Original Research Article

Does the Mass-Gap hint atthe existence of new types compact objects ?

Abstract:

The discovery of gravitational waves (GW) fromthe coalescence binary system reduces the mass-gap to (2.6 – 5) $M_{\odot}$. Hypothetical stars such as thesuper-Chandrasekhar mass white dwarf, quark star, boson star, electro weak star, grav-star, dark matter star , supermassive neutron star, etc may exist between neutron star (NS) and blackhole (BH) within this range. Comparing the observed results with numerical calculations this author suggests the detected unseen companion compact object with mass 3.3 $M_{\odot}$ in the binary J05215658 + 4359220 is a triaxially deformed quark star (or triaxial star). If this is correct, then themass gap will again reduce from (2.6–5) $M_{\odot}$to (3.3 – 5) $M_{\odot}$. Application of the solutions of theVaidya-TikekarAnsatz in modeling Einstein's general relativity hintsthat the maximum mass of a compact object ~ 4.178 $M_{\odot}$ (i.e., four times the solar mass) in realistic case. This author proposes that compact objects with higher masses are possible in a triaxially deformed compact state. As the lighter dark matter is five times more massive thanthe baryonic matter it is also suggested that further higher mass is possible when the compact object is triaxially deformed, consisting of lighter dark matter or unknown matter whose nature or form is not yet known to us. As the detection of gravitational waves can be a convenient method to collect evidence of quark stars (e.g. strange quark stars) or dark matter admixed with normal matter neutron stars we encourage the GW community to search for theirelectromagnetic counterparts during their observation of compact objects through LIGO or VIRGO.

Key Words: **mass-gap; compact object ; neutron star, neutron star; gravitational wave**

1. Introduction

In 1915 Einstein's Field Equations [1,2] of General Relativity introduced the idea that gravitation is not a force but rather a consequence of curvature in a four dimensional space-time, which relate the Einstein tensor to stress-energy tensor, representing the distribution of energy, momentum, and stress in the space-time manifold. The fact is that the Einstein's field equations alone are not enough to determine the evolution of a gravitational system in many cases where dependency on the dynamics of matter and energy are relevant, i.e.,on the gravitational field. Since the Einstein's field equations are a system of coupled non-linear partial differential equations, several effective techniquesand approximations offer different exact solutions that are comfortable to express physically realistic stress energy tensors,which are in general found in complex, non-spherical stress in space. This means that the types of materials that would go into the makeup of such spaces are seldom found in nature (practically impossible to create such asituation on earth), and it is frequently desired to understand the geometrical properties of such space of a given mass distribution [3].

In the year 1916 Schwarzschild [4]first gave the solution (interior and exterior) of Einstein's field equation with a prediction on the existence of ablack hole. This creates interest in studies of stellar compact objects in the realm of relativistic astrophysics. Searching for the exact solutions and the best of these solutions provides a limit (e.g., maximum) on certain physical parameters (such as mass, radius, etc) as well as the behavior of thegravitational field. For this reason, the construction of exact solutions, therefore, be used for physical modeling, and, thereafter, to predict the measureable quantities about the physical systems (compact objects such as white dwarf, neutron star, etc.).

The exact solution of Einstein's equation for a spherical star full of incompressible matter, calculated by Karl Schwarzschild [4], first indicated the idea of existence of a maximum mass (M_{max}) above which the star cannot be in hydrostatic equilibrium. The existence of a limiting mass for a degenerate star was first introduced in thewhite dwarf case. In 1930, Chandrasekhar[5] theoretically calculated that during the sequence of white dwarf the gravitational pressure is balanced by the degenerate pressure of the electrons and the limiting mass is 1.44 $M_\odot$ which is known as theChandrasekhar limit. The electrons were the latest atomic particles (neutrons were not discovered at that time). So, he treated electrons as an ideal Fermi gas and this limiting mass is reached when the electrons become ultra-relativistic.

In the case of ideal gas of fermions, i.e., neutrons, there will be a sequence of neutron stars in which gravity is balanced by the degeneracy pressure of neutron gas and for arbitrarily massive stars there will be an upper limit to the possible size of such a neutron star.

The Mass-Gap [6,7], i.e., a gap that lies between the maximum mass of aneutron star and the lowest mass ofa blackhole, has puzzled astronomers for a long decades. When more massive stars die through collapse under their own gravity, they leave behind black holes. In the case of less massive stars, they end their lives through the explosion of supernovae and leave behind compact remnants, called neutron stars (NS) . Our present understanding of compact object indicates that mass of the heaviest known NS is no more than 2.5$M_\odot$[8], and the lightest known BH mass is about 5 $M_\odot$[8, 9]. Analysis of some models of stellar evolution predicted that the BHs in the mass range (2-5)$M_\odot$, known as lower mass-gap, could not be formed through the gravitational collapse of massive stars [10,11]. This mass-gap , thus raises the question: "Does anything lie in this so called mass-gap ? In this study I have tried to find answer of this question.

2. Maximum mass of a Neutron Star

Assuming the average density inside the Neutron Star is comparable to that inside heavy atomic nuclei (i.e., $\rho \approx 10^{14}$ g.cm^{-3}) and using Schwarzschild's solution, Zwicky[13,14]showed the neutron star's maximum mass $\cong 11\ M_{\odot}$. But for the degeneracy pressure of an ideal neutron gas which balances gravity, the limiting mass will be ~ $5.76\ M_{\odot}$[15]. Here Zwicky pointed out that a star's gravitational mass should be distinguished from the baryon or rest mass (i.e., the sum of baryon masses in the star) because of the astrophysical importance of this difference, which represents the released energy during core collapse of massive stars. Considering a star full of ideal Fermi gas of neutrons, Oppenheimer and Volkoff[16] concluded the maximum mass of Neutron Star as $M_{max} \cong 0.7\ M_{\odot}$. In this calculation, presence of protons was included,but nuclear forces were ignored. Cameron suggested that nuclear forces should be considered while calculating the maximum mass of Neutron Star. Considering the nuclear forces among the constituents ,i.e., stiff Equation of State (EOS) Cameron [17] obtained the maximum mass of Neutron Star $\cong 2.0\ M_{\odot}$. Observations of pulsars with the help of advanced technology and various theoretical model calculations with the microscopic level neutron star matter predict that the maximum mass of spherical, non-rotating neutron star is $M_{max} \leq 2.5\ M_{\odot}$[18]. The heaviest Neutron Star, observed recently by Antoniadis etal [19], has a mass of $2.01 \pm 0.04\ M_{\odot}$. Using the relativistic Shapiro delay method the measurements of the component mass of the millisecond pulsar J0740+6620 claim that the host Neutron Star may have a mass $2.14^{+0.10}_{-0.09}\ M_{\odot}$ with the systematic error (68.3% credibility) [20]. Latest analysis of the sample of neutron star's mass distribution models (bimodal distribution) suggests the cut-off at Neutron Star's mass i.e.,$M_{max} = 2.26^{+0.12}_{-0.05}\ M_{\odot}$ (with 68% credible interval) [21].

