Stochastic Analysis Of Asset Returns Which Follows Multiplicative Effects Series

Abstract

The key importance of asset values and it return rates are geared towards investment funds which grows wealth over time. This paper considered stochastic models where asset values were examined. The problems were accurately solved analytical by means of Ito's theorem and a closed form solutions were obtained which governed asset price return rates through multiplicative effects series. However, the behaviour on the value of asset prices were analysed using Kolmogorov-Smirnov (KS). To this end, graphical solutions and the effects of the relevant stock variables were conferred accordingly.

Key words: Asset value, Kolmogorov –Smirnov (KS), Stochastic Analysis, Prices.

1.1 Introduction

Investing on assets results in using the returns to take care of the daily expenditures in capital assets. To improve on trading activities, an investor may need more capital investments. Thus, capital assets are put in to acquire more capital investments for growth which enables the investment to bring more yields in the unit production, produce new concepts on products or even introduce values to the business and to check modern technology in order to improve productivity and reduce cost and to replace the worn out assets. Trading investments, where capital assets are absent, will absolutely have challenging time getting off the ground. The results of stock market undertaking and performances have raised considerably in the growth of financial market as investors gain their normal returns to have a living.

Notwithstanding, stock business is usually known for its high returns. A return on investments for stock market is a medium of correctly estimating cash flow of an investment. It is the major parameter consistently applied by investors to handle the expenditure [1]. The dynamics of stock prices in the stock market are reproduced by unpredictable flow or movement of their value over time. For this reason, it is better to have an apt comprehension of the type of physical quantities to model so as to have pragmatic outcomes of the stock variables. For example, [2] studied on the stochastic model of the changes or variations of stock market prices. Conditions for finding out the equilibrium price, adequate circumstances for dynamic stability and convergence to equilibrium of the growth rate of the value function of shares. Furthermore, [3] worked on stochastic model of price changes on the floor of stock market. Here, the equilibrium price and the market growth rates of shares were determined. [4] considered the stochastic analysis of Markov Chain in Finite state. The transition Matrix replicated the use of 3-State transition probability matrix which enables them to present precise condition of obtaining expected mean rate of returns of each stock.

Similarly, [5] looked at the stochastic analysis of stock market expected returns and Growth-Rates. The exact conditions for obtaining the drifts, volatilities, Growth-Rates of four different stocks were also considered. [6] worked on the problem of stock price fluctuations using stochastic differential equations, principal component analysis and KS goodness of fit test were studied. Details of the analytical solutions were given; the computational and graphical results were presented and discussed respectively. [7] looked at a combination of deterministic and stochastic systems with its random parameters in the model. A detailed presentation of the analytical solution to the proposed model was made, which determined the insurance quantities or variables.

Nonetheless, [8] studied the unstable nature of stock market forces using proposed differential equation model. The work in [9] considered stability analysis of stochastic model of price change on the floor of a stock market. Precise conditions in this research were obtained which determined the equilibrium price and growth-rate of stock shares.

The research in [10] looked at stochastic analysis of the behaviour of stock prices. Results show that the proposed model if efficient for the prediction of stock prices. Likewise, [11] considered the stochastic model of some selected stocks in the Nigerian Stock Exchange (NSE), in this study, the drift and volatility coefficients for the stochastic differential equations were determined. [12], built the geometric Brownian Motion and studied the accuracy of the model with detailed analysis of simulated data. [13] worked on the solution of differential equations and stochastic differential equations of time-varying investment returns; where precise conditions were obtained which governs stock return rates through multiplicative effect and multiplicative inverse trends series.

However, this paper is aimed at solving a closed form solutions which represents asset price return rates through multiplicative effects series. It is obvious that the previous efforts have framed a dynamics which governed asset return rates through multiplicative effects in which changes in these prices of financial asset were systematically considered. The advantage of this paper over the work of [15] is that the present work models the impact of SDDE over SDE through multiplicative effect series of asset value changes. To the best of our knowledge this novel contribution compliments the previous discovery and extends the frontier of knowledge in this area of financial mathematics.

The plan of this paper is set as follows: Section 2.1 presents the mathematical preliminaries, Results and discussion are seen in Section 3.1 and paper is concluded in Section 4.1.

