A Comparative Study of Proposed Ranking Method and Robust Ranking Method in Fuzzy Transportation Problem

Abstract: Many methods have been discussed earlier for solving fuzzy transportation problem, where the cost coefficients, supply and demand quantities are considered in the form of trapezoidal fuzzy numbers. There are many ranking methods to covert trapezoidal fuzzy numbers into crisp numbers. In this paper we have study the different ranking methods and a comparison has given between two ranking methods. This is illustrated with a numerical example.

Keywords: Fuzzy transportation problem, Trapezoidal fuzzy number, Crisp number, Ranking method

1. Introduction

Transportation problem is used globally in solving certain concrete world problems. It plays a vital role in production industry and also in many other purposes. It is a special case of Linear programming problem, which permits us to regulate the optimum shipping patterns between origins and destinations. A transportation problem in which the cost of transportation, supply and demand quantities are uncertain i.e. fuzzy in nature, it is a fuzzy transportation problem; also when the demand in the market and supply are uncertain, it satisfies the condition of vagueness. Ranking fuzzy number is a necessary step in many mathematical models. The concepts of fuzzy sets were first introduced by Zadeh L. A. [20]. He defined fuzzy set as the class of objects with continuum of grades of membership and presented various operations of fuzzy set like that of crisp set. Ranking normal fuzzy number was first introduced by Jain R. [9], for decision making in fuzzy situations. Many authors presented various approaches for solving the fuzzy transportation problem. Few of these ranking approaches have been reviewed and compared by Bortolan and Degani [4]. They dealt problem of ranking *n* fuzzy subsets of the unit interval. The ranking was obtained from various authors and studied for finding the inference. Maliniand and Ananthanarayanan [12], studied a ranking procedure based on octagonal fuzzy numbers for fuzzy valued transportation problem. They converted fuzzy transportation problem to a crisp valued transportation problem using ranking method.

Abbasbandy S. et al. [1], in their paper, introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and the right spreads at some levels of trapezoidal fuzzy numbers. They proposed several strategies for ranking of fuzzy numbers. Comparative examples were used to illustrate the advantages of proposed ranking method. Cheng C.H. [6], proposed a new method for ranking fuzzy numbers by distance method, which is based on calculating the centroid point, where the distance means from original point to the centroid point (x_0, y_0). They illustrated numerical examples to compare their results with other ranking methods. A. Sahaya Sudha & S. Karunambigai [3], in their paper introduced a ranking method for a fuzzy transportation problem with heptagonal fuzzy numbers. They used Russell's Method, North West corner Method and Least Cost Method for optimal solution and then compared the results. They found that, for solving fuzzy transportation problem by introduced ranking method and least cost method give a easier and better result than the other methods.

Gaurav Sharma et al. [7] proposed new algorithm for finding an optimal solution for a fuzzy transportation problem. They have not used linear programming techniques & goal and parametric programming technique. The optimal solution is a fuzzy number. In their paper they have used the ranking method proposed by Abbasbandy, S.,et al. [1] to convert trapezoidal fuzzy numbers to crisp numbers. R. Jahirhussain & P. Jayaraman [17] used Robust's ranking method for ranking the fuzzy numbers. The fuzzy assignment problem transformed into crisp assignment problem in the linear programming problem form and solved by using Hungarian method. By Numerical examples they showed that the fuzzy ranking method offers an effective tool for handling the fuzzy assignment problem. Maliniand, P., and Ananthanarayanan, M. [12], proposed a new approach for ranking of generalized trapezoidal number, using trapezoid as reference point. Ranking methods map fuzzy number directly in to the real line. That is, M: $F \rightarrow R$ which associate every fuzzy number with a real number and then use the ordering $\geq$ on the real line.

In this paper fuzzy transportation problem is converted in to a crisp transportation problem, using new approach of fuzzy ranking method by Rajshri Gupta et al [8] and Robust Ranking method. Then we find the basic feasible solution and compare the results.

2. Objective

The objective of this paper is to study & compare the existing ranking methods to convert trapezoidal fuzzy numbers into crisp numbers for fuzzy transportation problems.

3. Fuzzy Ranking Methods

In many fuzzy decision problems, the final scores of alternatives are represented in terms of fuzzy numbers. In order to express a crisp preference of alternatives, we need a method for constructing a crisp total ordering from fuzzy numbers. Numerous methods for total ordering of fuzzy numbers have been suggested in the literature. Each method appears to have some advantages as well as disadvantages. Some methods seem more appropriate than others, in the context of each application. Choosing a proper ordering method is still a subject of active research. The methods are based upon defining the Hamming distance, on α-cuts, on the extension principle etc. In this paper, we focus on the methods that gained high importance in the literature.