3. Minimum Mass of a stellar Black-Hole

In the theory of general relativity, ablack-hole with any mass could exist. Theoretically, it is predicted that a stellar black-hole with mass $<\sim 5\ M_{\odot}$cannot be formed directly by the gravitational collapse of a star [22]. On the basis of observation, the detected black-holes are classified into two categories : (a) stellar black-holes ($5M_{\odot}$ - $80M_{\odot}$) and massive black-holes ($10^{6}M_{\odot}$–$10^{10}M_{\odot}$) that are found at the centers of galaxies [23]. Latest detection of an unseen companion with mass of $3.3M_{\odot}$in the rapidly rotating giant binary system J 05215658 + 4359220 [12] indicates it either be a low mass black-hole or an exceedingly massive neutron star. As the companion mass lies within the lower mass gap it creates uncertainty in the maximum mass of the neutron star as well as the lowest mass of the black-hole.

4. Observation and Data Source

4.1 Compact Objects

The discovery of a companion compact object with mass 2.50–2.67$M_\odot$[11] in the GW signal GW190814 opens the opportunity to place it in the mass gap (2-5)$M_\odot$. But it is not yet confirmed whether this secondary compact object is the most massive NS or the lightest BH ever discovered. Another important discovery of a massive unseen companion with mass 3.3$M_\odot$(detected by Thompson et al. [12]) in a bright rapidly rotating giant binary system J 05215658 + 4359220 helps us to place this companion object firmly in the mass-gap range (2-5)$M_\odot$. Here, uncertainty still remains on identifying this unseen companion as a low mass BH or an exceedingly massive NS.

From the discovery of Pulsar (as NS) in 1967 to date, thousands of NS (as pulsars) have been detected through advanced astronomical instruments. It is found that their masses are around (1.2–2.0) $M_\odot$ and the radii are close to 10 Km [24-26]. Neutron Star masses derived from the binary radio pulsars, particularly (NS – NS) binary radio pulsars, occupy an average mass near 1.35 ±0.04 $M_\odot$[27,28] whereas NS-Xray binary system 4U1700 – 37 shows a somewhat high mass, i.e., 2.4 ± 0.3$M_\odot$[29]. X-ray band observation of a central compact object within the supermassive remnant HESS J1731 – 347 indicated the mass and radius of NS to be M = 0.77 $\pm^{0.20}_{0.70}$ $M_\odot$ and R = 10.4 $\pm^{0.86}_{0.76}$ Km [30], respectively. But this object is either a lightest NS or a 'strange star' with a more exotic EOS. The estimated NS mass, if considered a NS, is further lower than the known least massive NS, which has a mass$\cong$ 1.174 ± 0.004$M_\odot$[31]. Table I shows all the important masses of compact objects obtained through observation and theoretical calculations. From this table it is seen that the übermassiveNS , i.e., supermassive NS, with mass 3.30$M_\odot$ in model NL3 EOS (see text of Espino&Paschalidis2019[32]) may have an important role in comparison with the observed companion mass 3.3$M_\odot$ in the rotating giant binary system J05215659 + 4359220 (Thompson et al. 2019[12]). The mass-gap Neutron Star (M > 3$M_\odot$) can be detected by observing its associated electromagnetic counterparts (Wei & Yu 2021[33]).

Table I: Some important Masses of compact objects (CO)

Compact object's Description	Value ($M_\odot$)	Type	Confirmation	Ref.
Lowest NS Mass	0.77 $M_\odot$	Lightest NS or Strange Star	Doubtful	Doroshenko et al. 2022[30]
Least massive NS known	~1.17$M_\odot$	Observation of central	Confirmed	Suwa et al.

to date		compact object withinthe SN Remnant HESS J1731-347		2021[31]
Super-Chandrasekhar White Dwarf Mass	~ 2.6$M_\odot$	Theoretically estimated	Not observed	Kalita et al. 2021[34]
Observed CO with masses 2.0 – 2.40 $M_\odot$	2 – 2.4 $M_\odot$	BE-Condensed Star	Proposed	ChavanisandHarko 2011 [35]
Observed NS mass in PSR4U1700-377	2.40$M_\odot$	Mass-gap Neutron star having Radius 12.57 Km and internal magnetic field ~ 10^{18-19} G	Confirmed	Eslam-Panah et al 2017[36]
Observed NS mass in PSR J1748-2021B. Companion compact object with mass ~ 2.6$M_\odot$ in the signal GW190814.	2.70$M_\odot$ ~ 2.6$M_\odot$	Neutron Star but its radius not known Either it be a massive NS or be a lowest BH	Confirmed Not Confirmed	Eslam-Panah et al. 2017 [36]
NS having large number of colors (N_c) i.e.2.1$M_\odot$<N_c<3$M_\odot$	2.1$M_\odot$<M<3 $M_\odot$	Quark phase transition to be take place leading to formation of unstable branch of hybrid stars	Theoretical, not observed	Giacosa&Pagliara 2017 [37]
Differentially rotating NS with realistic NL3 EOS e.g. übermassive NS	3.30$M_\odot$	Supramassive NS	Proposed by this author for confirmation through observation	Espino&Paschalidis 2019 [32]
Companion mass of rapidly rotating giant binary system J05215658 + 4359220	3.30 $M_\odot$	Either it be an exceedingly massive NS or a lowest BH	Not confirmed	Thompson et al. 2019 [12]
Mass limit for compact stars in the Inverse Chameleon Mechanism	3.03$M_\odot$, can be up to 5.0 $M_\odot$	Either be a Super Chandrasekhar WD or NS around it an exoplanet is orbiting	Theoretical, not confirmed	Wei & Yu 2021 [33]

4.2 Quark Stars / Strange (Quark) Stars

It is believed that strange stars contain strange matter in the form of s-quarks in hydrostatic equilibrium. According to Pandharipande- Smith model [38] the maximum mass of such a stellar configuration is 2.24$M_\odot$. The note-worthy point is strange stars with masses exceeding the maximum masses of neutron stars will not be in hydrostatic equilibrium [39]. In table II theoretical and observed masses of various strange stars are shown. The nature of observed compact stellar objects, i.e., in determining their nature as neutron stars or strange stars, is based on the " m – a" relationship ('m',' a' are the mass and radius of the object expressed in km) i.e., neutron stars are expected to have $m < 0.3a$ and strange stars $m > 0.3a$. For the observed strange stars and compact proposed strange stars, their masses vary rangingfrom

0.85$M_\odot$ - 2.01 $M_\odot$. In the case of Swift J1818.0 – 1607 and companion in the binary system GW190814 the calculated masses are 2.01 $M_\odot$ and 2.5 $M_\odot$, respectively. These two compact objects are not yet confirmed as strange stars. In the case of theobserved super-luminous supernova ASASSN – 15LH the radiation from the core indicates the signal of birth of a strange quark star[53].

