

The Minimum Superior Eccentric Dominating Energy of Graphs

ABSTRACT

For a graph $G = (V, E)$ of order k , the minimum superior eccentric dominating energy $\mathbb{SE}_{ed}(G)$ is the sum of the eigen values obtained from the minimum superior eccentric dominating $k \times k$ matrix $A_{sed}(G) = (se_{ij})$. In this paper $\mathbb{SE}_{ed}(G)$ of standard graphs are computed. Properties, upper and lower bounds for $\mathbb{SE}_{ed}(G)$ are established.

Keywords: Superior eccentricity, superior eccentric vertex, minimum superior eccentric dominating set, superior eccentric dominating eigen values, minimum superior eccentric dominating energy.

1. INTRODUCTION

In 2007, Kathiresan and Marimuthu[7] introduced superior distance in graphs. Let $D_{uv} = N[u] \cup N[v]$. A D_{uv} -walk is defined as a $u - v$ walk in G that contains every vertex of D_{uv} . The superior distance $d_D(u, v)$ from u to v is the length of a shortest D_{uv} walk. For each vertex v of a simple connected graph G , we define the superior eccentricity of v as $e_D(v) = \max\{d_D(u, v) : u \in V(G)\}$. We find the superior neighbour using the formula $d_D = \min\{d_D(u, v) : v \in V(G) - \{u\}\}$. A vertex $v (\neq u)$ is called a superior neighbour of u if $d_D(u, v) = d_D(u)$. In 2008 Kathiresan and Marimuthu[6] introduced superior domination. A vertex u is said to superior dominate a vertex v if v is a superior neighbour of u . A set S of vertices of G is called a superior dominating set of G if every vertex $V(G) - S$ is superior dominated by some vertex in S . A superior dominating set G of minimum cardinality is a minimum superior dominating set and its cardinality is called superior domination number of G and denoted by $\gamma_{sd}(G)$. A vertex v of a graph G is said to be a superior eccentric vertex of a vertex u if $d_D(u, v) = e_D(u)$. A vertex u is superior eccentric vertex of G if it is a superior eccentric vertex of some vertex v . M Bhanumathi and R Meenal Abirami[2] introduced superior eccentric domination in graphs in 2017. A superior dominating set S of vertices of G is called a superior eccentric dominating set if every vertex of $V(G) - S$ has some superior eccentric vertex in S . A superior eccentric dominating set of G of minimum cardinality is a minimum superior eccentric dominating set and its cardinality is called the superior eccentric domination number and is denoted by $\gamma_{sed}(G)$.

In 1978 I. Gutman[4, 3] introduced energy of a graph. Inspired by Gutman many authors have explored different types of energy in graph theory. M. R. Rajesh Kanna et al[5] found the minimum dominating energy of a graph. For a graph $G = (V, E)$, let $A = (a_{ij})$ be the minimum dominating matrix defined by

$$(a_{ij}) = \begin{cases} 1, & \text{if } v_i v_j \in E, \\ 1, & \text{if } i = j \text{ and } v_i \in D, \\ 0, & \text{otherwise} \end{cases}$$

and let $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$ are the eigen values of A , then minimum dominating energy is $E_D(G) = \sum_{i=1}^n |\lambda_i|$. In this paper we find $\mathbb{S}E_{ed}(G)$ of standard graphs, state and prove the properties of $\mathbb{S}E_{ed}(G)$ and find the bounds of $\mathbb{S}E_{ed}(G)$.

2. THE MINIMUM SUPERIOR ECCENTRIC DOMINATING ENERGY- $\mathbb{S}E_{ed}(G)$

In this section, minimum superior eccentric dominating matrix and minimum superior eccentric dominating energy are defined. The minimum superior eccentric dominating energy of some standard graphs are obtained.

Definition 2.1: Let $G = (V, E)$ be a simple graph where $V(G) = \{v_1, v_2, \dots, v_k\}$ where $k \in \mathbb{N}$ is the set of vertices and E is the set of edges. The superior eccentric vertex set of a vertex u is given by $S_{eD}(u) = \{v \in V(G) \mid d(u, v) = e_D(u)\}$. Let D be a minimum superior eccentric dominating set of G then the minimum superior eccentric dominating matrix of G is a $k \times k$ defined by $A_{sed}(G) = (se_{ij})$, where

$$(se_{ij}) = \begin{cases} 1, & \text{if } v_i \in S_{eD}(v_j) \text{ or } v_j \in S_{eD}(v_i), \\ 1, & \text{if } i = j \text{ and } v_i \in D, \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.2: The characteristic polynomial of a minimum superior eccentric dominating matrix $A_{sed}(G)$ is defined by $\mathcal{F}_k(G, \mu) = \det(A_{sed}(G) - \mu I)$.

