Efficient Classes of Estimators of Population Mean under various Allocation Schemes in Stratified Random Sampling

Abstract
The present paper is an extension of the work published in Kumar and Vishwakarma (Proceedings of the National Academy of Sciences, India, Section A: Physical Sciences, 90(5): 933-939, 2020). In this paper, various sample allocation schemes are utilized to derive the mathematical expressions for mean square errors (MSEs) of several well-known estimators of population mean in stratified random sampling. Moreover, the effects of various allocation schemes on the estimation of mean, are demonstrated theoretically as well as empirically. The findings of the study reveal that the Neyman allocation provides a smaller variance (or MSE, as the case may be) as compared to that of Equal and Proportional allocation schemes for the concerned estimators. Moreover, the proposed classes of estimators are dominant over the pre-existing estimators under the various allocation schemes considered in the study.

Keywords: Study variable, Auxiliary variable, Population mean, Mean square error, Percent relative efficiency.

2022 Mathematics Subject Classification: 62D05

1. Introduction

It is well known that the efficiency of an estimator, for the population parameter, is directly proportional to the sample size, i.e., the larger the sample size, the more efficient is the estimator. This result is widely used at the estimation stage when the sampling units are evenly distributed over the target population, and the samples are selected by using simple random sampling (SRS) design. However, in case of a heterogeneous target population, the efficiency can be greatly increased by dividing the target population into homogeneous sub-groups, known as strata, and then selecting samples from each stratum separately.

In stratified random sampling, the selection of samples from each stratum can be carried out by utilizing various allocation methods, for instance, equal allocation, proportional allocation, optimum allocation, and Neyman allocation.

There are many practical situations in which stratified random sampling is desired and preferred as compared to other sampling designs. It is widely used in studies dealing with the estimation of population parameters for sub-groups of a population, and to analyze the relationships between two or more sub-groups. For instance, persons of different ages tend to have different blood pressures, so in a blood pressure study it would be helpful to stratify the target population by age groups, and to estimate the blood pressures separately for each age group. A stratified sample may be more convenient to administer and may also result in a lower cost for the survey. For instance, sampling frames may be constructed differently in different strata (see Lohr [1]).

In most of the surveys, the target population consists of heterogeneous units, and in that situation the SRS design does not yield precise estimators for the population parameters (such as population mean, population variance, etc.) of the variable under study. Hence, it becomes indispensable to adopt stratified random sampling in that case.

In the past as well as in the recent times, several authors have given their noteworthy and innovative contributions towards the development of estimation strategies for estimating the population mean of the study variable under stratified random sampling. Some remarkable contributions in this direction have been made by Kadilar and Cingi [2], Singh and Vishwakarma [3, 4], Shabbir and Gupta [5, 6], Singh *et al.* [7], Tailor *et al.* [8], Vishwakarma and Singh [9], Vishwakarma and Kumar [10], Nidhi *et al.* [11], and Shahzad *et al.* [12]. Also, some recent significant contributions in stratified random sampling have been made by Cetin and Koyuncu [13], Kumar and Vishwakarma [14], Yadav and Tailor [15], Bhushan *et al.* [16], and Kumar *et al.* [17].

It is a well known fact that the sample allocation schemes have significant role in stratified random sampling. Considering the given fact, an attempt has been made in this paper to extend the work published in Kumar and Vishwakarma [14] for the cases of various sample allocation schemes and to describe, theoretically as well as empirically, the effects of various allocation schemes on the efficiency of well-known estimators of population mean ($\bar{Y}$) of the study variable Y in stratified random sampling. The works in the subsequent sections are organized as follows.

The Section 2 provides a detailed review of the estimation strategies in stratified random sampling by revealing the various well-known estimators for population mean ($\bar{Y}$) of the study variable Y. In Section 3, a methodology is developed for obtaining the mathematical expressions for mean square errors (MSEs) of the class of separate regression-cum-ratio estimators (T_s) as developed in Kumar and Vishwakarma [14] under various allocation schemes. Moreover, in Section 4, a methodology is developed for obtaining the mathematical expressions for MSEs of the class of combined regression-cum-ratio estimators (T_c) as developed in Kumar and Vishwakarma [14] under various allocation schemes. Furthermore, in Section 5, the variances and MSEs of several pre-existing estimators are elaborated for the concerned sample allocation schemes. Also, in Sections 6 and 7, the *necessary and sufficient conditions (NASCs)* for the dominance of T_s and T_c, respectively, over the well-known pre-existing estimators, are obtained for the concerned allocation schemes. In addition, an empirical analysis is carried out in Section 8 for demonstrating the relative efficiencies of the classes of estimators T_s and T_c over the well-known estimators, under various allocation schemes. Finally, the results and conclusion are given in Sections 9 and 10, respectively.

2. Some Pre-Existing Estimators of the Population Mean

In this section, we have revealed the various well-known estimators for population mean ($\bar{Y}$) of the study variable Y in stratified random sampling, along with their MSEs. To proceed further, we consider a finite population $U = \{U_1, U_2, ..., U_N\}$ consisting of N units, and the units being partitioned into L distinct strata with h^{th}-stratum containing N_h units, $(h = 1, 2, ..., L)$, such that $\sum_{h=1}^{L} N_h = N$. Let Y and X be the study and auxiliary variables, respectively, taking the values y_{hi} and x_{hi} on the i^{th} unit $(i = 1, 2, ..., N_h)$ of the h^{th}-stratum. Further, let n_h be the size of the sample drawn from the h^{th}-stratum by using *simple random sampling without replacement (SRSWOR)* scheme such that $\sum_{h=1}^{L} n_h = n$.

Moreover, the population means of the variables Y and X in the h^{th}-stratum are $\bar{Y}_h = \sum_{i=1}^{N_h} y_{hi}/N_h$ and $\bar{X}_h = \sum_{i=1}^{N_h} x_{hi}/N_h$. The corresponding sample means in the h^{th}-stratum are $\bar{y}_h = \sum_{i=1}^{n_h} y_{hi}/n_h$ and $\bar{x}_h = \sum_{i=1}^{n_h} x_{hi}/n_h$.

The sample means of the variables Y and X, in stratified random sampling, are given by

$$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h \text{ and } \bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h$$

where $W_h = N_h/N$ is the stratum weight. Also, $\bar{y}_{st}$ and $\bar{x}_{st}$ are the unbiased estimators of the population means $\bar{Y} = \sum_{h=1}^{L} W_h \bar{Y}_h$ and $\bar{X} = \sum_{h=1}^{L} W_h \bar{X}_h$, respectively.

