

**Does Mass-Gap hint the existence of new type compact objects ?**

**Abstract:**

Discovery of gravitational waves (GW) from coalescence binary system reduce the mass-gap to  $(2.6 - 5) M_{\odot}$ . Hypothetical stars such as super-Chandrasekhar mass white dwarf, quark star, boson star, electro weak star, gravstar, dark matter star, übermassive neutron star etc may exist between neutron star (NS) and black hole (BH) within this range, although all of them have not yet been discovered to date. Comparing with the observed results in numerical calculations this author suggests the detected unseen companion compact object with mass  $3.3 M_{\odot}$  in the binary J05215658 + 4359220 is a triaxially deformed quark star (or triaxial star). If this is correct, then mass gap will again reduce from  $(2.6-5) M_{\odot}$  to  $(3.3 - 5) M_{\odot}$ . Application of the solutions of Vaidya-Tikekar Ansatz in modeling Einstein's general relativity hints the maximum mass of a compact object  $\sim 4.178 M_{\odot}$  (i.e. four times the solar mass) is possible in realistic case. This author proposes that compact objects with higher masses is possible when it is a triaxially deformed compact state. As the lighter dark matter is 5 times massive the baryonic matter it is also suggested that further higher mass is possible when the compact object is triaxially deformed, consisting of lighter dark matter or unknown matter whose nature or form is not yet known to us. As the detection of gravitational waves can be a convenient method to collect evidence of quark stars (e.g. strange quark stars) or dark matter admixed normal matter Neutron Stars we encourage the GW community to search the electromagnetic counterparts during their observation of compact objects through LIGO or VIRGO.

**Key Words: mass-gap; compact object ; neutron star, neutron star; gravitational wave**

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**1. Introduction**

In 1915 Einstein's Field Equations [1,2] of General Relativity introduce the idea that gravitation is not a forces, rather it is a consequence of curvature in a four dimensional space-time which relate the Einstein tensor to stress-energy tensor representing the distribution of energy, momentum and stress in the space-time manifold. The fact is that the Einstein's field equations alone are not enough to determine the evolution of a gravitational system in many cases where dependency on the dynamics of matter and energy are relevant i.e. on the gravitational field. Since the Einstein's field equations are a system of coupled non-linear partial differential

equations, several effective techniques/ approximations offer different exact solutions that are comfortable to express a physically realistic stress energy tensors which are in general found in complex, non-spherical stress in space. This means that such types of materials that would go into the makeup of such spaces are seldom found in nature (practically impossible to create such situation on the earth) and it is frequently desired to understand the geometrical properties of such space of a given mass distribution [3].

In the year 1916 Schwarzschild [4] first gave the solution ( interior and exterior) of the Einstein's field equation with a prediction on the existence of black hole. This creates interest on studies of stellar compact objects in the realm of relativistic astrophysics. Searching of the exact solutions and the best of these solutions provide a limit (e.g. maximum) on certain physical parameters ( such as mass, radius, etc) as well as the behavior of gravitational field. For this reason, the construction of exact solutions, therefore, can be used for physical modeling and there after to predict the measureable quantities about the physical systems (compact object such as white dwarf, neutron star, ... ) .

The exact solution of Einstein's equation for a spherical star full of incompressible matter, calculated by Karl Schwarzschild [4], first indicated the idea of existence of maximum mass ( $M_{\max}$ ) above which the star can not be in hydrostatic equilibrium. The existence of a limiting mass for degenerate star was first introduced in white dwarf case. In 1930 Chandrasekhar [5] theoretically calculated that during the sequence of white dwarf the gravitational pressure is balanced by the degenerate pressure of the electrons and the limiting mass is  $1.44 M_{\odot}$  which is known as Chandrasekhar limit. The electrons were the latest atomic particles (neutrons were not discovered on that time). So, he treated electrons as an ideal Fermi gas and this limiting mass is reached when the electrons become ultra-relativistic.

In the case of ideal gas of fermions i.e. neutrons a sequence of neutron stars in which gravity is balanced by the degeneracy pressure of neutron gas and for arbitrarily massive stars there will be the upper limit the possible size of such a neutron star.

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The Mass-Gap [6,7] i.e., a gap that lies between the maximum mass of neutron star and the lowest mass of black hole, creates a puzzle to the astronomers for a long decades. When more massive stars die through collapse under their own gravity, they leave behind black holes. In the case of less massive stars they end their lives through the explosion of supernovae and they leave behind compact remnant, called neutron stars (NS) . Our present understanding of compact object indicates that mass of the heaviest known NS is no more than  $2.5 M_{\odot}$  [8] and the lightest known BH mass is about  $5 M_{\odot}$  [8, 9]. Analysis of some models of stellar evolution predicted that the BHs in the mass range  $(2-5) M_{\odot}$  , known as lower mass-gap, can not be formed through the gravitational collapse of massive stars [10,11]. This mass-gap , thus raises the question:

“Does anything lie in this so called mass-gap ? In this study I have tried to find answer of this question.

## 2. Maximum mass of a Neutron Star

Assuming the average density inside the Neutron Star which is comparable to that inside heavy atomic nuclei ( i.e.,  $\rho \approx 10^{14} \text{ g.cm}^{-3}$  ) and using Schwarzschild’s solution Zwicky [13,14] showed Neutron Star’s maximum mass  $\cong 11 M_{\odot}$ . But for the degeneracy pressure of an ideal neutron gas which balances the gravity the limiting mass will be  $\sim 5.76 M_{\odot}$  [15]. Here Zwicky pointed out that star’s gravitational mass should be distinguished from the baryon or rest mass ( i.e., the sum of baryon masses in the star ) because of the astrophysical importance of this difference which represents the released energy during core collapse of massive stars. Considering a star full of ideal Fermi gas of neutrons Oppenheimer and Volkoff [16] concluded the maximum mass of Neutron Star as  $M_{\text{max}} \cong 0.7 M_{\odot}$  . In this calculation presence of protons was included but nuclear forces were ignored. Cameron suggested that nuclear forces should be considered while calculating the maximum mass of Neutron Star. Considering the nuclear forces among the constituents i.e., stiff Equation of State (EOS) Cameron [17] obtained the maximum mass of Neutron Star  $\cong 2.0 M_{\odot}$  . Observations of pulsars with the help of advanced technology and various theoretical model calculations with the microscopic level neutron star matter predict that the maximum mass of spherical, non-rotating neutron star is  $M_{\text{max}} \leq 2.5 M_{\odot}$  [18]. The heaviest Neutron Star, observed recently by Antoniadis et al [19], has a mass of  $2.01 \pm 0.04 M_{\odot}$  . Using the relativistic Shapiro delay method the measurements of the component mass of the millisecond pulsar J0740+6620 claim that the host Neutron Star may have a mass  $2.14^{+0.10}_{-0.09} M_{\odot}$  with the systematic error (68.3% credibility) [20]. Latest analysis of the sample of neutron stars’ mass distribution models (bimodal distribution) suggests the cut-off at Neutron Star’s mass i.e.  $M_{\text{max}} = 2.26^{+0.12}_{-0.05} M_{\odot}$  (with 68% credible interval) [21].

