

**ANALYSIS OF THERMAL DIFFUSION AND
DIFFUSION THERMO EFFECTS ON AN
UNSTEADY HEAT AND MASS TRANSFER
MAGNETOHYDRODYNAMIC NATURAL
CONVECTION COUETTE FLOW**

ABSTRACT

This study examines the numerical solutions of unsteady hydromagnetic natural convection Couette flow of a viscous, incompressible and electrically conducting fluid between the two vertical parallel plates in the presence of thermal radiation, thermal diffusion, diffusion thermo and other physical flow parameters. The fundamental dimensionless governing partial differential equations for impulsive movement of the plate were solved by method of lines (MOL). Numerical results for the velocity, the temperature and the concentration are shown graphically for Thermal diffusion (Soret), Diffusion thermo (Dufour) flow parameters. Analysis indicates that the fluid velocity is an increasing function of Soret and Dufour numbers. Also, the concentration profile increases with increase in the Soret number and the temperature profile increases with increase in Dufour number.

Keywords: MHD flow, method of lines (MOL), Dufour, Soret, Couette Flow

1. INTRODUCTION

In fluid dynamics, Couette flow refers to the laminar, incompressible, steady flow between two infinitely long parallel plates, top plate moving steadily and sustains the flow, bottom plate stationary. Such flow is driven by virtue of viscous drag force acting on fluid and the applied pressure gradient parallel to the plates. The flow has application in hydro-static lubrication, viscosity pumps, turbine etc.

Electrically conducting viscous fluid flow between two parallel plates in the presence of a transversely applied magnetic field has several applications in many devices such as magnetohydrodynamic (MHD) power generators, MHD pumps, accelerators, aerodynamics, heating, electrostatic precipitation, polymer technology, petroleum industry, purification of molten metals from non-metallic inclusions and fluid droplets-sprays.

Many authors have worked on different aspects of heat transfer of Couette flow. Umavathi et. al. 2010, studied the generalized plain heat transfer of Couette flow in a composite channel. Rajput et.al. 2012 analyzed the exact solution of free convection in unsteady MHD Couette flow of a viscous incompressible, electrically conducting fluid between the two vertical parallel plates with the presence of thermal radiation, in the absence of thermal diffusion and diffusion thermo. The system of non-linear differential equations of a Newtonian magnetic lubricant squeeze film flow with magnetic induction effects were solved by Rashidi et.al. 2015, using the combination of the differential transform and Padé

approximation methods. Raju et. al.2016, worked on thermal diffusion and diffusion thermo effects on an unsteady heat and mass transfer magnetohydrodynamic natural convection Couette flow using FEM.

The objective of this paper is to study the thermal diffusion and diffusion thermo effects on an unsteady two-dimensional heat and mass transfer radiative MHD natural convective Couette flow of a viscous, incompressible, electrically conducting fluid between two vertical parallel plates with suction, embedded in a porous medium, under the influence of a uniform transverse magnetic field with suitable boundary conditions in the case of impulsive movement in the plate. The governing equations of the flow is described by a system of coupled nonlinear partial differential equations. Solution to the system is obtained using the method of lines (MOL) which is a semi-analytical is used to obtain the numerical solutions to the model.