The separate ratio estimator for population mean $\bar{Y}$ is defined by

$$\bar{y}_{RS} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h}{\bar{x}_h}\right) \tag{1}$$

Also, the separate regression estimator for $\bar{Y}$ is defined by

$$\bar{y}_{lrs} = \sum_{h=1}^{L} W_h [\bar{y}_h + b_h(\bar{X}_h - \bar{x}_h)] \tag{2}$$

Here, $b_h = s_{yxh}/s_{xh}^2$ denotes the sample regression coefficient of Y on X in the h^{th}-stratum, where $s_{xh}^2 = \frac{1}{n_h-1}\sum_{i=1}^{n_h}(x_{hi} - \bar{x}_h)^2$ and $s_{yxh} = \frac{1}{n_h-1}\sum_{i=1}^{n_h}(y_{hi} - \bar{y}_h)(x_{hi} - \bar{x}_h)$.

The combined ratio estimator (see Singh [18]) for population mean $\bar{Y}$ is defined by

$$\bar{y}_{RC} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right) \tag{3}$$

Also, the combined regression estimator for $\bar{Y}$ is defined by

$$\bar{y}_{lrc} = \bar{y}_{st} + b(\bar{X} - \bar{x}_{st}) \tag{4}$$

where $b = \left(\sum_{h=1}^{L} W_h^2 \lambda_h s_{yxh} \Big/ \sum_{h=1}^{L} W_h^2 \lambda_h s_{xh}^2\right)$, $\lambda_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right)$.

The variance of stratified sample mean $\bar{y}_{st}$ under *SRSWOR* scheme is given by

$$Var(\bar{y}_{st}) = \sum_{h=1}^{L} W_h^2 \lambda_h S_{yh}^2 = \sum_{h=1}^{L} W_h^2 \lambda_h \bar{Y}_h^2 C_{yh}^2 \tag{5}$$

where $C_{yh} = S_{yh}/\bar{Y}_h$.

To the first order of approximation, the *mean square errors (MSEs)* of $\bar{y}_{RS}$, $\bar{y}_{lrs}$, $\bar{y}_{RC}$, $\bar{y}_{lrc}$ and $\bar{y}_{KC}$ are given, respectively, by

$$MSE(\bar{y}_{RS}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 - 2R_h S_{yxh} + R_h^2 S_{xh}^2\right) \tag{6}$$

$$MSE(\bar{y}_{lrs}) = \sum_{h=1}^{L} W_h^2 \lambda_h S_{yh}^2 \left(1 - \rho_{yxh}^2\right) \tag{7}$$

$$MSE(\bar{y}_{RC}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 - 2R S_{yxh} + R^2 S_{xh}^2\right) \tag{8}$$

$$MSE(\bar{y}_{lrc}) = \sum_{h=1}^{L} W_h^2 \lambda_h S_{yh}^2 \left(1 - \rho_{yx}^2\right) \tag{9}$$

where

$$R_h = \frac{\bar{Y}_h}{\bar{X}_h}\ ,\ R = \frac{\bar{Y}}{\bar{X}}\ ,\ \rho_{yx} = \frac{\sum_{h=1}^{L} W_h^2 \lambda_h S_{yxh}}{\sqrt{\left(\sum_{h=1}^{L} W_h^2 \lambda_h S_{yh}^2\right)\left(\sum_{h=1}^{L} W_h^2 \lambda_h S_{xh}^2\right)}}\ ,\ \rho_{yxh} = \frac{S_{yxh}}{S_{yh} S_{xh}}$$

$$S_{yh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)^2\ ,\ S_{xh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (x_{hi} - \bar{X}_h)^2$$

$$S_{yxh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{hi} - \bar{Y}_h)(x_{hi} - \bar{X}_h)$$

3. Proposed Methodology for Separate Regression-cum-Ratio Estimators

In this section, we have developed the methodology for obtaining the mathematical expressions for MSEs of the class of separate regression-cum-ratio estimators (T_s) under various allocation schemes. To continue further, we consider the following class of separate regression-cum-ratio estimators for the population mean $\bar{Y}$ as developed in Kumar and Vishwakarma [14]:

$$T_s = \sum_{h=1}^{L} W_h [\bar{y}_h + b_h(\bar{X}_h - \bar{x}_h)] \left(\frac{\alpha_h \bar{X}_h + \gamma_h}{\alpha_h \bar{x}_h + \gamma_h}\right) \tag{10}$$

where α_h and γ_h are either real numbers or functions of some known parameters of auxiliary variable X, which are determined such that the *MSE* of T_s is minimum.

It is also worth noting that for $\alpha_h = 0$, the class T_s reduces to the separate regression estimator $\bar{y}_{lrs}$ in Eq. (2).

To the first order of approximation, the *MSE* of T_s is given by

$$MSE(T_s) = \sum_{h=1}^{L} W_h^2 \lambda_h \left[S_{yh}^2 + (\delta_h R_h + \beta_h)^2 S_{xh}^2 - 2(\delta_h R_h + \beta_h) S_{yxh}\right] \tag{11}$$

where $\beta_h = S_{yxh}/S_{xh}^2$ is the population regression coefficient of Y on X in the h^{th}-stratum, and $\delta_h = \left\{\alpha_h \bar{X}_h \Big/ (\alpha_h \bar{X}_h + \gamma_h)\right\}$.

The *MSE* of T_s at Eq. (11) is minimized for $\delta_h = 0$, which is possible only when $\alpha_h = 0$. Substituting $\delta_h = 0$ in Eq. (11) yields the minimum *MSE* of T_s as

$$MSE(T_s)_{min} = \sum_{h=1}^{L} W_h^2 \lambda_h S_{yh}^2 \left(1 - \rho_{yxh}^2\right) = MSE(\bar{y}_{lrs}) \tag{12}$$

Now, to apply various sample allocation schemes, we re-write the *MSE* of T_s in Eq. (11) as follows:

$$MSE(T_s) = \sum_{h=1}^{L} W_h^2 \lambda_h \psi_h = \sum_{h=1}^{L} W_h^2 \left(\frac{1}{n_h} - \frac{1}{N_h}\right) \psi_h \tag{13}$$

where

$$\psi_h = S_{yh}^2 + (\delta_h R_h + \beta_h)^2 S_{xh}^2 - 2(\delta_h R_h + \beta_h) S_{yxh} \quad \text{and} \quad W_h = N_h/N.$$

In the subsequent sub-sections, we will derive the mathematical expressions for *MSEs* of T_s under various allocation schemes.