## 3. Minimum Mass of a stellar Black-Hole

In the theory of general relativity a Black-hole with any mass could exist. Theoretically, it is

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predicted that the stellar Black-hole with mass  $< \sim 5 M_{\odot}$  cannot be formed directly by the gravitational collapse of a star [22]. On the basis of observation the detected Black-holes are classified into two categories – (a) Stellar Black-holes (  $5M_{\odot} - 80M_{\odot}$  ) and massive Black-holes (  $10^6 M_{\odot} - 10^{10} M_{\odot}$  ) that are found at the centers of galaxies [23]. Latest detection of an unseen companion with mass of  $3.3M_{\odot}$  in the rapidly rotating giant binary system J 05215658 +

4359220 [12] indicates it either be a low mass Black-hole or an exceedingly massive Neutron Star. As the companion mass lies within the lower mass gap it creates uncertainty in the maximum mass of Neutron Star as well as the lowest mass of the Black-hole.

#### 4. Observation and Data Source

##### 4.1 Compact Objects

Discovery of a companion compact object with mass  $2.50\text{--}2.67M_{\odot}$ [11] in the GW signal GW190814 opens opportunity to place it in the mass gap  $(2\text{--}5)M_{\odot}$ . But it is not yet confirmed either this secondary compact object is the most massive NS or the lightest BH ever discovered. Another important discovery of a massive unseen companion with mass  $3.3 M_{\odot}$  (detected by Thompson et al. [12]) in a bright rapidly rotating giant binary system J 05215658 + 4359220 help us to place this companion object firmly in the mass-gap range  $(2\text{--}5) M_{\odot}$ . Here uncertainty still remains on identifying this unseen companion as a low mass BH or an exceedingly massive NS.

From the discovery of Pulsar ( as NS) in 1967 to till date thousands of NS ( as pulsars) were detected through the advanced astronomical instruments. It is found that their masses are around  $1.2\text{--}2.0 M_{\odot}$  and the radii are close to 10 Km [24-26]. Neutron Star masses derived from the binary radio pulsars, particularly (NS – NS) binary radio pulsars, occupy an average mass near  $1.35 \pm 0.04 M_{\odot}$  [27,28] whereas NS-Xray binary system 4U1700 – 37 shows a somewhat high mass i.e.,  $2.4 \pm 0.3M_{\odot}$  [29]. X-ray band observation of a central compact object within the supermassive remnant HESS J1731 – 347 indicated the mass and radius of NS to be  $M = 0.77 \pm^{0.20}_{0.70} M_{\odot}$  and  $R = 10.4 \pm^{0.86}_{0.76} \text{ Km}$  [30], respectively. But this object is either a lightest NS or a ‘strange star’ with a more exotic EOS. The estimated NS mass, if consider it be a NS, is further lower than the known least massive NS having mass  $\cong 1.174 \pm 0.004 M_{\odot}$ [31]. Table I shows all the important masses of compact objects obtained through observation and theoretical calculations. From this table it is seen that the übermassive NS i.e. supermassive NS with mass  $3.30 M_{\odot}$  in model NL3 EOS (see text of Espino & Paschalidis 2019 [32]) may have an important role in comparison with the observed companion mass  $3.3M_{\odot}$  in the rotating giant binary system J05215659 + 4359220 (Thompson et al. 2019 [12]). The mass-gap Neutron Star (  $M > 3 M_{\odot}$ ) can be detected by observing its associated electromagnetic counterparts (Wei & Yu 2021 [33]).

Table I: Some important Masses of compact objects (CO)

Compact object's Description	Value ( $M_{\odot}$ )	Type	Confirmation	Ref.
Lowest NS Mass	$0.77 M_{\odot}$	Lightest NS or Strange Star	Doubtful	Doroshenko et al. 2022 [30]
Least massive NS known to date	$\sim 1.17 M_{\odot}$	Observation of central compact object within the SN Remnant HESS J1731-347	Confirmed	Suwa et al. 2021 [31]
Super-Chandrasekhar White Dwarf Mass	$\sim 2.6 M_{\odot}$	Theoretically estimated	Not observed	Kalita et al. 2021 [34]
Observed CO with masses $2.0 - 2.40 M_{\odot}$	$2 - 2.4 M_{\odot}$	BE-Condensed Star	Proposed	Chavanis and Harko 2011 [35]
Observed NS mass in PSR4U1700-377	$2.40 M_{\odot}$	Mass-gap Neutron star having Radius 12.57 Km and internal magnetic field $\sim 10^{18-19}$ G	Confirmed	Eslam-Panah et al 2017 <sup>36</sup>
Observed NS mass in PSR J1748-2021B. Companion compact object with mass $\sim 2.6 M_{\odot}$ in the signal GW190814.	$2.70 M_{\odot}$ $\sim 2.6 M_{\odot}$	Neutron Star but its radius not known Either it be a massive NS or be a lowest BH	Confirmed Not Confirmed	Eslam-Panah et al. 2017 [36]
NS having large number of colors ( $N_c$ ) i.e. $2.1M_{\odot} < N_c < 3 M_{\odot}$	$2.1M_{\odot}$ $< M < 3 M_{\odot}$	Quark phase transition to be take place leading to formation of unstable branch of hybrid stars	Theoretical, not observed	Giacosa & Pagliara 2017 [37]
Differentially rotating NS with realistic NL3 EOS e.g. übermassive NS	$3.30 M_{\odot}$	Supramassive NS	Proposed by this author for confirmation through observation	Espino & Paschalidis 2019 [32]
Companion mass of rapidly rotating giant binary system J05215658 + 4359220	$3.30 M_{\odot}$	Either it be an exceedingly massive NS or a lowest BH	Not confirmed	Thompson et al. 2019 [12]
Mass limit for compact stars in the Inverse Chameleon Mechanism	$3.03M_{\odot}$ can be up to $5.0 M_{\odot}$	Either be a Super Chandrasekhar WD or NS around it an exoplanet is orbiting	Theoretical, not confirmed	Wei & Yu 2021 [33]

#### 4.2 Quark Stars / Strange (Quark) Stars

It is believed that strange stars contain strange matter in the form of s-quarks in hydrostatic equilibrium. According to Pandharipande- Smith model [38] the maximum mass of such stellar configuration is  $2.24 M_{\odot}$ . The note-worthy point is strange stars with masses exceeding the maximum masses of neutron stars will not be in hydrostatic equilibrium [39]. In table II theoretical and observed masses of various strange stars are shown. The nature of observed compact stellar objects i.e. in determining their nature as neutron stars or strange stars is based on the '  $m - a$  ' relationship ('m', 'a' are the mass and radius of the object expressed in km )

i.e., neutron stars are expected to have  $m < 0.3a$  and for strange stars  $m > 0.3a$ . For the observed strange stars and compact proposed strange stars their masses vary ranging  $0.85 M_{\odot} - 2.01 M_{\odot}$ . In the case of Swift J1818.0 – 1607 and companion in the binary system GW190814 the calculated masses are  $2.01 M_{\odot}$  and  $2.5 M_{\odot}$ , respectively. These two compact objects are not yet confirmed as strange stars. In the case of observed super-luminous supernova ASASSN – 15LH the radiation from the core indicates the signal of birth of a strange quark star [53].

