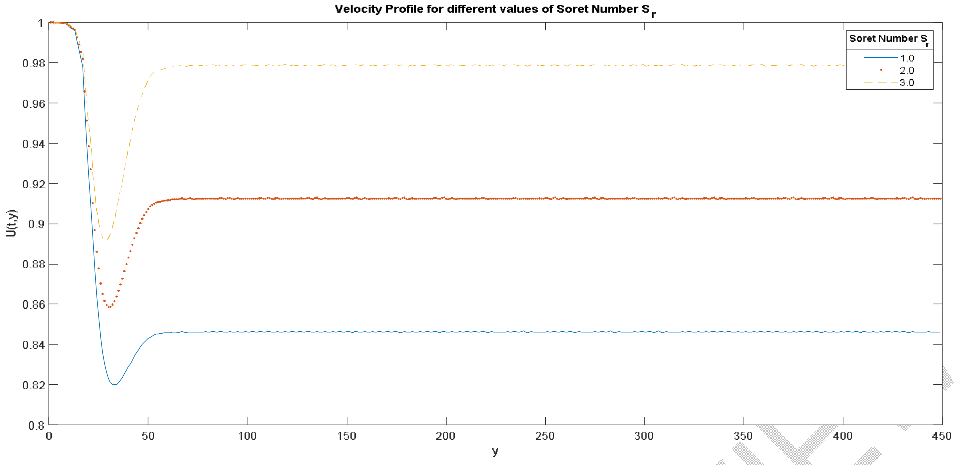

Figure 3: Velocity profile with variations in Soret Number (S_r)

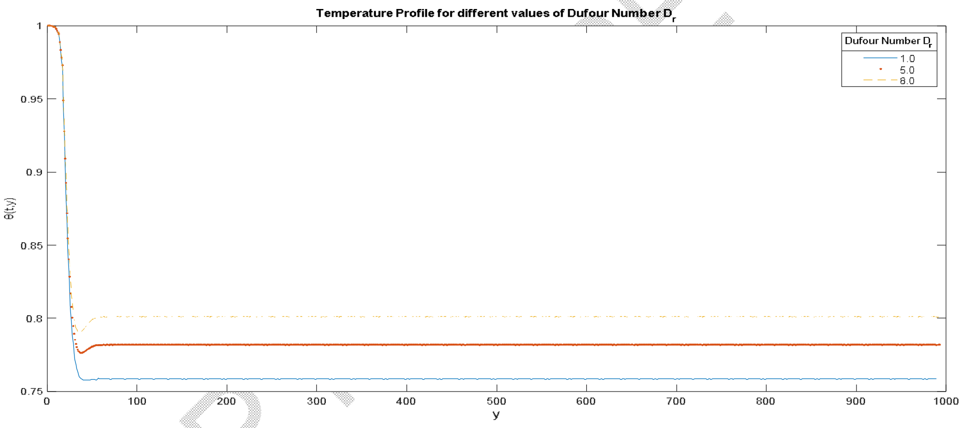

Figure 4: Temperature profile with variations in Dufour Number (D_r)

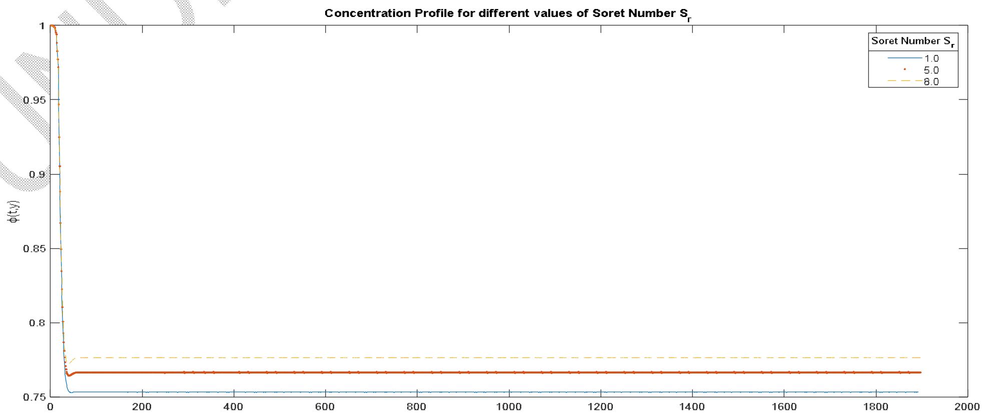

Figure 5: Concentration profile with variations in Soret Number (S_r)

5. CONCLUSION

In this paper, we use method of lines (MOL) in solvingthe governing equations of the fluid flow in dimensionless forms, and also in analyzing the effects of Dufourand Soret on velocity profile, temperature profile and concentrations profile of the flow. The results from the study show that:

1. Increase inDufour and Soret number causes increase in the velocity profile of the flow.
2. Increase in Dufour number causes increase in the temperature profile of the flow.
3. Increase inSoret number causes increase in the concentration profile of the flow.
4. Theoutputs of thegraphical simulations in Figure 2- Figure 5, of the present study agrees with those of previous study by Srinivasa et.al [1].