3.1: Mathematical Preliminaries

Here, we present few intricate definitions as stirring this dynamic area of study, hence we have as follows:

Definition 1: Stochastic process: A stochastic process $X(t)$ is a relations of random variables $\{X_t(\gamma), t \in T, \gamma \in \Omega\}$, i.e, for each t in the index set T, $X(t)$ is a random variable. Now we understand t as time and call $X(t)$ the state of the procedure at time t. In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

It can also be seen as a statistical event that evolves time in accordance to probabilistic laws. Mathematically, a stochastic process may be defined as a collection of random variables which are ordered in time and defines at a set of time points which may be continuous or discrete.

Definition 2: A stochastic process whose finite dimensional probability distributions are all Gaussian.(Normal distribution).

Definition 3: Random Walk: There are different methods to which we can state a stochastic process. Then relating the process in terms of movement of a particle which moves in discrete steps with probabilities from a point $x = a$ to a point $x = b$. A random walk is a stochastic sequence $\{S_n\}$ with $S_0 = 0$, defined by

$$S_n = \sum_{k-1}^{n} X_k \tag{1}$$

where X_k are independent and identically distributed random variables.

Definition 4: (Differential Equation): is an equation which has functions and their derivatives. In reality the functions is associated to real quantities whereas the derivatives denotes rate of change. Example of differential equation is follows

$$\frac{dS(t)}{dt} = \mu S(t) \tag{2}$$

$$S(0) = S_0 \tag{3}$$

where $S(t)$ represent asset price, μ rate of return, $\frac{dS(t)}{dt}$ is the rate of change of asset price and S_0 is the initial stock price; (2) and (3) can be obtained using variable separable which gives:

$$S(t) = S_0 e^{\mu t} \tag{4}$$

Therefore μ is not known completely which is subject to environmental effects. Therefore (2) can be written as

$$dS(t) = \mu S(t)\,\mathrm{dt} + \sigma S(t)\,\mathrm{dZ(t)} \tag{5}$$

Where σ is the volatility, dZ is the Brownian motion or Wierners process which is random term, the stochastic term added to (2) gives (5) which makes it stochastic differential equation

Definition 5: A Stochastic Differential Equation is a differential equation with stochastic term. Therefore assume that $(\Omega, \mathrm{F}, \wp)$ is a probability space with filteration $\{f_t\}_t \geq 0$ and $W(t) = (W_1(t), W_2(t), \ldots, W_m(t))^T$ $, t \geq 0$ an m-dimensional Brownian motion on the given probability space. We have SDE in coefficient functions of f and g *as* follows

$$dX(t) = f(t, X(t))dt + g(t, X(t))dZ(t),\ 0 \leq t \leq T, \tag{6}$$

$$X(0) = x_0, \tag{7}$$

where $T > 0,\ x_0$ is an n-dimensional random variable and coefficient functions are in the form $f:[0,\mathrm{T}]\times \mathbb{R}^n$ *and* $g:[0,T]\times \mathbb{R}^n \to \times \mathbb{R}^{n\times n}$. SDE can also be written in the form of integral as follows:

$$X(t) = x_0 + \int_0^t f(S, X(S))dS + \int_0^t g(S, X(S))dZ(S) \tag{8}$$

Where dX, dZ are known as stochastic differentials. The $\mathbb{R}^n$ is a valued stochastic process $X(t)$ satisfying (6).

Theorem 1.1: let $T > 0,$ be a given final time and assume that the coefficient functions $f:[0,\mathrm{T}]\times\mathbb{R}^n \to \mathbb{R}^n$ *and* $g:[0,T]\times\mathbb{R}^n \to \times\mathbb{R}^{n\times n}$ are continuous. Moreover, $\exists$ finite constant numbers λ *and* β such that $\forall t \in [0,\mathrm{T}]$ and for all $x, y \in \mathbb{R}^n$, the drift and diffusion term satisfly

$$\| f(t,x) - f(t,y) \| + \| \mathrm{g}(t,x) - g(t,y) \| \leq \lambda \| x - y \| \ , \tag{9}$$

$$\| f(t,x) \| + \| \mathrm{g}(t,x) \| - g(t,\mathrm{x}) \| \leq \beta(1 + \| x \|). \tag{10}$$

Suppose also that x_0 is any $\mathbb{R}^n$ -valued random variable such that $E\left(\| x_0 \|^2\right) < \infty.$ then the above SDE has a unique solution X in the interval $[0,T].$ Moreover, it satisfies $E\left(\sup_{0\leq t\leq T} \| X(t) \|^2\right) < \infty$. the proof of the theorem 1.1 is seen in [20].