**Table II:** Masses and Radii of various Strange Stars and other profound Strange Stars

Stange Stars	Theoretical Calculated		Observed		References
	Mass ( $M_{\odot}$ )	Radius (Km)	Mass ( $M_{\odot}$ )	Radius (Km)	
SAX J1808.4-3658	0.97	9.69	$0.9 \pm 0.3$	$7.951 \pm 1.0$	Elebert et al [40]
Her X-1	0.85	8.1	$0.85 \pm 0.15$	$8.1 \pm 0.45$	Abubkekerov et al [41]
4U1538-52	9.87	7.866	$0.87 \pm 0.07$	$7.866 \pm 0.21$	Rawls et al [42]
LMC –X-4	1.04	8.301	$1.04 \pm 0.09$	$8.301 \pm 0.2$	Rawls et al [42]
Cen X-5	1.49	9.178	$1.49 \pm 0.08$	$9.178 \pm 0.13$	Rawls et al [42]
Vela X-1	1.77	9.56	$1.77 \pm 0.08$	$9.56 \pm 0.08$	Rawls et al [42]
PSRJ 1614 – 2230	1.908	13	$1.97 \pm 0.04$	$9.69 \pm 0.2$	Demorest et al [43]
PSR J0348 + 0432	2.01	13	$2.01 \pm 0.04$	$13.0 \pm 2.0$	Demorest et al [43]
SMC X - 1	1.04	8.301			Rawls et al [42]
4U1820 - 30	1.58	9.1			Hansraj et al [44]
PSR J1903 + 327	1.667	9.438			Murad & Fatema [62]
4U1608 – 52	1.74	9.528			Murad & Fatema [62]
EXO 1785 - 248	1.3	8.849			Ozel et al [61]
Swift J 18118.0-1607	2.0	12.5			Thakore et al [45]
Companion of GW190814	2.5	.....			Horvard , Morres [40]
RXJ1856 – 37	0.9	6.0			Jotania, Tikekar [64]
Central Engine of Supernova ASASSN-15LH	Indication of birth of a strange quark star				Dong et al [63], Dai et al [53]

- $1 M_{\odot} = 1.475 \text{ km}$

Differentially rotating strange stars play important role in the mass gap region. The fact to be noted that for rigidly rotating neutron stars the maximum allowed mass can be larger by 14% - 22% than their non-rotating maximum mass whereas for strange stars this larger amount

is 44% [48]. Using two-fluid frame work a study [49] of dimensionless tidal deformability of dark matter admixed neutron stars shows a sharp change but narrow range for pure normal matter star and for pure dark matter star. Using the spectral code FLAT STAR and the MIT Bag model for strange stars a numerical calculations<sup>49</sup> shows the highest increase of the maximum mass compared to the value of non-rotating star arises in the state of low degree of differential

rotation resulting which strange stars can sustain masses much larger than the stars made from nuclear matter. In that case the highest mass is over four times larger than that of non-rotating configuration. For the given maximum densities  $\rho_{\max} = 1.70 \times 10^{15} \text{ g. cm}^{-3}$  the observed maximum masses of strange quark stars are  $1.96372 M_{\odot}$  and  $2.91624 M_{\odot}$  for non-rotating and differentially rotating cases, respectively. This study reinforces the hope of the existence of strange matter that can be detected through the observation of the gravitational waves, gamma rays produced from the merger of compact binary system.

### **4.3 Triaxially deformed star ( or simply Triaxial Star)**

Triaxial instability (i.e., triplanar symmetry w.r.t. three orthogonal x, y, z planes) has very important role for both neutron stars (NS) and quark stars (QS). As the quark matter is composed of deconfined quarks (i.e., u, d and s quarks) so quark clustering is another important parameter for bifurcation from axisymmetry to triaxial configuration. Although this bifurcation is very close to the mass shedding limit but for soft NS equation of states (EoSs) for NSs with large compactness the triaxial sequence could totally disappear ( or vanish). According to general relativity quark stars, in general, can have larger triaxial deformation (e.g.  $R_y / R_x$  ratio ) before terminating the sequence at the mass shedding limit. As the decomposed u (up), d (down) and s (strange) quarks could be stable [50, 51] the significance of triaxial deformation is that a small tidal deformability of quark star is favorable for observation [52, 53]. Using the new EoS which is based on quark clustering, proposed by Lai and Xu [54] ), Zhou et al [55] showed in a numerical simulation that in a stiff EoS a triaxial quark star can support its maximum allowed mass upto  $3.3 M_{\odot}$  . In this context, the detection of super-luminous supernova ASAssN-15LH provides the valuable information of a possible signature for the birth of an strange quark star (SQS) [56]. It leads this author [57] to suggest the unseen companion compact object with mass  $3.3 M_{\odot}$ , detected by Thompson et al [58], in the binary J05215658 + 4359220 is a triaxially deformed quark star, In other words, this could be a possible direct evidence of the existence of a triaxial star.

### **4.4 Dark Matter Stars**

The evolution of the universe as large scales indicates about 95% of the constituents of the universe is indeed completely unknown i.e. it can not be described in terms of known particles.

Modern cosmology suggests these unknown constituents of the universe are Dark Matter (DM) and Dark Energy (DE). The distribution of the observed structures in the universe reflects that the behavior of Dark Matter is a self-interacting dark matter candidate [65], i.e. form of DM seems to be self-bound [66]. Using two-fluid TOV equations individually for normal matter (NM) and for Dark Matter Wang et al [67] found in a numerical simulation study of the dark matter admixed neutron star that

- (a) Microscopic nature of DM particles have directly connection or link with its macroscopic mass existing in the dark matter admixed neutron stars (DANSs).
- (b) DM has obvious effect on the mass of compact star but comparing with the energy density in the Milky Way galaxy the existence of DM might hardly affect the mass of the compact stars situated in this galaxy.

The general scenario [68] indicates that neutron stars may contain a sizeable amount of dark matter which was accreted throughout their life time and / or was present in proto-star cloud. Not only that, light dark matter particles have a tendency to form an extended halo around NS while heavy dark matter particles are most likely create a dense DM core inside the NS. The meaning is that when halo formation case is active the outer most radius of the dark matter admixed neutron stars will exceed the radius of NS, while in the core formation case the outer-most radius will be almost same as baryon one [69]. In other words, the radius of compact star (neutron star) becomes affected by the presence of DM but its mass leads towards lower values in the case of core formation.

In an investigation of mass radius relationship of non-rotating and rotating pure hadronic matter (neutron stars) admixed with self-interacting fermionic asymmetric dark matter, with the two-fluid formalism as well as the conditions of equal and differential frequencies of nuclear matter and dark matter, Mukhopadhyay et al [69] found the maximum mass of differentially rotating stars having self-interacting dark matter  $\sim 1.94 M_{\odot}$  with radius  $\sim 10.4$  km.

