STUDY OF VARIOUS DOMINATIONS IN BIPOLAR INTUITIONISTIC ANTI FUZZY GRAPHS

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Abstract: Bipolar intuitionistic fuzzy graphs (BIFG) with perfectly connected domination are introduced in this study. The study also discusses the bipolar intuitionistic antifuzzy graphs (BIAFG) domination concepts. Additionally, the topic of 2-domination in BIAFG are explored. we developed the secure edge dominating set and its domination number in BIAFG also some results are derived with suitable examples.
Keywords and Phrases: Bipolar domination, Bipolar intuitionistic fuzzy graphs, 2-domination, secure edge domination.
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1. Introduction

Fuzzy set plays an important role in numerous disciplines such as decision-making problems, medicine, chemistry, computer science and engineering. Attanasov designed an extension of FS with its satisfaction and non-satisfaction values including its hesitancy part is known as intuitionistic fuzzy set (IFS) whose sum is 1 .The concept of bipolar fuzzy set (BFS) is established by Zhang [10] where its range is extended to $[-1, 1]$ When there is a counter judgment on an object. Recently, the notion of bipolar intuitionistic fuzzy set (BIFS) is established when the bipolar idea is extended and applied in IFS. The Fuzzy graph (FG) was originally introduced by Kaufmann based on Zadeh's fuzzy relation (FR). The concept of fuzzy graph (FG) has earned lot of applications such as traffic light modelling, time table scheduling, neural networks and cardiac function, etc. The structure of FG was defined by considering two FS and finding its relations by Rosenfield. Also, several theoretical ideas were obtained.

The notion of intuitionistic fuzzy graph (IFG) [17] was designed when Attanasov's IFS is combined with fuzzy graph. As it doesn't satisfy the complementary condition, Parvathi and Karunambigai reframed the view of IFG [22]. Based on Zhang's idea, the BFS concept was extended in FG [11].Akram et al. [1] gave an excellent source on BFG . Recently, the notion of bipolar intuitionistic fuzzy graph (BIFG) was designed by Ezhilmaran and Sankar [14] combining the bipolar idea with IFG. But, it doesn't satisfy the complementary condition. Then, it was reframed by Mandal and Pal. The notion of strong BIFG and their properties were also defined by Sankar and Ezhilmaran [15].

M. G. Karunambigai, M. Akram and K. Palanivel in (2013) investigated the concept of domination, independence and irredundance on BFG. The concept of Total strong (weak) domination in bipolar fuzzy graph was investigated by R. Muthuraj and Kanimozhi [21]. The perfect domination in BFG was introduced by R. Muthuraj in (2018) [20]. In this paper we introduced and investigate the concepts of connected perfect dominating in bipolar intutionistic fuzzy graphs and obtain many result related to this concepts and relationship between this concepts and the others in BIFG will be given with suitable examples.

2. Preliminaries

Definition 2.1. [12] Let $\mathcal{X}$ be a non empty set. A *bipolar fuzzy set* $\mathcal{M}$ in $\mathcal{X}$ is an object having the form $B = \{(x, \mu_B^+(x), \mu_B^-(x)) | x \in \mathcal{X}\}$ where, $\mu_B^+ : \mathcal{X} \to [0,1]$ and $\mu_B^- : \mathcal{X} \to [-1,0]$ are mapppings.

Definition 2.2. [12] A *Bipolar fuzzy graph* (BFG) is of the form $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where

(1) $\mathcal{V} = v_1, v_2, ...v_n$ such that $\mu_1^+ : X \to [0,1]$ and $\mu_1^- : X \to [-1,0]$
(2) $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ where $\mu_2^+ : \mathcal{V} \times \mathcal{V} \to [0,1]$ and $\mu_2^- : \mathcal{V} \times \mathcal{V} \to [-1,0]$ such that
$$\mu_{2ij}^+ = \mu_2^+(v_i, v_j) \leq min(\mu_1^+(v_i), \mu_1^+(v_j))$$
and
$$\mu_{2ij}^- = \mu_2^-(v_i, v_j) \geq max(\mu_1^-(v_i), \mu_1^-(v_j))$$
for all $(v_i, v_j) \in \mathcal{E}$.

Definition 2.3. [12] A BFG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is called *strong* if $\mu_2^+ = min(\mu_1^+(v_i), \mu_1^+(v_j))$ and $\mu_2^- = max(\mu_1^-(v_i), \mu_1^-(v_j))$ $\forall v_i, v_j \in \mathcal{V}$.