3.1. Adamo

When employing the method suggested by Adamo [11], one simply evaluates the fuzzy number based on the right most point of the α-cut for a given α by equation 3.1.1.They proposed to extend the decision trees method to the case when the involved data (probabilities, cost, profits, and losses) appeared as words belonging to the common language whose semantic representations are fuzzy sets.

$$AD_{\alpha}(A) = a_{\alpha}^{+} \qquad \textbf{(3.1.1)}$$

3.2. Center of maxima

The center of maxima [8] of a fuzzy number is calculated as the average value of the endpoints of the modal values interval (x is in the modal values interval if A(x) = 1

$$CoM\,(A) = \frac{a_1^- + a_1^+}{2} \quad \textbf{(3.2.1)}$$

3.3. Center of gravity

The center of gravity of a fuzzy number was introduced in [10] as

$$CoG\,(A) = \frac{\int_{-\infty}^{\infty} x\,A(x)\,dx}{\int_{-\infty}^{\infty} A(x)\,dx}$$

(3.3.1)

3.4. Chang's method

Chang [19] proposed a ranking method based on the index

$$C(A) = \int_{x \in \sup p\,A} x\,A(x)\,dx \quad \textbf{(3.4.1)}$$

It can be observed that

$$CoG(A) = \frac{C\,(A)}{\int_{-\infty}^{\infty} A(x)\,dx} \quad \textbf{(3.4.2)}$$

3.5. Possibilistic mean

The possibilistic mean value given by C. Carlsson, and R. Fullér [5] of a fuzzy number A ∈ F is the weighted average of the middle points of the α-cuts of a fuzzy number A. They introduced lower & upper possibilistic mean values and defined the interval valued possibilistic mean. They also investigated the relationship with interval valued possibilistic mean.

$$E_p\,(A) = \int_0^1 \alpha\,(a_\alpha^- + a_\alpha^+)\,d\alpha \quad \textbf{(3.5.1)}$$

The definition of possibilistic mean is based on the ordering proposed by Goetschel and Voxman [16].They studied the fuzzy number in different perspective i.e. topological vector space setting. Fullér and Majlender [15] extended the original definition by replacing the weight α with a general weighting function f (α).

Gaurav Sharma et al [7], studied fuzzy transportation problem for industries to reduce the transportation cost of commodity from one source to another source. They proposed algorithm to obtain the fuzzy optimal solution of fuzzy transportation problem with trapezoidal membership function. They considered transportation cost, demand & supply in fuzzy trapezoidal number. The proposed algorithm is given below

If A= (a_1 , a_2 , a_3 , a_4) is trapezoidal fuzzy number, then the defuzzified value or the ordinary crisp of A

$$a = \frac{(a_1 + 2a_2 + 2a_3 + a_4)}{6} \qquad \textbf{(3.5.2)}$$

3.6 Robust Ranking Method

Various features from fuzzy sets can be extracted for ordering of fuzzy quantities. These may be represented by a center of gravity, an area under the membership function, or various intersection points between fuzzy sets.

Robust ranking technique which satisfy compensation, linearity, and additively properties and provides results which are consist human intuition. If ã is a fuzzy number then the Robust Ranking is defined by

$$R(\tilde{a}) = \int_0^1 0.5 \left(a_\alpha^L \, a_\alpha^U\right) d\alpha \qquad \textbf{(3.6.1)}$$

Where, $\left(a_\alpha^L \, a_\alpha^U\right)$ is the α level cut of the fuzzy number ã.