With the assumption of DM and normal matter only couple through gravity Leung et al [71] found in their study of tidal deformation of DM admixed NS that

- i) A two-fluid star will only reach its maximum mass i.e.  $M_{\max} = 2.6 M_{\odot}$  at either the object is pure normal matter (NM) or pure dark matter (DM) i.e. the DM admixed

Neutron Star allows a maximum mass  $2.6 M_{\odot}$  only provided the DM EoS can reach =  $2.6 M_{\odot}$ .

- ii) For fermionic DM and bosonic DM cases this maximum mass  $M_{\max} = 2.6 M_{\odot}$  can be reached if  $\mu < 0.536 \text{ GeV}$  (for fermionic DM) and  $\rho_0 \hbar^3 < 3.69 \times 10^4 \text{ GeV}^4$  (for bosonic DM) where  $\mu$ ,  $\rho_0$  being the particle mass and energy density, respectively.
- iii) Pure normal matter neutron stars cannot reach  $2.6 M_{\odot}$
- iv) Higher mass limit i.e.  $M_{\max} > 2.6 M_{\odot}$  is possible if smaller (e.g. lighter) is considered.
- v) The other possibility is that any more massive compact objects ( that are not black hole ) could exist, if detected.

## 5 Compact stars --- New Degrees of Freedom

One of the most significant predictions of general relativity is the existence of maximum mass for any static matter configuration. The cores of massive stars are not completely relativistic. As a result, it reduces the pressure and requires an increase amount of mass to reach gravitational potential to collapse the star. Constraining the Equation of State (EoS) of massive stars, in particular, neutron stars ( than the others ) have remained one of the biggest unsolved problems for the nuclear astrophysics.

### White Dwarf Stars

For white dwarf stars the Chandrasekhar mass limit is  $M_{\text{ch}} = 1.4 M_{\odot}$  and the wider mass range of  $M_{\text{ch}}$  is  $\sim 1.17 - 1.75 M_{\odot}$  while the critical mass of  $M_{\text{ch}}$  is  $\sim 1.47 M_{\odot}$  [72].

### Neutron Stars

The cores of neutron stars cannot be directly observed. To obtain the properties of matter at the neutron star core one needs a suitable model of neutron star matter describing from its crust to its core. Due to the lack of exact theory for strongly interacting matter at densities beyond saturation density it is very difficult to model neutron star cores very accurately. Compact objects are actually the final fate of the stars. They are characterized by ultra-high matter densities. So, dense compact objects are relativistic. On the other hand, quark matter is

absolutely stable and it can be considered as the true ground state of hadronic matter. As a result, a new class of compact objects has been postulated to exist as an alternative to neutron star. for example, “Strange Quark Star” , a much more stable configuration compared to neutron star [73], that may explain the phenomenon of release of huge amount energy in super-luminous supernovae. Using nonlinear equation of state in a numerical solution study Panotopoulos and Rincon [73] found the maximum mass of strange quark star is  $2.55 M_{\odot}$  with radius  $R = 12.34$  Km for color superconductivity model CPL10. This value is slightly more than a) the value of the maximum mass limit of quark star  $\approx 2.33516 M_{\odot}$  with radius  $R = 10.04$  km obtained by Pant et al [74] considering the solutions representing charged fluid spheres joining smoothly with Reissner-Nordstrom metric at the pressure free surface and b) the value of the maximum mass of stable strange quark star  $M_{\max} \approx 2.4M_{\odot}$  obtained by Dong et al [75]. Note that, utilizing a specific choice with surface density  $\rho \sim (2.0 - 2.7) \times 10^{14}$  g/cm<sup>3</sup> and satisfying the inequalities  $0 \leq K \leq 0.24988$ , K being a parameter involves in electrical intensity, Pant et al [74] extended the upper limit of maximum mass of neutron star to  $M_{\max} = 3.57546 M_{\odot}$  (with radius  $R = 15.37$ Km).

Theoretical estimations (based on a number of observation) predicted the maximum mass of neutron star exists in the range  $(2.2 - 2.9) M_{\odot}$  i.e., up to  $\sim 3 M_{\odot}$  (uncleared whether the limit of GR is supported or not) [75]. Using gravitational wave detectors’ data (i.e., observed in the NS – NS , NS – BH binary systems) the value of the maximum mass for neutron star could be in the range of  $(2.5 - 2.67) M_{\odot}$  with the violation of the causal limit for  $R_{1.4} \leq 11.38$  Km [76]. On the other hand, Khadkikar et al [77] boost up this of maximum mass to  $3.60 M_{\odot}$  for a spinning neutron star with exotic equation of state at mass-shedding limit.

In the presence of somehow realistic, strong internal magnetic field  $\approx 10^{17}$  G the stellar charge particle population can be able to re-leptonize and de-hyperonize which finally generate more massive neutron star  $> 2.1 M_{\odot}$ . Considering the density dependent relativistic mean field (DD-RMF) parameters Rather et al [78] found the maximum masses of a heavy magnetic neutron star for pure nucleonic and hyperonic matter are  $2.575 M_{\odot}$  (with radius  $R = 12.465$  km ) and  $2.183 M_{\odot}$  (with radius  $R=12.238$  km) in the case of dimensionless tidal deformability  $\Lambda_{1.4} = 791.483$  and internal magnetic field  $1.79 \times 10^{17}$  G.

## Pulsars

The most dominant sub-class of neutron stars is Radio Pulsars i.e., isolated neutron stars with period of rotation ( $P$ ) of a few tens milliseconds to several seconds and dipole field  $B_d = 10^{11} - 10^{13}$  G.

The second sub-class is binary X-ray pulsars with  $B_d = (1 - 7) \times 10^{12}$  G and period  $P$  varies from 2 ms –  $\sim 10^4$  s.

The third sub-class lies at the low  $B_d$  and short  $P$  end i.e. a mixture of millisecond pulsars (MSPs : isolated or in wide binaries) and low mass X-ray binaries (LMXBs in which case neutron stars are considered to be old  $>10^9$  years). The observed fact is 50% of pulsars (PSRs) with characteristic age less than  $\sim 5 \times 10^4$  years have associated with supernova remnants (SNRs).

## Hybrid Stars

To explore the possible hadron-quark phase transition in the interior of neutron star Qin et al [74] studied the first order phase transition and crossover , the tidal deformability of hybrid star considering the 3- window construction model and found the possibility of attaining maximum mass of hybrid stars in excess of  $2 M_\odot$ . This value is consistent with the value proposed by Alaverdyand and Vartanyan [79] and the observed value for the pulsar PSRJ0348+0432 with a mass of  $2.1 \pm 0.04 M_\odot$ .

A significant maximum mass of hybrid star is resulted in the study of possible hadron-quark phase transition in the interior of neutron star . Using semi-analytical hybrid equation of states at fixed entropy per baryon for hot and cold hybrid stars Mariani et al [80] obtained the maximum mass of the hybrid star is  $(2.68 - 2.69) M_\odot$  with a quark matter core of  $2.67 M_\odot$ . On the other

Hand, multi-messenger observation of a binary Neutron Star merger (GW170817) provides a new constraint for the masses and radii of neutron stars such that a new class of hybrid equation of state (EoS) [81] could fulfill

- a) the NICER measured radius (less than 11 km i.e.,  $10.5 < R < 12.5$  km) for the high mass pulsar PSR J0740+6620 , and

- b) the maximum mass of the component star (hybrid nature)  $\sim 2.6 M_{\odot}$  in the case of asymmetric binary merger GW190814.