Definition 2.4. [12] A BFG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is called *complete* if
$$\mu_2^+(v_i, v_j) = min(\mu_1^+(v_i), \mu_1^+(v_j))$$
$$\mu_2^-(v_i, v_j) = min(\mu_1^-(v_i), \mu_1^-(v_j))$$
for all $v_i, v_j \in \mathcal{V}$

Definition 2.5. [12] Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a BFG, then cardinality of $\mathcal{G}$ is defined as
$$|\mathcal{G}| = \sum_{v_i \in \mathcal{V}} \frac{(1 + \mu_1^+(v_i) + \mu_1^-(v_i))}{2} + \sum_{(v_i, v_j) \in \mathcal{E}} \frac{(1 + \mu_2^+(v_i, v_j) + \mu_2^-(v_i, v_j))}{2}$$

Definition 2.6. [12] The cardinality of $\mathcal{V}$, i.e.; amount of nodes is termed as the order of a BFG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and is signified by $|\mathcal{V}|$(or $O(\mathcal{G})$) and determined by
$$O(\mathcal{G}) = |\mathcal{V}| = \sum_{v_i \in \mathcal{V}} \frac{(1 + \mu_1^+(v_i) + \mu_1^-(v_i))}{2}$$
The no. of elements in a set of S,i.e., amount of edges is termed as size of BFG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and signified as $|S|$(or $S(\mathcal{G})$) and determined by
$$S(\mathcal{G}) = |S| = \sum_{(v_i, v_j) \in \mathcal{E}} \frac{(1 + \mu_2^+(v_i, v_j) + \mu_2^-(v_i, v_j))}{2}$$

for all $(v_i, v_j) \in \mathcal{E}$

Definition 2.7. [12] The *degree* of a vertex v in a BFG, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is defined to be the sum of the weights of the strong edges incident at v. It is denoted by $d_{\mathcal{G}}(v)$. The minimum degree of $\mathcal{G}$ is $\bigtriangledown(\mathcal{G}) = \min(d_{\mathcal{G}}(v)|v \in V)$ The maximum degree of $\mathcal{G}$ is $\triangle(\mathcal{G}) = \max(d_{\mathcal{G}}(v)|v \in V)$

Definition 2.8. [12] Two vertices v_i and v_j are said to be *neighbors* in BFG, if either one of the following conditions hold

(1) $\mu_2^+(v_i, v_j) > 0$ and $\mu_2^-(v_i, v_j) = 0$
(2) $\mu_2^+(v_i, v_j) = 0$ and $\mu_2^-(v_i, v_j) < 0$
(3) $\mu_2^+(v_i, v_j) > 0$ and $\mu_2^-(v_i, v_j) < 0$, $v_i, v_j \in \mathcal{V}$

Definition 2.9. [12] The strength of connectedness between two nodes a and b is

$$\mu^\infty(a, b) = sup(\mu^k(a, b)|k = 1, 2, ...)$$

whereas $\mu^k(a, b) = sup(\mu(aa_1) \wedge \mu(a_1a_2)... \wedge \mu(a_{k-1}b)|a_1, ...a_{k-1} \in \mathcal{V}$

Definition 2.10. [12] An arc (a, b) is said to be strong edge in a BFG, if

$$\mu_2^+(a, b) \geq (\mu_2^+)^\infty(a, b) \text{ and } \mu_2^-(a, b) \geq (\mu_2^-)^\infty(a, b)$$

whereas $(\mu_2^+)^\infty(a, b) = max\{(\mu_2^+)^k(a, b)|k = 1, 2, ..., n\}$
and $(\mu_2^-)^\infty(a, b) = min\{(\mu_2^-)^k(a, b)|k = 1, 2, ..., n\}$.

Definition 2.11. [12] Let u be a vertex in a BFG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ then $N(u)$=$\{v : v \in \mathcal{V}\}$ and (u, v) is a strong edge in $\mathcal{G}$ is called *neighbourhood* of u in $\mathcal{G}$.

Theorem 2.12. *[12] Every arc in a complete BFG is a strong arc.*

Definition 2.13. [12] A vertex $u \in \mathcal{V}$ of a BFG $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is said to be an *isolated vertex* if $\mu_2^+(u, v) = 0$ and $\mu_2^-(u, v) = 0 \forall\, v \in \mathcal{V}, u \neq v$. That is, $N(u) = \phi$. Thus an isolated vertex does not dominate any other vertex of $\mathcal{G}$.