3.1 *MSE* of T_s under Equal Allocation Scheme

In case of equal allocation scheme, samples of equal sizes are selected from each stratum, i.e., $n_h = n/L$, where n denotes the overall sample size, and L represents the total number of strata.

Hence, on substituting $n_h = n/L$ in Eq. (13), the *MSE* of T_s under equal allocation scheme is obtained as

$$MSE(T_s)_{Equal} = \frac{1}{nN^2} \sum_{h=1}^{L} N_h (LN_h - n) \psi_h \tag{14}$$

3.2 *MSE* of T_s under Proportional Allocation Scheme

In case of proportional allocation scheme, the sample size in the h^{th}-stratum (i.e., n_h) is proportional to the respective stratum size (i.e., N_h). Symbolically, we have

$n_h = (n/N)N_h = nW_h$, where W_h is the stratum weight of the h^{th}-stratum.

Hence, on substituting $n_h = nW_h$ in Eq. (13), the *MSE* of T_s under proportional allocation scheme is obtained as

$$MSE(T_s)_{Prop} = \frac{(N-n)}{Nn}\sum_{h=1}^{L} W_h\psi_h \tag{15}$$

3.3 *MSE* of T_s under Optimum Allocation Scheme

In case of optimum allocation scheme, the optimum sample sizes in each stratum is obtained by considering a cost function of the form:

$$c = c_0 + \sum_{h=1}^{L} c_h n_h \tag{16}$$

where c = total sampling cost in stratified random sampling, c_0 = overhead cost, c_h = sampling cost per unit in the h^{th}-stratum, and n_h = sample size in the h^{th}-stratum.

Now, we shall obtain the optimum values of n_h such that the *MSE* of T_s is minimized for a specified cost $c \leq c^*$.

To proceed further, consider the Lagrangian function of the form:

$$L = MSE(T_s) + \tau(c_0 + \sum_{h=1}^{L} c_h n_h - c^*) \tag{17}$$

where τ is the Lagrange's multiplier.

Now substituting Eq. (13) in Eq. (17), we have

$$L = \sum_{h=1}^{L} W_h^2 \left(\frac{1}{n_h} - \frac{1}{N_h}\right)\psi_h + \tau(c_0 + \sum_{h=1}^{L} c_h n_h - c^*) \tag{18}$$

Differentiating Eq. (18) with respect to n_h, equating the result to zero, and then using the condition $\sum_{h=1}^{L} n_h = n$, we obtain the optimum value of n_h as

$$n_h = \frac{nW_h\sqrt{\psi_h}/\sqrt{c_h}}{\sum_{h=1}^{L} W_h\sqrt{\psi_h}/\sqrt{c_h}} \tag{19}$$

Hence, substituting the value of n_h from Eq. (19) in Eq. (13), the *MSE* of T_s under optimum allocation scheme is obtained as

$$MSE(T_s)_{Opt} = \frac{1}{n}\left(\sum_{h=1}^{L} W_h\sqrt{\psi_h}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h\sqrt{\psi_h}/\sqrt{c_h}\right) - \sum_{h=1}^{L}\frac{W_h^2\psi_h}{N_h} \tag{20}$$

3.4 MSE of T_s under Neyman Allocation Scheme

The Neyman allocation scheme is a particular case of optimum allocation scheme, in which the sampling cost in each stratum is considered to be the same (i.e., $c_1 = c_2 = ... = c_L = c^{'}$).

Under Neyman allocation scheme, the sample size in the h^{th}-stratum is obtained from Eq. (19) on replacing c_h with $c^{'}$, and is given by

$$n_h = \frac{nW_h\sqrt{\psi_h}}{\sum_{h=1}^{L} W_h\sqrt{\psi_h}} \tag{21}$$

Hence, substituting the value of n_h from Eq. (21) in Eq. (13), the MSE of T_s under Neyman allocation scheme is obtained as

$$MSE(T_s)_{Neyman} = \frac{1}{n}\left(\sum_{h=1}^{L} W_h\sqrt{\psi_h}\right)^2 - \sum_{h=1}^{L}\frac{W_h^2\psi_h}{N_h} \tag{22}$$

4. Proposed Methodology for Combined Regression-cum-Ratio Estimators

In this section, we have developed the methodology for obtaining the mathematical expressions for MSEs of the class of combined regression-cum-ratio estimators (T_c) under various allocation schemes. To proceed further, we consider the following class of combined regression-cum-ratio estimators for the population mean $\bar{Y}$ as developed in Kumar and Vishwakarma [14]:

$$T_c = [\bar{y}_{st} + b(\bar{X} - \bar{x}_{st})]\left(\frac{\bar{X}_M}{\bar{x}_M}\right) \tag{23}$$

where $\bar{X}_M = \sum_{h=1}^{L} W_h(\alpha_h\bar{X}_h + \gamma_h)$ and $\bar{x}_M = \sum_{h=1}^{L} W_h(\alpha_h\bar{x}_h + \gamma_h)$. Also, the scalars α_h and γ_h are determined such that the MSE of T_c is minimum.

It is worth mentioning that for $\alpha_h = 0$, the class T_c reduces to the combined regression estimator $\bar{y}_{lrc}$ in Eq. (4).

To the first order of approximation, the MSE of T_c is given by

$$MSE(T_c) = \sum_{h=1}^{L} W_h^2\lambda_h\left[S_{yh}^2 + (R_M\alpha_h + \beta)^2 S_{xh}^2 - 2(R_M\alpha_h + \beta)S_{yxh}\right] \tag{24}$$

where $\beta = \left(\sum_{h=1}^{L} W_h^2\lambda_h S_{yxh}\Big/\sum_{h=1}^{L} W_h^2\lambda_h S_{xh}^2\right)$ is the population regression coefficient of Y on X, and $R_M = \bar{Y}/\bar{X}_M$.