2. MATHEMATICAL MODEL AND ANALYSIS

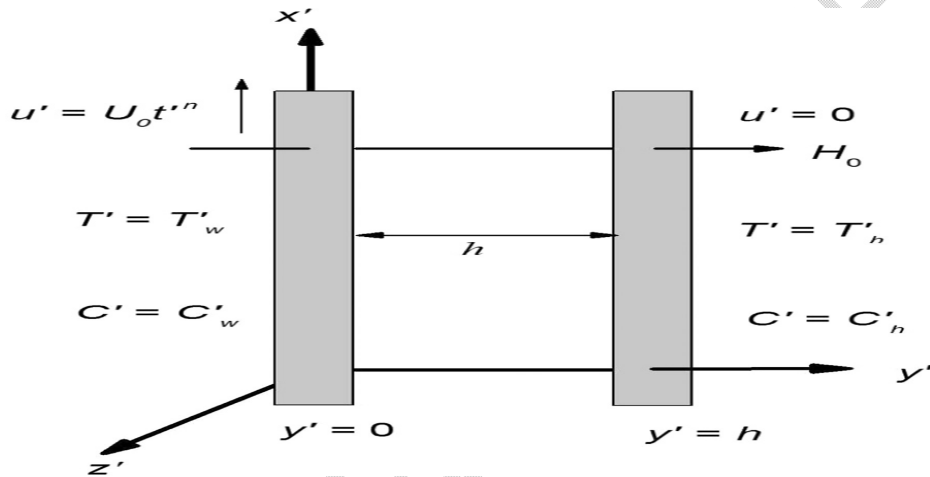


Figure 1: Physical Model of the Problem (Source: Raju et.al. 2016)

We consider the two-dimensional unsteady natural convective Couette flow of a viscous, electrically conducting fluid past a vertical porous plate with suction, under the influence of a uniform transverse magnetic field, thermal radiation, heat and mass transfer. The x' -axis and y' -axis are taken along the plate in the vertical upward and normal direction to the plate respectively. Let the plates be separated by a distance h . Initially, at time $t' \leq 0$, the fluid and the plates of the channel are assumed to be at rest and at same temperature T'_h and concentration C'_h . When time $t' > 0$, the plate (at $y' = 0$) start moving with time dependent velocity $U_0 t^n$ (U_0 being a constant and n being a non-negative integer) in its own plane and at the same time the plate temperature and concentration is raised to T'_w and C'_w respectively while the plate (at $y' = h$) is kept fixed. At the same time $t' > 0$, the wall at $y' = h$ is stationary and maintained at a constant temperature T'_h and constant concentration C'_h . An assumed transverse magnetic field of the uniform strength H_0 is to be applied normal to the plate. The hall effect, viscous dissipation and induced magnetic field are assumed to be negligible because of the magnetic Reynolds number of the flow is very small. It is assumed that voltage is not applied which implies the absence of an electric field. The homogeneous chemical reaction of first order with rate constant \bar{K} between the diffusion species and the fluid is neglected. The fluid has constant thermal conductivity and kinematic viscosity, and the Boussinesq's approximation is taken for the flow. In view of the assumptions above, the unsteady flow is governed by the following partial differential equations. Raju et.al. 2016.

Momentum equation:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_h) + g\beta^*(C' - C'_h) - \frac{\sigma\mu_0^2 H_0^2}{\rho} (u' - U_0 t'^m) \quad (2.1)$$

Energy equation

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{D_m k_T}{C_s c_p} \frac{\partial^2 C'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y'} \quad (2.2)$$

Species diffusion equation:

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T'}{\partial y'^2} \quad (2.3)$$

The corresponding boundary conditions are:

$$\begin{aligned} t' \leq 0: u' = 0, T' = T'_h, C' = C'_h \text{ for } 0 \leq y' \leq h \\ t' > 0: u' = U_0 t'^m, T' = T'_w, C' = C'_w \text{ at } y' = 0 \\ u' = 0, T' = T'_h, C' = C'_h \text{ at } y' = h \end{aligned} \quad (2.4)$$

where u' - velocity component in x' - direction(ms^{-1}), T' - temperature of fluid (K), C' - concentration of fluid ($mol m^{-3}$), C'_h - concentration of the fluid near to the stationary plate ($mol m^{-3}$), T'_h - Temperature of the fluid near to the stationary plate (K), β - Coefficient of volume expansion for heat transfer (K^{-1}), β^* - Coefficient of volume expansion for mass transfer ($m^3 kg^{-1}$), g - acceleration due to gravity, σ - electrical conductivity of the fluid (Sm^{-1}), U_0 - Reference velocity (ms^{-1}), ρ - density of the fluid (kgm^{-1}), H_0 - magnetic field component along y' -axis (Am^{-1}), μ - viscosity (m^2s^{-1}), t' - time in x', y' - coordinate systems (s), k - Thermal conductivity of the fluid ($Wm^{-1}K^{-1}$), C_p - Specific heat at constant pressure ($Jkg^{-1}K$), C_s - Concentration susceptibility ($m mol^{-1}$), D_m - Mass diffusivity (m^2s^{-1}), k_T - Thermal diffusion ratio, q_r – radiative heat flux, D - Chemical molecular diffusivity (m^2s^{-1}), T_m - Mean fluid temperature (K). $g\beta(T' - T'_h)$ - thermal buoyancy effect, $g\beta^*(C' - C'_h)$ - concentration buoyancy effect, $\frac{\sigma\mu_0^2 H_0^2}{\rho} (u' - U_0 t'^m)$ - magnetohydrodynamic effect due to the