Definition 2.3: The minimum superior eccentric dominating eigen values of G are the eigen values of minimum superior eccentric dominating matrix $A_{sed}(G)$. Since $A_{sed}(G)$ is symmetric and real, the eigen values of $A_{sed}(G)$ are real. We label the eigen values in non-increasing order $\mu_1 \geq \mu_2 \geq \dots \geq \mu_k$.

Definition 2.4: The minimum superior eccentric dominating energy of G is defined by $\mathbb{S}E_{ed}(G) = \sum_{i=1}^k |\mu_i|$.

Remark 2.1: The trace of $A_{sed}(G)$ = Superior eccentric domination number.

Example 2.1:

Fig 1. Fish graph

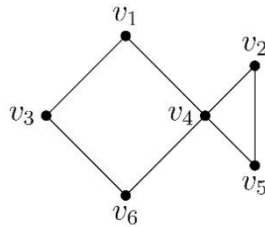


Table 1. Superior Eccentricity $e_D(v)$ and Superior Eccentric vertex set $S_{eD}(v)$

Vertex	Superior Eccentricity $e_D(v)$	Superior Eccentric vertex set $S_{eD}(v)$
v_1	6	v_4
v_2	6	v_3, v_4

v_3	7	v_4
v_4	7	v_3
v_5	6	v_3, v_4
v_6	6	v_4

The minimum superior eccentric dominating sets of fish graph are $D_1 = \{v_1, v_4\}$, $D_2 = \{v_3, v_4\}$ and $D_3 = \{v_4, v_6\}$.

1. $D_1 = \{v_1, v_4\}$,

$$A_{sed}(G) = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

The characteristic polynomial $\mathcal{F}_k(G, \mu) = \mu^6 - 2\mu^5 - 6\mu^4 + 4\mu^3 + 6\mu^2 - 2\mu$.

Minimum superior eccentric dominating eigen values are $\mu_1 \approx 3.3007$, $\mu_2 \approx 1.113$, $\mu_3 \approx 0.3024$, $\mu_4 \approx 0$, $\mu_5 \approx -1.149$, $\mu_6 \approx -1.567$.

Minimum superior eccentric dominating energy $\mathbb{SE}_{ed}(G) \approx 7.4321$.

2. $D_2 = \{v_3, v_4\}$,

$$A_{sed}(G) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

The characteristic polynomial $\mathcal{F}_k(G, \mu) = \mu^6 - 2\mu^5 - 6\mu^4 + 2\mu^3 + 4\mu^2$.

Minimum superior eccentric dominating eigen values are $\mu_1 \approx 3.4679$, $\mu_2 \approx 0.9128$, $\mu_3 \approx 0$, $\mu_4 \approx 0$, $\mu_5 \approx -0.7989$, $\mu_6 \approx -1.5818$.

Minimum superior eccentric dominating energy $\mathbb{SE}_{ed}(G) \approx 6.7614$.

3. $D_3 = \{v_4, v_6\}$,

$$A_{sed}(G) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

The characteristic polynomial $\mathcal{F}_k(G, \mu) = \mu^6 - 2\mu^5 - 6\mu^4 + 4\mu^3 + 6\mu^2 - 2\mu$.

Minimum superior eccentric dominating eigen values are $\mu_1 \approx 3.3007$, $\mu_2 \approx 1.113$, $\mu_3 \approx 0.3024$, $\mu_4 \approx 0$, $\mu_5 \approx -1.149$, $\mu_6 \approx -1.567$.

Minimum superior eccentric dominating energy $\mathbb{SE}_{ed}(G) \approx 7.4321$.

$\mathbb{SE}_{ed}(G)$ of D_1 and D_3 is 7.4321, but $\mathbb{SE}_{ed}(G)$ of D_2 is 6.7614. Therefore $\mathbb{SE}_{ed}(G)$ varies based on the superior eccentric dominating set.

Remark 2.2: The minimum superior eccentric dominating energy depends on the eccentric dominating set.

Theorem 2.1: For a star graph S_k where $k > 2$ the minimum superior eccentric dominating energy of a star is $\mathbb{SE}_{ed}(S_k) = \left| \frac{1+\sqrt{4k-3}}{2} \right| + \left| \frac{1-\sqrt{4k-3}}{2} \right|$.

