A Comparative Study of Proposed Ranking Method and Robust Ranking Method in Fuzzy Transportation Problem

Abstract: Different approaches have been explored in the past to solve fuzzy transportation or fuzzy shipping problems. The cost of shipping, supply, and demand quantities are all ambiguous in a shipping dilemma. Trapezoidal fuzzy numbers are used to represent all of the parameters in this situation. Trapezoidal fuzzy numbers can be converted into crisp numbers using a variety of ranking techniques. In this paper, we've studied the many ranking techniques and provided a comparison of two of them. A numerical example is used to illustrate this.

Keywords: Fuzzy Transportation Problem, Trapezoidal Fuzzy Number, Crisp Number, Ranking Method.

1. Introduction

Globally, the shipping problem is employed to address certain real-world issues. This particular linear programming problem is a special case. In many mathematical models, ranking fuzzy numbers is a crucial step. Fuzzy set theory was initially proposed by Zadeh L. A. [21]. Jain R. [9] was the first to identify ranking fuzzy numbers as a method for making decisions in ambiguous circumstances. Bortolan and Degani [4] have analysed and contrasted a few ranking methodologies. The ranking was collected from different authors and examined in order to draw the conclusion. Maliniand and Ananthanarayanan [12] investigated a ranking method for a fuzzy valued shipping problem based on octagonal fuzzy numbers. They transformed a fuzzy shipping issue into a clear-cut valued shipping issue.

In their study, Abbasbandy S. et al. [1] introduced a novel method for sorting trapezoidal fuzzy numbers. They put forth a number of methods for ranking fuzzy numbers. In order to demonstrate the benefits of the suggested ranking technique, comparative cases were used. A new approach for ordering fuzzy integers by distance was put out by Cheng C.H. [6] and is based on computing the centroid point, where the distance is the distance from the original point to the centroid point (x0, y0). In order to compare their findings to those of other ranking systems, they provided numerical examples. In their paper, A. Sahaya Sudha and S. Karunambigai [3] developed a ranking solution for a fuzzy transportation problem using heptagonal fuzzy integers. In order to find the optimum answer, they first employed the North West corner method, Russell's method, and least cost method.

A new approach was proposed by Gaurav Sharma et al. [7] for solving the optimal fuzzy shipping problem. For ranking the fuzzy numbers, R. Jahirhussain & P. Jayaraman [18] applied Robust's ranking algorithm. The linear programming problem form transforms the fuzzy assignment problem into a crisp assignment problem, which is then addressed using the Hungarian approach. They demonstrated using numerical examples how the fuzzy ranking method is a useful tool for resolving the fuzzy assignment problem. A novel method for rating generalised trapezoidal numbers was put out by Maliniand and Ananthanarayanan [12] utilising the trapezoid as a benchmark. Ranking techniques convert fuzzy numbers into real lines. In other words, $M: F \rightarrow R$ uses the ordering on each fuzzy number to associate it with a real number.

In this paper fuzzy shipping problem is converted in to a crisp shipping problem, by Rajshri Gupta et al [14] and Robust Ranking Method. Then we find the basic feasible solution & compare the results.

2. Objective

To learn & compare the existing ranking methods to convert trapezoidal fuzzy numbers into crisp numbers for fuzzy shipping problems is the objective of this paper.

3. Trapezoidal Fuzzy Number

For a Trapezoidal fuzzy number A(x), it has the membership function , it can be represented by A (a, b, c, d; 1) where a, b, c, d are real numbers.

Definition: A real fuzzy number A= (a, b, c, d) is a fuzzy subset of real line R with membership function $\mu_A(x)$ such that

$\mu_A(x) \rightarrow [0, 1]$

$\mu_A(x) = 0$ for a' = a

$\mu_A(x)$ is increasing in [a, b]

$\mu_A(x)$ =1 i.e. constant in [b, c]

$\mu_A(x)$ is decreasing in [c, d] &

$\mu_A(x)$ = 0 for a' = d

4. Fuzzy Ranking Methods

The literature has offered a number of techniques for rating fuzzy numbers overall. Every technique seems to have its benefits and drawbacks. The best ranking methodology to use is still being researched. The techniques are based on the extension concept, α-cuts, the Hamming distance, etc. The methods that are highly valued in the literature are the focus of this paper.

