

## **Original Research Article**

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Does the Mass-Gap hint at the existence of new types of compact objects? **Does Mass-Gap hint the existence of new type compact objects ?**

### **Abstract:**

The discovery of gravitational waves (GW) from the coalescence binary system reduces the mass-gap to  $(2.6 - 5) M_{\odot}$ . Hypothetical stars such as the super-Chandrasekhar mass white dwarf, quark star, boson star, electro weak star, gravstar, ~~dark matter star, subermassive neutron star, etc. may exist between neutron stars (NS) and black holes (BH) within this range~~ ~~dark matter star, übermassive neutron star etc may exist between neutron star (NS) and blackhole (BH) within this range, although all of them havenot yet been discovered to date.~~ Comparing the with the observed results with in numerical calculations, this author suggests the detected unseen companion compact object with mass  $3.3 M_{\odot}$  in the binary J05215658 + 4359220 is a triaxially deformed quark star (or triaxial star). If this is correct, then the mass gap will again reduce from  $(2.6-5) M_{\odot}$  to  $(3.3 - 5) M_{\odot}$ . Application of the solutions of the Vaidya-Tikekar Ansatz in modeling Einstein's general relativity hints that the maximum mass of a compact object  $\sim 4.178 M_{\odot}$  (i.e., four times the solar mass) in as possible in realistic case. ~~This author proposes that compact objects with higher masses are possible in a triaxially deformed compact state. As the lighter dark matter is five times more massive than the baryonic matter, This author proposes that compact objects with higher masses is possible when it is a triaxially deformed compact state. As the lighter dark matter is 5 times massive the baryonic matter~~ it is also suggested that further higher mass is possible when the compact object is triaxially deformed, consisting of lighter dark matter or unknown matter whose nature or form is not yet known to us. As the detection of gravitational waves can be a convenient method to collect evidence of quark stars (e.g. strange quark stars) or dark matter admixed with normal matter nNeutron sStars we encourage the GW community to search for their the electromagnetic counterparts during their observation of compact objects through LIGO or VIRGO.

Key Words: mass-gap; compact object; neutron star, neutron star; gravitational wave

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### **1. Introduction**

In 1915, Einstein's Field Equations [1, 2] of General Relativity introduced the idea that gravitation is not a force but rather a consequence of curvature in a four dimensional space-time, which relate the Einstein tensor to the stress-energy tensor, representing the distribution of energy, momentum, and stress in the space-time manifold. The fact is that Einstein's field equations alone are not enough to determine the evolution of a gravitational system in many cases where dependency on the dynamics of matter and energy is relevant, i.e., on the gravitational field. Since Einstein's field equations are a system of coupled non-linear partial differential equations, several effective techniques and approximations offer different exact solutions that are comfortable to express physically realistic stress energy tensors, which are in general found in complex, non-spherical stress in space. This means that the types of materials that would go into the makeup of such spaces are seldom found in nature (practically impossible to create such a situation on earth), and it is frequently desired to understand the geometrical properties of such spaces for a given mass distribution [3].

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In the year 1916, Schwarzschild [4] first gave the interior and exterior) of Einstein's field equation with a prediction on the existence of a black hole. This creates interest in studies of stellar compact objects in the realm of relativistic astrophysics. Searching for the exact solutions and the best of these solutions provides a limit (e.g., maximum) on certain physical parameters ( such as mass, radius, etc.) as well as the behavior of the gravitational field. For this reason, the construction of exact solutions, therefore be used for physical modeling and, thereafter, to predict the measureable quantities about the physical systems (compact objects such as white dwarfs, neutron stars, etc.).

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The exact solution of Einstein's equation for a spherical star full of incompressible matter, calculated by Karl Schwarzschild [4], first indicated the idea ~~of of the existence of a maximum existence of maximum~~ mass ( $M_{\max}$ ) above which the star cannot be in hydrostatic equilibrium. The existence of a limiting mass for ~~a~~ degenerate star was first introduced in ~~the~~ white dwarf case. In 1930, Chandrasekhar [5] theoretically calculated that during the sequence of white dwarf, the gravitational pressure is balanced by the degenerate pressure of the electrons, and the limiting mass is  $1.44 M_{\odot}$  which is known as ~~the~~ Chandrasekhar limit. The electrons were the latest atomic particles (neutrons were not discovered ~~at on~~ that time). So, he treated electrons as an ideal Fermi gas, and this limiting mass is reached when the electrons become ultra-relativistic.

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~~The mass gap [6, 7], i.e., a gap that lies between the maximum mass of a neutron star and the lowest mass of a black hole, has puzzled astronomers for decades. When more massive stars die through collapse under their own gravity, they leave behind black holes. In the case of less massive stars, they end their lives through the explosion of supernovae and leave behind compact remnants called neutron stars (NS). Our present understanding of compact objects indicates that the mass of the heaviest known NS is no more than  $2.5 M_{\odot}$  [8], and the lightest known BH mass is about  $5 M_{\odot}$  [8, 9]. Analysis of some models of stellar evolution predicted that the BHs in the mass range (2-5)  $M_{\odot}$ , known as the lower mass gap, could not be formed through the gravitational collapse of massive stars [10, 11]. This mass-gap, thus raises the question: "Does anything lie in this so called mass-gap? In this study I have tried to find answer of this question.~~

~~The Mass Gap [6,7] i.e., a gap that lies between the maximum mass of neutron star and the lowest mass of blackhole, creates a puzzle to the astronomers for a long decades. When more massive stars die through collapse under their own gravity, they leave behind black holes. In the case of less massive stars they end their lives through the explosion of supernovae and they leave behind compact remnant, called neutron stars (NS). Our present understanding of compact object indicates that mass of the heaviest known NS is no more than  $2.5M_{\odot}$ [8] and the lightest~~

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## 2. Maximum mass of a Neutron Star