The MSE of T_c at Eq. (24) is minimized for

$$\alpha_h = \frac{(\beta_h - \beta)}{R_M} = \alpha_{h(opt)} \text{ (say)} \tag{25}$$

Substitution of $\alpha_{h(opt)}$ in place of α_h in Eq. (24) yields the minimum MSE of T_c as

$$MSE(T_c)_{min} = \sum_{h=1}^{L} W_h^2\lambda_h S_{yh}^2\left(1 - \rho_{yxh}^2\right) = MSE(\bar{y}_{lrs}) \tag{26}$$

Now, to apply various sample allocation schemes, the *MSE* of T_c in Eq. (24) can be re-written as

$$MSE(T_c) = \sum_{h=1}^{L} W_h^2 \lambda_h \xi_h = \sum_{h=1}^{L} W_h^2 \left(\frac{1}{n_h} - \frac{1}{N_h}\right) \xi_h \tag{27}$$

where

$$\xi_h = S_{yh}^2 + (R_M\alpha_h + \beta)^2 S_{xh}^2 - 2(R_M\alpha_h + \beta)S_{yxh} \text{ and } W_h = N_h/N.$$

Now, proceeding in a manner similar to that in Section 3, the expressions for *MSEs* of T_c under various allocation schemes are obtained as follows:

$$MSE(T_c)_{Equal} = \frac{1}{nN^2} \sum_{h=1}^{L} N_h(LN_h - n)\xi_h \tag{28}$$

$$MSE(T_c)_{Prop} = \frac{(N-n)}{Nn} \sum_{h=1}^{L} W_h \xi_h \tag{29}$$

$$MSE(T_c)_{Opt} = \frac{1}{n}\left(\sum_{h=1}^{L} W_h\sqrt{\xi_h}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h\sqrt{\xi_h}/\sqrt{c_h}\right) - \sum_{h=1}^{L}\frac{W_h^2\xi_h}{N_h} \tag{30}$$

$$MSE(T_c)_{Neyman} = \frac{1}{n}\left(\sum_{h=1}^{L} W_h\sqrt{\xi_h}\right)^2 - \sum_{h=1}^{L}\frac{W_h^2\xi_h}{N_h} \tag{31}$$

5. MSEs of the Pre-Existing Estimators under various Allocation Schemes

In this section, the variances and *MSEs* of several well-known estimators considered in Section 2 are obtained by utilizing various sample allocation schemes as follows:

(i) Variances of $\bar{y}_{st}$ under various allocation schemes:

$$Var(\bar{y}_{st})_{Equal} = \frac{1}{nN^2} \sum_{h=1}^{L} N_h(LN_h - n)S_{yh}^2 \tag{32}$$

$$Var(\bar{y}_{st})_{Prop} = \frac{(N-n)}{Nn} \sum_{h=1}^{L} W_h S_{yh}^2 \tag{33}$$

$$Var(\bar{y}_{st})_{Opt} = \frac{1}{n}\left(\sum_{h=1}^{L} W_h S_{yh}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h S_{yh}/\sqrt{c_h}\right) - \sum_{h=1}^{L}\frac{W_h^2 S_{yh}^2}{N_h} \tag{34}$$

$$Var(\bar{y}_{st})_{Neyman} = \frac{1}{n}\left(\sum_{h=1}^{L} W_h S_{yh}\right)^2 - \sum_{h=1}^{L}\frac{W_h^2 S_{yh}^2}{N_h} \tag{35}$$

(ii) *MSEs* of $\bar{y}_{RS}$ under various allocation schemes:

$$MSE(\bar{y}_{RS})_{Equal} = \frac{1}{nN^2} \sum_{h=1}^{L} N_h(LN_h - n)\zeta_h \; ; \; \zeta_h = S_{yh}^2 - 2R_h S_{yxh} + R_h^2 S_{xh}^2 \tag{36}$$

$$MSE(\bar{y}_{RS})_{Prop} = \frac{(N-n)}{Nn}\sum_{h=1}^{L} W_h\zeta_h \tag{37}$$

$$MSE(\bar{y}_{RS})_{Opt} = \frac{1}{n}\left(\sum_{h=1}^{L} W_h\sqrt{\zeta_h}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h\sqrt{\zeta_h}/\sqrt{c_h}\right) - \sum_{h=1}^{L}\frac{W_h^2\zeta_h}{N_h} \tag{38}$$

$$MSE(\bar{y}_{RS})_{Neyman} = \frac{1}{n}\left(\sum_{h=1}^{L} W_h\sqrt{\zeta_h}\right)^2 - \sum_{h=1}^{L}\frac{W_h^2\zeta_h}{N_h} \tag{39}$$

(iii) *MSEs* of $\bar{y}_{lrs}$ under various allocation schemes:

$$MSE(\bar{y}_{lrs})_{Equal} = \frac{1}{nN^2}\sum_{h=1}^{L} N_h(LN_h-n)S_{yh}^2\left(1-\rho_{yxh}^2\right) \tag{40}$$

$$MSE(\bar{y}_{lrs})_{Prop} = \frac{(N-n)}{Nn}\sum_{h=1}^{L} W_hS_{yh}^2\left(1-\rho_{yxh}^2\right) \tag{41}$$

$$MSE(\bar{y}_{lrs})_{Opt} = \frac{1}{n}\left[\sum_{h=1}^{L} W_hS_{yh}\sqrt{\left(1-\rho_{yxh}^2\right)}\sqrt{c_h}\right]\left[\sum_{h=1}^{L} W_hS_{yh}\sqrt{\left(1-\rho_{yxh}^2\right)}/\sqrt{c_h}\right] - \sum_{h=1}^{L}\frac{W_h^2S_{yh}^2\left(1-\rho_{yxh}^2\right)}{N_h} \tag{42}$$

$$MSE(\bar{y}_{lrs})_{Neyman} = \frac{1}{n}\left[\sum_{h=1}^{L} W_hS_{yh}\sqrt{\left(1-\rho_{yxh}^2\right)}\right]^2 - \sum_{h=1}^{L}\frac{W_h^2S_{yh}^2\left(1-\rho_{yxh}^2\right)}{N_h} \tag{43}$$

(iv) *MSEs* of $\bar{y}_{RC}$ under various allocation schemes:

$$MSE(\bar{y}_{RC})_{Equal} = \frac{1}{nN^2}\sum_{h=1}^{L} N_h(LN_h-n)\Omega_h \;;\; \Omega_h = S_{yh}^2 - 2RS_{yxh} + R^2S_{xh}^2 \tag{44}$$

$$MSE(\bar{y}_{RC})_{Prop} = \frac{(N-n)}{Nn}\sum_{h=1}^{L} W_h\Omega_h \tag{45}$$

$$MSE(\bar{y}_{RC})_{Opt} = \frac{1}{n}\left(\sum_{h=1}^{L} W_h\sqrt{\Omega_h}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h\sqrt{\Omega_h}/\sqrt{c_h}\right) - \sum_{h=1}^{L}\frac{W_h^2\Omega_h}{N_h} \tag{46}$$

$$MSE(\bar{y}_{RC})_{Neyman} = \frac{1}{n}\left(\sum_{h=1}^{L} W_h\sqrt{\Omega_h}\right)^2 - \sum_{h=1}^{L}\frac{W_h^2\Omega_h}{N_h} \tag{47}$$