Definition 2.14. [12] Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a BFG on $\mathcal{V}$, Let $u, v \in \mathcal{V}$,we say that u dominates v in $\mathcal{G}$ if there exists a strong edge between them.

Remark 2.15. [12]

(1) For any $u, v \in \mathcal{V}$, if u dominates v then v dominates u and hence domination is a symmetric relation on $\mathcal{V}$.
(2) For any $v \in \mathcal{V}, N(v)$ is precisely the set of all vertices in $\mathcal{V}$ which are dominated by v.
(3) If $(\mu_2^+)(u, v) < (\mu_2^+)^\infty(u, v)$ and $(\mu_2^-)(u, v) > (\mu_2^-)^\infty(u, v)$ for all $u, v \in \mathcal{V}$, then the dominating set of $\mathcal{G}$ is $\mathcal{V}$.

3. Connected Perfect Domination in Bipolar Intuitionistic Fuzzy Graphs

Definition 3.1. [14] Let $\mathcal{X}$ be a non empty set. A BIF set
$\mathcal{B} = \{(x, \mu^P(x), \mu^N(x), \gamma^P(x), \gamma^N(x)) \mid x \in \mathcal{X}\}$ where $\mu^P : \mathcal{X} \to [0, 1]$, $\mu^N : \mathcal{X} \to [-0, 1]$ $\gamma^P : X \to [0, 1], \gamma^N : X \to [-0, 1]$ are the mappings such that $0 \leq \mu^P(x) + \gamma^P(x) \leq 1$ and $-1 \leq \mu^N(x) + \gamma^N(x) \leq 0$.

We use the positive membership degree $\mu^P(x)$, which denotes satisfaction of the property corresponding to a bipolar intuitionistic fuzzy set $\mathcal{B}$ and the negative membership degree $\mu^N(x)$, which denotes the satisfaction implicit counter property corresponding to a bipolar intuitionistic fuzzy set. we use the positive non membership degree $\gamma^P(x)$, which is one minus the positive membership degree and negative nonmembership degree $\gamma^N(x)$, which is one minus the negative membership degree.

Definition 3.2. [14] Let $\mathcal{X}$ be a non empty set. Then we call a mapping $\left(\mu_1^P, \mu_1^N, \gamma_1^P, \gamma_1^N\right) : \mathcal{X} \times \mathcal{X} \to [0,1] \times [-1,0] \times [0,1] \times [-1,0]$ a bipolar intuitionistic fuzzy relation on $\mathcal{X}$ such that

$$\mu_1^P(u,v) \in [0,1], \mu_1^N(u,v) \in [-1,0], \gamma_1^P(u,v) \in [0,1], \gamma_1^N(u,v) \in [-1,0].$$

Definition 3.3. [14] A Bipolar Intuitionistic Fuzzy Graph (BIFG) is a pair $\mathbb{G}_I(P,Q)$ where $P = (\mu_1^P, \mu_1^N, \gamma_1^P, \gamma_1^N)$ is a BIF set in $\tilde{V}$ and $Q = (\mu_2^P, \mu_2^N, \gamma_2^P, \gamma_2^N)$ is a BIF set in $\tilde{E}$ such that

$$\begin{aligned} \mu_2^P(uv) &\leq min(\mu_1^P(u), \mu_1^P(v)) \\ \mu_2^N(uv) &\geq max(\gamma_1^N(u), \gamma_1^N(v)) \\ \mu_2^P(uv) &\geq max(\mu_1^P(u), \mu_1^P(v)) \\ \mu_2^N(uv) &\leq min(\gamma_1^N(u), \gamma_1^N(v)) \end{aligned}$$

for all u, v in $\tilde{V}$.