Table II: Masses and radii of various strange stars and other profound strange stars

Stange Stars	Theoretical Calculated		Observed		References
	Mass ($M_\odot$)	Radius (Km)	Mass ($M_\odot$)	Radius (Km)	
SAX J1808.4-3658	0.97	9.69	0.9 ± 0.3	7.951 ± 1.0	Elebert et al [40]
Her X-1	0.85	8.1	0.85 ± 0.15	8.1 ± 0.45	Abubkekerov et al[41]
4U1538-52	9.87	7.866	0.87 ± 0.07	7.866 ± 0.21	Rawls et al [42]
LMC –X-4	1.04	8.301	1.04 ± 0.09	8.301 ± 0.2	Rawls et al [42]
Cen X-5	1.49	9.178	1.49 ± 0.08	9.178± 0.13	Rawls et al [42]
Vela X-1	1.77	9.56	1.77 ± 0.08	9.56 ± 0.08	Rawls et al [42]
PSRJ 1614 – 2230	1.908	13	1.97 ± 0.04	9.69 ± 0.2	Demorest et al [43]
PSR J0348 + 0432	2.01	13	2.01 ± 0.04	13.0 ± 2.0	Demorest et al [43]
SMC X - 1	1.04	8.301			Rawls et al [42]
4U1820 - 30	1.58	9.1			Hansraj et al [44]
PSR J1903 + 327	1.667	9.438			Murad&Fatema [62]
4U1608 – 52	1.74	9.528			Murad&Fatema [62]
EXO 1785 - 248	1.3	8.849			Ozel et al [61]
Swift J 18118.0-1607	2.0	12.5			Thakore et al [45]
Companion of GW190814	2.5	…..			Horvard , Morres [40]
RXJ1856 – 37	0.9	6.0			Jotania, Tikekar [64]
Central Engine of Supernova ASASSN-15LH	Indication of birth of a strange quark star				Dong et al [63], Dai et al [53]

- 1$M_\odot$ = 1.475 km

Differentially rotating strange stars play important role in the mass gap region. The fact to be noted that for rigidly rotating neutron stars, the maximum allowed mass can be larger by 14% - 22% than their non-rotating maximum mass, whereas for strange stars, this larger amount

is 44% [48] .Using two-fluid frame work, a study [49] of thedimensionless tidal deformability of dark matter admixed neutron stars shows a sharp change but anarrow range for pure normal matter star and for pure dark matter stars. Using the spectral code FLAT STAR and the MIT Bag model for strange stars a numerical calculations[49]showthat the highest increase in the maximum mass compared to the value of anon-rotating star arises in the state of alow degree of differential rotation, resulting in strange stars that can sustain masses much larger than those made from nuclear matter. In that case, the highest mass is over four times larger than that of anon-rotating configuration. For the given maximum densities ρ_{max} = 1.70 x 10^{15} g. cm^{-3} the observed maximum masses of strange quark stars are 1.96372 $M_\odot$ and 2.91624 $M_\odot$ for non-rotating and differentially rotating cases, respectively. This study reinforces the hope of the existence of strange matter that can be detected through the observation of gravitational waves, gamma rays produced from the merger of compact binary systems.

4.3 Triaxially deformed star (or simply Triaxial Star)

Triaxial instability (i.e.,triplanar symmetry w.r.t. three orthogonal x, y, z planes) hasavery important role for both neutron stars (NS) and quark stars (QS). As the quark matter is composed of deconfined quarks (i.e., u, d and s quarks) quark clustering is another important parameter for bifurcation from axisymmetry to triaxial configuration. Although this bifurcation is very close to the mass shedding limit, for soft NS equation of states (EoSs) with large compactness, the triaxial sequence could totally disappear. According to general relativity, quark stars, in general, can have larger triaxial deformation (e.g., R_y/R_x ratio) before terminating the sequence at the mass shedding limit. As the decomposed u (up), d (down) and s (strange) quarks could be stable [50, 51], the significance of triaxial deformation is that a small tidal deformability of aquark star is favorable for observation [52, 53]. Using the new EoS, which is based on quark clustering, proposed by Lai and Xu[54]), Zhou et al [55]showed in a numerical simulation that in a stiff EoS a triaxial quark star can support its maximum allowed mass upto 3.3$M_\odot$. In this context, the detection of super-luminous supernova ASASSN-15LH provides the valuable information about a possible signature for the birth of a strange quark star (SQS)[56]. It leads this author[57] to suggest the unseen companion compact object with mass 3.3$M_\odot$, detected by Thompson et al [58], in the binary J05215658 + 4359220 is a triaxially deformed quark star, In other words, this could be a possible direct evidence of the existence of a triaxial star.

4.4 Dark Matter Stars

The evolution of the universe on large scales indicates about 95% of the constituents of the universe are indeed completely unknown, i.e., they cannot be described in terms of known particles. Modern cosmologysuggests these unknown constituents of the universe are dark matter (DM) and dark energy (DE). The distribution of the observed structures in the universe reflects that the behavior of dark matter is a self-interacting dark matter candidate [65], i.e.,theform of DM seems to be self-bound [66]. Using two-fluid TOV equations individually for normal matter (NM) and for dark matter(M) Wang et al [67] found in a numerical simulation study of the dark matter admixed neutron star that microscopic nature of DM particles has directly connection or link with its macroscopicmass existing in the dark matter admixed neutron stars (DANSs).

DM has obvious effect on the mass of compact star, butcompared with the energy density in the Milky Way galaxy, the existence of DM might hardly affect the mass of the compact stars situated in this galaxy.

The general scenario[68] indicates that neutron stars may contain a sizeable amount of dark matter that was accreted throughout their lives time and / or was present in proto-star cloud. Not only that, light dark matter particles have a tendency to form an extended halo around NS, while heavy dark matter particles are most likelyto create a dense DM core inside the NS. The meaning is that whenthe halo formation case is active, the outer most radius of the dark matter admixed neutron stars will exceed the radius of NS, while in the core formation case, the outermost radius will be almost thesame as baryon one[69]. In other words, the radius of compact star (neutron star) becomes affected by the presence of DM, but its mass leads towards lower values in the case of core formation.

In an investigation of mass-radius relationship of non-rotating and rotating pure hadronic matter (neutron stars) admixed with self-interacting fermionic asymmetric dark matter, with the two-fluid formalism as well as the conditions of equal and differential frequencies of nuclear matter and dark matter, Mukhopadhyay et al [69] found the maximum mass of differentially rotating stars having self-interacting dark matter ~ 1.94 $M_\odot$ with radius ~ 10.4 km.