(v) *MSEs* of $\bar{y}_{lrc}$ under various allocation schemes:

$$MSE(\bar{y}_{lrc})_{Equal} = \frac{1}{nN^2}\sum_{h=1}^{L} N_h(LN_h-n)S_{yh}^2\left(1-\rho_{yx}^2\right) \tag{48}$$

$$MSE(\bar{y}_{lrc})_{Prop} = \frac{(N-n)}{Nn}\sum_{h=1}^{L} W_hS_{yh}^2\left(1-\rho_{yx}^2\right) \tag{49}$$

$$MSE(\bar{y}_{lrc})_{Opt} = \left(1-\rho_{yx}^2\right)\left[\frac{1}{n}\left(\sum_{h=1}^{L} W_h S_{yh}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h S_{yh}/\sqrt{c_h}\right) - \sum_{h=1}^{L}\frac{W_h^2 S_{yh}^2}{N_h}\right] \tag{50}$$

$$MSE(\bar{y}_{lrc})_{Neyman} = \left(1-\rho_{yx}^2\right)\left[\frac{1}{n}\left(\sum_{h=1}^{L} W_h S_{yh}\right)^2 - \sum_{h=1}^{L}\frac{W_h^2 S_{yh}^2}{N_h}\right] \tag{51}$$

Remark 5.1 From Eqs. (48), (49), (50), and (51), we have

$$MSE(\bar{y}_{lrc})_{Equal} = \left(1-\rho_{yx}^2\right)\left[Var(\bar{y}_{st})_{Equal}\right] \tag{52}$$

$$MSE(\bar{y}_{lrc})_{Prop} = \left(1-\rho_{yx}^2\right)\left[Var(\bar{y}_{st})_{Prop}\right] \tag{53}$$

$$MSE(\bar{y}_{lrc})_{Opt} = \left(1-\rho_{yx}^2\right)\left[Var(\bar{y}_{st})_{Opt}\right] \tag{54}$$

$$MSE(\bar{y}_{lrc})_{Neyman} = \left(1-\rho_{yx}^2\right)\left[Var(\bar{y}_{st})_{Neyman}\right] \tag{55}$$

6. Efficiency Comparisons for Separate Regression-cum-Ratio Estimators

In this section, we have obtained the *necessary and sufficient conditions (NASCs)* for revealing the dominance of the class of separate regression-cum-ratio estimators (T_s) over the well-known pre-existing estimators on utilizing the *MSE* criterion, under various allocation schemes, as follows:

Case-I: Case of Equal Allocation

From Eqs. (14), (32), (36), and (40), we have

(i) $MSE(T_s)_{Equal} < Var(\bar{y}_{st})_{Equal}$ if

$$\sum_{h=1}^{L} N_h(LN_h - n)\left(\psi_h - S_{yh}^2\right) < 0 \tag{56}$$

(ii) $MSE(T_s)_{Equal} < MSE(\bar{y}_{RS})_{Equal}$ if

$$\sum_{h=1}^{L} N_h(LN_h - n)\left(\psi_h - \zeta_h\right) < 0 \tag{57}$$

(iii) $MSE(T_s)_{Equal} = MSE(\bar{y}_{lrs})_{Equal}$ if

$$\psi_h = S_{yh}^2\left(1-\rho_{yxh}^2\right) \tag{58}$$

$$\text{i.e.,}\quad \rho_{yxh}^2 S_{yh}^2 + (\delta_h R_h + \beta_h)^2 S_{xh}^2 - 2(\delta_h R_h + \beta_h)S_{yxh} = 0 \tag{59}$$

Case-II: Case of Proportional Allocation

From Eqs. (15), (33), (37), and (41), we have

(i) $MSE(T_s)_{Prop} < Var(\bar{y}_{st})_{Prop}$ if

$$\psi_h < S_{yh}^2 \tag{60}$$

$$\text{i.e.,}\quad (\delta_h R_h + \beta_h) < \frac{2S_{yxh}}{S_{xh}^2} \tag{61}$$

(ii) $MSE(T_s)_{Prop} < MSE(\bar{y}_{RS})_{Prop}$ if

$$\psi_h < \zeta_h \tag{62}$$

$$\text{i.e.,}\quad (\delta_h R_h + \beta_h + R_h) < \frac{2S_{yxh}}{S_{xh}^2} \tag{63}$$

(iii) $MSE(T_s)_{Prop} = MSE(\bar{y}_{lrs})_{Prop}$ if

$$\psi_h = S_{yh}^2\left(1 - \rho_{yxh}^2\right) \tag{64}$$

$$\text{i.e.,}\quad \rho_{yxh}^2 S_{yh}^2 + (\delta_h R_h + \beta_h)^2 S_{xh}^2 - 2(\delta_h R_h + \beta_h) S_{yxh} = 0 \tag{65}$$

Case-III: Case of Optimum Allocation

From Eqs. (20), (34), (38), and (42), we have

(i) $MSE(T_s)_{Opt} < Var(\bar{y}_{st})_{Opt}$ if

$$\frac{1}{n}\left\{\left(\sum_{h=1}^{L} W_h\sqrt{\psi_h}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h\sqrt{\psi_h}/\sqrt{c_h}\right) - \left(\sum_{h=1}^{L} W_h S_{yh}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h S_{yh}/\sqrt{c_h}\right)\right\}$$
$$< \sum_{h=1}^{L}\frac{W_h^2\left(\psi_h - S_{yh}^2\right)}{N_h} \tag{66}$$

(ii) $MSE(T_s)_{Opt} < MSE(\bar{y}_{RS})_{Opt}$ if

$$\frac{1}{n}\left\{\left(\sum_{h=1}^{L} W_h\sqrt{\psi_h}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h\sqrt{\psi_h}/\sqrt{c_h}\right) - \left(\sum_{h=1}^{L} W_h\sqrt{\zeta_h}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h\sqrt{\zeta_h}/\sqrt{c_h}\right)\right\}$$
$$< \sum_{h=1}^{L}\frac{W_h^2\left(\psi_h - \zeta_h\right)}{N_h} \tag{67}$$