Lorentz force, $\frac{1}{\rho c_p} \frac{\partial q_r}{\partial y'}$ - radiation term, $\frac{D_m k_T}{C_s c_p} \frac{\partial^2 C'}{\partial y'^2}$ – dufour effect, $\frac{D_m k_T}{T_m} \frac{\partial^2 T'}{\partial y'^2}$ – Soret effect, $D \frac{\partial^2 C'}{\partial y'^2}$ – molecular diffusivity term.

The radiative heat flux q_r in radiation term in equation (2) is simplified by making use of the Rosseland approximation as:

$$q_r = - \frac{4\bar{\sigma}}{3k^*} \frac{\partial T'^4}{\partial y'} \quad (2.5)$$

where $\bar{\sigma}$ - Stefan–Boltzmann constant ($Wm^{-2}K^{-4}$), k^* - mean absorption coefficient (m^{-1})

Following Rajput and Sahu, [19], we assume that the differences in temperature within the flow are sufficiently small such that q_r may be expressed as a linear function of T' . Hence on expanding T'^4 in a Taylor Series about T'_h up to first order approximation, we have:

$$T'^4 \cong T_h'^4 + 4(T' - T'_h)T_h'^3 = 4T'T_h'^3 - 3T_h'^4 \quad (2.6)$$

Using equations (5) and (6) in the last term of equation (2), we obtain:

$$\frac{\partial q_r}{\partial y'} = - \frac{16\bar{\sigma}T_h'^3}{3k^*} \frac{\partial^2 T'}{\partial y'^2} \quad (2.7)$$

Substituting equation (7) in the equation (2), the energy equation becomes:

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{16\bar{\sigma}T_h'^3}{3k^*} \frac{\partial^2 T'}{\partial y'^2} + \frac{D_m k_T}{C_s c_p} \frac{\partial^2 C'}{\partial y'^2} \quad (2.8)$$

To transform the governing equations and boundary conditions into dimensionless form, the following non-dimensional quantities are introduced. Raju et.al., [18]

$$\begin{aligned} u = \frac{u'}{h}, y = \frac{y'}{h}, t = \frac{t'v}{h^2}, \theta = \frac{T' - T'_h}{T'_w - T'_h}, \phi = \frac{C' - C'_h}{C'_w - C'_h}, G_r = \frac{g\beta h(T'_w - T'_h)}{\nu} \\ G_c = \frac{g\beta^* h(C'_w - C'_h)}{\nu}, F = \frac{U_0 h^{2n-1}}{\nu^n}, M^2 = \frac{\sigma\mu_0^2 H_0^2 h^2}{\rho\nu}, R = \frac{3kk^*}{4\bar{\sigma}T_h'^3}, \\ P_r = \frac{\rho\nu c_p}{K}, S_c = \frac{\nu}{D}, D_r = \frac{D_m k_T h^2 (C'_w - C'_h)}{\nu c_s c_p (T'_w - T'_h)}, S_r = \frac{D_m k_T (T'_w - T'_h)}{\nu T_m (C'_w - C'_h)}, Re_x^{-1} = \frac{U_0 x'}{\nu} \end{aligned} \quad (2.9)$$

In view of non-dimensional quantities in equation (2.9), the equations (2.1), (2.3) and (2.8) reduce to the following non-dimensional form:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_c \phi - M^2(u - Ft^n) \quad (2.10)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \left(\frac{3R+4}{3R} \right) \frac{\partial^2 \theta}{\partial y^2} + D_r \frac{\partial^2 \phi}{\partial y^2} \quad (2.11)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} + S_r \frac{\partial^2 \theta}{\partial y^2} \quad (2.12)$$