Maximum mass of a neutron star Assuming the average density inside the neutron star is comparable to that inside heavy atomic nuclei ( i.e.,  $\rho \approx 10^{14} \text{ g.cm}^{-3}$  ) and using Schwarzschild's solution, Zwicky [13, 14] showed the neutron star's maximum mass  $\cong 11 M_{\odot}$ . But for the degeneracy pressure of an ideal neutron gas, which balances gravity, the limiting mass will be  $\sim 5.76 M_{\odot}$  [15]. Here Zwicky pointed out that a star's gravitational mass should be distinguished from the baryon or rest mass (i.e., the sum of baryon masses in the star) because of the astrophysical importance of this difference, which represents the released energy during the core collapse of massive stars. Considering a star full of ideal Fermi gas of neutrons, Oppenheimer and Volkoff [16] Assuming the average density inside the Neutron Star which is comparable to that inside heavy atomic nuclei ( i.e.,  $\rho \approx 10^{14} \text{ g.cm}^{-3}$  ) and using Schwarzschild's solution Zwicky [13,14] showed Neutron Star's maximum mass  $\cong 11 M_{\odot}$ . But for the degeneracy pressure of an ideal neutron gas which balances the gravity the limiting mass will be  $\sim 5.76 M_{\odot}$  [15]. Here Zwicky pointed out that star's gravitational mass should be distinguished from the baryon or rest mass ( i.e., the sum of baryon masses in the star ) because of the astrophysical importance of this difference which represents the released energy during core collapse of massive stars. Considering a star full of ideal Fermi gas of neutrons Oppenheimer and Volkoff [16] concluded the maximum mass of a neutron star as  $M_{\text{max}} \cong 0.7 M_{\odot}$ . In this calculation, the presence of protons was included, but nuclear forces were ignored. Cameron suggested that nuclear forces should be considered while calculating the maximum mass of the neutron star. Considering the nuclear forces among the constituents, i.e., stiff Equation of State (EOS) Cameron [17] obtained the maximum mass of Neutron Star  $\cong 2.0 M_{\odot}$ . Observations of pulsars with the help of advanced technology and various theoretical model calculations with the microscopic level neutron star matter predict that the maximum mass of spherical, non-rotating neutron star is  $M_{\text{max}} \leq 2.5 M_{\odot}$  [18]. The heaviest Neutron Star, observed recently by Antoniadis et al [19], has a mass of  $2.01 \pm 0.04 M_{\odot}$ . Using the relativistic Shapiro delay method the measurements of the component mass of the millisecond pulsar J0740+6620 claim that the host Neutron Star may have a mass  $2.14^{+0.10}_{-0.09} M_{\odot}$  with the systematic error (68.3% credibility) [20]. e concluded the maximum mass of Neutron Star as  $M_{\text{max}} \cong 0.7 M_{\odot}$ . In this calculation presence of protons was included but nuclear forces were ignored. Cameron suggested that nuclear forces should be considered while calculating the maximum mass of Neutron Star. Considering the nuclear forces among the constituents i.e., stiff Equation of State (EOS) Cameron [17] obtained the maximum mass of Neutron Star  $\cong 2.0 M_{\odot}$ . Observations of pulsars with the help of advanced technology and various theoretical model calculations with the microscopic level neutron star matter predict that the maximum mass of spherical, non-rotating neutron star is  $M_{\text{max}} \leq 2.5 M_{\odot}$  [18]. The heaviest Neutron Star, observed recently by Antoniadis et al [19], has a mass of  $2.01 \pm 0.04 M_{\odot}$ . Using the relativistic Shapiro

delay method the measurements of the component mass of the millisecond pulsar J0740+6620 claim that the host Neutron Star may have a mass  $2.14^{+0.10}_{-0.09} M_{\odot}$  with the systematic error (68.3% credibility) [20]. Latest analysis of the sample of neutron stars' mass distribution models (bimodal distribution) suggests the cut-off at Neutron Star's mass i.e.  $M_{\max} = 2.26^{+0.12}_{-0.05} M_{\odot}$  (with 68% credible interval) [21]. Latest analysis of the sample of neutron stars' mass distribution models (bimodal distribution) suggests the cut-off at Neutron Star's mass i.e.  $M_{\max} = 2.26^{+0.12}_{-0.05} M_{\odot}$  (with 68% credible interval) [21].

### 3. Minimum Mass of a stellar Black-Hole

—In the theory of general relativity, a black hole with any mass could exist. Theoretically, it is predicted that a stellar black hole with a mass  $< 5 M_{\odot}$  cannot be formed directly by the gravitational collapse of a star [22]. On the basis of observation, the detected black holes are classified into two categories: (a) stellar black holes ( $5 M_{\odot} - 80 M_{\odot}$ ) and massive black holes ( $10^6 M_{\odot} - 10^{10} M_{\odot}$ ) that are found at the centers of galaxies [23]. The latest detection of an unseen companion with a mass of  $3.3 M_{\odot}$  in the rapidly rotating giant binary system J 05215658 + 4359220 [12] indicates it either be a low-mass black hole or an exceedingly massive neutron star. As the companion mass lies within the lower mass gap, it creates uncertainty in the maximum mass of the neutron star as well as the lowest mass of the black hole. In the theory of general relativity a Black hole with any mass could exist. Theoretically, it is

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### 4.3 Observation and Data Source

#### 4.1 Compact Objects

The discovery of a companion compact object with mass  $2.50\text{--}2.67M_{\odot}$  [11] in the GW signal GW190814 opens the opportunity to place it in the mass gap (2–5) . But it is not yet confirmed whether this secondary compact object is the most massive NS or the lightest BH ever discovered. Another important discovery of a massive unseen companion with mass  $3.3 M_{\odot}$  (detected by Thompson et al. [12]) in a bright rapidly rotating giant binary system J 05215658 + 4359220 helps us to place this companion object firmly in the mass-gap range (2–5)  $M_{\odot}$ . Here, uncertainty still remains on identifying this unseen companion as a low mass BH or an exceedingly massive NS. Discovery of a companion compact object with mass  $2.50\text{--}2.67M_{\odot}$  [11] in the GW signal GW190814 opens opportunity to place it in the mass gap (2–5)  $M_{\odot}$ . But it is not yet confirmed either this secondary compact object is the most massive NS or the lightest BH ever discovered. Another important discovery of a massive unseen companion with mass  $3.3M_{\odot}$  (detected by Thompson et al. [12]) in a bright rapidly rotating giant binary system J 05215658 + 4359220 help us to place this companion object firmly in the mass gap range (2–5)  $M_{\odot}$ . Here uncertainty still remains on identifying this unseen companion as a low mass BH or an exceedingly massive NS.