With the assumption that DM and normal matter only couple through gravity, Leung et al [71] found in their study of tidal deformation of DM admixed withNS that

i) A two-fluid star will only reach its maximum mass i.e. $M_{max} = 2.6M_\odot$at either the object is pure normal matter (NM) or pure dark matter (DM) i.e. the DM admixed Neutron Star allows a maximum mass $2.6M_\odot$only provided the DM EoS can reach = $2.6M_\odot$.

ii) For fermionic DM and bosonic DM cases this maximum mass $M_{max} = 2.6M_\odot$ can be reached if $\mu < 0.536$ GeV (for fermionic DM) and $\rho_o\hbar^3 < 3.69$ x 10^{-4} GeV^4 (for bosonic DM) where μ,ρ_obeing the particle mass and energy density, respectively.

iii) Pure normal matter neutron stars cannot reach $2.6M_\odot$

iv) Higher mass limit i.e. $M_{max} > 2.6M_\odot$is possible if smaller (e.g.lighter) is considered.

v) The other possibility is that any more massive compact objects (that are not black hole) could exist, if detected.

5 Compact stars: New Degrees of Freedom

One of the most significant predictions of general relativity is the existence of a maximum mass for any static matter configuration. The cores of massive stars are not completely relativistic. As a result, it reduces the pressure and requires an increase in mass to reach thegravitational potential to collapse the star. Constraining the Equation of State (EoS) of massive stars , in particular, neutron star has remained one of the biggest unsolved problems in nuclear astrophysics.

White Dwarf Stars

For white dwarf stars the Chandrasekhar mass limit is $M_{ch} = 1.4\ M_\odot$ and the wider mass range of M_{ch} is ~ 1.17 – 1.75 $M_\odot$ while the critical mass of M_{ch} is ~ 1.47 $M_\odot$ [72].

Neutron Stars

The cores of neutron stars cannot be directly observed. To obtain the properties of matter at the neutron star core, one needs a suitable model of neutron star matter describingit from its crust to its core. Due to the lack of anexact theory for strongly interacting matter at densities beyond saturation density it is very difficult to model neutron star cores very accurately. Compact objects are actually the final fate of the stars. They are characterized by ultra-high matter densities. So, dense, compact objects are relativistic. On the other hand, quark matter is absolutely stable, and it can be considered the true ground state ofhadronic matter. As a result, a new class of compact objects has been postulated to exist as an alternative to neutron star. Forexample, “Strange Quark Star” , a much more stable configuration compared to neutron star

[73], that may explain the phenomenon of release of huge amount energy in super-luminous supernovae.

Using nonlinear equation of state in a numerical solution study,Panotopoulos and Rincon [73] found the maximum mass of strange quark star is 2.55 $M_{\odot}$ witharadius of R = 12.34 Km for color superconductivity model CPL10. This value is slightly more than a) the value of the maximum mass limit of quark star ≈ 2.33516 $M_{\odot}$ with radius R = 10.04 km obtained by Pant et al [74], considering the solutions representing charged fluid spheres joining smoothly with Reissner-Nordstrom metric at the pressure free surface, and b) the value of the maximum mass of stable strange quark star $M_{max} \approx 2.4 M_{\odot}$ obtained by Dong et al [75]. Note that, utilizing a specific choice with surface density ~ (2.0 – 2.7) x 10^{14} g/cm^{3} and satisfying the inequalities $0 \leq K \leq 0.24988$, K being a parameter involves n electrical intensity, Pant et al [74] extended the upper limit of maximum mass of neutron star to $M_{max} = 3.57546\ M_{\odot}$(with radius R = 15.37Km).

Theoretical estimations (based on a number of observations) predicted the maximum mass of neutron star exists in the range (2.2 – 2.9) $M_{\odot}$ i.e., up to ~ 3 $M_{\odot}$ (uncleared whether the limit of GR is supported or not) [75]. Using gravitational wave detectors' data (i.e., observed in the NS – NS , NS – BH binary systems) the value of the maximum mass for neutron star could be in the range of (2.5 – 2.67) $M_{\odot}$ with the violation of the causal limit for $R_{1.4} \leq 11.38$ Km [76]. On the other hand, Khadkikar et al [77] boost this of maximum mass to 3.60 $M_{\odot}$for a spinning neutron star with exotic equation of state at mass-shedding limit.

In the presence ofa somehow realistic, strong internal magnetic field (≈10^{17} G), the stellar charge particle population can be able to re-leptonize and de-hyperonize which finally generate more massive neutron star > 2.1 $M_{\odot}$. Considering the density dependent relativistic mean field (DD-RMF) parameters, Rather et al.[78] found the maximum masses of a heavy magnetic neutron star for pure nucleonic and hyperonicmatter are 2.575 $M_{\odot}$ (with radius R = 12.465 km) and 2.183 $M_{\odot}$(with radius R=12.238 km) in the case of dimensionless tidal deformability $\Lambda_{1.4}$ =791.483 and internal magnetic field 1.79 x 10^{17} G.

Pulsars

The most dominant sub-class of neutron stars is radio pulsars i.e., isolated neutron stars with aperiod of rotation (P) of a few tens ofmilliseconds to several seconds and dipole field $B_d = 10^{11} – 10^{13}$ G.

The second sub-class is binary X-ray pulsars with $B_d = (1 – 7) \times 10^{12}$ G and period P varies from 2 ms – ~10^4 s.

The third sub-class lies at the low B_d and short P end i.e. a mixture of millisecond pulsars (MSPs : isolated or in wide binaries) and low mass X-ray binaries (LMXBs in which case neutron stars are considered to be old >10^9 years). The observed fact is 50% of pulsars (PSRs) with characteristic age less than ~ 5×10^4 years have associated with supernova remnants (SNRs).

Hybrid Stars

To explore the possible hadron-quark phase transition in the interior of neutron star Qin et al [74] studied the first order phase transition and crossover , the tidal deformability of hybrid stars considering the 3-windowconstruction model and found the possibility of attaining themaximum mass of hybrid stars in excess of 2 $M_\odot$. This value is consistent with the value proposed by Alaverdyand and Vartanyan [79] and the observed value for the pulsar PSRJ0348+0432 with a mass of 2.1 ± 0.04 $M_\odot$.

A significant maximum mass of hybrid star has resulted in the study of possible a hadron-quark phase transition in the interior of neutron star . Using thesemi-analytical hybrid equation of states at fixed entropy per baryon for hot and cold hybrid stars,Mariani et al [80] obtainedthatthe maximum mass of the hybrid star is (2.68 – 2.69) $M_\odot$ with a quark matter core of 2.67 $M_\odot$. On the other hand, multi-messenger observation of a binary Neutron Star merger (GW170817) provides a new constraint for the masses and radii of neutron stars such that a new class of hybrid equation of state (EoS) [81] could fulfill

a) the NICER measured radius (less than 11 km i..e., 10.5 < R < 12.5 km) for the high mass pulsar PSR J0740+6620 , and
b) the maximum mass of the component star (hybrid nature) ~ 2.6 $M_\odot$ in the case of asymmetric binary merger GW190814.