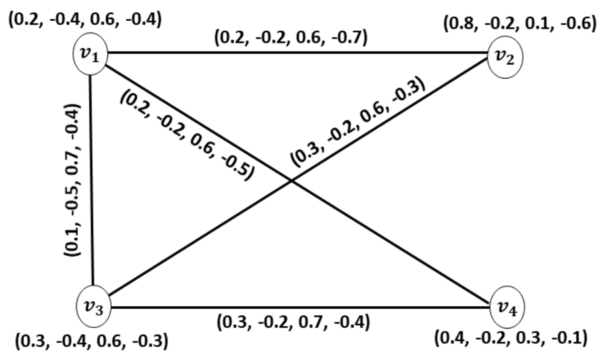

FIGURE 1. Bipolar Intuitionistic Fuzzy Graph

Definition 3.4. [23] A perfect dominating set $\mathcal{D}_p$ of a Bipolar intuitionistic fuzzy graph $\mathbb{G}_I$ is said to be a minimal perfect dominating set, if for each vertex u in $\mathcal{D}_p, \mathcal{D}_p - \{u\}$ is not a perfect dominating set of a BIFG $\mathbb{G}_I$.

Definition 3.5. [23] The minimum fuzzy cardinality of a minimal perfect dominating set of a BIFG $\mathbb{G}_I$ is called the perfect domination number of BIFG. It is denoted by $\gamma_p(\mathbb{G}_I)$. The maximum fuzzy cardinality of a minimal perfect dominating set of $\mathbb{G}_I$ is called the upper perfect domination number of a BIFG. It is denoted by $\Gamma_p(\mathbb{G}_I)$.

Definition 3.6. The perfect dominating set $\mathcal{D}_p$ of a bipolar intuitionistic fuzzy graph $\mathbb{G}_I$ is connected perfect dominating set of $\mathbb{G}_I$ if $< \mathcal{D} >$ the bipolar intuitionistic fuzzy subgraph induced by $\mathcal{D}$ is connected.

Definition 3.7. The connected perfect dominating set $\mathcal{D}_{cp}$ in a bipolar intuitionistic fuzzy graph BIFG $\mathbb{G}_I$ is called minimal connected perfect dominating set of $\mathbb{G}_I$ if for every $r \in \mathcal{D}_{cp}$, $\mathcal{D}_{cp} - \{r\}$ is not connected perfect dominating set of $\mathbb{G}_I$.

Definition 3.8. The minimum fuzzy cardinality of a minimal connected perfect dominating set of a BIFG $\mathbb{G}_I$ is called the connected perfect domination number of a BIFG $\mathbb{G}_I$. It is denoted by $\gamma_{cp}(\mathbb{G}_I)$ The maximum fuzzy cardinality of a minimal connected perfect dominating set of a BIFG $\mathbb{G}_I$ is called the upper perfect domination number of BIFG. It is denoted by $\Gamma_{cp}(\mathbb{G}_I)$.

Theorem 3.9. *Every connected perfect dominating set in a bipolar intiuitionistic fuzzy graph $\mathbb{G}_I$ is a perfect dominating set of a BIFG $\mathbb{G}_I$.*

Proof. We know that a dominating set $\mathcal{D}$ is connected perfect dominating set in a BIFG $\mathbb{G}_I$. if for each vertex $v \notin \mathcal{D}$ and v dominated by exactly one vertex of $\mathcal{D}$ and also the induced bipolar intuitionistic fuzzy subgraph $< \mathcal{D} >$ is connected it is clear that every vertex $v \notin \mathcal{D}$ and v is dominated by exactly one vertex of $\mathcal{D}$ which is a perfect dominating set of BIFG. Therefore every connected perfect dominating set of $\mathbb{G}_I$ is a perfect dominating set of $\mathbb{G}_I$. □

Theorem 3.10. *Let $\mathbb{G}_I$ be a connected bipolar intutionistic fuzzy graph. Let $\mathcal{D}_{cp}$ be a minimal connected perfect dominating set of $\mathbb{G}_I$. Then $V - \mathcal{D}_{cp}$ is not a connected perfect dominating set of $\mathbb{G}_I$.*

Proof. Let $\mathcal{D}$ be a minimal connected perfect dominating set in BIFG $\mathbb{G}_I$. Let v be any vertex of $\mathcal{D}$. Since $\mathbb{G}_I$ is connected. Then by theorem 3.9, $\mathbb{G}_I$ has no isolated vertices, there exists a vertex $u \in N(v)$. u must be dominated by at least one vertex in $\mathcal{D} - \{v\}$, (i.e) $\mathcal{D} - \{v\}$ is a dominating set. Therefore, every vertex in $\mathcal{D}$ is dominated by at least one vertex in $V - \mathcal{D}$ and $V - \mathcal{D}$ is a dominating set. But, every vertex in $\mathcal{D}$ is not dominated by exactly one vertex in $V - P$. So, $V - \mathcal{D}$ is not a perfect dominating set. Further, since $< \mathcal{D} >$ is connected. Thus $V - \mathcal{D}$ is not connected. □

Remark 3.11. Let $\mathcal{G}^I = K_p$ be a is a complete bipolar intuitionistic fuzzy graph. Let $\mathcal{D}$ be a minimal connected perfect dominating set of $\mathbb{G}_I$. Then $V - \mathcal{D}$ has a connected perfect dominating set of $\mathbb{G}_I$.