Proof: Let S_k be a star graph with the vertex set $V = \{v_1, v_2, \dots, v_k\}$. The minimum superior eccentric dominating set is $D = \{v_k\}$, where v_k is the central vertex of star graph then

$$A_{sed}(S_k) = \begin{pmatrix} 0 & 0 & 0 & \dots & 1 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 1 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \dots & 1 & \dots & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 1 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 1 & \dots & 0 & 0 & 0 \end{pmatrix}$$

Characteristic polynomial is $\mathcal{F}_k(S_k, \mu) = \det(A_{sed}(S_k) - \mu I)$.

$$\begin{vmatrix} -\mu & 0 & 0 & \dots & 1 & \dots & 0 & 0 & 0 \\ 0 & -\mu & 0 & \dots & 1 & \dots & 0 & 0 & 0 \\ 0 & 0 & -\mu & \dots & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \dots & 1 - \mu & \dots & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & \dots & -\mu & 0 & 0 \\ 0 & 0 & 0 & \dots & 1 & \dots & 0 & -\mu & 0 \\ 0 & 0 & 0 & \dots & 1 & \dots & 0 & 0 & -\mu \end{vmatrix}$$

The characteristic equation is $\mathcal{F}_k(S_k, \mu) = (-1)^k \mu^k - (-1)^k \mu^{k-1} - (-1)^k (k-1) \mu^{k-2}$.

The minimum superior eccentric dominating eigen values are

$$\begin{aligned} \mu &= 0, \\ \mu &= \frac{1+\sqrt{4k-3}}{2} \text{ and} \\ \mu &= \frac{1-\sqrt{4k-3}}{2}. \end{aligned}$$

The minimum superior eccentric dominating energy of the star graph S_k is given by

$$\begin{aligned} \mathbb{S}E_{ed}(S_k) &= |(0)|(k-2) + \left| \frac{1+\sqrt{4k-3}}{2} \right| + \left| \frac{1-\sqrt{4k-3}}{2} \right|. \\ \mathbb{S}E_{ed}(S_k) &= \left| \frac{1+\sqrt{4k-3}}{2} \right| + \left| \frac{1-\sqrt{4k-3}}{2} \right|. \end{aligned}$$

Theorem 2.2: For a complete graph K_k where $k \geq 2$ the minimum superior eccentric dominating

$$\text{energy of a complete graph } \mathbb{S}E_{ed}(K_k) = |-1|(k-2) + \left| \frac{(k-1)+\sqrt{(k-1)^2+4}}{2} \right| + \left| \frac{(k-1)-\sqrt{(k-1)^2+4}}{2} \right|.$$

Proof: Let K_k be a complete graph with the vertex set $V = \{v_1, v_2, \dots, v_k\}$. The minimum superior eccentric dominating set is $D = \{v_1\}$ then

$$A_{sed}(K_k) = \begin{pmatrix} 1 & 1 & 1 & \dots & 1 & 1 & 1 \\ 1 & 0 & 1 & \dots & 1 & 1 & 1 \\ 1 & 1 & 0 & \dots & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \dots & 0 & 1 & 1 \\ 1 & 1 & 1 & \dots & 1 & 0 & 1 \\ 1 & 1 & 1 & \dots & 1 & 1 & 0 \end{pmatrix}_{k \times k}$$

The characteristic equation is $\mathcal{F}_k(K_k, \mu) = \det(A_{sed}(K_k) - \mu I)$.

$$\begin{vmatrix} 1 - \mu & 1 & 1 & \dots & 1 & 1 & 1 \\ 1 & -\mu & 1 & \dots & 1 & 1 & 1 \\ 1 & 1 & -\mu & \dots & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \dots & -\mu & 1 & 1 \\ 1 & 1 & 1 & \dots & 1 & -\mu & 1 \\ 1 & 1 & 1 & \dots & 1 & 1 & -\mu \end{vmatrix}$$

The characteristic equation is $\mathcal{F}_k(K_k, \mu) = (\mu + 1)^{k-2}(\mu^2 - (k-1)\mu - 1)$.

The minimum superior eccentric dominating eigen values are

$$\begin{aligned} \mu &= -1 \text{ (} k-2 \text{ times)}, \\ \mu &= \frac{(k-1) + \sqrt{(k-1)^2 + 4}}{2} \text{ and} \\ \mu &= \frac{(k-1) - \sqrt{(k-1)^2 + 4}}{2}. \end{aligned}$$

The minimum superior eccentric dominating energy of the complete graph K_k is given by

$$SE_{ed}(K_k) = |(-1)|(k-2) + \left| \frac{(k-1) + \sqrt{(k-1)^2 + 4}}{2} \right| + \left| \frac{(k-1) - \sqrt{(k-1)^2 + 4}}{2} \right|.$$

3. Properties of Minimum Superior Eccentric Dominating Eigen Values

In this section we discuss the properties of eigen values of $A_{sed}(G)$ for complete and star graphs. Bounds for minimum superior eccentric dominating energy of some standard graphs are obtained.