(iii) $MSE(T_s)_{Opt} = MSE(\bar{y}_{lrs})_{Opt}$ if

$$\frac{1}{n}\left[\left(\sum_{h=1}^{L} W_h\sqrt{\psi_h}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h\sqrt{\psi_h}/\sqrt{c_h}\right)\right.$$
$$\left. - \left\{\sum_{h=1}^{L} W_h S_{yh}\sqrt{\left(1-\rho_{yxh}^2\right)}\sqrt{c_h}\right\}\left\{\sum_{h=1}^{L} W_h S_{yh}\sqrt{\left(1-\rho_{yxh}^2\right)/\sqrt{c_h}}\right\}\right]$$
$$= \sum_{h=1}^{L}\frac{W_h^2\left\{\psi_h - S_{yh}^2\left(1-\rho_{yxh}^2\right)\right\}}{N_h} \tag{68}$$

Case-IV: Case of Neyman Allocation

From Eqs. (22), (35), (39), and (43), we have

(i) $MSE(T_s)_{Neyman} < Var(\bar{y}_{st})_{Neyman}$ if

$$\frac{1}{n}\left\{\left(\sum_{h=1}^{L} W_h\sqrt{\psi_h}\right)^2 - \left(\sum_{h=1}^{L} W_h S_{yh}\right)^2\right\} < \sum_{h=1}^{L}\frac{W_h^2\left(\psi_h - S_{yh}^2\right)}{N_h} \tag{69}$$

(ii) $MSE(T_s)_{Neyman} < MSE(\bar{y}_{RS})_{Neyman}$ if

$$\frac{1}{n}\left\{\left(\sum_{h=1}^{L} W_h\sqrt{\psi_h}\right)^2 - \left(\sum_{h=1}^{L} W_h\sqrt{\zeta_h}\right)^2\right\} < \sum_{h=1}^{L}\frac{W_h^2\left(\psi_h - \zeta_h\right)}{N_h} \tag{70}$$

(iii) $MSE(T_s)_{Neyman} = MSE(\bar{y}_{lrs})_{Neyman}$ if

$$\frac{1}{n}\left[\left(\sum_{h=1}^{L} W_h\sqrt{\psi_h}\right)^2 - \left\{\sum_{h=1}^{L} W_h S_{yh}\sqrt{\left(1-\rho_{yxh}^2\right)}\right\}^2\right] = \sum_{h=1}^{L}\frac{W_h^2\left\{\psi_h - S_{yh}^2\left(1-\rho_{yxh}^2\right)\right\}}{N_h} \tag{71}$$

7. Efficiency Comparisons for Combined Regression-cum-Ratio Estimators

In this section, we have obtained the *necessary and sufficient conditions (NASCs)* for revealing the superiority of the class of combined regression-cum-ratio estimators (T_c) over the well-known pre-existing estimators on utilizing the *MSE* criterion, under various allocation schemes, as follows:

Case-I: Case of Equal Allocation

From Eqs. (28), (32), (44), and (48), we have

(i) $MSE(T_c)_{Equal} < Var(\bar{y}_{st})_{Equal}$ if

$$\sum_{h=1}^{L} N_h(LN_h - n)\left(\xi_h - S_{yh}^2\right) < 0 \tag{72}$$

(ii) $MSE(T_c)_{Equal} < MSE(\bar{y}_{RC})_{Equal}$ if

$$\sum_{h=1}^{L} N_h(LN_h - n)\left(\xi_h - \Omega_h\right) < 0 \tag{73}$$

(iii) $MSE(T_c)_{Equal} < MSE(\bar{y}_{lrc})_{Equal}$ if

$$\sum_{h=1}^{L} N_h(LN_h - n)\left\{\xi_h - S_{yh}^2\left(1-\rho_{yx}^2\right)\right\} < 0 \tag{74}$$

Case-II: Case of Proportional Allocation

From Eqs. (29), (33), (45), and (49), we have

(i) $MSE(T_c)_{Prop} < Var(\bar{y}_{st})_{Prop}$ if

$$\xi_h < S_{yh}^2 \tag{75}$$

$$\text{i.e.,}\ \ (R_M\alpha_h + \beta) < \frac{2S_{yxh}}{S_{xh}^2} \tag{76}$$

(ii) $MSE(T_c)_{Prop} < MSE(\bar{y}_{RC})_{Prop}$ if

$$\xi_h < \Omega_h \tag{77}$$

$$\text{i.e.,}\ \ (R_M\alpha_h + \beta + R) < \frac{2S_{yxh}}{S_{xh}^2} \tag{78}$$

(iii) $MSE(T_c)_{Prop} < MSE(\bar{y}_{lrc})_{Prop}$ if

$$\xi_h < S_{yh}^2 \left(1 - \rho_{yx}^2\right) \tag{79}$$

$$\text{i.e.,}\;\; \rho_{yx}^2 S_{yh}^2 + (R_M \alpha_h + \beta)^2 S_{xh}^2 - 2(R_M \alpha_h + \beta) S_{yxh} < 0 \tag{80}$$

Case-III: Case of Optimum Allocation

From Eqs. (30), (34), (46), and (50), we have

(i) $MSE(T_c)_{Opt} < Var(\bar{y}_{st})_{Opt}$ if

$$\frac{1}{n}\left\{\left(\sum_{h=1}^{L} W_h \sqrt{\xi_h}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h \sqrt{\xi_h}/\sqrt{c_h}\right) - \left(\sum_{h=1}^{L} W_h S_{yh}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h S_{yh}/\sqrt{c_h}\right)\right\} < \sum_{h=1}^{L} \frac{W_h^2\left(\xi_h - S_{yh}^2\right)}{N_h} \tag{81}$$

(ii) $MSE(T_c)_{Opt} < MSE(\bar{y}_{RC})_{Opt}$ if

$$\frac{1}{n}\left\{\left(\sum_{h=1}^{L} W_h \sqrt{\xi_h}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h \sqrt{\xi_h}/\sqrt{c_h}\right) - \left(\sum_{h=1}^{L} W_h \sqrt{\Omega_h}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h \sqrt{\Omega_h}/\sqrt{c_h}\right)\right\} < \sum_{h=1}^{L} \frac{W_h^2\left(\xi_h - \Omega_h\right)}{N_h} \tag{82}$$