The membership function of the trapezoidal fuzzy number (3, 5, 6, 7) is

$$\mu_A(x) = \begin{cases} \frac{x-3}{2}, & 3 \le x \le 5 \\ 1, & 5 \le x \le 6 \\ \frac{7-x}{1}, & 6 \le x \le 7 \\ 0, & otherwise \end{cases} \qquad \textbf{(3.6.2)}$$

The rank of α level cut of the fuzzy number ã = (3, 5, 6, 7) is

$$\left(a_\alpha^L \, a_\alpha^U\right) = (2\alpha + 3, 7 - \alpha)$$

$$R(\tilde{a}) = \int_0^1 0.5\left(a_\alpha^L\ a_\alpha^U\right) d\alpha$$

$$= \int_0^1 0.5\left(a_\alpha^L + a_\alpha^U\right) d\alpha$$

$$= \int_0^1 0.5\left(2\alpha + 3 + 7 - \alpha\right) d\alpha$$

$$= \int_0^1 0.5\left(\alpha + 10\right) d\alpha = 5.25$$

3.7 New Approach For Ranking Of Fuzzy Numbers by Rajshri Gupta et al [14]

We have considered the transportation cost, demand & supply in fuzzy trapezoidal number with new approach for ranking as described below

If $A = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ is trapezoidal fuzzy number where $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in R$ then the defuzzified value or the crisp value of A is given as

$$R(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \frac{2\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4}{8} \qquad \textbf{(3.7.1)}$$

4. Numerical Example

Consider the following balanced fuzzy transportation problem where three sources S_1, S_2, S_3 and four destinations D_1, D_2, D_3, D_4, supply and demand are given as trapezoidal fuzzy numbers.

	D_1	D_2	D_3	D_4	**Demand**
S_1	(1,2,3,4)	(1,3,6,8)	(-1,0,1,2)	(3,5,6,8)	(0,2,4,6)
S_2	(4,8,12,16)	(6,7,11,12)	(2,4,6,8)	(1,3,5,7)	(2,5,9,13)
S_3	(1,5,9,13)	(0,4,8,12)	(0,6,8,14)	(4,7,9,12)	(2,4,6,7)
Supply	(1,3,5,7)	(0,2,4,6)	(1,3,5,7)	(1,3,5,7)	

Table 1: Transportation Problem with Trapezoidal Numbers

By using definition, Fuzzy Transportation Problem is balanced

i.e. Sum of supply = Sum of demand

(3, 11, 19, 27) = (4, 11, 19, 26)

Step1: Using our new approach for ranking the trapezoidal fuzzy numbers given in equation (3.7.1)

(i) $R(1, 2, 3, 4) = \frac{2\times1+2+3+2\times4}{8} = 1.9$

(ii) $R(1, 3, 6, 8) = \frac{2\times1+3+6+2\times8}{8} = 3.4$

(iii) $R(-1, 0, 1, 2) = \frac{2\times(-1)+0+1+2\times2}{8} = 0.4$

(iv) $R(3, 5, 6, 8) = \frac{2\times3+5+6+2\times8}{8} = 4.1$

(v) $R(0, 2, 4, 6) = \frac{2\times0+2+4+2\times6}{8} = 2.3$

(vi) $R(4, 8, 12, 16) = \frac{2\times4+8+12+2\times16}{8} = 7.5$

(vii) $R(6, 7, 11, 12) = \frac{2\times6+7+11+2\times12}{8} = 6.8$

(viii) $R(2, 4, 6, 8) = \frac{2\times2+4+6+2\times8}{8} = 3.8$

(ix) $R(1, 3, 5, 7) = \frac{2\times1+3+5+2\times7}{8} = 3.0$

(x) $R(2, 5, 9, 13) = \frac{2\times2+5+9+2\times13}{8} = 5.5$

(xi) $R(1, 5, 9, 13) = \frac{2\times1+5+9+2\times13}{8} = 5.3$

(xii) $R(0, 4, 8, 12) = \frac{2\times0+4+8+2\times12}{8} = 4.5$

(xiii) $R(0, 6, 8, 14) = \frac{2\times0+6+8+2\times14}{8} = 5.3$

(xiv) $R(4, 7, 9, 12) = \frac{2\times4+7+9+2\times12}{8} = 6.0$

(xv) $R(2, 4, 6, 7) = \dfrac{2\times 2+4+6+2\times 7}{8} = 3.5$

Thus, the fuzzy transportation problem is changed in to a crisp transportation problem as in table 2.

	D_1	D_2	D_3	D_4	Demand
S_1	1.9	3.4	0.4	4.1	2.3
S_2	7.5	6.8	3.8	3	5.5
S_3	5.3	4.5	5.3	6	3.5
Supply	3	2.3	3	3	11.3

Table 2: After Ranking by Rajshri Gupta et al [8]

Step 2: Applied Least Cost Method for the basic feasible solution

Using Least cost method the basic feasible solution of above crisp transportation problem is shown in table 3.