Theorem 3.2: (Ito's lemma). Let $f(S,t)$ be a twice continuous differential function on $[0,\infty) \times \mathsf{A}$ and let S_t denotes an Ito's process

$$dS_t = a_t dt + b_t dz(t), t \geq 0 \ ,$$

Applying Taylor series expansion of F gives:

$$dF_t = \frac{\partial F}{\partial S_t} dS_t + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}(dS_t)^2 + \text{higer order terms } (h.\text{o}, \text{t}) \ ,$$

So, ignoring h.o.t and substituting for dS_t we obtain

$$\begin{aligned} dF_t &= \frac{\partial F}{\partial S_t}(a_t dt + bdz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2}(a_t dt + bdz(t))^2 \\ &= \frac{\partial F}{\partial S_t}(a_t dt + bdz(t)) + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} b_t^2 dt, \\ &= \left(\frac{\partial F}{\partial S_t} a_t + \frac{\partial F}{\partial t} dt + \frac{1}{2}\frac{\partial^2 F}{\partial S_t^2} b_t^2 \right) dt + \frac{\partial F}{\partial S_t} b_t dz(t) \end{aligned}$$

More so, given the variable $S(\mathrm{t})$ denotes stock price, then following GBM implies (5) and hence, the function $F(S,t)$,Ito's lemma gives:

$$dF = \left(\mu S \frac{\partial F}{\partial S} + \frac{\partial F}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 F}{\partial S^2} \right) dt + \sigma S \frac{\partial F}{\partial S} dz(t) \tag{11}$$

2.1 .1 Formulation of the Problem

We consider stochastic differential equations imposed with the dynamics of return rates which is said to have a complete probability space $(\Omega, \mathrm{F}, \wp)$ such that a finite time investment horizon $T > 0$. Time delay is paramount in assessing asset value So adding delay term in a model gives more realistic and better model. Therefore delay term is added in the stochastic differential equation below;

$$dX_i(t) = (t-\tau) X_i(t)dt + \sigma X_1(t)dZ(t) \tag{12}$$

where μ is an expected rate of returns on stock, σ is the volatility of the stock , dt is the relative change in the price during the period of time and Z is a Wiener process .Following the method of [1] and [15] on rate of returns gives as:

$$\begin{aligned} &R_t := (\lambda_1\lambda_2)^2, \ldots \\ &\text{where } t = 1, 2\ldots \end{aligned} \tag{13}$$

Using (13) in (12) gives the following system of delay stochastic differential equations :

$$dX_1(t) = (t-\tau)(\theta_1\theta_2)^2 \mathrm{X}_1(t)dt + \sigma X_1(t)dZ^{(1)}(\mathrm{t}) \tag{14}$$

$$dX_3(t) = (\theta_1\theta_2)^2 \mathrm{X}_3(t)dt + \sigma X_3(t)dZ^{(3)}(\mathrm{t}) \tag{15}$$

Where $\mathrm{X}_1(\mathrm{t})$ and $S_3(\mathrm{t})$ are underlying stocks with the following initial conditions:

$$\left.\begin{aligned} \mathrm{X}_1(0) = X_0 \;\; t > 0 \\ \mathrm{X}_3(0) = X_0 \;\; t > 0 \end{aligned}\right\} \tag{16}$$

where, $X_1(t), X_3(t),$ are asset prices, The expression dZ , which contains the randomness that is certainly a characteristic of asset prices is called a Wiener process or Brownian motion. λ_1 *and* λ_2 represents rate of returns of first and second investments respectively, $\lambda_1\lambda_2$ is multiplicative effects.