Theorem 3.12. *A bipolar intuitionistic fuzzy graph $\mathbb{G}_I$ has connected perfect dominating set $\mathcal{D}$ if and only if $\mathbb{G}_I$ is a connected bipolar intuitionistic fuzzy graph.*

Proof. Let $\mathbb{G}_I$ be BIFG with connected perfect dominating set $\mathcal{D}$, since $\mathcal{D}$ is a perfect dominating set and $< \mathcal{D} >$ is connected, and since every vertex v in $V - \mathcal{D}$ is dominated by exactly one vertex in $\mathcal{D}$. Thus $\mathbb{G}_I$ is a connected BIFG. Conversely, let $\mathbb{G}_I$ be a connected BIFG if $\mathbb{G}_I$ is separable graph the $\mathcal{D} = V(\mathcal{G}^I) - \{u\}$ is a connected perfect dominating set of $\mathbb{G}_I$, for every non bipolar fuzzy cut node u in $V(\mathcal{G}^I)$. Hence every connected BIFG has a connected perfect dominating set $\mathcal{D}$. $\square$

4. 2-Domination in Bipolar Intuitionistic Anti Fuzzy Graph

Definition 4.1. [24] A *Bipolar Anti fuzzy graph* (BAFG) is of the form $\mathbb{G}^* = (V', E')$ where $V' = (\mu_1^+, \mu_1^-)$ and $E' = (\mu_2^+, \mu_2^-)$ are bipolar fuzzy sets in which

(1) $V' = v_1, v_2, ...v_n$ such that $\mu_1^+ : V' \to [0, 1]$ and $\mu_1^- : V' \to [-1, 0]$ and $E' \subset V' \times V'$ where $\mu_2^+ : V' \times V' \to [0, 1]$ and $\mu_2^- : V' \times V' \to [-1, 0]$ are bipolar fuzzy mappings such that

$$\mu_{2ij}^+ = \mu_2^+(v_i, v_j) \geq max(\mu_1^+(v_i), \mu_1^+(v_j))$$

and

$$\mu_{2ij}^- = \mu_2^-(v_i, v_j) \leq min(\mu_1^-(v_i), \mu_1^-(v_j))$$

for all $(v_i, v_j) \in E'$.

(2) V' is known as bipolar anti fuzzy vertex set and E' is known as bipolar anti fuzzy edge set.

Definition 4.2. [24] Let $\mathbb{G}^*$ be a BAFG. Then the order of $\mathbb{G}^*$ or cardinality of V' is

$$s = |V'| = \sum_{v_i \in V'} \frac{1 + \mu_1^+(v_i) + \mu_1^-(v_i)}{2}$$

Definition 4.3. [24] A Bipolar Anti Fuzzy Graph (BAFG) $\mathbb{G}^* = (V', E')$ is called *complete* if

$$\mu_2^+(v_i, v_j) = min(\mu_1^+(v_i), \mu_1^+(v_j))$$
$$\mu_2^-(v_i, v_j) = min(\mu_1^-(v_i), \mu_1^-(v_j))$$

for all $v_i, v_j \in V'$

Definition 4.4. [24] Let $\mathbb{G}^* = (V', E')$ be a BAFG, then cardinality of $\mathbb{G}^*$ is defined as

$$|\mathbb{G}^*| = \sum_{v_i \in V'} \frac{(1 + \mu_1^+(v_i) + \mu_1^-(v_i))}{2} + \sum_{(v_i, v_j) \in E'} \frac{(1 + \mu_2^+(v_i, v_j) + \mu_2^-(v_i, v_j))}{2}$$