From the discovery of pulsars ( as NS) in 1967 to date, thousands of NS ( as pulsars) have been detected through advanced astronomical instruments. It is found that their masses are around  $1.2\text{--}2.0 M_{\odot}$  and the radii are close to 10 Km [24–26]. Neutron star masses derived from the binary radio pulsars, particularly (NS-NS) binary radio pulsars, occupy an average mass near  $1.35 \pm 0.04 M_{\odot}$  [27,28] whereas the NS-Xray binary system 4U1700–37 shows a somewhat high mass, i.e.,  $2.4 \pm 0.3M_{\odot}$  [29]. X-ray band observation of a central compact object within the supermassive remnant HESS J1731–347 indicated the mass and radius of NS to be  $M = 0.77 \pm 0.200.70 M_{\odot}$  and  $R = 10.4 \pm 0.860.76 \text{ Km}$  [30], respectively. But this object is either the lightest NS or a ‘strange star’ with a more exotic EOS. The estimated NS mass, if considered a NS, is further lower than the known least massive NS, which has a mass of  $1.174 \pm 0.004 M_{\odot}$  [31]. Table I shows all the important masses of compact objects obtained through observation and theoretical calculations. From this table, it is seen that the übermassive NS, i.e., supermassive NS, with

mass  $3.30 M_{\odot}$  in model NL3 EOS (see text of Espino & Paschalidis 2019 [32]) may have an important role in comparison with the observed companion mass  $3.3M_{\odot}$  in the rotating giant binary system J05215659 + 4359220 (Thompson et al. 2019 [12]). The mass-gap neutron star ( $M > 3 M_{\odot}$ ) can be detected by observing its associated electromagnetic counterparts (Wei & Yu 2021 [33]). From the discovery of Pulsar (as NS) in 1967 to till date thousands of NS (as pulsars) were detected through the advanced astronomical instruments. It is found that their masses are around  $1.2 - 2.0 M_{\odot}$  and the radii are close to  $10 \text{ Km}$  [24-26]. Neutron Star masses derived from the binary radio pulsars, particularly (NS-NS) binary radio pulsars, occupy an average mass near  $1.35 \pm 0.04 M_{\odot}$  [27,28] whereas NS X-ray binary system 4U1700-37 shows a somewhat high mass i.e.,  $2.4 \pm 0.3 M_{\odot}$  [29]. X-ray band observation of a central compact object within the supermassive remnant HESS J1731-347 indicated the mass and radius of NS to be  $M = 0.77 \pm^{0.20}_{0.70} M_{\odot}$  and  $R = 10.4 \pm^{0.86}_{0.76} \text{ Km}$  [30], respectively. But this object is either a lightest NS or a 'strange star' with a more exotic EOS. The estimated NS mass, if consider it be a NS, is further lower than the known least massive NS having mass  $\cong 1.174 \pm 0.004 M_{\odot}$  [31]. Table I shows all the important masses of compact objects obtained through observation and theoretical calculations. From this table it is seen that the übermassive NS i.e. supermassive NS with mass  $3.30 M_{\odot}$  in model NL3 EOS (see text of Espino & Paschalidis 2019 [32]) may have an important role in comparison with the observed companion mass  $3.3M_{\odot}$  in the rotating giant binary system J05215659 + 4359220 (Thompson et al. 2019 [12]). The mass gap Neutron Star ( $M > 3M_{\odot}$ ) can be detected by observing its associated electromagnetic counterparts (Wei & Yu 2021 [33]).

Table I: Some important 'Masses of compact objects' ( $M_{\odot}$ )

Compact object's Description	Value ( $M_{\odot}$ )	Type	Confirmation	Ref.
Lowest NS Mass	$0.77 M_{\odot}$	Lightest NS or Strange Star	Doubtful	Doroshenko et al. 2022 [30]
Least massive NS known to date	$\sim 1.17 M_{\odot}$	Observation of central compact object within the SN Remnant HESS J1731-347	Confirmed	Suwa et al. 2021 [31]
Super-Chandrasekhar White Dwarf Mass	$\sim 2.6 M_{\odot}$	Theoretically estimated	Not observed	Kalita et al. 2021 [34]
Observed CO with masses $2.0 - 2.40 M_{\odot}$	$2 - 2.4 M_{\odot}$	BE-Condensed Star	Proposed	Chavanis and Harko 2011 [35]
Observed NS mass in PSR4U1700-377	$2.40 M_{\odot}$	Mass-gap Neutron star having Radius $12.57 \text{ Km}$ and internal	Confirmed	Eslam-Panah et al 2017 <sup>36</sup>

		magnetic field $\sim 10^{18-19}$ G		
Observed NS mass in PSR J1748-2021B. Companion compact object with mass $\sim 2.6M_{\odot}$ in the signal GW190814.	$2.70M_{\odot}$  $\sim 2.6M_{\odot}$	Neutron Star but its radius not known  Either it be a massive NS or be a lowest BH	Confirmed  Not Confirmed	Eslam-Panah et al. 2017 [36]
NS having large number of colors ( $N_c$ ) i.e. $2.1M_{\odot} < N_c < 3M_{\odot}$	$2.1M_{\odot} < M < 3M_{\odot}$	Quark phase transition to be take place leading to formation of unstable branch of hybrid stars	Theoretical, not observed	Giacosa & Pagliara 2017 [37]
Differentially rotating NS with realistic NL3 EOS e.g. übermassive NS	$3.30M_{\odot}$	Supramassive NS	Proposed by this author for confirmation through observation	Espino & Paschalidis 2019 [32]
Companion mass of rapidly rotating giant binary system J05215658 + 4359220	$3.30 M_{\odot}$	Either it be an exceedingly massive NS or a lowest BH	Not confirmed	Thompson et al. 2019 [12]
Mass limit for compact stars in the Inverse Chameleon Mechanism	$3.03M_{\odot}$ can be up to $5.0 M_{\odot}$	Either be a Super Chandrasekhar WD or NS around it an exoplanet is orbiting	Theoretical, not confirmed	Wei & Yu 2021 [33]

#### 4.23.2 Quark Stars / Strange (Quark) Stars

Stars: It is believed that strange stars contain strange matter in the form of s-quarks in hydrostatic equilibrium. According to the Pandharipande-Smith model [38], the maximum mass of such a stellar configuration is  $2.24 M_{\odot}$ . The note-worthy point is strange stars with masses exceeding the maximum masses of neutron stars will not be in hydrostatic equilibrium [39]. In Table II, the theoretical and observed masses of various strange stars are shown. The nature of observed compact stellar objects, i.e., in determining their nature as neutron stars or strange stars, is based on the 'm-a' relationship ("m" and "a" are the mass and radius of the object expressed in km) i.e., neutron stars are expected to have  $m < 0.3a$  and strange stars  $m > 0.3a$ . For the observed strange stars and compact proposed strangestars, their masses vary, ranging from  $0.85 M_{\odot}$  -  $2.01 M_{\odot}$ . In the case of Swift J1818.-1607 and its companion in the binary system GW190814, the calculated masses are  $2.01 M_{\odot}$  and  $2.5 M_{\odot}$ , respectively. These two compact objects are not yet confirmed as strange stars. In the case of the observed super-luminous supernova ASASSN-15LH, the radiation from the core indicates the birth of a strange quark star [53]. It is believed that strange stars contain strange matter in the form of s-quarks in hydrostatic equilibrium. According to Pandharipande-Smith model [38] the maximum mass of such stellar configuration is  $2.24M_{\odot}$ . The note-worthy point is strange stars with masses exceeding the maximum masses of neutron stars will not be in hydrostatic equilibrium [39]. In table II theoretical and observed masses of various strange stars are shown. The nature of