Theorem 3.1: Let D be a minimum superior eccentric dominating set and $\mu_1, \mu_2, \dots, \mu_k$ are the eigen values of minimum superior eccentric dominating matrix $A_{sed}(G)$, if G is

1. For any graph then $\sum_{i=1}^k \mu_i = |D|$,
2. For a complete graph K_k , $\sum_{i=1}^k \mu_i^2 = |D| + \sum_{i=1}^k |Se_D(v_i)| + (k-1)$,
3. For a star graph S_k then $\sum_{i=1}^k \mu_i^2 = |D| + \sum_{i=1}^k |Se_D(v_i)|$.

Proof:

1. We know that the sum of eigen values of $A_{sed}(G)$ is the trace of $A_{sed}(G)$.

$$\sum_{i=1}^k \mu_i = \sum_{i=1}^k se_{ii} = |D|.$$

2. In a complete graph, sum of square of eigen values of $A_{sed}(G)$ is trace of $[A_{sed}(G)]^2$

$$\begin{aligned} \text{Now } \sum_{i=1}^k \mu_i^2 &= \sum_{i=1}^k \sum_{j=1}^k se_{ij} se_{ij} \\ \sum_{i=1}^k \mu_i^2 &= \sum_{i=1}^k (se_{ii})^2 + \sum_{i \neq j} se_{ij} se_{ij} \\ \sum_{i=1}^k \mu_i^2 &= \sum_{i=1}^k (se_{ii})^2 + 2 \sum_{i < j} (se_{ij})^2 \end{aligned}$$

$$\sum_{i=1}^k \mu_i^2 = |D| + \sum_{i=1}^k |Se_D(v_i)| + (k-1) \text{ [since, } 2 \sum_{i < j} (se_{ij})^2 = \sum_{i=1}^k |Se_D(v_i)| + (k-1)]$$

3. In a star graph, sum of square of eigen values of $A_{sed}(G)$ is trace of $[A_{sed}(G)]^2$

$$\begin{aligned} \text{Now } \sum_{i=1}^k \mu_i^2 &= \sum_{i=1}^k \sum_{j=1}^k se_{ij} se_{ij} \\ \sum_{i=1}^k \mu_i^2 &= \sum_{i=1}^k (se_{ii})^2 + \sum_{i \neq j} se_{ij} se_{ij} \\ \sum_{i=1}^k \mu_i^2 &= \sum_{i=1}^k (se_{ii})^2 + 2 \sum_{i < j} (se_{ij})^2 \end{aligned}$$

$$\sum_{i=1}^k \mu_i^2 = |D| + \sum_{i=1}^k |Se_D(v_i)| \text{ [since, } 2 \sum_{i < j} (se_{ij})^2 = \sum_{i=1}^k |Se_D(v_i)|]$$

Theorem 3.2: For complete graph K_k , if D be the minimum superior eccentric dominating set and $W = |\det A_{sed}(G)|$ then $\sqrt{|D| + \sum_{i=1}^k |Se_D(v_i)| + (k-1) + k(k-1)W^{2/k}} \leq \mathbb{SE}_{ed}(G) \leq \sqrt{k(\sum_{i=1}^k |Se_D(v_i)| + (k-1) + |D|)}$.

Proof: By Cauchy schwarz inequality $(\sum_{i=1}^k a_i b_i)^2 \leq (\sum_{i=1}^k a_i^2)(\sum_{i=1}^k b_i^2)$. If $a_i = 1$ and $b_i = \mu_i$ then

$$\begin{aligned} \left(\sum_{i=1}^k |\mu_i|\right)^2 &\leq \left(\sum_{i=1}^k 1\right) \left(\sum_{i=1}^k \mu_i^2\right) \\ (\mathbb{SE}_{ed}(G))^2 &\leq k \left(|D| + \sum_{i=1}^k |Se_D(v_i)| + (k-1)\right) \\ \Rightarrow \mathbb{SE}_{ed}(G) &\leq \sqrt{k \left(|D| + \sum_{i=1}^k |Se_D(v_i)| + (k-1)\right)} \end{aligned}$$