Blaschke and Cierniak [81] suggested that this  $2.6 M_{\odot}$  object in the binary merger GW190814 could have been a hyper-massive hybrid neutron star with a large quark matter core. As the hadronic picture changes at high densities i.e. quarks may be deconfined or percolate among tightly packed baryons a problem ---- “ whether there is a phase transition or a cross over between hadronic and quark matter “ remains, not yet solved [82,83].

### Quark Star or Strange Quark Star

The theoretical idea ---- there exists a process of conversion of a hadronic star into a quark star is energetically convenient even if the radius of the final configuration is larger than the radius of the initial configuration. In the two families scenario compact stars, i.e., hadronic stars and quark stars ( precisely light stars are hadronic stars whereas large and massive stars are strange quark stars), do coexist and the transition between the two branches will appear in a hadronic star when a sizable fraction of strangeness is present through hyperons i.e. the metastable stellar system that can convert into a pure strange quark star [84]. Note that the highest mass measured through observation of PSR J0348 + 0432 is  $M_{\max} = 2.01 \pm 0.04 M_{\odot}$  but the estimated mass of this black widow pulsar is  $2.4 M_{\odot}$  [85]. This means strange quark stars could have maximum masses larger than  $2 M_{\odot}$ . In other words, measurements of masses larger than  $2 M_{\odot}$  would favor the strange quark star scenario.

A study [83] of the thermodynamic properties of asymmetric quark matter and large mass quark stars under the confined isospin-density-dependent-quark mass model suggests that neutron stars could be converted to hybrid stars or strange quark stars. These stars are usually made up of deconfined absolutely stable ‘u’, ‘d’ and ‘s’ quarks with lepton in  $\beta$ -equilibrium i.e., can be

Considered as strange quark matter. By fitting the radial velocity lines and the three-band light curves in the irradiated compact star models Chu et al [83] also showed that the recently detected three heavy compact stars PSR J0348+0432 ( $2.01 \pm 0.04 M_{\odot}$ ), MSP J0740+6620 (with mass  $2.14^{+0.010}_{-0.09} M_{\odot}$  of 68.3% credibility interval and  $2.14^{+0.20}_{-0.18} M_{\odot}$  of 95.4% credibility interval) and PSR J2215+5135 ( with mass  $2.27^{+0.10}_{-0.09} M_{\odot}$ ) as heavy quark stars. In other

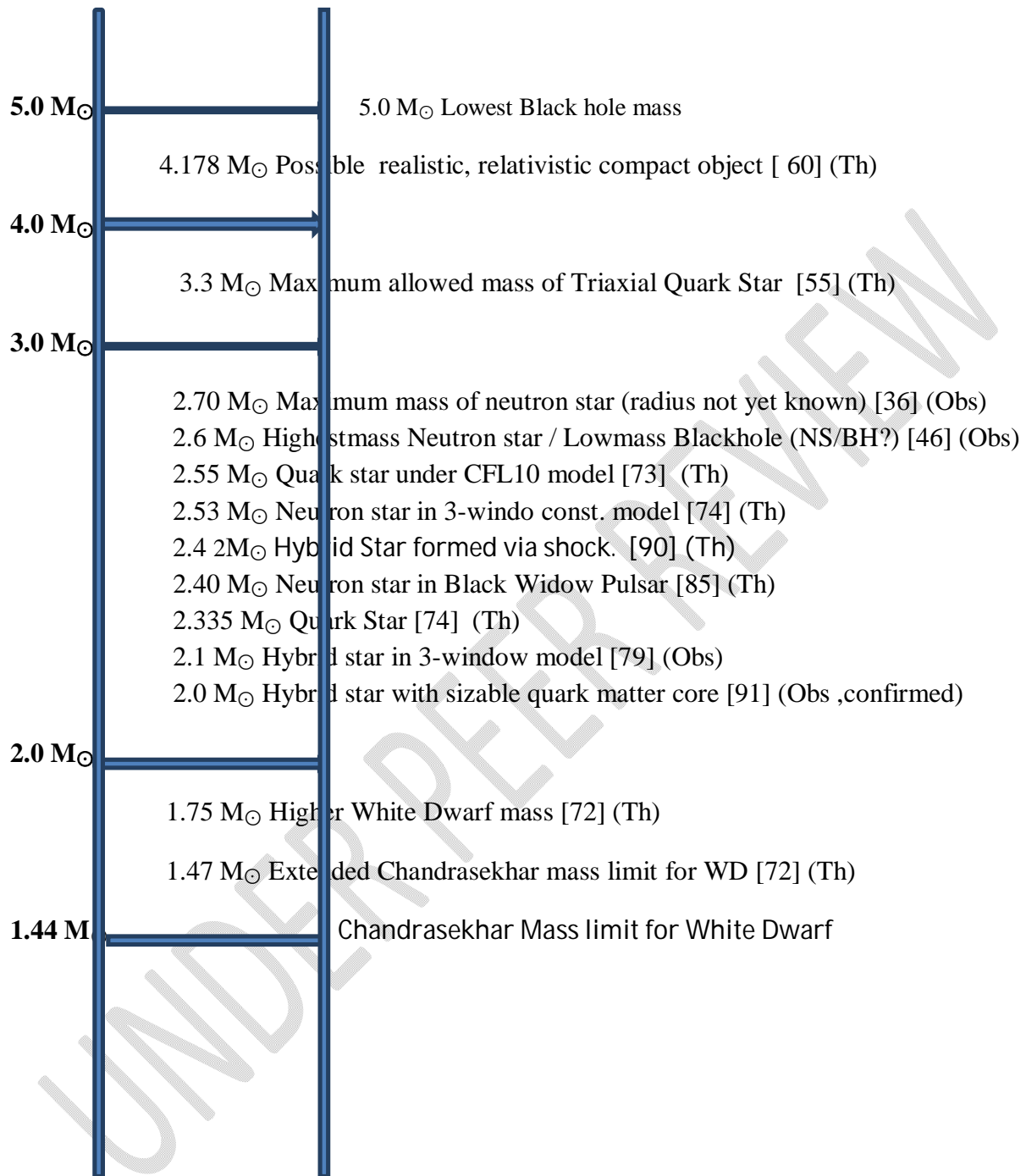
words, the detected heavy compact star e.g. the heavy quark star have maximum mass  $2.27 M_{\odot}$  till date.

### Magnetars, Triaxial Stars

Till date 31 magnetars have been detected. Among them only six magnetars are passing through their triaxial phases [86] whereas rest numbers magnetars have already crossed their triaxial phases. Recently detected cosmic baby Swift J1818.0 – 1607 is a very young magnetar having characteristic age  $\sim 240 - 300$  years which can be considered as the best source for detecting and confirming it as a triaxial star [87]. Its estimated mass is  $\sim 2.0 M_{\odot}$ . On the other hand, numerical simulation using a stiff equation of state suggests that a triaxial quark star can support its maximum allowed mass up to  $3.3 M_{\odot}$  [55].