(iii) $MSE(T_c)_{Opt} < MSE(\bar{y}_{lrc})_{Opt}$ if

$$\frac{1}{n}\left[\left(\sum_{h=1}^{L} W_h \sqrt{\xi_h}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h \sqrt{\xi_h}/\sqrt{c_h}\right) - \left(1 - \rho_{yx}^2\right)\left(\sum_{h=1}^{L} W_h S_{yh}\sqrt{c_h}\right)\left(\sum_{h=1}^{L} W_h S_{yh}/\sqrt{c_h}\right)\right] < \sum_{h=1}^{L} \frac{W_h^2\left\{\xi_h - S_{yh}^2\left(1 - \rho_{yx}^2\right)\right\}}{N_h} \tag{83}$$

Case-IV: Case of Neyman Allocation

From Eqs. (31), (35), (47), and (51), we have

(i) $MSE(T_c)_{Neyman} < Var(\bar{y}_{st})_{Neyman}$ if

$$\frac{1}{n}\left\{\left(\sum_{h=1}^{L} W_h \sqrt{\xi_h}\right)^2 - \left(\sum_{h=1}^{L} W_h S_{yh}\right)^2\right\} < \sum_{h=1}^{L} \frac{W_h^2\left(\xi_h - S_{yh}^2\right)}{N_h} \tag{84}$$

(ii) $MSE(T_c)_{Neyman} < MSE(\bar{y}_{RC})_{Neyman}$ if

$$\frac{1}{n}\left\{\left(\sum_{h=1}^{L} W_h \sqrt{\xi_h}\right)^2 - \left(\sum_{h=1}^{L} W_h \sqrt{\Omega_h}\right)^2\right\} < \sum_{h=1}^{L} \frac{W_h^2\left(\xi_h - \Omega_h\right)}{N_h} \tag{85}$$

(iii) $MSE(T_c)_{Neyman} < MSE(\bar{y}_{lrc})_{Neyman}$ if

$$\frac{1}{n}\left[\left(\sum_{h=1}^{L} W_h\sqrt{\xi_h}\right)^2 - \left(1-\rho_{yx}^2\right)\left(\sum_{h=1}^{L} W_h S_{yh}\right)^2\right] < \sum_{h=1}^{L}\frac{W_h^2\left\{\xi_h - S_{yh}^2\left(1-\rho_{yx}^2\right)\right\}}{N_h} \quad (86)$$

8. Empirical Study

In order to demonstrate the relative performances of the classes of estimators T_s and T_c over the pre-existing estimators subjected to various allocation schemes, we have considered three natural population data sets. The descriptions of the populations with the values of various parameters, are described below:

Population I- (Source: Murthy [19])
Y= factories in region
X= fixed capital
(see Table 1)

Population II- (Source: Kadilar and Cingi [20])
Y= amount of apple production
X= number of apple trees
(see Table 2)

Population III- (Source: Koyuncu and Kadilar [21])
Y= number of teachers
X= number of students
(see Table 3)

Table 1: Parameters of Population I

	Stratum →	1	2	3	4
$N = 80$, $n = 45$,	N_h	19	32	14	15
$\bar{Y} = 5182.64$,	$\bar{Y}_h$	2967.95	4657.63	6537.21	7843.67
$\bar{X} = 1126.46$,	$\bar{X}_h$	349.684	706.594	1539.57	2620.53
$R = 4.6008$,	S_{yh}	757.089	669.127	416.113	645.688
$\rho_{yx} = 0.7769$.	S_{xh}	109.449	109.222	277.181	370.972
	ρ_{yxh}	0.9364	0.9260	0.9835	0.9692

Table 2: Parameters of Population II

	Stratum →	1	2	3	4	5	6
$N = 854$, $n = 140$,	N_h	106	106	94	171	204	173
$\bar{Y} = 2930$,	$\bar{Y}_h$	1536	2212	9384	5588	967	404
$\bar{X} = 37600$,	$\bar{X}_h$	24375	27421	72409	74365	26441	9844
$R = 0.0779$,	S_{yh}	6425	11552	29907	28643	2390	946
$\rho_{yx} = 0.8267$.	S_{xh}	49189	57461	160757	285603	45403	18794
	ρ_{yxh}	0.82	0.86	0.90	0.99	0.71	0.89

Table 3: Parameters of Population III

	Stratum →	1	2	3	4	5	6
$N = 923$, $n = 180$,	N_h	127	117	103	170	205	201
$\bar{Y} = 436.433$,	$\bar{Y}_h$	703.74	413	573.17	424.66	267.03	393.84
$\bar{X} = 11440.50$,	$\bar{X}_h$	20804.59	9211.79	14309.30	9478.85	5569.95	12997.59
$R = 0.0381$,	S_{yh}	883.897	645.106	1033.43	810.676	403.749	711.669
$\rho_{yx} = 0.9552$.	S_{xh}	30478.70	15181	27545.40	18218.30	8499.74	23096.70
	ρ_{yxh}	0.936	0.996	0.994	0.983	0.989	0.965

The *mean square errors (MSEs)* are computed under various allocation schemes for the well-known estimators of $\bar{Y}$, and the findings are depicted in Table 4.

Table 4: *MSEs* under various allocation schemes for the estimators of $\bar{Y}$

Estimator	Allocation Scheme	Population I	Population II	Population III
$\bar{y}_{st}$	Equal	*	*	2378.45
	Proportional	4119.27	1707723.08	2446.14
	Neyman	3844.34	697338.37	2229.14
$\bar{y}_{RS}$	Equal	*	*	147.17
	Proportional	2048.6	276768	153.32
	Neyman	1309.81	137999	99.63
$\bar{y}_{lrs}$	Equal	*	*	124.22
	Proportional	466.89	171278	123.54
	Neyman	353.15	76260.7	85.35
$\bar{y}_{RC}$	Equal	*	*	247.79
	Proportional	4023.67	380982	245.45
	Neyman	1502.95	168925	162.26
$\bar{y}_{lrc}$	Equal	*	*	208.34
	Proportional	1632.99	540609	214.26
	Neyman	1524	220754	195.26
T_s	Equal	*	*	124.22
	Proportional	466.89	171278	123.54
	Neyman	353.15	76260.7	85.35
T_c	Equal	*	*	124.22
	Proportional	466.89	171278	123.54
	Neyman	353.15	76260.7	85.35

* Data is not applicable.