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**Table II:** Masses and Radii of various Strange Stars and other profound Strange Stars

Strange Stars	Theoretical Calculated		Observed		References
	Mass ( $M_{\odot}$ )	Radius (Km)	Mass ( $M_{\odot}$ )	Radius (Km)	
SAX J1808.4-3658	0.97	9.69	$0.9 \pm 0.3$	$7.951 \pm 1.0$	Elebert et al [40]
Her X-1	0.85	8.1	$0.85 \pm 0.15$	$8.1 \pm 0.45$	Abubkekerov et al [41]
4U1538-52	9.87	7.866	$0.87 \pm 0.07$	$7.866 \pm 0.21$	Rawls et al [42]
LMC -X-4	1.04	8.301	$1.04 \pm 0.09$	$8.301 \pm 0.2$	Rawls et al [42]
Cen X-5	1.49	9.178	$1.49 \pm 0.08$	$9.178 \pm 0.13$	Rawls et al [42]
Vela X-1	1.77	9.56	$1.77 \pm 0.08$	$9.56 \pm 0.08$	Rawls et al [42]
PSRJ 1614 - 2230	1.908	13	$1.97 \pm 0.04$	$9.69 \pm 0.2$	Demorest et al [43]
PSR J0348 + 0432	2.01	13	$2.01 \pm 0.04$	$13.0 \pm 2.0$	Demorest et al [43]
SMC X - 1	1.04	8.301			Rawls et al [42]
4U1820 - 30	1.58	9.1			Hansraj et al [44]
PSR J1903 + 327	1.667	9.438			Murad & Fatema [62]
4U1608 - 52	1.74	9.528			Murad & Fatema [62]
EXO 1785 - 248	1.3	8.849			Ozel et al [61]
Swift J 18118.0-1607	2.0	12.5			Thakore et al [45]
Companion of GW190814	2.5	.....			Horvard , Morres [40]
RXJ1856 - 37	0.9	6.0			Jotania, Tikekar [64]
Central Engine of Supernova ASASSN-15LH	Indication of birth of a strange quark star				Dong et al [63], Dai et al [53]

- $1M_{\odot} = 1.475 \text{ km}$

Differentially rotating strange stars play an important role in the mass gap region. The fact to be noted is that for rigidly rotating neutron stars, the maximum allowed mass can be larger by 14%-22% than their non-rotating maximum mass, whereas for strange stars, this larger amount is 44% [48]. Differentially rotating strange stars play important role in the mass gap region. The fact to be noted that for rigidly rotating neutron stars the maximum allowed mass can be larger by 14% - 22% than their non rotating maximum mass whereas for strange stars this larger amount

is 44% [48]. Using a two-fluid framework, a study [49] of the dimensionless tidal deformability of dark matter admixed neutron stars shows a sharp change but a narrow range for pure normal matter stars and for pure dark matter stars. Using the spectral code FLAT STAR and the MIT Bag model for strange star, a numerical calculation<sup>49</sup> shows that the highest increase in the maximum mass compared to the value of a non-rotating star arises in the state of a low degree of differential rotation, resulting in strange stars that can sustain masses much larger than those made from nuclear matter. In that case, the highest mass is over four times larger than that of a non-rotating configuration. For the given maximum densities  $\rho_{\max} = 1.70 \times 10^{15} \text{ g. cm}^{-3}$  the observed maximum masses of strange quark stars are 1.96372  $M_{\odot}$  and 2.91624  $M_{\odot}$  for non-rotating and differentially rotating cases, respectively. This study reinforces the hope of the existence of strange matter that can be detected through the observation of gravitational waves, gamma rays produced from the merger of compact binary systems. Using two fluid framework a study [49] of dimensionless tidal deformability of dark matter admixed neutron stars shows a sharp change but narrow range for pure normal matter star and for pure dark matter star. Using the spectral code FLAT STAR and the MIT Bag model for strange stars a numerical calculation<sup>49</sup> shows the highest increase of the maximum mass compared to the value of non rotating star arises in the state of low degree of differential rotation resulting which strange stars can sustain masses much larger than the stars made from nuclear matter. In that case the highest mass is over four times larger than that of non rotating configuration. For the given maximum densities  $\rho_{\max} = 1.70 \times 10^{15} \text{ g. cm}^{-3}$  the observed maximum masses of strange quark stars are 1.96372  $M_{\odot}$  and 2.91624  $M_{\odot}$  for non rotating and differentially rotating cases, respectively. This study reinforces the hope of the existence of strange matter that can be detected through the observation of the gravitational waves, gamma rays produced from the merger of compact binary system.

### 4.3 Triaxially deformed star ( or simply Triaxial Star)

Triaxial instability (i.e., triplanar symmetry w.r.t. three orthogonal x, y, and z planes) has a very important role for both neutron stars (NS) and quark stars (QS). As quark matter is composed of deconfined quarks (i.e., u, d, and s quarks), quark clustering is another important parameter for bifurcation from axisymmetry to triaxial configuration. Although this bifurcation is very close to the mass shedding limit, for soft NS equations of states (EoSs) with large compactness, the triaxial sequence could totally disappear. According to general relativity, quark stars, in general, can have larger triaxial deformation (e.g.,  $R_y / R_x$  ratio) before terminating the sequence at the mass shedding limit. As the

decomposed u (up), d (down), and s (strange) quarks could be stable [50, 51], the significance of triaxial deformation is that a small tidal deformability of a quark star is favorable for observation [52, 53]. Using the new EoS, which is based on quark clustering, proposed by Lai and Xu [54], Zhou et al. [55] showed in a numerical simulation that in a stiff EoS, a triaxial quark star can support its maximum allowed mass up to 3.3  $M_{\odot}$ . In this context, the detection of the super-luminous supernova ASAssN-15LH provides valuable information about a possible signature for the birth of a strange quark star (SQS) [56]. It leads this author [57] to suggest the unseen companion compact object with mass 3.3  $M_{\odot}$ , detected by Thompson et al [58], in the binary J05215658 + 4359220 is a triaxially deformed quark star. In other words, this could be a possible direct evidence of the existence of a triaxial star. Triaxial instability (i.e., triplanar symmetry w.r.t. three orthogonal x, y, z planes) has very important role for both neutron stars (NS) and quark stars (QS). As the quark matter is composed of deconfined quarks (i.e., u, d and s quarks) so quark clustering is another important parameter for bifurcation from axisymmetry to triaxial configuration. Although this bifurcation is very close to the mass shedding limit but for soft NS equation of states (EoSs) for NSs with large compactness the triaxial sequence could totally disappear (or vanish). According to general relativity quark stars, in general, can have larger triaxial deformation (e.g.  $R_y/R_x$  ratio) before terminating the sequence at the mass shedding limit. As the decomposed u (up), d (down) and s (strange) quarks could be stable [50, 51] the significance of triaxial deformation is that a small tidal deformability of quark star is favorable for observation [52, 53]. Using the new EoS which is based on quark clustering, proposed by Lai and Xu [54], Zhou et al [55] showed in a numerical simulation that in a stiff EoS a triaxial quark star can support its maximum allowed mass upto 3.3 $M_{\odot}$ . In this context, the detection of super luminous supernova ASAssN 15LH provides the valuable information of a possible signature for the birth of an strange quark star (SQS)[56]. It leads this author[57] to suggest the unseen companion compact object with mass 3.3 $M_{\odot}$ , detected by Thompson et al [58], in the binary J05215658 + 4359220 is a triaxially deformed quark star. In other words, this could be a possible direct evidence of the existence of a triaxial star.