2.1.2 Method of Solution

The model (14) and (15) is made up of a system of stochastic delay differential equations and stochastic differential equation whose solutions are not trivial. We implement the methods of Ito's lemma in solving for $\mathrm{X}_1(t)$, *and* $X_3(t)$ To seize this problem we note that we can forecast the future worth of the asset with sureness.

From (14) Let $f(X_1, t) = \ln X_1$ so differentiating partially gives

$$\frac{\partial f}{\partial X_1} = \frac{1}{X_1}, \;\; \frac{\partial^2 f}{\partial X_1^2} = -\frac{-1}{X_1^2}, \;\; \frac{\partial f}{\partial t} = 0 \tag{17}$$

According to Ito's gives

$$df\left(X_1,t\right)=\sigma X_1\frac{\partial f}{\partial X_1}dZ(t)+\left((\theta_1\theta_2)^2X_1(t)\frac{\partial f}{\partial X_1}+\frac{1}{2}\sigma^2X_1^2\frac{\partial^2 f}{\partial X_1^2}+\frac{\partial f}{\partial t}\right)dt \qquad (18)$$

Substituting (17) and (18) into (14) gives

$$=\sigma X_1\frac{1}{X_1}dZ(t)+\left(\left(t-\tau\right)(\theta_1\theta_2)^2X_1(t)\frac{1}{X_1}+\frac{1}{2}\sigma^2X_1^2(-\frac{1}{X_1^2})+0\right)dt$$

$$=\sigma dZ(t)+\left(\left(t-\tau\right)(\theta_1\theta_2)^2-\frac{1}{2}\sigma^2\right)dt$$

Integrating both sides , talking upper and lower limits gives

$$\int_0^t d\ln X_1=\int_0^t df\left(X_u,u\right)=\left(t-\tau\right)\int\left((\theta_1\theta_2)^2-\frac{1}{2}\sigma^2\right)du+\int_0^t\sigma dZ(t)$$
$$\ln X_1-\ln X_0=\left(t-\tau\right)\left((\theta_1\theta_2)^2u-\frac{1}{2}\sigma^2u\right)|_0^t+\left(\sigma Zu\right)|_0^t \qquad (19)$$
$$\ln\left(\frac{X_1}{X_0}\right)=\left(t-\tau\right)\left((\theta_1\theta_2)^2-\frac{1}{2}\sigma^2\right)t+\sigma Z(t)$$

Taking the ln of the both sides

$$X_1(t)=X_0\exp\left(t-\tau\right)\left((\theta_1\theta_2)^2-\frac{1}{2}\sigma^2\right)t+\sigma Z(t) \qquad (20)$$

From (15) Let $f\left(X_3,t\right)=\ln X_3$ so differentiating partially gives

$$\frac{\partial f}{\partial X_3}=\frac{1}{X_3},\ \frac{\partial^2 f}{\partial X_3^2}=-\frac{-1}{X_3^2},\ \frac{\partial f}{\partial t}=0 \qquad (21)$$

According to Ito's gives

$$df\left(X_3,t\right)=\sigma X_3\frac{\partial f}{\partial X_3}dZ(t)+\left((\theta_1\theta_2)^2X_3(t)\frac{\partial f}{\partial X_3}+\frac{1}{2}\sigma^2X_2^2\frac{\partial^2 f}{\partial X_3^2}+\frac{\partial f}{\partial t}\right)dt \qquad (22)$$

Substituting (22) and (21) into (15) gives

$$= \sigma X_3 \frac{1}{X_3} dZ(t) + \left((\theta_1\theta_2)^2 X_3(t) \frac{1}{X_2} + \frac{1}{2}\sigma^2 X_3^2 (-\frac{1}{X_3^2}) + 0 \right) dt$$

$$= \sigma dZ(t) + \left((\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2 \right) dt$$

Integrating both sides , talking upper and lower limits gives

$$\int_0^t d\ln X_3 = \int_0^t df\left(X_3, u\right) = \left(t-\tau\right)\int \left((\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2 \right) du + \int_0^t \sigma dZ(t)$$

$$\ln X_3 - \ln X_0 = \left((\theta_1\theta_2)^2 u - \frac{1}{2}\sigma^2 u \right) |_0^t + \left(\sigma Z u\right) |_0^t \qquad (23)$$

$$\ln\left(\frac{X_3}{X_0}\right) = \left((\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2 \right) t + \sigma Z(t)$$