The *percent relative efficiencies (PREs)* are obtained for the well-known estimators of $\bar{Y}$ with respect to the stratified sample mean $\bar{y}_{st}$, under various allocation schemes, and the findings are demonstrated in Tables 5, 6 and 7. The *PREs* have been computed using the below mentioned formulae:

$$PRE(\phi, \bar{y}_{st})_{Equal} = \frac{Var(\bar{y}_{st})_{Equal}}{MSE(\phi)_{Equal}} \times 100 \ ,$$

$$PRE(\phi, \bar{y}_{st})_{Prop} = \frac{Var(\bar{y}_{st})_{Prop}}{MSE(\phi)_{Prop}} \times 100 \ ,$$

$$PRE(\phi, \bar{y}_{st})_{Neyman} = \frac{Var(\bar{y}_{st})_{Neyman}}{MSE(\phi)_{Neyman}} \times 100 \ ,$$

where $\phi = \bar{y}_{st}, \ \bar{y}_{RS}, \ \bar{y}_{lrs}, \ \bar{y}_{RC}, \ \bar{y}_{lrc}, \ T_s, \ T_c$.

Table 5: *PREs* of various estimators of $\bar{Y}$ under Equal Allocation Scheme

Estimator	Population I	Population II	Population III
$\bar{y}_{st}$	*	*	100
$\bar{y}_{RS}$	*	*	1616.15
$\bar{y}_{lrs}$	*	*	**1914.66**
$\bar{y}_{RC}$	*	*	959.87
$\bar{y}_{lrc}$	*	*	1141.64
T_s	*	*	**1914.66**
T_c	*	*	**1914.66**

Bold values represent the maximum *PREs*.

* Data is not applicable.

Table 6: *PREs* of various estimators of $\bar{Y}$ under Proportional Allocation Scheme

Estimator	Population I	Population II	Population III
$\bar{y}_{st}$	100	100	100
$\bar{y}_{RS}$	201.08	617.02	1595.48
$\bar{y}_{lrs}$	**882.28**	**997.05**	**1980.02**
$\bar{y}_{RC}$	102.38	448.24	996.61
$\bar{y}_{lrc}$	252.25	315.89	1141.64
T_s	**882.28**	**997.05**	**1980.02**
T_c	**882.28**	**997.05**	**1980.02**

Bold values represent the maximum *PREs*.

Table 7: *PREs* of various estimators of $\bar{Y}$ under Neyman Allocation Scheme

Estimator	Population I	Population II	Population III
$\bar{y}_{st}$	100	100	100
$\bar{y}_{RS}$	293.50	505.32	2237.35
$\bar{y}_{lrs}$	**1088.58**	**914.41**	**2611.61**
$\bar{y}_{RC}$	255.79	412.81	1373.79
$\bar{y}_{lrc}$	252.25	315.89	1141.64
T_s	**1088.58**	**914.41**	**2611.61**
T_c	**1088.58**	**914.41**	**2611.61**

Bold values represent the maximum *PREs*.

9. Results

From Table 4, it is revealed that:

(i) In all the three populations, we have

$$Var(\bar{y}_{st})_{Neyman} < Var(\bar{y}_{st})_{Prop}$$

$$\text{and } MSE(\phi)_{Neyman} < MSE(\phi)_{Prop}$$

where $\phi = \bar{y}_{RS},\ \bar{y}_{lrs},\ \bar{y}_{RC},\ \bar{y}_{lrc},\ T_s,\ T_c$.

(ii) In population III, we observe that

$$Var(\bar{y}_{st})_{Neyman} < Var(\bar{y}_{st})_{Equal}$$

$$\text{and } MSE(\phi)_{Neyman} < MSE(\phi)_{Equal}$$

(iii) The case of equal allocation is applied only in population III for the reason that $n_h = n/L = 180/6 = 30$, which is an integer value. Hence, in order to apply the equal allocation scheme in population III, the samples of equal sizes (i.e., 30) are selected from each of the six stratum to make an overall sample of size 180.

(iv) In all the three populations, the class of separate regression-cum-ratio estimators (T_s) is dominant over the stratified sample mean ($\bar{y}_{st}$), and the separate ratio estimator ($\bar{y}_{RS}$) under various allocation schemes, as was expected from the theoretical results of Section 6. Moreover, the performance of T_s is same as that of the separate regression estimator ($\bar{y}_{lrs}$) for various allocation schemes.

(v) In all the three populations, the class of combined regression-cum-ratio estimators (T_c) is dominant over the stratified sample mean ($\bar{y}_{st}$), the combined ratio estimator ($\bar{y}_{RC}$), and the combined regression estimator ($\bar{y}_{lrc}$) under various allocation schemes, as was expected from the theoretical results of Section 7. Furthermore, the performance of T_c is same as that of the separate regression estimator ($\bar{y}_{lrs}$) for various allocation schemes.

Again, from Tables 5, 6, and 7, we observe that:

(i) In all the three allocation schemes (i.e., equal, proportional, and Neyman), the *PREs* of the class of separate regression-cum-ratio estimators (T_s) are more as compared to $\bar{y}_{st}$, and $\bar{y}_{RS}$ in the respective populations.

(ii) In all the three allocation schemes, the *PREs* of the class of combined regression-cum-ratio estimators (T_c) are more as compared to $\bar{y}_{st}$, $\bar{y}_{RC}$, and $\bar{y}_{lrc}$ in the respective populations.

(iii) In all the three allocation schemes, the *PREs* of the classes of estimators T_s, and T_c are equivalent to that of the separate regression estimator ($\bar{y}_{lrs}$) in the respective populations, as was expected from the theoretical results of Sections 3 and 4.

10. Conclusion

In this paper, the theory of estimation of mean in stratified random sampling has been enhanced by incorporating various sample allocation schemes, viz. equal allocation, proportional allocation, optimum allocation, and Neyman allocation. Moreover, the relevance and significance of various allocation schemes in the estimation procedure have been explored. For instance, the case of equal allocation can be applied only when the ratio n/L is an integer value. In a similar manner, the case of optimum allocation can be applied if we have a prior information on the cost involved in the survey. Furthermore, if the sampling cost incurred is the same in each stratum of the population under consideration, then the case of Neyman allocation is applied.

It can be observed from the empirical results that if all the allocation schemes are applicable to a given data, then the Neyman allocation provides a smaller variance (or MSE as the case may be) for the concerned estimator as compared to the other allocation schemes.

In view of the theoretical and empirical results, we conclude that the classes of estimators T_s and T_c, as developed by Kumar and Vishwakarma [14], are dominant over the stratified sample mean ($\bar{y}_{st}$) and the other well-known pre-existing estimators for the estimation of population mean ($\bar{Y}$) of the study variable Y under various allocation schemes. Moreover, it is also revealed that the performances of T_s and T_c are equivalent to that of the separate regression estimator ($\bar{y}_{lrs}$).

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Competing Interests

Authors have declared that no competing interests exist.

Authors′ contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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