Using equation (9), the boundary conditions (4) reduce to:

$$\begin{aligned} t \leq 0 : u = 0, \quad \theta = 0, \quad \phi = 0 \text{ for } 0 \leq y \leq 1 \\ t > 0: u = Ft^n, \quad \theta = 1, \quad \phi = 1, \text{ at } y = 0 \\ u = 0, \quad \theta = 0, \quad \phi = 0 \text{ at } y = 1 \end{aligned} \quad (2.13)$$

Considering the impulsive movement of the plate at $y' = 0$ i.e. $n = 0$, equation (2.10) becomes:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_c \phi - M^2(u - F) \quad (2.14)$$

and the corresponding boundary conditions (13) reduce to:

$$\begin{aligned} t \leq 0 : u = 0, \quad \theta = 0, \quad \phi = 0 \text{ for } 0 \leq y \leq 1 \\ t > 0: u = F, \quad \theta = 1, \quad \phi = 1, \text{ at } y = 0 \\ u = 0, \quad \theta = 0, \quad \phi = 0 \text{ at } y = 1 \end{aligned} \quad (2.15)$$

where G_r -Grashof number for heat transfer, G_c -Grashof number for mass transfer, M -Hartmann number P_r -Prandtl number, S_c -Schmidt number, R -Thermal radiation parameter, F -Accelerating parameter, S_r - Soret number, D_r -Dufour number, u -velocity of the fluid, θ -temperature of the fluid, ϕ -concentration of the fluid, t -time in dimensionless forms.

For practical engineering applications and the design of chemical engineering systems, quantities of interest include the following Skin-friction, Nusselt number and Sherwood number. The skin-friction or the shear stress at the moving plate of the channel in non-dimensional form is given by:

$$\tau = -\left(\frac{\tau'_w}{\rho u_o v} \right)_{y'=0} = -\left(\frac{\partial u}{\partial y} \right)_{y=0} \quad (2.16)$$

The rate of heat transfer at the moving hot plate of the channel in non-dimensional form is given by:

$$Nu_o = -x' \frac{\left(\frac{\partial T'}{\partial y'} \right)_{y'=0}}{T'_w - T'_\infty}, \quad Nu_o Re_x^{-1} = -\left(\frac{\partial \theta}{\partial y} \right)_{y=0} \quad (2.17)$$

The rate of heat transfer on the stationary plate is given by:

$$Nu_1 = -x' \frac{\left(\frac{\partial T'}{\partial y'} \right)_{y'=h}}{T'_w - T'_\infty}, \quad Nu_1 Re_x^{-1} = -\left(\frac{\partial \theta}{\partial y} \right)_{y=1} \quad (2.18)$$

The Sherwood number at the moving plate of the channel in non-dimensional form is given by:

$$Sh = -x' \frac{\left(\frac{\partial C'}{\partial y'} \right)_{y'=0}}{C'_w - C'_\infty}, \quad Sh Re_x^{-1} = -\left(\frac{\partial \phi}{\partial y} \right)_{y=0} \quad (2.19)$$

where Re_x is the Reynold's number. Raju et.al.2016.

3. METHOD OF LINES (MOL)

The basic idea of the MOL is to replace the spatial (boundary value) derivatives in the PDE with algebraic approximations (Biazar and Nomidi, 2013, Shiesser , Knapp, 2008). Once this is done, only the initial value variable, typically time in a physical problem, remains. Then, with only one remaining independent variable, we have a system of ODEs that approximates the original PDE. Any suitable integration algorithm for initial value ODEs can now be used to compute an approximate numerical solution to the PDE.

For computation and linearization purpose, and to explicitly decouple equations (2.11), (2.12) and (2.14), the following approximations are adopted: $\frac{\partial^2 \phi}{\partial y^2} \cong 1$ in equation (2.11), $\frac{\partial^2 \theta}{\partial y^2} \cong 1$ in equation (2.12), $\theta = \phi \cong 1$ in equation (2.14) (Chung, 2002).

