gr-connectedness in topological spaces

Abstract

In this paper, we introduce the new concepts gr-connectedness in topological spaces and obtain some of their properties using gr-closed sets.

Keywords: gr-closed sets, gr-continuous maps and gr-connectedness.

1 Introduction

In topology, Connectedness [1] is a known concept. Many researchers [2-11] have analyzed the basic properties of connectedness. The notions of connectedness resulted in motivating mathematicians to generalize these notions further.

Bhattacharya S. [12] introduced and studied the properties of gr-closed sets in topological spaces. The aim of this paper is to study gr-connectedness using gr-closed set and also discuss some of their properties

2 Preliminaries

Throughout this paper (X, τ), (Y, σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X, τ), cl(A) and Int(A) denote the closure of A and interior of A respectively.

Definition 2.1. Let (X, τ) be a topological space. Then, a subset A of (X, τ) is called gr-closed set [12] if rcl$(A)\subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

The complement of the above mentioned gr-closed set is gr-open set.

Definition 2.2. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

(i) gr-continuous [13] if the inverse image of every closed set in (Y, σ) is gr-closed in (X, τ).

(ii) gr-irresolute [13] if the inverse image of every gr-closed set in (Y, σ) is gr-closed in (X, τ).

Definition 2.3. A topological space X is said to be T_{gr}-space [13] if every gr-closed subset of X is closed subset of X.

3 gr-connectedness

Definition 3.1. A topological space X is said to be gr-connected if X can not be expressed as a disjoint union of two non-empty gr-open sets. A subset of X is gr-connected if it is gr-connected as a subspace.

Example 3.1. *Let $X = \{a, b\}$ and let $\tau = \{X, \phi, \{a\}\}$. Then, (X, τ) is gr-connected.*

Remark 3.1. Every gr-connected space is connected. But, the converse need not be true in general, which follows from the following example.

Example 3.2. *Let $X = \{a, b\}$ and let $\tau = \{X, \phi\}$. Clearly, (X, τ) is connected. The gr-open sets of X are $\{X, \phi, \{a\}, \{b\}\}$. Therefore, (X, τ) is not a gr-connected space, because $X = \{a\} \cup \{b\}$ where $\{a\}$ and $\{b\}$ are non-empty gr-open sets.*

Theorem 3.3. *For a topological space X, the following are equivalent:*

(i) X is gr-connected.

(ii) X and ϕ are the only subsets of X which are both gr-open and gr-closed.

(iii) Each gr-continuous map of X into a discrete with at least two points is a constant map.

Proof. $(i) \Rightarrow (ii)$: Let F be any gr-open and gr-closed subset of X. Then, F^c is both gr-open and gr-closed. Since X is disjoint union of the gr-open sets F and F^c implies from the hypothesis of (i) that either $F = \phi$ or $F = X$.

$(ii) \Rightarrow (i)$: Suppose that $X = A \cup B$ where A and B are disjoint non-empty gr-open subsets of X. Then, A is both gr-open and gr-closed. By assumption, $A = \phi$ or X. Therefore, X is gr-connected.

$(ii) \Rightarrow (iii)$: Let $f : X \to Y$ be a gr-continuous map.Then, X is covered by gr-open and gr-closed covering $\{f^{-1}(y) : y \in Y\}$. By assumption, $f^{-1}(y) = \phi$ or X for each $y \in Y$. If $f^{-1}(y) = \phi$ for all $y \in Y$, then f fails to be a map. Then, there exists only one point $y \in Y$ such that $f^{-1}(y) = \phi$ and hence $f^{-1}(y) = X$. This shows that f is a constant map.