Blaschke and Cierniak [81] suggested that this 2.6 $M_\odot$ object in the binary merger GW190814 could have been a hyper-massive hybrid neutron star with a large quark matter core. As the hadronic picture changes at high densities, i.e., quarks may be deconfined or percolate among tightly packed baryons, a problemremains : " whether there is a phase transition or a cross over between hadronic and quark matter " [82,83].

Quark Star or Strange Quark Star

The theoretical idea —there exists a process of conversion of a hadronic star into a quark star

is energetically convenient even if the radius of the final configuration is larger than the radius of

the initial configuration. In the two families scenario compact stars, i.e., hadronic stars and quark stars (precisely light stars are hadronic stars whereas large and massive stars are strange quark stars), do coexist and the transition between the two branches will appear in a hadronic star when a sizable fraction of strangeness is present through hyperons i.e. the metastable stellar system that can convert into a pure strange quark star [84]. Note that the highest mass measured through observation of PSR J0348 + 0432 is $M_{max} = 2.01 \pm 0.04\ M_\odot$ but the estimated mass of this black widow pulsar is 2.4 $M_\odot$ [85]. This means strange quark stars could have maximum masses larger than 2 $M_\odot$. In other words, measurements of masses larger than 2 $M_\odot$ would favor the strange quark star scenario.

A study [83] of the thermodynamic properties of asymmetric quark matter and large mass quark stars under the confined isospin-density-dependent-quark mass model suggests that neutron stars could be converted to hybrid stars or strange quark stars. These stars are usually made up of deconfined absolutely stable 'u', 'd' and 's' quarks with lepton in β-equilibrium i.e., can be

Considered as strange quark matter. By fitting the radial velocity lines and the three-band light curves in the irradiated compact star models Chu et al [83] also showed that the recently detected three heavy compact stars PSR J0348+0432 ($2.01 \pm 0.04\ M_\odot$), MSP J0740+6620 (with mass $2.14^{+0.010}_{-0.09} M_\odot$ of 68.3% credibility interval and $2.14^{+0.20}_{-0.18} M_\odot$ of 95.4% credibility interval) and PSR J2215+5135 (with mass $2.27^{+0.10}_{-0.09} M_\odot$) as heavy quark stars. In other words, the detected heavy compact star e.g. the heavy quark star have maximum mass 2.27 $M_\odot$ till date.

Magnetars, Triaxial Stars

To date, 31 magnetars have been detected. Among them, only six magnetars are passing through their triaxial phases [86], whereas rest numbers magnetars have already crossed their triaxial phases. Recently detected cosmic baby Swift J1818.0 – 1607 is a very young magnetarwith a characteristic age ~ 240 – 300 years, which can be considered the best source for detecting and confirming it as a triaxial star [87]. Its estimated mass is ~ 2.0 $M_\odot$. On the other hand, numerical simulation using a stiff equation of state suggests that a triaxial quark star can support its maximum allowed mass up to 3.3 $M_\odot$ [55].

Black Holes

Based on a number of observational and theoretical models, the mass distribution between the highest mass of neutron star (2 $M_\odot$) and the lowest mass of black hole (5 $M_\odot$) exists in the range 2 – 5 $M_\odot$ [88-90]. Observation of GW 190814 by the LIGO and VIRGO detectors indicated that the gravitational wave signal was originated from the merger of a $\approx$ 23 $M_\odot$ black hole (BH) with a $\cong$ 2.6$M_\odot$ compact object. This compact binary source, GW190814, is atypical both in its highly asymmetric masses and in its lower-mass component lying between the heaviest known neutron star and the lightest known black hole. Therefore, GW190814 offers an unprecedented probe into the mass gap, making the mass of binary's secondary lighter component (2.5 – 2.67) $M_\odot$ as the heaviest neutron star or lightest black hole ever identified in a compact object binary.