Definition 4.5. [24] The amount of nodes is termed as the order of a BAFG $\mathbb{G}^* = (V', E')$ and is signified by $|V'|$(or $O(\mathbb{G}^*)$) and cardinality of V' is determined by

$$O(\mathbb{G}^*) = |V'| = \sum_{v_i \in V'} \frac{(1 + \mu_1^+(v_i) + \mu_1^-(v_i))}{2}$$

The amount of edges is termed as size of BAFG $\mathbb{G}^* = (V', E')$ and signified as $|E'|$(or $S(\mathbb{G}^*)$) and determined by

$$S(\mathbb{G}^*) = |E'| = \sum_{(v_i,v_j)\in E'} \frac{(1 + \mu_2^+(v_i, v_j) + \mu_2^-(v_i, v_j))}{2}$$

for all $(v_i, v_j) \in E'$

Definition 4.6. Let $\mathbb{G}^* = (V', E')$ be a BAFG is said to be strong if

$$\mu_2^+(v_i, v_j) = min(\mu_1^+(v_i), \mu_1^+(v_j))$$
$$\mu_2^-(v_i, v_j) = min(\mu_1^-(v_i), \mu_1^-(v_j))$$

for all $v_iv_j \in E'$

Definition 4.7. A Bipolar fuzzy graph is said to be a *Bipolar Intuitionistic Anti Fuzzy Graph (BIAFG)* $\mathbb{G}_I^* = (\tilde{V'}, \tilde{E'})$ where $\tilde{V'} = (\mu_1^P, \mu_1^N, \gamma_1^P, \gamma_1^N)$ is a Bipolar Intuitionistic Fuzzy set and $\tilde{E'} = (\mu_2^P, \mu_2^N, \gamma_2^P, \gamma_2^N)$ is a Bipolar Intutionistic Fuzzy Relation, such that

$$\mu_2^P(uv) \geq max(\mu_1^P(u), \mu_1^P(v))$$
$$\mu_2^N(uv) \leq min(\gamma_1^N(u), \gamma_1^N(v))$$
$$\mu_2^P(uv) \geq min(\mu_1^P(u), \mu_1^P(v))$$
$$\mu_2^N(uv) \leq max(\gamma_1^N(u), \gamma_1^N(v))$$

for all u, v in $\tilde{V'}$.

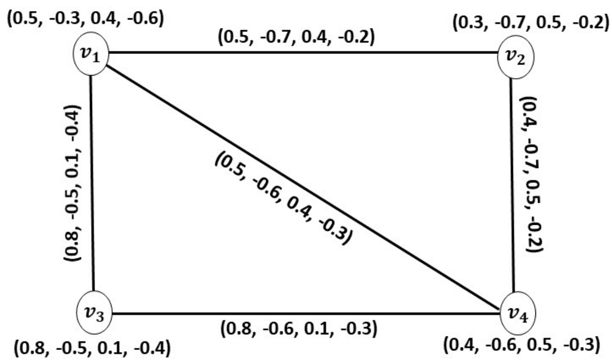

FIGURE 2. Bipolar Intuitionistic Anti Fuzzy Graph

Definition 4.8. An edge $e = (uv)$ is an effective edge in $\mathbb{G}_I^*$ if $\mu^P(uv) = \mu^P(u) \vee \mu^P(v)$; $\mu^N(uv) = \mu^N(u) \wedge \mu^N(v)$; $\gamma^P(uv) = \gamma^P(u) \wedge \gamma^P(v)$; $\gamma^N(uv) = \gamma^N(u) \vee \gamma^N(v)$ $\forall u, v \in \mathbb{G}_I^*$.

Definition 4.9. In a BIAFG $\mathbb{G}_I^*$, The neighbourhood of a vertex u is $N(u) = \{v \in \tilde{V'} | \mu^P(uv) = \mu^N(uv) = \gamma^P(uv) = \gamma^N(uv) \neq 0\}$.

Definition 4.10. In a BIAFG $\mathbb{G}_I^*$, The closed neighbourhood of a vertex u is $N[u] = N(u) \cup u$.