## **Dark Matter Stars**

The evolution of the universe on large scales indicates that about 95% of the constituents of the universe are indeed completely unknown, i.e., they cannot be described in terms of known particles. Modern cosmology suggests these unknown constituents of the universe are dark matter (DM) and dark

energy (DE). The distribution of the observed structures in the universe reflects that the behavior of dark matter is a self-interacting dark matter candidate [65], i.e., the form of DM seems to be self-bound [66]. Using two-fluid TOV equations individually for normal matter (NM) and dark matter (DM), Wang et al. [67] found in a numerical simulation study of the dark matter admixed neutron star that the microscopic nature of DM particles has a direct connection or link with the macroscopic mass existing in the dark matter admixed neutron stars (DANSs).

DM has an obvious effect on the mass of compact stars, but compared with the energy density in the Milky Way galaxy, the existence of DM might hardly affect the mass of the compact stars situated in this galaxy. The evolution of the universe as large scales indicates about 95% of the constituents of the universe is indeed completely unknown i.e. it can not be described in terms of known particles. Modern cosmology suggests this unknown constituents of the universe are Dark Matter (DM) and Dark Energy (DE). The distribution of the observed structures in the universe reflects that the behavior of Dark Matter is a self-interacting dark matter candidate [65], i.e. form of DM seems to be self-bound [66]. Using two fluid TOV equations individually for normal matter (NM) and for Dark Matter Wang et al [67] found in a numerical simulation study of the dark matter admixed neutron star that

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- (a) DM has obvious effect on the mass of compact star but comparing with the energy density in the Milky Way galaxy the existence of DM might hardly affect the mass of the compact stars situated in this galaxy.

The general scenario [68] indicates that neutron stars may contain a sizeable amount of dark matter that was accreted throughout their lives and/or was present in the proto-star cloud. Not only that, light dark matter particles have a tendency to form an extended halo around the NS, while heavy dark matter particles are most likely to create a dense DM core inside the NS. The meaning is that when the halo formation case is active, the outermost radius of the dark matter admixed neutron stars will exceed the radius of NS, while in the core formation case, the outermost radius will be almost the same as baryon one [69]. In other words, the radius of a compact star (neutron star) becomes affected by the presence of DM, but its mass leads to lower values in the case of core formation.

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In an investigation of the mass-radius relationship of non-rotating and rotating pure hadronic matter (neutron stars) admixed with self-interacting fermionic asymmetric dark matter, with the two-fluid formalism as well as the conditions of equal and differential frequencies of nuclear matter and dark matter, Mukhopadhyay et al. [69] found the maximum mass of differentially rotating stars having self-interacting dark matter - 1.94  $M_{\odot}$  with a radius - 10.4 km.  
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- i) A two-fluid star will only reach its maximum mass i.e.  $M_{\max} = 2.6M_{\odot}$  at either the object is pure normal matter (NM) or pure dark matter (DM) i.e. the DM admixed Neutron Star allows a maximum mass  $2.6M_{\odot}$  only provided the DM EoS can reach =  $2.6M_{\odot}$ .
- ii) For fermionic DM and bosonic DM cases this maximum mass  $M_{\max} = 2.6M_{\odot}$  can be reached if  $\mu < 0.536 \text{ GeV}$  (for fermionic DM) and  $\rho_0 \hbar^3 < 3.69 \times 10^{-4} \text{ GeV}^4$  (for bosonic DM) where  $\mu, \rho_0$  being the particle mass and energy density, respectively.

- iii) Pure normal matter neutron stars cannot reach  $2.6M_{\odot}$
- iv) Higher mass limit i.e.  $M_{\max} > 2.6M_{\odot}$  is possible if smaller (e.g. lighter) is considered.
- v) The other possibility is that any more massive compact objects ( that are not black hole ) could exist, if detected.

#### 4 Compact stars:—New Degrees of Freedom

One of the most significant predictions of general relativity is the existence of a maximum mass for any static matter configuration. The cores of massive stars are not completely relativistic. As a result, it reduces the pressure and requires an increase amount of mass to reach gravitational potential to collapse the star. Constraining the Equation of State (EoS) of massive stars, in particular, neutron stars, has remained one of the biggest unsolved problems in nuclear astrophysics.

##### White Dwarf Stars

For white dwarf stars, the Chandrasekhar mass limit is  $M_{\text{ch}} = 1.4 M_{\odot}$  and the wider mass range of  $M_{\text{ch}}$  is  $\sim 1.17 - 1.75 M_{\odot}$  while the critical mass of  $M_{\text{ch}}$  is  $\sim 1.47 M_{\odot}$  [72].

##### Neutron Stars

The cores of neutron stars cannot be directly observed. To obtain the properties of matter at the neutron star core, one needs a suitable model of neutron star matter describing it from its crust to its core. Due to the lack of an exact theory for strongly interacting matter at densities beyond saturation density, it is very difficult to model neutron star cores very accurately. Compact objects are actually the final fate of the stars. They are characterized by ultra-high matter densities. So, dense, compact objects are relativistic. On the other hand, quark matter is absolutely stable, and it can be considered the true ground state of hadronic matter. As a result, a new class of compact objects has been postulated to exist as an alternative to neutron stars. For example, the "Strange Quark Star", a much more stable configuration compared to neutron stars [73], that may explain the phenomenon of the release of huge amounts of energy in super-luminous supernovae. Using the nonlinear equation of state in a numerical solution study, Panotopoulos and Rincon [73] found the maximum mass of a strange quark star is  $2.55 M_{\odot}$  with a radius of  $R = 12.34 \text{ Km}$  for the color superconductivity model CPL10. This value is slightly