Rewriting equations (2.11), (2.12), (2.14), with approximation above, we have:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r + G_c - M^2(U - F) \quad (3.1)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r} \left(\frac{3R+4}{3R} \right) \frac{\partial^2 \theta}{\partial y^2} + D_r \quad (3.2)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{S_c} \frac{\partial^2 \phi}{\partial y^2} + S_r \quad (3.3)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r} \left(\frac{3R+4}{3R} \right) \frac{\partial^2 \theta}{\partial y^2} + D_r \frac{\partial^2 \phi}{\partial y^2}$$

Then, we solve equations (2.20) – (2.22) subject to the transformed boundary conditions (2.15) by Method of Lines (MOL).

Discretizing equation (20) in space variable y while leaving time variable t continuous, we have the system of ODEs:

$$\left(\frac{du}{dt} \right)_i = \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + G_r + G_c - M^2 u_i + M^2 F \quad (3.4)$$

$$\begin{aligned} &= \frac{1}{h^2} u_{i-1} - \left(\frac{2}{h^2} + M^2 \right) u_i + \frac{1}{h^2} u_{i+1} + G_r + G_c + M^2 F \\ &= \alpha_1 u_{i-1} - \alpha_2 u_i + \alpha_3 u_{i+1} + \alpha_4 \end{aligned} \quad (3.5)$$

$$\text{where } \alpha_1 = \alpha_3 = \frac{1}{h^2}, \quad \alpha_2 = \frac{2}{h^2} + M^2, \quad \alpha_4 = G_r + G_c + M^2 F \quad (3.6)$$

Now, equations (3.5) – (3.6) with conditions $u(0, t) = u_0(y, t) = F$ and $u(N + 1, t) \approx u(1, t) = 0$, can be solved iteratively.

For $i = 1, 2, \dots, N$, $u(0, t) = u_0(y, t) = F$ and $u(N + 1, t) \approx u(1, t) = 0$, equation (3.5) can be written in matrix form:

$$\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \vdots \\ \dot{u}_{N-1} \\ \dot{u}_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & \alpha_1 & \alpha_2 & \alpha_3 & 0 & \dots & 0 & 0 \\ 0 & 0 & \alpha_1 & \alpha_2 & \alpha_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} F \\ u_1 \\ u_2 \\ \vdots \\ u_{N-1} \\ u_N \end{bmatrix} + \begin{bmatrix} \alpha_4 \\ \alpha_4 \\ \alpha_4 \\ \vdots \\ \alpha_4 \end{bmatrix} \quad (3.7)$$

where the coefficients $\alpha_1, \alpha_2, \alpha_3$ and α_4 are given by equation (25) and $\dot{u}_i = \left(\frac{du}{dt}\right)_i$

In the same way, equation (21) becomes:

$$\left(\frac{d\theta}{dt}\right)_i = \frac{3R+4}{3RP_r} \left(\frac{\theta_{i+1}-2\theta_i+\theta_{i-1}}{h^2}\right) + D_r \quad (3.8)$$

$$\begin{aligned} &= \frac{3R+4}{3Rh^2P_r} \theta_{i-1} - 2 \left(\frac{3R+4}{3Rh^2P_r}\right) \theta_i + \frac{3R+4}{3Rh^2P_r} \theta_{i+1} + D_r \\ &= \beta_1 \theta_{i-1} - \beta_2 \theta_i + \beta_3 \theta_{i+1} + \beta_4 \end{aligned} \quad (3.9)$$

$$\text{where } \beta_1 = \frac{3R+4}{3Rh^2P_r}, \beta_2 = 2 \left(\frac{3R+4}{3Rh^2P_r}\right), \beta_3 = \frac{3R+4}{3Rh^2P_r}, \beta_4 = D_r \quad (3.10)$$

Now, equation (28) – (29) with conditions $\theta(0, t) = \theta_0(y, t) = 1$ and $\theta(N + 1, t) \approx \theta(1, t) = 0$, can be solved iteratively.