Taking the ln of the both sides gives the following close form solution of SDE.

$$X_3(t) = X_0 \exp\left((\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2 \right) t + \sigma Z(t) \qquad (24)$$

Table 1: The impact of time delay in the assessment of multiplicative Asset values through the solution below: $X_3\left(t\right) = X_0 \exp\left\{ \left(t-\tau\right)\left(\left(\theta_1\theta_2\right)^2 - \frac{1}{2}\sigma^2 \right) t + \sigma dz\left(t\right) \right\}$

$t = 2, \;\; dz = 1$

τ	S_0	σ	$(\theta_1\theta_2)$	$X_3(t)$	$(\theta_1\theta_2)$	$X_3(t)$
0.25	5.00	0.2	1.0000	188.5641	2.0000	6847799.464
	6.00	0.2	1.0000	226.2769	2.0000	8217359.356
	5.70	0.2	1.0000	214.9631	2.0000	7806491.388
0.35	5.20	0.2	1.0000	161.2021	2.0000	3212816.824
	4.60	0.2	1.0000	142.6018	2.0000	2842107.19
	4.00	0.2	1.0000	124.0016	2.0000	2471397.557
0.45	3.70	0.2	1.0000	94.2860	2.0000	1031302.186
	3.90	0.2	1.0000	99.3826	2.0000	1087048.25
	3.50	0.2	1.0000	89.1895	2.0000	975556.122
0.55	3.31	0.2	1.0000	69.3349	2.0000	416211.2361
	3.75	0.2	1.0000	78.5516	2.0000	471538.4094
	4.00	0.2	1.0000	83.7884	2.0000	502974.3034

It can be observed the various levels of asset value increase as effect of delay parameter in the model. This is quite consistent because as you delay in selling- off your landed properties, Estates, companies etc over time; will ultimately produce additional cash associated to when it is sold for today. Investors buy these assets in speculation and allow it for period of times to grow in value before they can be made use of in time varying investments. More so, a little increase in return rates of 1.0000 and 2.0000 respectively increases the value of assets, see Table 1.

Table 2: The assessment of multiplicative Asset values through the solution below:

$$X_1(t) = X_0 \exp\left\{\left((\theta_1\theta_2)^2 - \frac{1}{2}\sigma^2\right)t + \sigma dz(t)\right\} \ \mathrm{t} = 2, \ \ dz = 1$$

S_0	σ	$(\theta_1\theta_2)$	$X_1(t)$	$(\theta_1\theta_2)$	$X_1(t)$
5.00	0.2	1.0000	43.3557	2.0000	17490.9330
6.00	0.2	1.0000	52.0268	2.0000	20989.1196
5.70	0.2	1.0000	49.4255	2.0000	19939.6636
5.20	0.2	1.0000	45.0899	2.0000	18190.5703
4.60	0.2	1.0000	39.8872	2.0000	16091.6584
4.00	0.2	1.0000	34.6846	2.0000	13992.7464
3.70	0.2	1.0000	32.0832	2.0000	12943.2904
3.90	0.2	1.0000	33.8174	2.0000	13642.9278
3.50	0.2	1.0000	30.3490	2.0000	12243.6531
3.31	0.2	1.0000	28.7015	2.0000	11578.9977
3.75	0.2	1.0000	32.5168	2.0000	13118.1998
4.00	0.2	1.0000	34.8846	2.0000	13992.7464

Clearly in Table 2, the amount of initial stock prices when return rates are fixed for 1.0000 and 2.0000 determines the high level of asset values an investor will gain throughout the period. This financial benefit informs investors or traders on the best ways to take effective decisions.