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more than a) the value of the maximum mass limit of quark star  $2.33516 M_{\odot}$  with radius  $R = 10.04$  km obtained by Pant et al. [74], considering the solutions representing charged fluid spheres joining smoothly with the Reissner-Nordstrom metric at the pressure free surface, and b) the value of the maximum mass of stable strange quark star  $M_{\max} 2.4M_{\odot}$  obtained by Dong et al. [75]. Note that, utilizing a specific choice with surface density  $\sim (2.0 - 2.7) \times 10^{14}$  g/cm<sup>3</sup> and satisfying the inequalities  $0 \leq K \leq 0.24988$ ,  $K$  being a parameter involved in electrical intensity, Pant et al [74] extended the upper limit of the maximum mass of a neutron star to  $M_{\max} = 3.57546 M_{\odot}$  (with radius  $R = 15.37$  Km). The cores of neutron stars cannot be directly observed. To obtain the properties of matter at the neutron star core one needs a suitable model of neutron star matter describing from its crust to its core. Due to the lack of exact theory for strongly interacting matter at densities beyond saturation density it is very difficult to model neutron star cores very accurately. Compact objects are actually the final fate of the stars. They are characterized by ultra-high matter densities. So, dense compact objects are relativistic. On the other hand, quark matter is absolutely stable and it can be considered as the true ground state of hadronic matter. As a result, a new class of compact objects has been postulated to exist as an alternative to neutron star. for example, "Strange Quark Star", a much more stable configuration compared to neutron star [73], that may explain the phenomenon of release of huge amount energy in super luminous supernovae. Using nonlinear equation of state in a numerical solution study Panotopoulos and Rineon [73] found the maximum mass of strange quark star is  $2.55 M_{\odot}$  with radius  $R = 12.34$  Km for color superconductivity model CPL10. This value is slightly more than a) the value of the maximum mass limit of quark star  $\approx 2.33516 M_{\odot}$  with radius  $R = 10.04$  km obtained by Pant et al [74] considering the solutions representing charged fluid spheres joining smoothly with Reissner-Nordstrom metric at the pressure free surface and b) the value of the maximum mass of stable strange quark star  $M_{\max} \approx 2.4M_{\odot}$  obtained by Dong et al [75]. Note that, utilizing a specific choice with surface density  $\rho \sim (2.0 - 2.7) \times 10^{14}$  g/cm<sup>3</sup> and satisfying the inequalities  $0 \leq K \leq 0.24988$ ,  $K$  being a parameter involves in electrical intensity, Pant et al [74] extended the upper limit of maximum mass of neutron star to  $M_{\max} = 3.57546 M_{\odot}$  (with radius  $R = 15.37$  Km).

Theoretical estimations (based on a number of observations) predicted the maximum mass of a neutron star exists in the range ( 2.2 - 2.9)  $M_{\odot}$  i.e., up to  $\sim 3 M_{\odot}$  (uncleared whether the limit of GR is supported or not) [75]. Using gravitational wave detectors' data (i.e., observed in the NS-NS and NS-BH binary systems), the value of the maximum mass for a neutron star could be in the range of (2.5-2.67)

$M_{\odot}$  with the violation of the causal limit for  $R \leq 11.38$  Km [76]. On the other hand, Khadkikar et al. [77] boost this maximum mass to  $3.60 M_{\odot}$  for a spinning neutron star with an exotic equation of state at the mass-shedding limit. Theoretical estimations (based on a number of observation) predicted the maximum mass of neutron star exists in the range  $(2.2 - 2.9) M_{\odot}$  i.e., up to  $3 M_{\odot}$  (uncleared whether the limit of GR is supported or not) [75]. Using gravitational wave detectors' data (i.e., observed in the NS-NS, NS-BH binary systems) the value of the maximum mass for neutron star could be in the range of  $(2.5 - 2.67) M_{\odot}$  with the violation of the causal limit for  $R_{1.4} \leq 11.38$  Km [76]. On the other hand, Khadkikar et al [77] boost up this of maximum mass to  $3.60 M_{\odot}$  for a spinning neutron star with exotic equation of state at mass shedding limit.

In the presence of a somehow realistic, strong internal magnetic field ( $10^{17}$  G), the stellar charge particle population can be able to re-leptonize and de-hyperonize, which finally generates a more massive neutron star  $> 2.1 M_{\odot}$ . Considering the density dependent relativistic mean field (DD-RMF) parameters, Rather et al. [78] found the maximum masses of a heavy magnetic neutron star for pure nucleonic and hyperonic matter are  $2.575 M_{\odot}$  (with radius  $R = 12.465$  km) and  $2.183 M_{\odot}$  (with radius  $R = 12.238$  km) in the case of dimensionless tidal deformability  $\Lambda_{1.4} = 791.483$  and internal magnetic field  $1.79 \times 10^{17}$  G.

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### Pulsars

The most dominant sub-class of neutron stars is rRadio pPulsars i.e., isolated neutron stars with a period of rotation (P) of a few tens of milliseconds to several seconds and dipole field  $B_d = 10^{11} - 10^{13}$  G.

The second sub-class is binary X-ray pulsars, with  $B_d = (1 - 7) \times 10^{12}$  G and period P varies from 2 ms  $- \sim 10^4$  s.

The third sub-class lies at the low  $B_d$  and short P end i.e. a mixture of millisecond pulsars (MSPs : isolated or in wide binaries) and low mass X-ray binaries (LMXBs in which case neutron stars are considered to be old  $>10^9$  years). The observed fact is 50% of pulsars (PSRs) with characteristic age less than  $\sim 5 \times 10^4$  years have associated with supernova remnants (SNRs).

### Hybrid Stars

To explore the possible hadron-quark phase transition in the interior of neutron star, Qin et al. [74] studied the first order phase transition and crossover, the tidal deformability of hybrid stars considering the 3-window construction model, and found the possibility of attaining the maximum mass of hybrid stars in excess of  $2 M_\odot$ . This value is consistent with the value proposed by Alaverdyand and Vartanyan [79] and the observed value for the pulsar PSRJ0348+0432 with a mass of  $2.1 \pm 0.04 M_\odot$ .

A significant maximum mass of hybrid star hasis resulted in the study of possible a hadron-quark phase transition in the interior of neutron star . Using the semi-analytical hybrid equation of states at fixed entropy per baryon for hot and cold hybrid stars, Mariani et al. [80] obtained that the maximum mass of the hybrid star is (2.68–2.69)  $M_\odot$  with a quark matter core of 2.67  $M_\odot$ . On the otherhand, multi-messenger observation of a binary Neutron Star merger (GW170817) provides a new constraint for the masses and radii of neutron stars such that a new class of hybrid equation of state (EoS) [81] could fulfill Using semi-analytical hybrid equation of states at fixed entropy per baryon for hot and cold hybrid stars Mariani et al [80] obtained the maximum mass of the hybrid star is (2.68–2.69)  $M_\odot$  with a quark matter core of 2.67  $M_\odot$ . On the other

Hand, multi messenger observation of a binary Neutron Star merger (GW170817) provides a new constraint for the masses and radii of neutron stars such that a new class of hybrid equation of state (EoS) [81] could fulfill

- a) the NICER measured radius (less than 11 km i.e.,  $10.5 < R < 12.5$  km) for the high mass pulsar PSR J0740+6620 , and
- b) the maximum mass of the component star (hybrid nature)  $\sim 2.6 M_\odot$  in the case of asymmetric binary merger GW190814.