4.1. Adamo

When using the method recommended by Adamo [11], one simply evaluates the fuzzy number based on the right most point of the α-cut for a given α by equation 4.1.1.

$$AD_{\alpha}(A) = a_{\alpha}^{+} \quad \textbf{(4.1.1)}$$

4.2. Center of Maxima

The center of maxima [8] of a fuzzy number is calculated as the average value of the endpoints of the modal values interval (x is in the modal values interval if A(x) = 1)

$$CoM(A) = \frac{a_1^{-} + a_1^{+}}{2} \quad \textbf{(4.2.1)}$$

4.3. Center of Gravity

The center of gravity of a fuzzy number was introduced in [10] as

$$CoG(A) = \frac{\int_{-\infty}^{\infty} x\, A(x)\, dx}{\int_{-\infty}^{\infty} A(x)\, dx} \qquad (4.3.1)$$

4.4. Chang's Method

Chang [20] proposed a ranking method based on the index

$$C(A) = \int_{x \in \operatorname{supp} A} x\, A(x)\, dx \qquad (4.4.1)$$

It can be observed that

$$CoG(A) = \frac{C(A)}{\int_{-\infty}^{\infty} A(x)\, dx} \qquad (4.4.2)$$

4.5. Possibilistic Mean

The Possibilistic Mean value specified by C. Carlsson, and R. Fullér [5] of a Fuzzy number A ∈ F is the weighted average of the middle points of the α-cuts of a Fuzzy number A. They introduced lower & upper Possibilistic Mean values and defined the interval valued Possibilistic Mean.

$$E_p(A) = \int_0^1 \alpha \, (a_\alpha^- + a_\alpha^+)\, d\alpha \qquad (4.5.1)$$

The definition of Possibilistic Mean is based on the ranking introduced by Goetschel and Voxman [17].They considered the Fuzzy number in different perspective i.e. topological vector space setting. Fullér and Majlender [16] unmitigated the original definition.

For enterprises looking to lower the cost of delivering goods from one source to another, Gaurav Sharma et al. [7] explored the fuzzy shipping problem. In order to solve the fuzzy shipping problem with a trapezoidal membership function, they proposed an algorithm. They took into account shipping costs, supply and demand using a fuzzy trapezoidal number. Below is the proposed algorithm.

If A= (a_1 , a_2 , a_3 , a_4) is trapezoidal fuzzy number, then the defuzzified value or the ordinary crisp of A

$$a = \frac{(a_1 + 2a_2 + 2a_3 + a_4)}{6} \quad \textbf{(4.5.2)}$$

4.6Robust Ranking Method

The findings of the robust ranking method are consistent with human intuition and satisfy the qualities of compensation, linearity, and additiveness. The Robust Ranking is determined by the following if ã is a fuzzy number:

$$R(\tilde{a}) = \int_0^1 0.5\ \left(a_\alpha^L\ a_\alpha^U\right) d\alpha \quad \textbf{(4.6.1)}$$

Where, $\left(a_\alpha^L\ a_\alpha^U\right)$ is the α level cut of the fuzzy number ã.

The membership function of the trapezoidal fuzzy number (3, 5, 6, 7) is

$$\mu_A(x) = \begin{cases} \frac{x-3}{2}, & 3 \le x \le 5 \\ 1, & 5 \le x \le 6 \\ \frac{7-x}{1}, & 6 \le x \le 7 \\ 0, & otherwise \end{cases} \quad \textbf{(4.6.2)}$$

The rank of α level cut of the fuzzy number ã = (3, 5, 6, 7) is

$$\left(a_\alpha^L\ a_\alpha^U\right) = (2\alpha + 3, 7 - \alpha)$$

$$R(\tilde{a}) = \int_0^1 0.5\ \left(a_\alpha^L\ a_\alpha^U\right) d\alpha$$

$$= \int_0^1 0.5\ \left(a_\alpha^L + a_\alpha^U\right) d\alpha$$

$$= \int_0^1 0.5\ (2\alpha + 3 + 7 - \alpha)\ d\alpha$$

$$= \int_0^1 0.5\ (\alpha + 10)\ d\alpha = 5.25$$

4.7 Ranking Of Fuzzy Numbers by Rajshri Gupta et al [14]

We have considered the shipping cost, demand & supply in fuzzy trapezoidal number with new approach for ranking as described below