H_0 : The asset value functions $X_1(t)$ and $X_3(t)$ comes from *a* common distribution
H_1 : They are not from *a* common distribution

Table 3:Goodness of fit test for assessment of two asset values with and without delay when its return rates is: 1.0000

$(\theta_1\theta_2)$	$X_3(t)$	$X_1(t)$	Mean	STD	KSStat	P-value	Decision
1.0000	43.3557	188.5641	115.9599	102.6778	1.0000	0.2890	Accept
1.0000	52.0268	226.2769	139.1518	123.2134	1.0000	0.2890	Accept
1.0000	49.4255	214.9631	132.1943	1117.0528	1.0000	0.2890	Accept
1.0000	45.0899	161.2021	103.1460	82.1037	1.0000	0.2890	Accept
1.0000	39.8872	142.6018	91.2445	72.6302	1.0000	0.2890	Accept
1.0000	34.6846	124.0016	79.3431	63.1567	1.0000	0.2890	Accept
1.0000	32.0832	94.2860	63.1846	43.9840	1.0000	0.2890	Accept
1.0000	33.8174	99.3826	66.6000	46.3616	1.0000	0.2890	Accept
1.0000	30.3490	89.1895	59.7692	41.6065	1.0000	0.2890	Accept
1.0000	28.7015	69.3349	49.0182	28.7322	1.0000	0.2890	Accept
1.0000	32.5168	78.5516	55.5342	32.5515	1.0000	0.2890	Accept
1.0000	34.8846	83.7884	59.3365	34.5802	1.0000	0.2890	Accept

In Tables 3 and 4 the alternative hypothesis of KS was obviously accepted hence the p-values are non -significant. The results shows that at $\alpha = 0.01$ the two asset values does not come from the same distribution which has financial inferences in respect to asset returns; with this result traders or investors will be well informed on how to take some vital decisions based on the levels of their investments. However, we can say that there is significant difference between the two asset values $\left(X_1(t)\ ,X_3\ (t)\right)$ in time varying investments.

On the other hand, the mean rates seen in columns 5 of Tables 3 and 4 respectively describes the mean levels of asset values when multiplicative rate of return is 1.0000 and 2.0000. The standard deviation indicates each level of changes in respect to asset values.

Table 4 : Goodness of fit test for assessment of two asset values with and without delay when its return rates is: 2.0000

$(\theta_1\theta_2)$	$X_1(t)$	$X_1'(t)$	Mean	STD	KSStat	P-value	Decision
2.0000	17490.9330	6847799.464	3.1326e+06	4.8298e+06	1.0000	0.2890	Accept
2.0000	20989.1196	8217359.356	4.1192e+06	5.7957e+06	1.0000	0.2890	Accept
2.0000	19939.6636	7806491.388	3.9132e+06	5.5059e+06	1.0000	0.2890	Accept
2.0000	18190.5703	3212816.824	1.6155e+06	2,2589e+06	1.0000	0.2890	Accept
2.0000	16091.6584	2842107.19	1.4291e+06	1.9983e+06	1.0000	0.2890	Accept
2.0000	13992.7464	2471397.557	1.2427e+06	1.7376e+06	1.0000	0.2890	Accept
2.0000	12943.2904	1031302.186	5.2212e+05	7.2009e+05	1.0000	0.2890	Accept
2.0000	13642.9278	1087048.25	5.5035e+05	7.5901e+05	1.0000	0.2890	Accept
2.0000	12243.6531	975556.122	4.9390e+05	6.8116e+05	1.0000	0.2890	Accept
2.0000	11578.9977	416211.2361	2.1390e+05	2.8612e+05	1.0000	0.2890	Accept
2.0000	13118.1998	471538.4094	2.4233e+05	3.2415e+05	1.0000	0.2890	Accept
2.0000	13992.7464	502974.3034	2.5848e+05	3.4576e+05	1.0000	0.2890	Accept