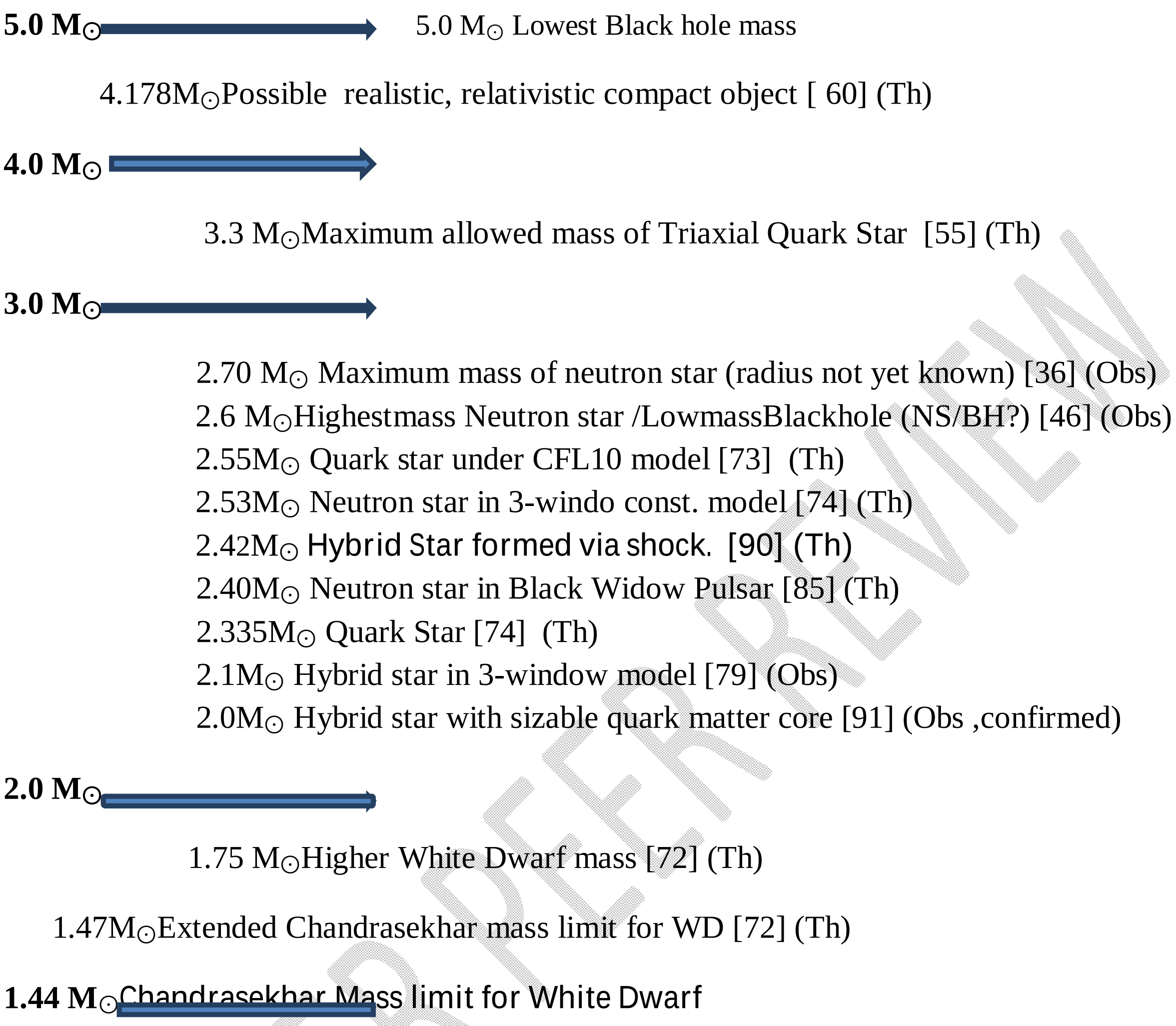

Fig.1: Mass Gap Ladder for different compact objects with their maximum masses (not scaled). Theoretical or estimated values of maximum mass are marked by (Th / Estimated), Observed or detected maximum mass by (Obs /Confirmed). For 2.6$M_\odot$ companion mass of binary system is marked by (NS/BH ?) i.e. not yet solved.

6 Conclusion

Analytical solutions of Einstein’s gravitational field equations provide a key for thedetermination ofthe maximum mass of very compact astrophysical objects. The central density of these compact objects could be ~ 10^{15} $g.cm^{-3}$ and calculations for obtaining their maximum mass require reliable information such as structure, behavior of matter at such anultra-high density. Analytical solutions describe the stable mass (in hydrostatic equilibrium) of normal matter, i.e., maximum mass of aneutron star is possible, which means that neutron stars are expected to have their mass $m < 0.3a$, (‘m’, ‘a’ being the mass and radius of the star expressed in km), while for strange stars $m > 0.3a$. The noteworthy point is that strange stars with masses exceeding the maximum mass of neutron stars will not be in hydrostatic equilibrium [39].

The highest massin a strange star (see table II) observed in PSR J1614 – 2230 is ~ $1.978M_\odot$ (≈ $2.0M_\odot$). In the absence of reliable information regarding the structure as well as behavior of matter inside the compact object,Vidya-Tikekarsuperdense star model [59] is very effective in modeling the astrophysical compact objects (such as neutron stars, quark stars, etc.). Using general solutions to Vaidya-Tikekar metric (with densityof2 x 10^{14} $g.cm^{-3}$) in modeling Einstein’s field equation,Sasidharan and Sabu [60] found a possible realistic, relativistic compact object with a mass of $4.1738M_\odot$.

Triaxial star is another realistic compact object which could be placed in the mass range (3 – 5) $M_\odot$ in the mass gap (2.6 – 5)$M_\odot$. We suggest the detected unseen companion compact object with mass ~ 3.3 $M_\odot$ in the binary J 05215658 + 4359220 as a triaxialstar, i.e.,triaxially deformed quark star, as it matches the result found in numerical simulation study. If it is correct, then the highest detected compact object mass will be ~ 3.3 $M_\odot$resulting in a mass gap of(3.3 – 5) $M_\odot$. Lighter dark matter is ~ 5 times the mass of baryonic matter [70], which means thatatriaxially deformed star consisting of lighter dark matter particles may have the possibility to possessing a higher mass, i.e.,> 3.3 $M_\odot$. Another possibility is the triaxially deformed massive compact object (not a blackhole) whose nature or type is not yet known to us and whosedetection we have to search for. This author thus suggests that possibly it be a triaxialstar, which consists of unknown matter whose form is not yet known to us and remains undetectable to the astronomers.

As the detection of gravitational waves can be a convenient method to collect evidence of quark stars (e.g., strange quark stars) or dark matter admixed normal matter neutron stars [92,93] we encourage the GW community to search for their electromagnetic counterparts during their observation of compact objects through LIGO or VIRGO.

References :

1. Einstein A.Sitzungsber. Preuss. Acad. Wiss.1916 ; 1 : 688
2. Einstein A.Die Grundlage der allgemeinenRelativitätstheorie.Annalen der Phys.1916 ;354 : 354. Doi:10.1002/andp.19163540702

3. Pandharipande V R, Pines D, Smith R. Neutron star structure: theory, observation, and speculation. Astrophys. J.1976 ; 208 : 550. Doi: 10.1086/154637
4. Schwarzschild K.Sitzungherichte der KniglichPreussischenAcademie derWissenschaftenzu Berlin , Phys Math, Klasse1916 ; 424 : 1
5. Chandrasekhar S. The maximum mass of an ideal white dwarf.Astrophys. J. 1931 ; 74 : 81
6. Farr W M,Sravan N.etal. The mass distribution of stellar-mass blackholes.Astrophys. J. 2011 ; 741: 103
7. California Institute of Tech*Science Daily* (23rd June 2020 Issue)
8. Özel F, Paltis D, Narayan R, McClintock J E. The Blackhole mass distribution in the Galaxy. Astrophys. J. 2010 ; 751:1918
9. Abbott B P, Abbott R, Abbott T D, AbrahamS ,Acernese F. et al. Binary Black Hole Population Properties Inferred from the First and Second Observing Runs of Advanced LIGO and Advanced Virgo.Astrophys. J.2019 ; 882 :L24
10. L. Kreidberg, C. D. Baulyan, W. M. Farr and V. Kalogera,Mass Measurements of Black Holes in x-ray transients: Isthere a mass gap?Astrophys. J. 2012 ; 757 : 36
11. Abbott R, Abbott T D,Abraham S, Acernese F, Ackley K, Adams C, Adhikari R X.et al. GW190814: Gravitational Waves from the Coalescence of a 23 $M_{\odot}$ Black Hole with a 2.6 $M_{\odot}$ Compact Object. Astrophys. J. Lett. 2020 ; 896 : L44.
12. Thompson T A, Kochanek C S. etal.,A noninteracting low-mass black hole-giant star binary system. Science 2019 ; 366 : 637
13. Zwicky F. On Collapsed Neutron Star.Astrophys. J. 1938 ; 88 :522
14. Zwicky F. On the Theory and Observation of Highly Collapsed Stars.Phys. Rev. 1939; 55:726
15. Srinivasan G. The maximum mass of neutron stars. Astron. Astrophys. Rev. 2002 ; 11: 67
16. Oppenheimer J R,Volkoff G M. On Massive Neutron CorePhys. Rev. 1939 ;55:374
17. Cameron A G W.Neutron Star Models .Astrophys. J. 1959 ; 130:884
18. Chamel N, Haensel P, Zdunik J L,Fantina A F.On the Maximum Mass of Neutron Stars. Int. J. Mod. Phys. E 2013 ; 22 : 130018