For $i = 1, 2, \dots, N$, $\theta(0, t) = \theta_0(y, t) = 1$ and $\theta(N + 1, t) \approx \theta(1, t) = 0$, equation (3.9) can be written in matrix form:

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_{N-1} \\ \dot{\theta}_N \end{bmatrix} = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & \beta_1 & \beta_2 & \beta_3 & 0 & \dots & 0 & 0 \\ 0 & 0 & \beta_1 & \beta_2 & \beta_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} 1 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{N-1} \\ \theta_N \end{bmatrix} + \begin{bmatrix} \beta_4 \\ \beta_4 \\ \beta_4 \\ \vdots \\ \beta_4 \end{bmatrix} \quad (3.11)$$

where the coefficients $\beta_1, \beta_2, \beta_3$ and β_4 are given by equation (3.10) and $\dot{\theta}_i = \left(\frac{d\theta}{dt}\right)_i$

Similarly, equation (3.3) becomes:

$$\left(\frac{d\phi}{dt}\right)_i = \frac{1}{S_c} \left(\frac{\phi_{i+1}-2\phi_i+\phi_{i-1}}{h^2}\right) + S_r \quad (3.12)$$

$$\begin{aligned} &= \frac{1}{h^2S_c} \phi_{i-1} - \frac{2}{h^2S_c} \phi_i + \frac{1}{h^2S_c} \phi_{i+1} + S_r \\ &= \gamma_1 \phi_{i-1} - \gamma_2 \phi_i + \gamma_3 \phi_{i+1} + \gamma_4 \end{aligned} \quad (3.13)$$

$$\text{where } \gamma_1 = \frac{1}{h^2S_c}, \gamma_2 = \frac{-2}{h^2S_c}, \gamma_3 = \frac{1}{h^2S_c}, \gamma_4 = S_r \quad (3.14)$$

Now, equation (32) – (33) with conditions $\phi(0, t) = \phi_0(y, t) = 1$ and $\phi(N + 1, t) \approx \phi(1, t) = 0$, can be solved iteratively.

For $i = 1, 2, \dots, N$, $\phi(0, t) = \phi_0(y, t) = 1$ and $\phi(N + 1, t) \approx \phi(1, t) = 0$, equation (3.12) can be written in matrix form:

$$\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \vdots \\ \dot{\phi}_{N-1} \\ \dot{\phi}_N \end{bmatrix} = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & \gamma_1 & \gamma_2 & \gamma_3 & 0 & \dots & 0 & 0 \\ 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & \gamma_1 & \gamma_2 \end{bmatrix} \begin{bmatrix} 1 \\ \phi_1 \\ \phi_2 \\ \vdots \\ \phi_{N-1} \\ \phi_N \end{bmatrix} + \begin{bmatrix} \gamma_4 \\ \gamma_4 \\ \gamma_4 \\ \vdots \\ \gamma_4 \end{bmatrix} \quad (3.15)$$

where the coefficients $\gamma_1, \gamma_2, \gamma_3$ and γ_4 are given by equation (3.13) and $\dot{\phi}_i = \left(\frac{d\phi}{dt}\right)_i$.

4.RESULT AND DISCUSSION

In this paper, the thermal diffusion and diffusion thermo effects on an unsteady two-dimensional heat and mass transfer radiative MHD natural convective Couette flow of a viscous, incompressible, electrically conducting fluid between the two vertical parallel plates with suction, embedded in a porous medium, under the influence of a uniform transverse magnetic field had been analysed. In the analysis, method of lines (MOL) is used to solve the governing equations of the flow and MATLAB codes is used for the computations system of ODEs in equations (3.7), (3.11), (3.15) for $i = 1, 2, 3$ and graphical simulations.

The flow parameters' values $G_r = 5.0, G_c = 5.0, M = 2.0, h = 0.1, P_r = 0.71, D_r = 1, S_c = 0.22, S_r = 1, R = 2.0, F = 0.5$, had been used for computations unless otherwise stated.

In Figure 2, the simulated graph of effect of diffusion thermo (Dufour) on velocity of the flow is shown. As the Dufour number increases, the velocity of the flow also increases.

Figure 3 shows that increase in thermal diffusion (Soret) causes increase in velocity of the flow. Thus, velocity distribution in the boundary layer is greatly affected by greater thermal diffusion.

Figure 4 shows that the temperature increases in the entire boundary region as Dufour number is increasing.