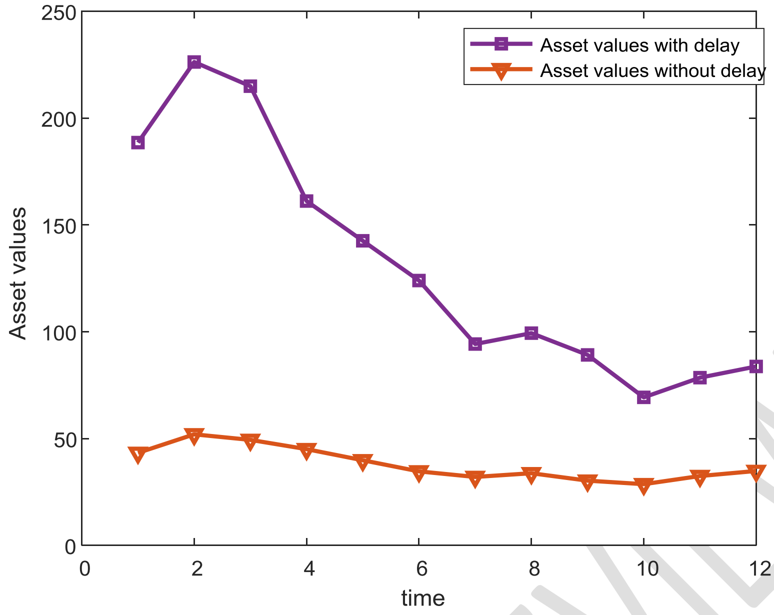

Figure 1: An assessment of asset value function when return rates is 1.0000

Figure 1 clearly describes the financial assessment of asset values with delay and without delay respectively when return rates is 1.0000. The two plots are independently separated and also not correlated. On the delay aspect of asset value implies that when there is a little delay in terms of asset value over time; its return rates increases which is profit maximizing and hence indexed in millions of naira during the period of trading. While the asset value without delay does not yield enough returns as can be seen in the plot above.

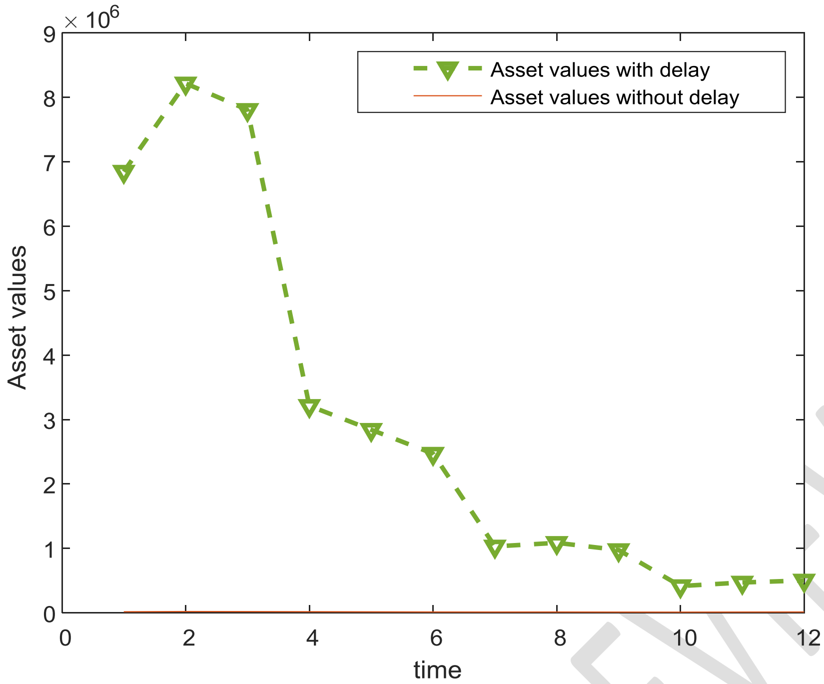

Figure 2: An assessment of asset value function when return rates is 2.0000

In Figure 2 , it can be seen that the two asset value function are quite not related in terms of flow in capital investments. The delay plot shows upward trend despite the of its movements. It implies an increase in respect to asset value and its return rates which is of good benefit to investors. On the other hand, plot without delay moves along origin within the horizontal axis showing low level returns within the season. With the plot above, it is enough for investors to be properly guided when taken decisions on level of this investment .

Conclusion

In finance generally the attainment of any investments lies on the value of assets which motivates the entire financial authority of every trader or investor and SDDE and SDE are well known mathematical tools used extensively for estimation of asset values over time. This paper considered stochastic systems where asset values were examined and compared between the two proposed models which shows: increase in delay parameter increases the value of asset, the solution of SDDE has more influence over SDE in terms of asset valuations, the high initial stock price has significant effects on the value of asset over time and the two asset values under-study do not come from a common distribution.

We shall be considering deterministic and stochastic systems in valuation of assets in the next study.

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