If $A = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ is trapezoidal fuzzy number where $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in R$ then the defuzzified value or the crisp value of A is given as

$$R(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \frac{2\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4}{8} \quad \textbf{(4.7.1)}$$

5. Numerical Example

The supply and demand are supplied as trapezoidal fuzzy numbers in the following balanced fuzzy shipping issue, which has three sources (s1, s2, s3) and four destinations (d1, d2, d3, d4).

	d1	**d2**	**d3**	**d4**	**demand**
s1	(1,2,3,4)	(1,3,6,8)	(-1,0,1,2)	(3,5,6,8)	(0,2,4,6)
s2	(4,8,12,16)	(6,7,11,12)	(2,4,6,8)	(1,3,5,7)	(2,5,9,13)
s3	(1,5,9,13)	(0,4,8,12)	(0,6,8,14)	(4,7,9,12)	(2,4,6,7)
supply	(1,3,5,7)	(0,2,4,6)	(1,3,5,7)	(1,3,5,7)	

Table 1: Shipping Problem

The Fuzzy Shipping Problem is balanced,
that is, (3, 11, 19, 27) = (4, 11, 19, 26)
Sum of Supply = Sum of Demand.

Step1: We use the method given in equation (4.7.1) for ranking the trapezoidal fuzzy numbers.

(i) $R(1, 2, 3, 4) = \frac{2\times1+2+3+2\times4}{8} = 1.9$

(ii) $R(1, 3, 6, 8) = \frac{2\times1+3+6+2\times8}{8} = 3.4$

(iii) $R(-1, 0, 1, 2) = \frac{2\times(-1)+0+1+2\times2}{8} = 0.4$

(iv) R (3 , 5 , 6 , 8) = $\frac{2\times3+5+6+2\times8}{8} = 4.1$

(v) R (0 , 2 , 4 , 6) = $\frac{2\times0+2+4+2\times6}{8} = 2.3$

(vi) R (4, 8, 12, 16) = $\frac{2\times4+8+12+2\times16}{8} = 7.5$

(vii) R (6, 7, 11, 12) = $\frac{2\times6+7+11+2\times12}{8} = 6.8$

(viii) R (2, 4, 6, 8) = $\frac{2\times2+4+6+2\times8}{8} = 3.8$

(ix) R (1, 3, 5, 7) = $\frac{2\times1+3+5+2\times7}{8} = 3.0$

(x) R (2, 5, 9, 13) = $\frac{2\times2+5+9+2\times13}{8} = 5.5$

(xi) R (1, 5, 9, 13) = $\frac{2\times1+5+9+2\times13}{8} = 5.3$

(xii) R (0, 4, 8, 12) = $\frac{2\times0+4+8+2\times12}{8} = 4.5$

(xiii) R (0, 6, 8, 14) = $\frac{2\times0+6+8+2\times14}{8} = 5.3$

(xiv) R (4, 7, 9, 12) = $\frac{2\times4+7+9+2\times12}{8} = 6.0$

(xv) R (2, 4, 6, 7) = $\frac{2\times2+4+6+2\times7}{8} = 3.5$

Thus, the fuzzy shipping problem is changed in to a crisp shipping problem as in table 2.

	d1	**d2**	**d3**	**d4**	**demand**
s1	1.9	3.4	0.4	4.1	2.3
s2	7.5	6.8	3.8	3	5.5
s3	5.3	4.5	5.3	6	3.5
supply	3	2.3	3	3	11.3

Table 2: After Ranking by Rajshri Gupta et al [14]

Step 2: Applied Least Cost Method

The Solution is shown in table 3.

	d1	**d2**	**d3**	**d4**	**demand**
s1	1.9	3.4	**(2.3)** 0.4	4.1	2.3
s2	**(1.8)** 7.5	6.8	**(0.7)** 3.8	**(3)** 3	5.5
s3	**(1.2)** 5.3	**(2.3)** 4.5	5.3	6	3.5
supply	3	2.3	3	3	