REFERENCES

[1] R. SrinivasaRaju, G. Jithender Reddy., J. Anand Rao, M.M. Rashidi: Thermal diffusion and diffusion thermo effects on an unsteady heat and mass transfer magnetohydrodynamic natural convection Couette flow using FEM. Journal ofComputer Design and Engineering. **3**(2016),349–362. http://dx.doi.org/10.1016/j.jcde.2016.06.003

[2] F.I. Alao, A.I. Fagbade, B.O. Falodun. Effects of thermal radiation, Soret and Dufour on an unsteady heatand mass transfer flow of a chemically reacting fluid past asemi-infinite vertical plate with viscous dissipation. Journal of the Nigerian Mathematical Society. 35(2016) 142-158. http://dx.doi.org/10.1016/j.jnnms.2016.01.002

[3] Hayat T., Shehzad S.A, Asaedi A. Soret and Dufour effects on magnetohydrodynamic (MHD) flow of casson fluid. Appl Math mech 2012;10:1301–12.

[4] Rao Madhusudhana, Viswanatha G. Soret and Dufour effects on hydromagnetic heat and mass transfer over a vertical plate in a porousmedium with a convective surface boundary condition and chemical reaction. Int J Eng Res Appl 2012;2:056–76.

[5] Bhavana M, ChennaKesavarah D, Sudhakariah A. The Soret effect on free convective unsteady MHD flow over a vertical plate with heatsource. Int J Innov Res SciEng Tech 2013;2:1617–28.

[6] Umavathi J.C, Chamkha A.J, Sridhar K.S.R. Generalized plain Couette flow and heat transfer in a composite channel. Transp. Porous Media, 2010; 85: 157–69.

[7] U. S. Rajput & P. K. Sahu. Effects of Thermal Radiation and Heat Source / Sink on The Natural Convection in Unsteady HydromagneticCouette Flow between Two Vertical Parallel Plates With Constant Heat Flux At One Boundary. International Journal of Mathematical Archive-3(6), 2012, 2211-2224.

[8] Mohammad Mehdi Rashidi, NavidFreidoonimehr, EbrahimMomoniat, BehnamRostami (2015). Study of nonlinear MHD tribological squeeze film at generalized magnetic Reynolds numbers using DTM. https://doi.org/10.1371/journal.pone.0135004

[9] Victor M. Job and Sreedhara Rao Gunakala. Unsteady hydromagnetic natural convection couette flow through a vertical channel in the presence of thermal radiation under an exponentially decaying pressure gradient with viscous and joule dissipation effects using galerkin's finite element method. International Journal of Application or Innovation in Engineering & Management (IJAIEM) 2(10), 2013. ISSN 2319 – 4847

[10] Seth, G.S., Sharma, R., Kumbhakar, B. Effects of Hall current on unsteady MHD convective Couetteflow of heat absorbing fluid due to accelerated movement of one of the plates of the channel in a porousmedium. J. Porous Media **19**, 13–20 (2016)

[11] AbderrahimWakif, ZoubairBoulahia, Farhad Ali, Mohamed R. Eid, RachidSehaqui. Numerical Analysis of the Unsteady Natural Convection MHD CouetteNanofluid Flow in the Presence of Thermal Radiation Using Single and Two-Phase Nanofluid Models for Cu-Water Nanofluids. *Int. J. Appl. Comput. Math* (2018) 4:81https://doi.org/10.1007/s40819-018-0513y

[12] M.O. Durojaye, K.A. Jamiu and I.O. Ajala. The Effects of Some Thermo-physical Properties ofFluid on Heat and Mass Transfer Flow Past Semi-infiniteMoving Vertical Plate with ViscousDissipation*Journal of Engineering Research and Reports8(2): 1-11, 2019; Article no.JERR.50730*, *ISSN: 2582-2926*

[13] M.O. Durojaye, K.A. Jamiu and F. O. Ogunfiditimi. Effects of Some Flow Parameters on Unsteady MHD Fluid FlowPast a Moving Vertical Plate Embedded in Porous Medium in the Presence of Hall Current and Rotating System. Asian Research Journal of Mathematics 16(6): 15-29, 2020; Article no.ARJOM.55543, *ISSN: 2456-477X*

[14] Adegbie K.S., Fagbade A.I. Influence of variable fluid properties and radiative heat loss on Magnetohydrodynamic forced convection flow in a fluid saturated porous medium. Journal of the Nigerian Association of Mathematical Physics. 2015;30:101–116.

[15] G.W. Griffths and W.E. Schiesser. Traveling Wave Analysis of Partial Differential Equations: Numerical and Analytical Methods with MATLABand Maple, Academic Press, 2010.

[16] G.W. Griffiths and W.E. Schiesser. A Compendium of Partial Differential Equation Models: Method of Lines Analysis with Matlab, CambridgeUniversity Press, 2009. https://doi.org/10.1017/cbo9780511576270

[17] G.W. Grffths and W.E. Schiesser, The Numerical Method of Lines:Integration of Partial Differential Equations, Elsevier, 2012.

[18]Schiesser W.E. The numerical Method of Lines- Integration of partial differential equations. Academic Press, San Diego;1991.

[19] Knapp R. A method of Lines Framework in Mathematica. Journal of Numerical Analysis, Industrial and Applied Mathematics. 2008;3(1):43-59.

[20] Biazar J, Nomidi N. Solution of theEmdem-Fowler equation by the method oflines. Journal of Nature Science andSustainable Technology. 2013;7(2):45-55.

[21] Chung T.J (2002). Computational Fluid Dynamics. Cambridge University Press.Pp:6-11