19. Antoniadis J, Freire P C C. etal.A Massive Pulsar in a Compact Relativistic BinaryScience 2013 ; 340:448
20. Cromartie H-T, Fonseca E. etal.Relativistic Shapiro delay measurements of an extremely massive millisecond pulsar.Nature Astronomy 2020 ; 4:72
21. Shao D-S, Tang S-P. etal.Maximum mass cutoff in the neutron star mass distribution and the prospect of forming supramassive objects in the double neutron star mergersPhys. Rev. D 2020 ; 102: 063006
22. Hughes S A.Trust but verify: The Case for astrophysical black holes.Proc.33rd SLAC Summer InstituteSSI2005; 2006 : L006
23. Kulkarni S R.Lecture Notes- “ How to find stellar Black-holes” 2019
24. Stair I H.Pulsars in Binary Systems: Probing Binary Stellar Evolution and General Relativity. Science 2004 ; 304 :547
25. J. E. McClintock and R. E. Remilland. Compact stellar X-ray Sources (Cambridge Univ. Press, UK, 2006)
26. Hu H, BaoS, Zhang Y. et al. PTEP 2020 ; 2020 : 043001
27. Thorsett S E, Chakraborty D.Neutron Star Mass Measurements. I. Radio PulsarsAstrophys. J. 1999 ; 512 :288
28. L. Kaper , A. van der Meer et al. Measuring the masses of Neutron Stars. The Messenger 2006 ; 126**:** 27
29. Clark J S,Goodwin S P,Crowther P A,Kaper L, Fairbairn M, Langer N,Brocksopp C. Physical parameters of the high-mass X-ray binary 4U1700-37 .Astron. Astrophys. 2002 ;392 :909
30. Doroshenko V, SuleimanovV, Puhlhofer G,Santangel A.A strangely light neutron star within a supernova remnant.Nature Astron. (24 Oct. 2022) doi: 10.1038/s41550-022-01800-1
31. Suwa Y, Yoshida T Y. et al.On the minimum mass of neutron stars. MNRAS 2021 ; 481 :3305
32. Espino P L, Paschalidis V,Thomas W. Baumgarte, Stuart T W, Shapiro L.Dynamical stability of quasi-toroidal differentially rotating neutron stars.Phys. Rev. D 2019 ; 99 : 083017

33. Wei H, Yu Zh-X.Inverse Chameleon Mechanism and Mass Limits for Compact StarsJCAP 2021 ; 2108 : 011
34. Kalita S, Mukhopadhyay B, Govindarajan T R. Super-Chandrasekhar limiting mass white dwarfs as emergent phenomena of noncommutative squashed fuzzy spheres. Int. J. Mod. Phys. D 2021 ; 30 :2150034
35. Chavanis P-H,Harko T.Bose-Einstein Condensate general relativistic stars. Phys. Rev. D 2012 ; 86 : 064011
36. Eslam-Panah B, Bordbar G. et al.Expansion of Magnetic Neutron Stars in an Energy (in)Dependent Spacetime. Astrophys. J. 2017 ; 848 :24
37. Giacosa F,Pagliara G.Neutron stars in the large-Nc limit.Nucl. Phys. A 2017; 968 :366
38. Pandharipande V R, Pines D, Smith R A.Neutron star structure: theory, observation, and speculation. Astrophys. J. 1976 ; 208 : 550-566
39. Shapiro S L, Teukolsky S A.Black holes, Whitedwarfs and Neutron stars: The Physics of Compact Objects*(* John Wiley, NY,1983)
40. Elebert P, Reynolds M T, Callanan P J, Hurley D J. et al.Optical spectroscopy and photometry of SAX J1808.4-3658 in outburst.MNRAS 2009 ; 395 : 885
41. Abubekerov M K, Antokhina E M, Cherepashchuk A M,Shimanskii V V..The Mass of the Compact Object in the X-Ray Binary Her X-1/HZ Her. Astron. Reports 2008 ; 52 : 379
42. Rawls M R, Orosz J A, McClintock J E, Torres M A P, Bailyn C D, Buxton M M.Refined Neutron Star Mass Determinations for Six Eclipsing X-Ray Pulsar BinariesAstrophys. J. 2011 ; 730 : 25
43. Demorest P, Pennucci T, Ransom S, Robert M, Hessels J. Shapiro delay measurement of a two solar mass neutron star. Nature 2010 ; 467:1081
44. Hansraj S, Govender M,Moody L, Singh K N. Strange stars in the framework of higher curvature gravity. Phys. Rev. D 2022 ; 105: 044030
45. Thakore B, Goti R, Shah S, PandyaH,PandyaD M. Finch-Skea Solutions of Anisotropic Stellar Models in f(R) Gravity. Astrophys. Space Sci. 2021 ; 366 ;95
46. Horvath J E,Morres E P.Modeling a 2.5$M_{\odot}$ compact star with quark matter. Int. J. Mod. Phys. D 2021 ; 30 :2150016

47. .R. Sharma, S. Das, M. Govender and D. M. Pandya. Revisiting Vaidya-Tikekar stellar model in the linearregime. Annals Phys. 2020 ; 414 :168079
48. Gondek-Rosinska D, Haensel P,Zdunik J L,Gourgulhon E.Rapidly rotating strange stars. in Proc. IAU colloq.- 177: Pulsar Astronomy- 2000 and Beyond,2000 ; 202 : 661
49. Szkudlarek M, Gondek-Rosinka D, Villain L, Ansorg M.. Maximum Mass of Differentially Rotating Strange Quark Stars. Astrophys. J. 2019 ; 879 ; 44
50. Bodmer A R.Collapsed Nuclei. Phys.Rev. D 1971 ; 4: 1601
51. E. Witten. Cosmic separation of phases. Phys. Rev. D 1984 ; 30: 272
52. Lai XY, Yu Y W, Zhou E P, Li Y, Wu R X.Pulsar glitches in a strangeon star model . Res. Astrophys. Astron. 2018 ; 18 : 24
53. Dai ZG,Wang SQ, Wang J S, Wang L J, Yu Y W.Themost Luminous Supernova ASASSN-15LH.Astrophys. J.2016 ; 817 : 132
54. Lai X Y, XuR X, Lennard-Jones quark matter and massive quark stars. MNRAS 2009 ; 398 : L31
55. E. Zhou, A. Tsokaros, L. Rezzola, R. Xuand K. Uryu . Uniformly rotating, axisymmetric, and triaxial quark stars ingeneral relativity. Phys. Rev. D 2018; 97 :23013
56. Dai Z G,Wang SQ, Wang J S, Wang L J, Yu Y W. The most Luminous Supernova ASASSN-15LH .Astrophys. J.2016 ; 817 : 132 (page 4)
57. Parui R K. Evidence of a “Triaxial Star” in BinarySystem , Submitted to *EPJ C* (2022)
58. Thompson T A, Kochanek C S, Stanek K Z, Bodemes C, Post R D, Jaysinghe T. et al. A noninteracting low-mass black hole-giant star binary system. Science 2019 ; 366 : 637 ; van den Heuvel E P J., Tauris, T M. Comment on “A noninteracting low-mass black hole–giant star binary system. Science 2020 ; 368 : id. eaba3282.
59. Vaidya P C, Tikekar R.Exact relativistic model for a superdensestarJ. Astrophys. Astron. 1982 ; 3 : 325
60. A. S. Sasidharan and M. C. Sabu. General Solution to VAIDYA-TIKEKAR METRIC with Charged Distributions on Spheroidal Space time. Int. J. Maths. Trends and Tech. 2021 ; 67 : 15
61. Ozel F, Guver T,Psaltiz D. The Mass and radius of neutron star in the buldge low mass X-ray Binary KS 1731−260. Astrophys. J. 2009 ; 693 : 1775