Figure 5 indicates how Soret number affects the concentration of the flow. As the Soret number increases, the concentration profile of the flow increases in the entire region.

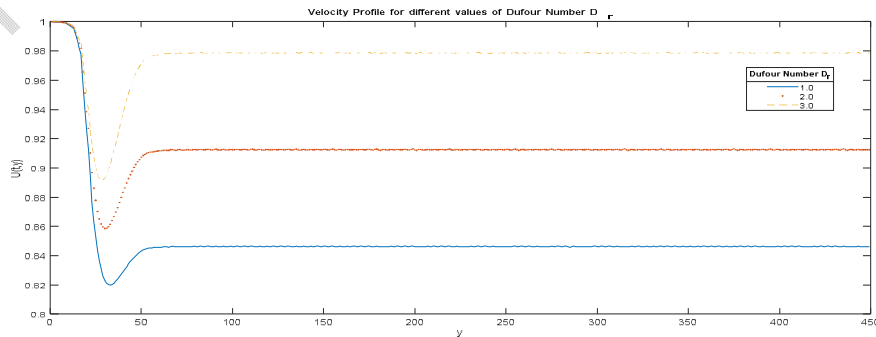


Figure 2: Velocity profile with variations in Dufour Number (D_r)

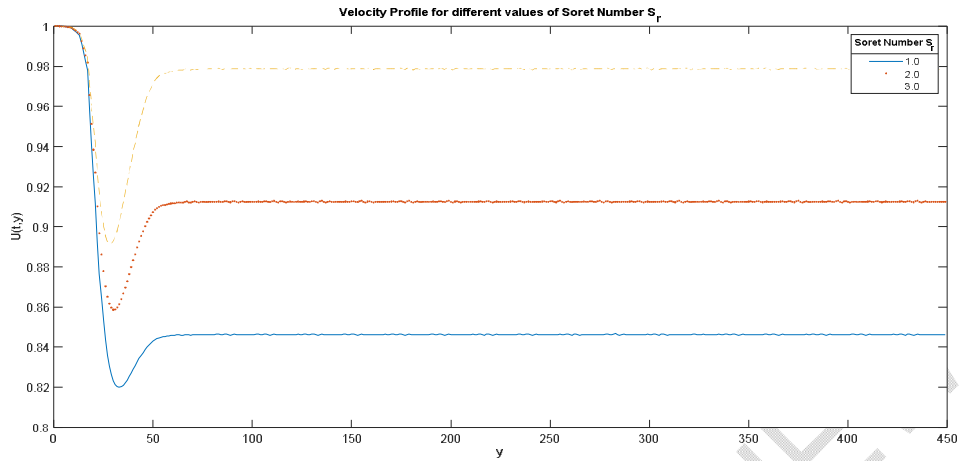


Figure 3: Velocity profile with variations in Soret Number (S_r)

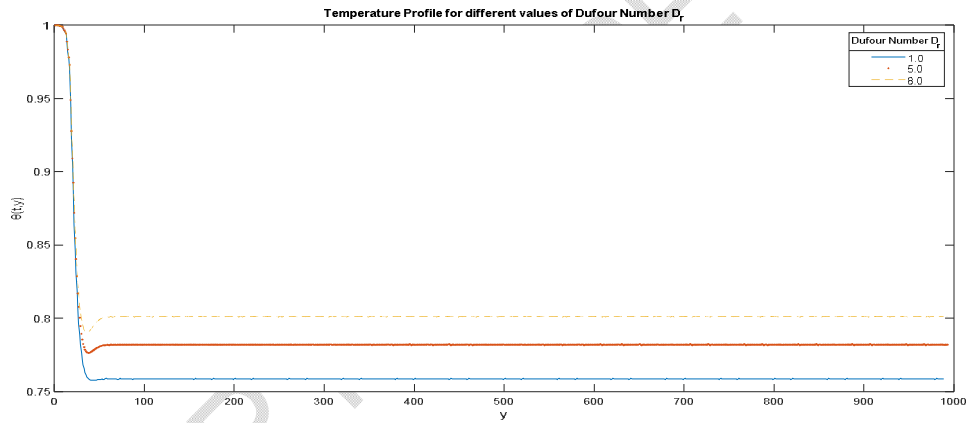


Figure 4: Temperature profile with variations in Dufour Number (D_r)