	D_1	D_2	D_3	D_4	Demand
S_1	1.9	3.4	**(2.3)** 0.4	4.1	2.3
S_2	**(1.8)** 7.5	6.8	**(0.7)** 3.8	**(3)** 3	5.5
S_3	**(1.2)** 5.3	**(2.3)** 4.5	5.3	6	3.5
Supply	3	2.3	3	3	

Table 3: Basic Feasible Solution by Least Cost Method (Ranking by First Method)

The transportation cost is (2.3) (0.4) + (1.8) (7.5) + (0.7) (3.8) + (3) (3) + (1.2) (5.3) + (2.3) (4.5) = 42.79

Step3: Using Robust Ranking Method for ranking the trapezoidal fuzzy numbers given in equation (3.6.1)

	D_1	D_2	D_3	D_4	Demand
S_1	2.5	4.5	0.5	5.5	3.0
S_2	10	9	5	4	7.25
S_3	7	6	7	8	4.75
Supply	4	3	4	4	15

Table 4: After Ranking by Robust Ranking Method

Step 4: Applied Least Cost Method for the basic feasible solution

Using Robust Ranking Method the basic feasible solution of above crisp transportation problem is shown in table 5.

	D_1	D_2	D_3	D_4	Demand
S_1	2.5	4.5	**(3.0)** 0.5	5.5	3
S_2	**(2.25)** 10	9	**(1)** 5	**(4)** 4	7.25
S_3	**(1.75)** 7	**(3)** 6	7	8	4.75
Supply	4	3	4	4	

Table 5: Basic Feasible Solution by Least Cost Method (Ranking by Second Method)

The transportation cost is (3) (0.5) + (2.25) (10) + (1) (5) + (4) (4) + (1.75) (7) + (3) (6) = 75.25

5. Result & Discussion

The basic feasible solution of the said fuzzy transportation problem after ranking by the two methods was derived by using least cost method. The basic feasible by the ranking method 1 i.e. by Rajshri Gupta et al [8] is 42.79, whereas the problem solved by the ranking method 2 i.e. by Robust Ranking Method is 75.25. This proves that the first method i.e. our proposed method is more economical and yields better solution.

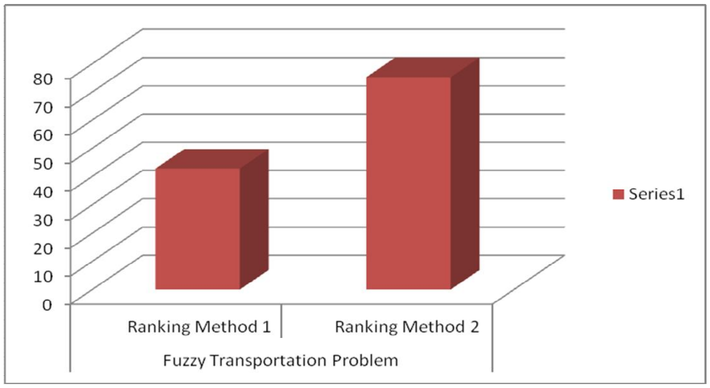

Figure 1: Graphical Comparison of Ranking Methods

Fuzzy Transportation Problem	
Ranking Method 1	Ranking Method 2
42.79	**75.25**
by Rajshri Gupta et al [8]	by Robust Ranking Method

Table 6: Comparison of Ranking Methods

6. Conclusion

In this paper we have study the different ranking methods and a comparison has given between two ranking methods. This is illustrated with a numerical example. Basic Feasible Solution obtained by the ranking method 1 i.e. by Rajshri Gupta et al [8] is least as compared to the ranking method 2 i.e. by Robust Ranking Method. The ranking method 1 i.e. by Rajshri Gupta et al [8] is very easy to understand and to apply for solving fuzzy transportation problems in real life situation. This new concept of ranking method can be used to all types of transportation problems which would give effective solutions for any uncertain data.

7. Future Scope

Ranking fuzzy numbers is a critical task in a fuzzy decision making process. Each ranking method represents a different point of view on fuzzy numbers. It is not possible to give a final answer to the question on which fuzzy ranking method is the best. Most of the time choosing a method rather than another is a matter of preference. However, believing that the results obtained in this paper gives us the optimum cost for the fuzzy transportation problems. In future some addition or deletion can be incorporated. A wide spectrum of methods has been proposed in the literature to rank fuzzy numbers. We hope that this paper could stimulate the debate on ranking methods and further research on the topic.

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