### Black Holes

Based on a number of observational and theoretical models the mass distribution between the highest mass of neutron star ( $2 M_{\odot}$ ) and the lowest mass of black hole ( $5 M_{\odot}$ ) exists in the range  $2 - 5 M_{\odot}$  [88-90]. Observation of GW 190814 by the LIGO and VIRGO detectors indicated that the gravitational wave signal was originated from the merger of a  $\approx 23 M_{\odot}$  black hole (BH) with a  $\cong 2.6 M_{\odot}$  compact object. This compact binary source of GW190814 is atypical both in its highly asymmetric masses and in its lower-mass component lying between the heaviest known neutron star and the lightest known black hole. Therefore, GW190814 offers an unprecedented probe into the mass gap making the mass of binary's secondary lighter component ( $2.5 - 2.67$ )  $M_{\odot}$ . as the heaviest neutron star or lightest black hole ever identified in a compact object binary.



**Fig.1:** Mass Gap Ladder for different compact objects with their maximum masses (not scaled). Theoretical or estimated values of maximum mass are marked by (Th / Estimated), Observed or detected maximum mass by (Obs / Confirmed). For 2.6 M<sub>⊙</sub> companion mass of binary system is marked by (NS/BH ? ) i.e. not yet solved.

## 6 Conclusion

Analytical solutions of Einstein's gravitational field equations provide a key for determination of maximum mass of very compact astrophysical objects. The central density of these compact objects could be  $\sim 10^{15} \text{ g.cm}^{-3}$  and calculations for obtaining their maximum mass require reliable information such as structure, behavior of matter at such ultra-high density. Analytical solutions describe the stable mass (in hydrostatic equilibrium) of normal matter i.e. maximum mass of neutron star is possible which follows that neutron stars are expected to have their mass  $m < 0.3a$ , ('m', 'a' being the mass and radius of the star expressed in km), while for strange stars  $m > 0.3a$ . The noteworthy point is that strange stars with masses exceeding the maximum mass of neutron stars will not be in hydrostatic equilibrium [39]. Highest mass in strange star (see table II) observed in PSR J1614 – 2230 which is  $\sim 1.978 M_{\odot}$  ( $\approx 2.0 M_{\odot}$ ). In the absence of reliable information regarding the structure as well as behavior of matter inside the compact object Vidya -Tikekar superdense star model [59] is very effective in modeling the astrophysical compact objects (such as neutron star, quark star ....). Using general solutions to Vaidya-Tikekar metric (with density  $2 \times 10^{14} \text{ g.cm}^{-3}$ ) in modeling Einstein's field equation Sasidharan and Sabu [60] found a possible realistic, relativistic compact object with mass =  $4.1738 M_{\odot}$ . Triaxial star is another realistic compact object which could be placed in the mass range (3 – 5)  $M_{\odot}$  in the mass gap (2.6 – 5)  $M_{\odot}$ . We suggest the detected unseen companion compact object with mass  $\sim 3.3 M_{\odot}$  in the binary J 05215658 + 4359220 as a triaxial star i.e. triaxially deformed quark star as it matches with the result found in numerical simulation study. If it is correct, then the highest detected compact object mass will be  $\sim 3.3 M_{\odot}$  resulting which the mass gap will also reduce to (3.3 – 5)  $M_{\odot}$ . Lighter dark matter is  $\sim 5$  times the mass of baryonic matter [70] which means that triaxially deformed star consisting of lighter dark matter particles may have the possibility to possess higher mass i.e.  $> 3.3 M_{\odot}$ . Another possibility is the triaxially deformed massive compact object (not blackhole) whose nature or type is not yet known to us that we have to search for detection. This author thus suggests that possibly it be a Triaxial Star which consists of unknown matter whose form not yet known to us and remains undetectable to the astronomers.

As the detection of gravitational waves can be a convenient method to collect evidence of quark stars (e.g. strange quark stars) or dark matter admixed normal matter Neutron Stars [92,93] we encourage the GW community to search the electromagnetic counterparts during their observation of compact objects through LIGO or VIRGO.

## References :

1. Einstein A. Sitzungsber. Preuss. Acad. Wiss .1916 ; 1 : 688

2. Einstein A. Die Grundlage der allgemeinen Relativitätstheorie. *Annalen der Phys.* 1916 ; 354 : 354. Doi: 10.1002/andp.19163540702
3. Pandharipande V R, Pines D, Smith R. Neutron star structure: theory, observation, and speculation. *Astrophys. J.* 1976 ; 208 : 550. Doi: 10.1086/154637
4. Schwarzschild K. Sitzungberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin , Phys Math, Klasse 1916 ; 424 : 1
5. Chandrasekhar S. The maximum mass of an ideal white dwarf. *Astrophys. J.* 1931 ; 74 : 81
6. Farr W M, Sravan N. et al. The mass distribution of stellar-mass blackholes. *Astrophys. J.* 2011 ; 741 : 103
7. California Institute of Tech *Science Daily* ( 23<sup>rd</sup> June 2020 Issue)
8. Özel F, Paltis D, Narayan R, McClintock J E. The Blackhole mass distribution in the Galaxy. *Astrophys. J.* 2010 ; 751: 1918
9. Abbott B P, Abbott R, Abbott T D, Abraham S , Acernese F. et al. Binary Black Hole Population Properties Inferred from the First and Second Observing Runs of Advanced LIGO and Advanced Virgo. *Astrophys. J.* 2019 ; 882 : L24
10. L. Kreidberg, C. D. Baulyan, W. M. Farr and V. Kalogera, Mass Measurements of Black Holes in x-ray transients: Is there a mass gap? *Astrophys. J.* 2012 ; 757 : 36
11. Abbott R, Abbott T D, Abraham S, Acernese F, Ackley K, Adams C, Adhikari R X. et al. GW190814: Gravitational Waves from the Coalescence of a 23  $M_{\odot}$  Black Hole with a 2.6  $M_{\odot}$  Compact Object. *Astrophys. J. Lett.* 2020 ; 896 : L44.
12. Thompson T A, Kochanek C S. et al., A noninteracting low-mass black hole-giant star binary system. *Science* 2019 ; 366 : 637
13. Zwicky F. On Collapsed Neutron Star. *Astrophys. J.* 1938 ; 88 : 522
14. Zwicky F. On the Theory and Observation of Highly Collapsed Stars. *Phys. Rev.* 1939; 55 : 726
15. Srinivasan G. The maximum mass of neutron stars. *Astron. Astrophys. Rev.* 2002 ; 11: 67
16. Oppenheimer J R, Volkoff G M. On Massive Neutron Core *Phys. Rev.* 1939 ;55: 374
17. Cameron A G W. Neutron Star Models . *Astrophys. J.* 1959 ; 130 : 884