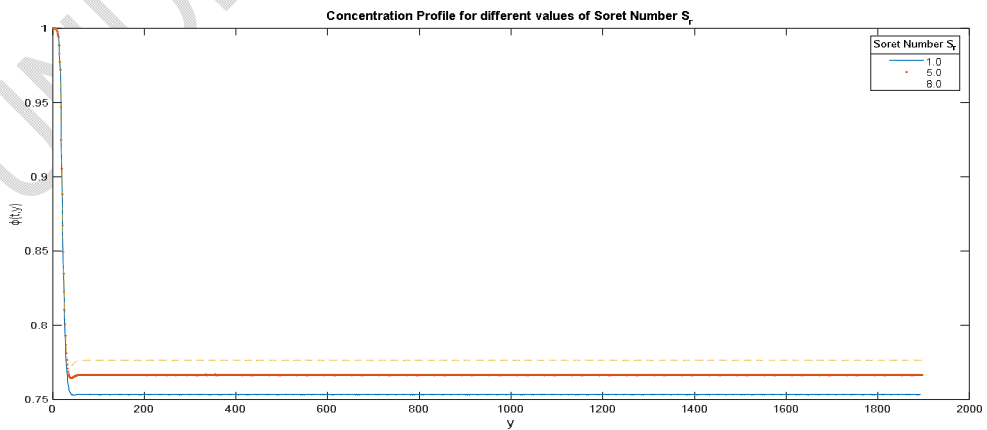


Figure 5: Concentration profile with variations in Soret Number (S_r)

5. CONCLUSION

In this paper, we have used method of lines (MOL) in solving coupled differential equations of the flow model. The Dufour, and Soret on velocity profile, temperature profile and concentrations profile are discussed. Result from the study shows that:

1. As Dufour and Soret number increase the velocity profile of the flow increases.
2. As Dufour number increases the temperature profile of the flow increases.
3. As Soret number increases the concentration profile of the flow increases.
4. It is also observed that the results of computations and graphical simulations by method of lines (MOL) used agrees with previous results of Raju et.al 2016.

REFERENCES

1. Biazar J. and Nomidi N., Solution of the Emdem-Fowler equation by the method of lines, *Journal of nature science and sustainable Technology*, 7(2): (2013) 45-55
2. Chung T.J (2002). *Computational Fluid Dynamics*. Cambridge University Press. Pp:6-11
3. Knapp R. A method of Lines Framework in Mathematica" *Journal of Numerical Analysis, Industrial and Applied Mathematics*, 3(1): (2008) 43-59.
4. Rashidi M.M, Freidoonimehr N., Momoniati E., Rostami B., Study of nonlinear MHD tribological squeeze film at generalized magnetic Reynolds numbers using DTM. *PLOS ONE* 2015; 10(8) e0135004 <http://dx.doi.org/10.1371/journal.pone.0135004>.
5. Rajput U.S., and Sahu P.K., Natural convection in unsteady hydromagnetic Couette flow through a vertical channel in the presence of thermal radiation. *Int. J. Appl. Math. Mech* 2012;8(3)35–56.
6. Rashidi M.M, HayatT, Erfani E., Mohimani Pour S.A, Hendi Awatif A. Simultaneous effects of partial slip and thermal-diffusion and diffusion-thermo on steady MHD convective flow due to a rotating disk. *Commun. Nonlinear Sci. Numer. Simul.* 2011;16(11)4303–17.
7. Rajput U.S and Sahu P.K. Natural convection in unsteady hydromagnetic Couette flow through a vertical channel in the presence of thermal radiation. *Int. J. Appl. Math. Mech*, (2012) 8(3)35–56.
8. Schiesser W.E. *The numerical Method of Lines- Integration of Partial Differential Equations*, Academic Press, San Diego. (1991).
9. Umavathi J.C, Chamkha A.J, Sridhar K.S.R. Generalized plain Couette flow and heat transfer in a composite channel. *Transp. Porous Media*, 2010; 85: 157–69.