~~Blaschke and Cierniak [81] suggested that this  $2.6 M_{\odot}$  object in the binary merger GW190814 could have been a hyper-massive hybrid neutron star with a large quark matter core. As the hadronic picture changes at high densities, i.e., quarks may be deconfined or percolate among tightly packed baryons, a problem remains: “whether there is a phase transition or a cross over between hadronic and quark matter” [82, 83]. Blaschke and Cierniak [81] suggested that this  $2.6 M_{\odot}$  object in the binary merger GW190814 could have been a hyper-massive hybrid neutron star with a large quark matter core. As the hadronic picture changes at high densities i.e. quarks may be deconfined or percolate among tightly packed baryons a problem — “ whether there is a phase transition or a cross over between hadronic and quark matter “ remains, not yet solved [82,83].~~

### Quark Star or Strange Quark Star

The theoretical idea ---- there exists a process of conversion of a hadronic star into a quark star is energetically convenient even if the radius of the final configuration is larger than the radius of the initial configuration. In the two families scenario compact stars, i.e., hadronic stars and quark stars ( precisely light stars are hadronic stars whereas large and massive stars are strange quark stars), do coexist and the transition between the two branches will appear in a hadronic star when a sizable fraction of strangeness is present through hyperons i.e. the metastable stellar system that can convert into a pure strange quark star [84]. Note that the highest mass measured through observation of PSR J0348 + 0432 is  $M_{\max} = 2.01 \pm 0.04 M_{\odot}$  but the estimated mass of this black widow pulsar is  $2.4 M_{\odot}$  [85]. This means strange quark stars could have maximum masses larger than  $2 M_{\odot}$ . In other words, measurements of masses larger than  $2 M_{\odot}$  would favor the strange quark star scenario.

A study [83] of the thermodynamic properties of asymmetric quark matter and large mass quark stars under the confined isospin-density-dependent-quark mass model suggests that neutron stars could be converted to hybrid stars or strange quark stars. These stars are usually made up of deconfined absolutely stable ‘u’, ‘d’ and ‘s’ quarks with lepton in  $\beta$ -equilibrium i.e., can be

Considered as strange quark matter. By fitting the radial velocity lines and the three-band light curves in the irradiated compact star models Chu et al [83] also showed that the recently detected three heavy compact stars PSR J0348+0432 ( $2.01 \pm 0.04 M_{\odot}$ ), MSP J0740+6620 (with mass

$2.14^{+0.010}_{-0.09}M_{\odot}$  of 68.3% credibility interval and  $2.14^{+0.20}_{-0.18}M_{\odot}$  of 95.4% credibility interval) and PSR J2215+5135 (with mass  $2.27^{+0.10}_{-0.09}M_{\odot}$ ) as heavy quark stars. In other words, the detected heavy compact star e.g., the heavy quark star have maximum mass  $2.27 M_{\odot}$  till date.

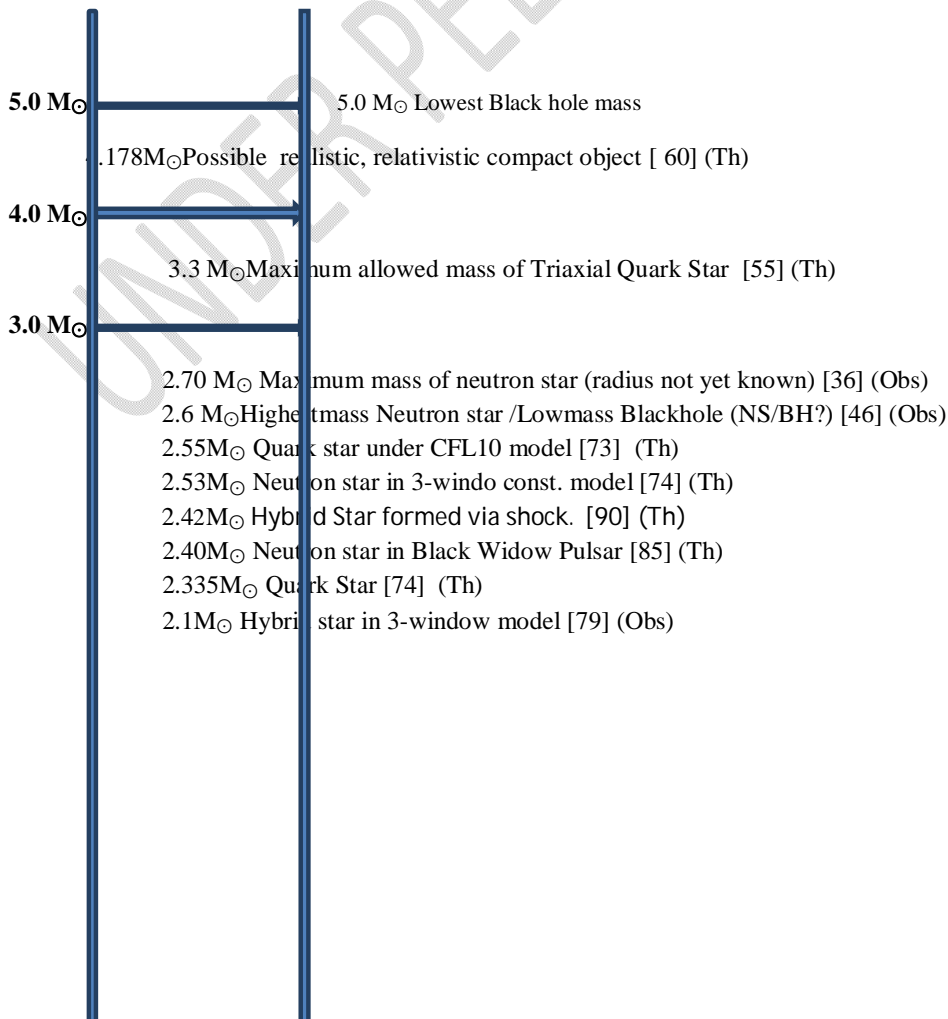
### Magnetars, Triaxial Stars

To date, 31 magnetars have been detected. Among them, only six magnetars are passing through their triaxial phases [86], whereas the rest of the magnetars have already crossed their triaxial phases. Recently detected cosmic baby Swift J1818.0-1607 is a very young magnetar with a characteristic age  $\sim 240$ – $300$  years, which can be considered the best source for detecting and confirming it as a triaxial star [87]. Its estimated mass is  $\sim 2.0 M_{\odot}$ . On the other hand, numerical simulation using a stiff equation of state suggests that a triaxial quark star can support its maximum allowed mass up to  $3.3 M_{\odot}$  [55]. Till date 31 magnetars have been detected. Among them only six magnetars are passing through their triaxial phases [86] whereas rest numbers magnetars have already crossed their triaxial phases. Recently detected cosmic baby Swift J1818.0-1607 is a very young magnetar having characteristic age  $\sim 240$ – $300$  years which can be considered as the best source for detecting and confirming it as a triaxial star [87]. Its estimated mass is  $\sim 2.0 M_{\odot}$ . On the other hand, numerical simulation using a stiff equation of state suggests that a triaxial quark star can support its maximum allowed mass up to  $3.3 M_{\odot}$  [55].