Table 3: Ranking by First Method

The shipping cost is (2.3) (0.4) + (1.8) (7.5) + (0.7) (3.8) + (3) (3) + (1.2) (5.3) + (2.3) (4.5) = 42.79

Step3: Using Robust Ranking Method for ranking the trapezoidal fuzzy numbers given in equation (4.6.1)

	d1	**d2**	**d3**	**d4**	**demand**
s1	2.5	4.5	0.5	5.5	3.0
s2	10	9	5	4	7.25
s3	7	6	7	8	4.75
supply	4	3	4	4	15

Table 4: After Ranking by Robust Ranking Method

Step 4: Applied Least Cost Method.

The solution is shown in table 5.

	d1	d2	d3	d4	Demand
s1	2.5	4.5	**(3.0)** 0.5	5.5	3
s2	**(2.25)** 10	9	**(1)** 5	**(4)** 4	7.25
s3	**(1.75)** 7	**(3)** 6	7	8	4.75
Supply	4	3	4	4	

Table 5: Ranking by Second Method

The shipping cost is (3) (0.5) + (2.25) (10) + (1) (5) + (4) (4) + (1.75) (7) + (3) (6) = 75.25

6. Result & Discussion

The solution of the taken fuzzy shipping problem after ranking by the two methods was derived by using least cost method. The basic feasible by the ranking method 1 i.e. by Rajshri Gupta et al [14] is 42.79, whereas the problem solved by the ranking method 2 i.e. by Robust Ranking Method is 75.25. This states that the first method is more efficient and yields superior solution.

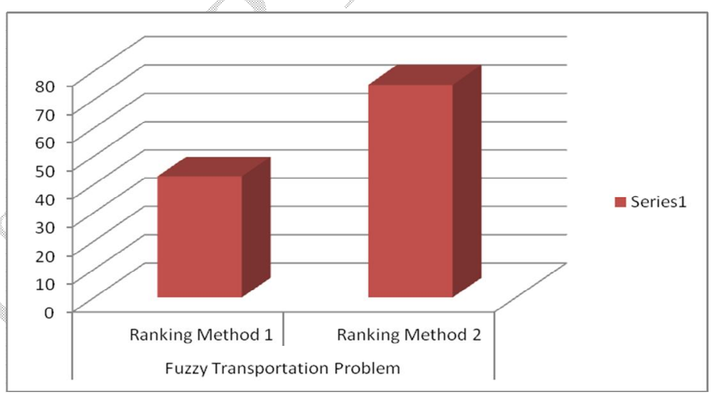

Figure 1: Graphical Comparison of Ranking Methods

Fuzzy Transportation Problem	
Ranking Method 1	Ranking Method 2
42.79	**75.25**
by Rajshri Gupta et al [14]	by Robust Ranking Method

Table 6: Comparison of Ranking Methods

7. Conclusion

In this paper we have study the different ranking methods and a comparison has given between two ranking methods. This is illustrated with a numerical example. Basic Feasible Solution obtained by the ranking method 1 i.e. by Rajshri Gupta et al [14] is least as compared to the ranking method 2 i.e. by Robust Ranking Method. The ranking method 1 i.e. by Rajshri Gupta et al [14] is more effective to solve fuzzy shipping problems in actual life situation.

8. Future Scope

Ranking fuzzy numbers is a critical task in a fuzzy decision making process. Each ranking method represents a different point of view on fuzzy numbers. It is not possible to give a final answer to the question on which fuzzy ranking method is the best. Most of the time choosing a method rather than another is a matter of preference. However, believing that the results obtained in this paper gives us the optimum cost for the fuzzy transportation problems. In future some addition or deletion can be incorporated. A wide spectrum of methods has been proposed in the literature to rank fuzzy numbers. We hope that this paper could stimulate the debate on ranking methods and further research on the topic.

9. References

1. Abbasbandy, S., and Hajjari, T., (2009), "A new approach for ranking of trapezoidal fuzzy numbers," Computers and Mathematics with Applications., 57, pp.413-419.