18. Chamel N, Haensel P, Zdunik J L, Fantina A F. On the Maximum Mass of Neutron Stars. *Int. J. Mod. Phys. E* 2013 ; 22 : 130018
19. Antoniadis J, Freire P C C. et al. A Massive Pulsar in a Compact Relativistic Binary *Science* 2013 ; 340 : 448
20. Cromartie H-T, Fonseca E. et al. Relativistic Shapiro delay measurements of an extremely massive millisecond pulsar. *Nature Astronomy* 2020 ; 4 :72
21. Shao D-S, Tang S-P. et al. Maximum mass cutoff in the neutron star mass distribution and the prospect of forming supramassive objects in the double neutron star mergers *Phys. Rev. D* 2020 ; 102 : 063006
22. Hughes S A. Trust but verify: The Case for astrophysical black holes. Proc.33<sup>rd</sup> SLAC Summer Institute SSI2005 ; 2006 : L006
23. Kulkarni S R. Lecture Notes- “ How to find stellar Black-holes” 2019
24. Stair I H. Pulsars in Binary Systems: Probing Binary Stellar Evolution and General Relativity. *Science* 2004 ; 304 : 547
25. J. E. McClintock and R. E. Remillard. *Compact stellar X-ray Sources* (Cambridge Univ. Press, UK, 2006)
26. Hu H, Bao S, Zhang Y. et al. *PTEP* 2020 ; 2020 : 043001
27. Thorsett S E, Chakraborty D. Neutron Star Mass Measurements. I. Radio Pulsars *Astrophys. J.* 1999 ; 512 : 288
28. L. Kaper , A. van der Meer et al. Measuring the masses of Neutron Stars. *The Messenger* 2006 ; 126 : 27
29. Clark J S, Goodwin S P, Crowther P A, Kaper L, Fairbairn M, Langer N, Brocksopp C. Physical parameters of the high-mass X-ray binary 4U1700-37 . *Astron. Astrophys.* 2002 ; 392 : 909
30. Doroshenko V, Suleimanov V, Puhhofer G, Santangel A. A strangely light neutron star within a supernova remnant. *Nature Astron.* (24 Oct. 2022) doi: 10.1038/s41550-022-01800-1
31. Suwa Y, Yoshida T Y. et al. On the minimum mass of neutron stars. *MNRAS* 2021 ; 481 : 3305

32. Espino P L, Paschalidis V, Thomas W. Baumgarte, Stuart T W, Shapiro L. Dynamical stability of quasi-toroidal differentially rotating neutron stars. *Phys. Rev. D* 2019 ; 99 : 083017
33. Wei H, Yu Zh-X. Inverse Chameleon Mechanism and Mass Limits for Compact Stars *JCAP* 2021 ; 2108 : 011
34. Kalita S, Mukhopadhyay B, Govindarajan T R. Super-Chandrasekhar limiting mass white dwarfs as emergent phenomena of noncommutative squashed fuzzy spheres. *Int. J. Mod. Phys. D* 2021 ; 30 : 2150034
35. Chavanis P-H, Harko T. Bose-Einstein Condensate general relativistic stars. *Phys. Rev. D* 2012 ; 86 : 064011
36. Eslam-Panah B, Bordbar G. et al. Expansion of Magnetic Neutron Stars in an Energy (in)Dependent Spacetime. *Astrophys. J.* 2017 ; 848 : 24
37. Giacosa F, Pagliara G. Neutron stars in the large- $N_c$  limit. *Nucl. Phys. A* 2017; 968 : 366
38. Pandharipande V R, Pines D, Smith R A. Neutron star structure: theory, observation, and speculation. *Astrophys. J.* 1976 ; 208 : 550- 566
39. Shapiro S L, Teukolsky S A. *Black holes, Whitedwarfs and Neutron stars: The Physics of Compact Objects* ( John Wiley, NY,1983)
40. Elebert P, Reynolds M T, Callanan P J, Hurley D J. et al. Optical spectroscopy and photometry of SAX J1808.4-3658 in outburst. *MNRAS* 2009 ; 395 : 885
41. Abubekkerov M K, Antokhina E M, Cherepashchuk A M, Shimanskii V V.. The Mass of the Compact Object in the X-Ray Binary Her X-1/Hz Her. *Astron. Reports* 2008 ; 52 : 379
42. Rawls M R, Orosz J A, McClintock J E, Torres M A P, Bailyn C D, Buxton M M. Refined Neutron Star Mass Determinations for Six Eclipsing X-Ray Pulsar Binaries *Astrophys. J.* 2011 ; 730 : 25
43. Demorest P, Pennucci T, Ransom S, Robert M, Hessels J. Shapiro delay measurement of a two solar mass neutron star. *Nature* 2010 ; 467: 1081
44. Hansraj S, Govender M, Moody L, Singh K N. Strange stars in the framework of higher curvature gravity. *Phys. Rev. D* 2022 ; 105: 044030
45. Thakore B, Goti R, Shah S, Pandya H, Pandya D M. Finch-Skea Solutions of Anisotropic Stellar Models in  $f(R)$  Gravity. *Astrophys. Space Sci.* 2021 ; 366 ; 95

46. Horvath J E, Morres E P. Modeling a  $2.5M_{\odot}$  compact star with quark matter. *Int. J. Mod. Phys. D* 2021 ; 30 : 2150016
47. .R. Sharma, S. Das, M. Govender and D. M. Pandya. Revisiting Vaidya-Tikekar stellar model in the linear regime. *Annals Phys.* 2020 ; 414 :168079
48. Gondek-Rosinska D, Haensel P, Zdunik J L, Gourgulhon E. Rapidly rotating strange stars. in *Proc. IAU colloq.- 177: Pulsar Astronomy- 2000 and Beyond*, 2000 ; 202 : 661
49. Szkudlarek M, Gondek-Rosinka D, Villain L, Ansorg M. . Maximum Mass of Differentially Rotating Strange Quark Stars. *Astrophys. J.* 2019 ; 879 ; 44
50. Bodmer A R. Collapsed Nuclei. *Phys.Rev. D* 1971 ; 4 : 1601
51. E. Witten. Cosmic separation of phases. *Phys. Rev. D* 1984 ; 30 : 272
52. Lai XY, Yu Y W, Zhou E P, Li Y, Wu R X. Pulsar glitches in a strangeon star model . *Res. Astrophys. Astron.* 2018 ; 18 : 24
53. Dai Z G,Wang SQ, Wang J S, Wang L J, Yu Y W. The most Luminous Supernova ASASSN-15LH . *Astrophys. J.*2016 ; 817 : 132
54. Lai X Y, Xu R X, Lennard-Jones quark matter and massive quark stars. *MNRAS* 2009 ; 398 : L31
55. E. Zhou, A. Tsokaros, L. Rezzola, R. Xu and K. Uryu . Uniformly rotating, axisymmetric, and triaxial quark stars in general relativity. *Phys. Rev. D* 2018; 97 :23013
56. Dai Z G,Wang SQ, Wang J S, Wang L J, Yu Y W. The most Luminous Supernova ASASSN-15LH . *Astrophys. J.*2016 ; 817 : 132 (page 4)
57. Parui R K. Evidence of a “Triaxial Star” in Binary System , Submitted to *EPJ C* (2022)
58. Thompson T A, Kochanek C S, Stanek K Z, Bodemes C, Post R D, Jaysinghe T. et al. A noninteracting low-mass black hole-giant star binary system. *Science* 2019 ; 366 : 637 ; van den Heuvel E P J., Tauris, T M. Comment on “A noninteracting low-mass black hole–giant star binary system. *Science* 2020 ; 368 : id. eaba3282.
59. Vaidya P C, Tikekar R. Exact relativistic model for a superdense star *J. Astrophys. Astron.* 1982 ; 3 : 325
60. A. S. Sasidharan and M. C. Sabu. General Solution to VAIDYA-TIKEKAR METRIC with Charged Distributions on Spheroidal Space time. *Int. J. Maths. Trends and Tech.* 2021 ; 67 : 15
61. Ozel F, Guver T, Psaltiz D. The Mass and radius of neutron star in the buldge low mass X-ray Binary KS 1731–260. *Astrophys. J.* 2009 ; 693 : 1775

