

gr-connectedness in topological spaces

Abstract

We discuss about gr-connectedness and different properties of it, such as remarks, examples and theorems.

Keywords: gr-closed sets, gr-continuous maps and gr-connectedness.

1 Introduction

In topology, Connectedness [1] is a known concept and it is one of the principal topological property corresponding to the intuitive thought of having no breaks, which is used to distinguish topological spaces. Any space is homeomorphic to the topological spaces which are connected. Intermediate Value Theorem, Maximum Value Theorem and Uniform Continuity Theorem are formulated by the properties of connectedness, about continuous functions. Connectedness are to be discussed in digital spaces. Data structure is important in the applications on the computer which relates real topological situation. Connectedness and compactness were studied by Whyburn G. T [2], considering the assumption of Hausdorff. The basic properties of connectedness had been discussed through many researchers [3-12]. Mathematicians are motivated through the notions of connectedness to extend the existing concepts.

Stone M. [13] investigated the regular closed sets and properties of gr-closed sets in topological spaces was analyzed by Bhattacharya S. [14]. We plan to study gr-connectedness in this research as it has not been discussed yet even gr-closed set was introduced long years ago.

2 Preliminaries

$(Y; \zeta); (Z; \mathcal{A})$ (or simply Y and Z) denotes topological spaces with no separation axioms assumed. For every subset D in $(Y; \zeta)$, closure of D and interior of D are given by $cl(D)$ and $Int(D)$.

Definition 2.1. A subset D of a topological space $(Y; \zeta)$ is called gr-closed set [14] when $rc(D) \mu U$ with $D \mu U$ and U is open in $(Y; \zeta)$.

gr-closed set will be the complement of the gr-open set.

Definition 2.2. A function $h : (Y; \zeta) \rightarrow (Z; \mathcal{A})$ is called

(i) gr-continuous [15] if the inverse image is gr-closed in $(Y; \zeta)$ for every closed set in $(Z; \mathcal{A})$.

(ii) gr-irresolute [15] if the inverse image is gr-closed in $(Y; \zeta)$ for every gr-closed set in $(Z; \mathcal{A})$.

Definition 2.3. A topological space Y is T_{gr} -space [15] if every gr-closed subset in Y is closed subset in Y .

3 Concepts in gr-connectedness

Definition 3.1. When Y can not be expressed as a disjoint union of two non-empty gr-open sets, a topological space Y is said to be gr-connected. A subset of Y is gr-connected when it is gr-connected as a subspace.

Example 3.1. Let $Y = fa; bg$ and $\zeta = fY; \acute{A}$; fagg. So, $(Y; \zeta)$ is gr-connected.

Remark 3.1. Every gr-connected space is connected. But, the following example gives in general, the converse is not true.

Example 3.2. Let $Y = fa; bg$ and $\zeta = fY; \acute{A}$; fag. Clearly, $(Y; \zeta)$ is connected. As gr-open sets of Y are $fY; \acute{A}$; fag; fbg and $Y = fag \cup fbg$ where fag and fbg are non-empty gr-open sets; $(Y; \zeta)$ is not a gr-connected space.

Theorem 3.3. In a topological space Y , the following statements are equivalent:

(i) Each gr-continuous map of Y into a discrete with at least two points is a constant map.

(ii) Only subsets of Y which are both gr-open and gr-closed are Y and \acute{A} .

(iii) Y is gr-connected.

Proof. (i) \Rightarrow (ii) : Let D be both gr-open and gr-closed in Y . Assume that $D \neq \acute{A}$. Let $h : Y \rightarrow Z$ be a gr-continuous map defined by $h(D) = fxg$ and $h(D^c) = fyg$ for some distinct points x and y in Z . Hence by using the assumption h is constant, we can say $D = Y$.

(ii) \Rightarrow (i) : Let $h : Y \rightarrow Z$ be a gr-continuous map. Hence Y is covered by gr-open and gr-closed covering $fh_1(z) : z \in Z$. By the assumption, $h_1(z) = \acute{A}$ or Y for each $z \in Z$. If $h_1(z) = \acute{A}$ for all $z \in Z$, then h fails to be a map. Therefore, there exists only one point $z \in Z$ such that $h_1(z) = \acute{A}$ and hence $h_1(z) = Y$. This shows that h is a constant map.

(ii) \Rightarrow (iii) : Let $Y = C \cup D$ where C and D are disjoint non-empty gr-open subsets of Y . Then, C is both gr-open and gr-closed. By the assumption, $C = \acute{A}$ or Y . Hence Y is gr-connected.

(iii) \Rightarrow (ii) : Let D be any gr-open and gr-closed subset of Y . Then, D^c is both gr-open and gr-closed. Then, Y is disjoint union of the gr-open sets D and D^c . Hence from the hypothesis of (iii), $D = \acute{A}$ or $D = Y$.

Theorem 3.4. Z is connected when $h : Y \rightarrow Z$ is a gr-continuous, onto and Y is gr-connected.

Proof. Let Z be not connected and $Z = C \cup D$ where C and D are the disjoint non-empty open set

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