### Black Holes

Based on a number of observational and theoretical models, the mass distribution between the highest mass of a neutron star ( $2 M_{\odot}$ ) and the lowest mass of black hole ( $5 M_{\odot}$ ) exists in the range  $2 - 5 M_{\odot}$  [88–90]. Observation of GW 190814 by the LIGO and VIRGO detectors indicated that the gravitational wave signal originated from the merger of a  $23 M_{\odot}$  black hole (BH) with a  $M_{\odot}$  compact object. This compact binary source, GW190814, is atypical both in its highly asymmetric masses and in its lower-mass component, lying between the heaviest known neutron star and the lightest known black hole. Therefore, GW190814 offers an unprecedented probe into the mass gap, making the mass of binary's secondary lighter component ( $2.5$ – $2.67 M_{\odot}$ ) as the heaviest neutron star or lightest black hole ever identified in a compact object binary.  $5.0 M_{\odot}$   $5.0 M_{\odot}$  Based on a number of observational and theoretical models the mass distribution between the highest mass of neutron star ( $2 M_{\odot}$ ) and the lowest mass of black hole ( $5 M_{\odot}$ ) exists in the range  $2 - 5 M_{\odot}$  [88–90]. Observation of GW

190814 by the LIGO and VIRGO detectors indicated that the gravitational wave signal was originated from the merger of a  $\approx 23 M_{\odot}$  black hole (BH) with a  $\approx 2.6 M_{\odot}$  compact object. This compact binary source of GW190814 is atypical both in its highly asymmetric masses and in its lower mass component lying between the heaviest known neutron star and the lightest known black hole. Therefore, GW190814 offers an unprecedented probe into the mass gap making the mass of binary's secondary lighter component (2.5–2.67)  $M_{\odot}$  as the heaviest neutron star or lightest black hole ever identified in a compact object binary.



2.0M<sub>⊙</sub> Hybrid star with sizable quark matter core [91] (Obs ,confirmed)

2.0 M<sub>⊙</sub> 

1.75 M<sub>⊙</sub> Higher White Dwarf mass [72] (Th)

1.47M<sub>⊙</sub> Extended Chandrasekhar mass limit for WD [72] (Th)

1.44 M<sub>⊙</sub> Chandrasekhar Mass limit for White Dwarf

**Fig. 1:** Mass Gap Ladder for different compact objects with their maximum masses (not scaled). Theoretical or estimated values of maximum mass are marked by (Th / Estimated), Observed or detected maximum mass by (Obs /Confirmed). For 2.6M<sub>⊙</sub> companion mass of binary system is marked by (NS/BH ? ) i.e. not yet solved.

## 5 Conclusion

Analytical solutions of Einstein's gravitational field equations provide a key for the determination of the maximum mass of very compact astrophysical objects. The central density of these compact objects could be  $\sim 10^{15} \text{ g.cm}^{-3}$  and calculations for obtaining their maximum mass require reliable information such as the structure, behavior of matter at such an ultra-high density. Analytical solutions describe the stable mass (in hydrostatic equilibrium) of normal matter, i.e., the maximum mass of a neutron star is possible, which means that neutron stars are expected to have their mass  $m < 0.3a$ , ('m', 'a' being the mass and radius of the star expressed in km), while for strange stars,  $m > 0.3a$ . The noteworthy point is that strange stars with masses exceeding the maximum mass of neutron stars will not be in hydrostatic equilibrium [39].

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$m > 0.3a$ . The noteworthy point is that strange stars with masses exceeding the maximum mass of neutron stars will not be in hydrostatic equilibrium [39].

The highest mass in a strange star (see table II) observed in PSR J1614–2230 is  $\sim 1.978 M_{\odot}$  ( $2.0 M_{\odot}$ ). In the absence of reliable information regarding the structure and behavior of matter inside the compact object, the Vaidya-Tikekar superdense star model [59] is very effective in modeling astrophysical compact objects (such as neutron stars, quark stars, etc.). Using general solutions to the Vaidya-Tikekar metric (with a density of  $2 \times 10^{14}$  g/cm<sup>3</sup>) in modeling Einstein's field equation, Sasidharan and Sabu [60] found a possible realistic, relativistic compact object with a mass of  $4.1738 M_{\odot}$ .

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A triaxial star is another realistic compact object that could be placed in the mass range  $(3 - 5) M_{\odot}$  in the mass gap  $(2.6 - 5) M_{\odot}$ . We suggest the detected unseen companion compact object with mass  $\sim 3.3 M_{\odot}$  in the binary J 05215658 + 4359220 as a triaxial star, i.e., a triaxially deformed quark star, as it matches the result found in the numerical simulation study. If it is correct, then the highest detected compact object mass will be  $\sim 3.3 M_{\odot}$  resulting in a mass gap of  $(3.3 - 5) M_{\odot}$ . Lighter dark matter is  $\sim 5$  times the mass of baryonic matter [70], which means that a triaxially deformed star consisting of lighter dark matter particles may have the possibility of possessing a higher mass, i.e.,  $> 3.3 M_{\odot}$ . Another possibility is the triaxially deformed massive compact object (not a black hole) whose nature or type is not yet known to us and whose detection we have to search for. This author thus suggests that possibly it is a triaxial star, which consists of unknown matter whose form is not yet known to us and remains undetectable to astronomers.

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~~As the detection of gravitational waves can be a convenient method to collect evidence of quark stars (e.g., strange quark stars ) or dark matter admixed with normal matter neutron stars [92, 93], we encourage the GW community to search for their electromagnetic counterparts during their observation of compact objects through LIGO or VIRGO. As the detection of gravitational waves can be a convenient method to collect evidence of quark stars (e.g. strange quark stars ) or dark matter admixed normal matter Neutron Stars [92,93] we encourage the GW community to search the electromagnetic counterparts during their observation of compact objects through LIGO or VIRGO.~~

#### References :

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