Since the arithmetic mean is not smaller than geometric mean we have

$$\begin{aligned} \frac{1}{k(k-1)} \sum_{i \neq j} |\mu_i| |\mu_j| &\geq \left[\prod_{i \neq j} |\mu_i| |\mu_j| \right]^{\frac{1}{k(k-1)}} \\ \frac{1}{k(k-1)} \sum_{i \neq j} |\mu_i| |\mu_j| &= \left[\prod_{i=1}^k |\mu_i|^{2(k-1)} \right]^{\frac{1}{k(k-1)}} \\ \frac{1}{k(k-1)} \sum_{i \neq j} |\mu_i| |\mu_j| &= \left[\prod_{i=1}^k |\mu_i| \right]^{\frac{2}{k}} \\ \frac{1}{k(k-1)} \sum_{i \neq j} |\mu_i| |\mu_j| &= \left[\prod_{i=1}^k \mu_i \right]^{\frac{2}{k}} \\ \frac{1}{k(k-1)} \sum_{i \neq j} |\mu_i| |\mu_j| &= |\det A_{sed}(G)|^{\frac{2}{k}} = W^{\frac{2}{k}} \\ \sum_{i \neq j} |\mu_i| |\mu_j| &\geq k(k-1)W^{\frac{2}{k}} \end{aligned}$$

Now consider

$$\begin{aligned} (\mathbb{SE}_{ed}(G))^2 &= \left(\sum_{i=1}^k |\mu_i|\right)^2 \\ (\mathbb{SE}_{ed}(G))^2 &= \left(\sum_{i=1}^k |\mu_i|\right)^2 + \sum_{i \neq j} |\mu_i| |\mu_j| \\ (\mathbb{SE}_{ed}(G))^2 &= \left(|D| + \sum_{i=1}^k |Se_D(v_i)| + (k-1)\right) + k(k-1)W^{\frac{2}{k}} \\ \mathbb{SE}_{ed}(G) &\geq \sqrt{\left(|D| + \sum_{i=1}^k |Se_D(v_i)| + (k-1)\right) + k(k-1)W^{\frac{2}{k}}} \end{aligned}$$

Theorem 3.3: For a star graph S_k , if D be the minimum superior eccentric dominating set and $W = |\det \mathbb{A}_{sed}(S_k)|$ then

$$\sqrt{|D| + \sum_{i=1}^k |Se_D(v_i)| + k(k-1)W^{2/k}} \leq \mathbb{S}E_{ed}(S_k) \leq \sqrt{k(\sum_{i=1}^k |Se_D(v_i)| + |D|)}.$$

Proof: The proof follows on the similar lines of theorem-3.2.

Theorem 3.4: If $\mu_1(G)$ is the largest minimum superior eccentric dominating eigen value of $\mathbb{A}_{sed}(G)$ then

1. For a complete graph K_k , $\mu_1(G) \geq \frac{|D| + \sum_{i=1}^k |Se_D(v_i)| + (k-1)}{k}$,
2. For a star graph S_k , $\mu_1(S_k) \geq \frac{|D| + \sum_{i=1}^k |Se_D(v_i)|}{k}$.

Proof:

1. In a complete graph K_k , let Y be a non-zero vector, then by ref.[1], we have

$$\mu_1(\mathbb{A}_{sed}(G)) = \max_{Y \neq 0} \frac{Y^T \mathbb{A}_{sed}(G) Y}{Y^T Y}.$$

$$\mu_1(\mathbb{A}_{sed}(G)) \geq \frac{U^T \mathbb{A}_{sed}(G) U}{U^T U} = \frac{|D| + \sum_{i=1}^k |Se_D(v_i)| + (k-1)}{k} \text{ where } U \text{ is the unit matrix.}$$

2. In a star graph S_k , let Y be a non-zero vector, then by ref.[1], we have

$$\mu_1(\mathbb{A}_{sed}(G)) = \max_{Y \neq 0} \frac{Y^T \mathbb{A}_{sed}(G) Y}{Y^T Y}.$$

$$\mu_1(\mathbb{A}_{sed}(S_k)) \geq \frac{U^T \mathbb{A}_{sed}(S_k) U}{U^T U} = \frac{|D| + \sum_{i=1}^k |Se_D(v_i)|}{k} \text{ where } U \text{ is the unit matrix.}$$

4. CONCLUSION

In this paper minimum superior eccentric dominating energy of graph is introduced. The superior eccentric dominating energy of some standard graphs are calculated. Results related to the upper and lower bound of the energy of standard graphs is stated and proved.

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