2. Amit Kumar, Pushpindersingh, and Jagdeep Kaur, (2010), "Two phase Method for solving Fuzzy linear programming problems using Ranking of generalized Fuzzy numbers," International Journal of Applied Science and Engineering., 8, pp.124-147.

3. A. Sahaya Sudha, and S. Karunambigai, (2017). Solving a Transportation Problem Using a Heptagonal Fuzzy Number. International Journal of Advanced Research in Science, Engineering and Technology, ISSN: 2350-0328, Vol. 4, Issue 1.

4. Bortolan,G., and Degani,R.,(1985)," A review of some methods for ranking fuzzy subsets," Fuzzy Set and Systems., 15, pp.1-19.

5. C. Carlsson, and R. Fullér, (2001). On Possibilistic Mean Value and Variance of Fuzzy Numbers. Fuzzy Sets and Systems, 122 (2), pp. 315 - 326.

6. Cheng, C. H., (1998), "A new approach for ranking fuzzy numbers by distance method," Fuzzy Sets and Systems., 95, pp. 307-317.

7. Gaurav Sharma, S. H. Abbas, Vijay Kumar Gupta, (2015), "A new Approach to solve Fuzzy Transportation Problem for Trapezoidal Number", J. of Progressive Research in Mathematics, ISSN 2395-0218,Vol.4,Issue 3, published online July 30.

8. G.J. Klir, and B. Yuan, (1995). Fuzzy Sets and Fuzzy Logic: Theory and Applications, Prentice Hall, New Jersey.

9. Jain, R., (1976), “Decision making in the presence of fuzzy variables,” IEEE Transactions on Systems, Man and Cybernetics., 6, pp. 698-703.
10. J. J. Østergaard, (1976). Fuzzy Logic Control of a Heat Exchanger Process. Stærkstrømsafdelingen, Danmarks Tekniske Højskole.
11. J. M. Adamo, (1980). Fuzzy Decision Trees: Fuzzy Sets and Systems, 4 (3), pp. 207–219.
12. Maliniand, P., and Ananthanarayanan, M., (2016), “Solving Fuzzy Transportation Problem using Ranking of Trapezoidal Fuzzy Numbers”, Int. J. of Mathematics Research. ISSN 0976-5840, Vol. 8, no. 2, pp. 127-132.
13. Pandian, P. and Natarajan, G., (2010), “A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems”, Applied Mathematical Sciences, Vol. 4, pp. 79-90.
14. Rajshri Gupta, O. K. Chaudhari and Nita Dhawade, (2018),A New Approach for Ranking Fuzzy Numbers to Solve Transportation Problem,International Journal of Mathematical Archive, 9(3), pp.186-190.
15. Rajshri Gupta, O K Chaudhari and T A Thakre (2021), A survey on the techniques to find elementary feasible solutions for fuzzy transportation problems, IOP Publishing, Journal of Physics:Conference Series, doi:10.1088/1742-6596/1913/1/012135
16. R. Fullér, and P. Majlender, (2003), On Weighted Possibilistic Mean and Variance of Fuzzy Numbers. Fuzzy Sets and Systems, 136, 3, pp. 363-374.
17. R. Goetschel, and W. Voxman, (1986),Elementary Fuzzy Calculus. Fuzzy Sets and Systems, Vol., 18, 1, pp. 31–43.
18. R. Jahirhussain and P. Jayaraman, (2013). Fuzzy Optimal Assignment Problems Via Robust Ranking Techniques,International Journal of Mathematical Archive, Vol 4, No 11.
19.Shugani Poonam, Abbas, S. H., and Gupta, V. K., (2012), “Fuzzy Transportation Problem of Trapezoidal Numbers with α- Cut and Ranking Technique”, Int. J. of Fuzzy Mathematics and Systems, ISSN 2248-9940 Vol. 2, no.3, pp. 263-267.
20.W. Chang, (1981). Ranking of Fuzzy Utilities with Triangular Membership Functions. Proceedings of International Conference on Policy Analysis and System, pp. 263–272.
21. Zadeh, L. A., (1965), “Fuzzy sets”, Information and Control, Vol. 8, pp. 338-353.