62. Murad H M, Fatema S. Some exact static charged perfect fluid spheres and relativistic compact Electrically charged stellar models in General Relativity. Euro. Phys. J. C 2015 ; 75 : 533
63. Dong S B,Shappee B J, Prieto J L. et al.ASASSN-15lh: A highly super-luminous supernova. Science 2016 ; 6270 : 257
64. Jotania K,Tikekar R.On relativistic models of strange stars. Int. J. Mod. Phys. D 2006 ; 15 : 1175
65. Iaani A, Mannarelli M, Rossi N. A new approach to dark matter from the mass–radius diagram of the Universe. Results in Phys. 2022 ; 38 : 105544
66. Tulin S, Yu H-B. Dark matter self-interactions and small scale structure. Phys. Reports 2018 ; 730 : 1
67. Wang X D,Qi B, Yang G L, Zhang N B, Wang S Y. Possible maximum mass of dark matter existing in compact stars based on the self-interacting fermionic model. Int. J. Mod. Phys. D 2019 ; 28 : 1950148
68. Sagun V,Giangrandi E,Ivanytski O, Lopes I, Bugaev K-A. Constraints on the fermionic dark matter from observations of neutron stars. *in Proc.* PANIC2021 (2021) p.313
69. Mukhopadhyay S, Atta D,Basu D N. Gravitational waves from isolated neutron stars: Mass dependence of -mode instability. in JPS Conf. Proc. 2020 ; 32**:** 010073
70. Zurek K M.Asymmetric Dark Matter: Theories, Signatures, and Constraints. Phys. Reports 2013 ; 537 : 91
71. Leung K-L, Chu M-C, Lin L-M. Tidal deformability of dark matter admixed neutron starsPhys. Rev. D 2022 ; 105 : 123010.
72. Kumar N, Sokolov V V. Mass distribution and "Mass Gap" of compact stellar remnants in binary systems. Astrophys. Bulletin. 2022; 77 : 197
73. Panotopoulos G, Rincon A. Electrically charged strange quark stars with a non-linear equation of state. Euro. Phys. J. C 2019 ; 79 : 524
74. Pant N, Pradhan N, Murad M H. A family of exact solutions of Einstein-Maxwell field equations in isotropic coordiantes: an application to optimization of quark star mass. Astrophys. Space Sci. 2014 ; 352 : 135
75. Dong H, Kuo T T S, Lee H. K, Machleidt R, Rho M. Half Skyrmions and the equation of state for compact star matter. Phys. Rev. C 2013 ; 87 : 054332

76. Godzieba D A, Radice D, Bernuzzi S. On the maximum mass of neutron stars and GW190814. Astrophys. J. 2021 ; 908 : 122
77. Khadhikar S, Mangat C S, Banik S. Quasi-stationary sequences of hypermassive neutron stars with exotic equation of state. J. Astrophys. Astron. 2022 ; 43 : 57
78. Rather I A, Rahaman U, Dexheimer V, Usmani A A, Patra S K. Heavy Magnetic Neutron Stars. Astrophys. J. 2021 ; 77 : 917
79. Alaverdyan G B, Vartanyan Yu L. Maximum mass of hybrid stars in the quark bag model. Astrophys. 2017 ; 60 :563
80. Mariani M, Orsaria M, Vucetich H. Constant Hybrid Stars : a first approximation of cooling evolution. Astron. &Astrophys. 2017; 601 : A 21
81. Blaschke D, Cierniak M. Studying the onset of deconfinement with mul-messenger astronomy of neutron stars. AstronomischeNachrichten. 2021 ;342 : 13909
82. Papakonstantinou P, Hyun C H. Energy density modeling of strongly interacting matter : Atomic Nuclei and Dense star. Symmetry 2023 ; 15 : 683
83. Chu P C, Zhou Y, Jiang Y-Y, Ma H-Y, Liu H, Zhang X-M. Quark star matter in heavy quark stars. Euro. Phys. J. C 2021 ; 81 : 93
84. Dondi N A, Drago A, Pagliara G. Conditions for the Co-existence of stable strange quark matter. EPJ Web Conference Series: XII th Quark Confinement and the Hadron Spectrum. 2017 ; 137 : 09004
85. Van Kerkwijik, Breton R, Kulkarni S R. Evidence for a Massive Neutron Star from a Radial-velocity study of the companion to the Black Widow Pulsar B1957+20Astrophys. J. 2011 ; 728 : 95
86. Parui R K.A Remark on “Do Triaxial Supermassive Compact Star Exist?” Int. Astron. Astrophys. Res. J. 2023 ; 5 : 33
87. Parui R K. A New Compact Star—the "Triaxial Star"—and the Detection of a Cosmic Baby: A Possibility. Int. Astron. Astrophys. Res. J. 2023 ; 5 : 38
88. Zevin M, Spera M, Berry C P L, Kalogera V. Exploring the lower mass-gap and unequal mass regime in compact Binary evolution Astrophys. J. Lett. 2020 ; 899 : L1
89. Abbott R, Abbott T D, Abbott S, Abraha F, Acernese K, Ackley A, Adams C.et al. Observation of Gravitational Waves from Two Neutron Star–Black Hole Coalescences Astrophys. J. Lett. 2021 ; 915 : L5

90. Mallick R, Singh S, Nandi R. Maximum mass of hybrid stars formed via shock induced phase transition in cold neutron stars. MNRAS 2021 ; 503 : 4829
91. Liu H, Yang Y-H, Han Y, Chu P-C. properties of quark matter and hybrid stars from a quasiparticle model. Arxiv: 2305.01246v.i [nucl-th] (2023)
92. Sotani H, Harada T. Nonradial oscillations of quark stars.Phys. Rev. D 2003 ; 68 : 024019
93. Sotani H,Kohri K, Harada T.Restricting quark matter models by gravitational wave observation.Phys. Rev. D 2004 ; 69 : 084008