62. Murad H M, Fatema S. Some exact static charged perfect fluid spheres and relativistic compact Electrically charged stellar models in General Relativity. *Euro. Phys. J. C* 2015 ; 75 : 533
63. Dong S B, Shappee B J, Prieto J L. et al. ASASSN-15lh: A highly super-luminous supernova. *Science* 2016 ; 6270 : 257
64. Jotania K, Tikekar R. On relativistic models of strange stars. *Int. J. Mod. Phys. D* 2006 ; 15 : 1175
65. Iaani A, Mannarelli M, Rossi N. A new approach to dark matter from the mass–radius diagram of the Universe. *Results in Phys.* 2022 ; 38 : 105544
66. Tulin S, Yu H-B. Dark matter self-interactions and small scale structure. *Phys. Reports* 2018 ; 730 : 1
67. Wang X D, Qi B, Yang G L, Zhang N B, Wang S Y. Possible maximum mass of dark matter existing in compact stars based on the self-interacting fermionic model. *Int. J. Mod. Phys. D* 2019 ; 28 : 1950148
68. Sagun V, Giangrandi E, Ivanytski O, Lopes I, Bugaev K-A. Constraints on the fermionic dark matter from observations of neutron stars. *in Proc. PANIC2021 (2021)* p.313
69. Mukhopadhyay S, Atta D, Basu D N. Gravitational waves from isolated neutron stars: Mass dependence of -mode instability. *in JPS Conf. Proc.* 2020 ; 32 : 010073
70. Zurek K M. Asymmetric Dark Matter: Theories, Signatures, and Constraints. *Phys. Reports* 2013 ; 537 : 91
71. Leung K-L, Chu M-C, Lin L-M. Tidal deformability of dark matter admixed neutron stars *Phys. Rev. D* 2022 ; 105 : 123010.
72. Kumar N, Sokolov V V. Mass distribution and “Mass Gap” of compact stellar remnants in binary systems. *Astrophys. Bulletin.* 2022; 77 : 197
73. Panotopoulos G, Rincon A. Electrically charged strange quark stars with a non-linear equation of state. *Euro. Phys. J. C* 2019 ; 79 : 524
74. Pant N, Pradhan N, Murad M H. A family of exact solutions of Einstein-Maxwell field equations in isotropic coordinates: an application to optimization of quark star mass. *Astrophys. Space Sci.* 2014 ; 352 : 135
75. Dong H, Kuo T T S, Lee H. K, Machleidt R, Rho M. Half Skyrmions and the equation of state for compact star matter. *Phys. Rev. C* 2013 ; 87 : 054332

76. Godzieba D A, Radice D, Bernuzzi S. On the maximum mass of neutron stars and GW190814. *Astrophys. J.* 2021 ; 908 : 122
77. Khadhikar S, Mangat C S, Banik S. Quasi-stationary sequences of hypermassive neutron stars with exotic equation of state. *J. Astrophys. Astron.* 2022 ; 43 : 57
78. Rather I A, Rahaman U, Dexheimer V, Usmani A A, Patra S K. Heavy Magnetic Neutron Stars. *Astrophys. J.* 2021 ; 77 : 917
79. Alaverdyan G B, Vartanyan Yu L. Maximum mass of hybrid stars in the quark bag model. *Astrophys.* 2017 ; 60 :563
80. Mariani M, Orsaria M, Vucetich H. Constant Hybrid Stars : a first approximation of cooling evolution. *Astron. & Astrophys.* 2017; 601 : A 21
81. Blaschke D, Cierniak M. Studying the onset of deconfinement with mul-messenger astronomy of neutron stars. *Astronomische Nachrichten.* 2021 ;342 : 13909
82. Papakonstantinou P, Hyun C H. Energy density modeling of strongly interacting matter : Atomic Nuclei and Dense star. *Symmetry* 2023 ; 15 : 683
83. Chu P C, Zhou Y, Jiang Y-Y, Ma H-Y, Liu H, Zhang X-M. Quark star matter in heavy quark stars. *Euro. Phys. J. C* 2021 ; 81 : 93
84. Dondi N A, Drago A, Pagliara G. Conditions for the Co-existence of stable strange quark matter. *EPJ Web Conference Series: XII<sup>th</sup> Quark Confinement and the Hadron Spectrum.* 2017 ; 137 : 09004
85. Van Kerkwijk, Breton R, Kulkarni S R. Evidence for a Massive Neutron Star from a Radial-velocity study of the companion to the Black Widow Pulsar B1957+20 *Astrophys. J.* 2011 ; 728 : 95
86. Parui R K. A Remark on “Do Triaxial Supermassive Compact Star Exist?” *Int. Astron. Astrophys. Res. J.* 2023 ; 5 : 33
87. Parui R K. A New Compact Star—the "Triaxial Star"—and the Detection of a Cosmic Baby: A Possibility. *Int. Astron. Astrophys. Res. J.* 2023 ; 5 : 38
88. Zevin M, Spera M, Berry C P L, Kalogera V. Exploring the lower mass-gap and unequal mass regime in compact Binary evolution *Astrophys. J. Lett.* 2020 ; 899 : L1
89. Abbott R, Abbott T D, Abbott S, Abrahama F, Acernese K, Ackley A, Adams C. et al. Observation of Gravitational Waves from Two Neutron Star–Black Hole Coalescences *Astrophys. J. Lett.* 2021 ; 915 : L5

90. Mallick R, Singh S, Nandi R. Maximum mass of hybrid stars formed via shock induced phase transition in cold neutron stars. *MNRAS* 2021 ; 503 : 4829
91. Liu H, Yang Y-H, Han Y, Chu P-C. properties of quark matter and hybrid stars from a quasiparticle model. Arxiv: 2305.01246v.i [nucl-th] (2023)
92. Sotani H, Harada T. Nonradial oscillations of quark stars. *Phys. Rev. D* 2003 ; 68 : 024019
93. Sotani H, Kohri K, Harada T. Restricting quark matter models by gravitational wave observation. *Phys. Rev. D* 2004 ; 69 : 084008

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