$(iii) \Rightarrow (ii)$: Let F be both gr-open and gr-closed in X. Suppose $F \neq \phi$. Let $f : X \to Y$ be a gr-continuous map defined by $f(F) = \{x\}$ and $f(F^c) = \{y\}$ for some distinct points x and y in Y. By assumption f is constant, we have $F = X$. □

Theorem 3.4. *If $f : X \to Y$ is a gr-continuous, onto and X is gr-connected, then Y is connected.*

Proof. Suppose that Y is not connected. Let $Y = A \cup B$ where A and B are disjoint non-empty open set in Y. Since f is gr-continuous and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty gr-open sets in X. This contradicts the fact that X is gr-connected. Hence Y is connected. □

Theorem 3.5. *If $f : X \to Y$ is a gr-irresolute surjection and X is gr-connected, then Y is gr-connected.*

Proof. Suppose that Y is not gr-connected. Let $Y = A \cup B$ where A and B are disjoint non-empty gr-open set in Y. Since f is gr-irresolute and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty gr-open sets in X. This contradicts the fact that X is gr-connected. Hence Y is connected. □

Theorem 3.6. *Suppose that X is a T_{gr}-space. Then, X is connected if and only if it is gr-connected.*

Proof. Suppose that X is connected. Then, X can not be expressed as disjoint union of two non-empty open subsets of X. Suppose X is not a gr-connected space. Let A and B be any two gr-open subsets of X such that $X = A \cup B$, where $A \cap B = \phi$ and $A \subset X$, $B \subset X$. Since X is T_{gr}-space and A, B are gr-open, A, B are open subsets of X, which contradicts to the fact that X is connected. Therefore, X is gr-connected. Conversely, assume that X is gr-connected. Then, X can not be expressed as disjoint union of two non-empty gr-open subsets of X. Since, X is a T_{gr}-space, every gr-open subset of X is open subset of X. Hence X can not be expressed as disjoint union of two non-empty open subsets of X. That is, X is connected. □

4 Conclusion

In this paper, we have introduced gr-connectedness in the topological spaces by using gr-closed sets and their properties were studied.

References

[1] Arhangelskii A. V, and Wiegandt R. Connectedness and disconnectedness in topology. Top. App. 1975;5:411-428.

[2] Brij K. Tyagi, Sumit Singh and Manoj Bhardwaj. P_β-connectedness in topological spaces. Demonstr. Math. 2017;50:299-308.

[3] Gnanambal Y, and Balachandran K. On gpr-continuous functions in topological spaces. Indian J. Pure Appl. Math. 1999;30(6):581-593.

[4] Vadivel A, and Sivashanmugaraja C. Pre-γ-Connectedness in topological spaces. Journal of Advanced Research in Scientific Computing. 2015;7(2):30-38.

[5] MEENAKUMARI N, and INDIRA T. $(r^*g^*)^*$ CONNECTEDNESS AND $(r^*g^*)^*$ COMPACTNESS IN TOPOLOGICAL SPACES. International Journal of Applied Mathematics $\&$ Statistical Sciences (IJAMSS). 2016;5(5):71-80.

[6] Sarika M, Patil and Rayanagoudar T. D. αg^*s-Compactness and αg^*s- Connectedness in Topological Spaces. Global Journal of Pure and Applied Mathematics. 2017;13(7):3549-3559.

[7] Rajeswari R, Darathi S, and Deva Margaret Helen D. Regular Strongly Compactness and Regular Strongly Connectedness in Topological Space. International Journal of Engineering Research and Technology (IJERT). 2020;9(2):547-550.

[8] Vivekananda Dembre, and Pankaj B Gavali. Compactness and Connectedness in Weakly Topological Spaces. International Journal of Trend in Research and Development. 2018;5(2):606-608.

[9] TYAGI B. K. MANOJ BHARDWAJ, and SUMIT SINGH. S_α-CONNECTEDNESS IN TOPOLOGICAL SPACES. Jordan Journal of Mathematics and Statistics (JJMS). 2019;12(3):409-429.

[10] Vivekannanda Dembre, and Sanjay M Mali. New Compactness and Connectedness in topological Spaces. International Journal of Applied Research. 2018;4(4):286-289.

[11] Vithyasangaran K. α^*-compactness and α^*-connectedness in topological spaces. JETIR. 2020;7(11):407-409.

[12] Bhattacharya S. Study of On generalized regular closed sets. Int. J. Contemp. Math. Sciences. 2011;6(3):145-152.

[13] Mahmood S. I. On Generalized Regular Continuous Functions In Topological Spaces. Ibn Al-Haitham Journal for Pure and Applied Science. 2012;25(3):377-385.