Definition 4.11. Let $\mathbb{G}_I^* = (\tilde{V}', \tilde{E}')$ be a BIAFG. The complement of the BIAFG is defined as $\overline{\mathbb{G}_I^*} = \left(\bar{\tilde{V}'}(\mathbb{G}_I^*), \overline{\tilde{E}'(\mathbb{G}_I^*)}\right)$ such that

- $\bar{\bar{\tilde{V}'}} = \tilde{V}'$.
- $\bar{\mu}^P(u) = \mu^P(u), \bar{\mu}^N(u) = \mu^N(u), \bar{\gamma}^P(u) = \gamma^P(u)$ and $\bar{\gamma}^N(u) = \gamma^N(u)$, for all u.
- $\bar{\mu}^P(uv) = \mu^P(uv) - \mu^P(u) \vee \mu^P(v)$ and $\bar{\mu}^N(uv) = \mu^N(uv) - \mu^N(u) \wedge \mu^N(v)$.
- $\bar{\gamma}^P(uv) = \gamma^P(uv) - \gamma^P(u) \wedge \gamma^P(v)$ and $\bar{\gamma}^N(uv) = \gamma^N(uv) - \gamma^N(u) \vee \gamma^N(v)$.

Definition 4.12. Let $\mathbb{G}_I^*$ be a BIAFG. Then the graph is said to be complete if $\mu^P(v_iv_j) = \mu^P(v_i) \vee \mu^P(v_j), \mu^N(v_iv_j) = \mu^N(v_i) \wedge \mu^N(v_j), \gamma^P(v_iv_j) = \gamma^P(v_i) \wedge \gamma^P(v_j)$ and $\gamma^N(v_iv_j) = \gamma^N(v_i) \vee \gamma^N(v_j)$ for all pair of $v_i, v_j \in \tilde{V}'$.

Definition 4.13. Let $\mathbb{G}_I^*$ be a BIAFG. Then the graph is said to be strong if $\mu^P(v_iv_j) = \mu^P(v_i) \vee \mu^P(v_j), \mu^N(v_iv_j) = \mu^N(v_i) \wedge \mu^N(v_j), \gamma^P(v_iv_j) = \gamma^P(v_i) \wedge \gamma^P(v_j)$ and $\gamma^N(v_iv_j) = \gamma^N(v_i) \vee \gamma^N(v_j)$ for all pair of $(v_iv_j) \in \tilde{E}'$.

Definition 4.14. Let $\mathbb{G}_I^*$ be a BIAFG.Then the degree of u in $\mathbb{G}_I^*$ is defined as $d(u) = \left(d_\mu^P(u), d_\mu^N(u), d_\gamma^P(u), d_\gamma^N(u)\right)$, where

$$d_\mu^P(u) = \sum_{u \neq v, u \in \tilde{V}'} \mu^P(uv), \quad d_\mu^N(u) = \sum_{u \neq v, u \in \tilde{V}'} \mu^N(uv)$$

$$d_\gamma^P(u) = \sum_{u \neq v, u \in \tilde{V}'} \gamma^P(uv), \quad d_\gamma^N(u) = \sum_{u \neq v, u \in \tilde{V}'} \gamma^N(uv).$$

for all $u, v \in \tilde{V}'$.

Definition 4.15. Let $\mathbb{G}_I^*$ be a BIAFG. Then the total degree of the node u is defined as $D(u) = \left(D_\mu^P(u), D_\mu^N(u), D_\gamma^P(u), D_\gamma^N(u)\right)$, where

$$D_\mu^P(u) = \sum_{u \neq v, u \in \tilde{V}'} \mu^P(uv) + \mu^P(u), \quad D_\mu^N(u) = \sum_{u \neq v, u \in \tilde{V}'} \mu^N(uv) + \mu^N(u)$$

$$D_\gamma^P(u) = \sum_{u \neq v, u \in \tilde{V}'} \gamma^P(uv) + \gamma^P(u), \quad D_\gamma^N(u) = \sum_{u \neq v, u \in \tilde{V}'} \gamma^N(uv) + \gamma^N(u).$$

for all $u, v \in \tilde{V}'$

Definition 4.16. If every node of $\mathbb{G}_I^*$ has equal degree, then $\mathbb{G}_I^*$ is called a regular BIAFG. i.e., $d(u) = d(v)$, for every $u, v \in \tilde{V}'$.

Definition 4.17. If every node of $\mathbb{G}_I^*$ has equal total degree, then $\mathbb{G}_I^*$ is called a totally regular BIAFG. i.e., $D(u) = D(v)$, for every $u, v \in \tilde{V}'$.

Definition 4.18. If a node adjacent to other nodes of $\mathbb{G}_I^*$ has different degrees, then $\mathbb{G}_I^*$ is called a Irregular BIAFG. i.e., $d(u) \neq d(v)$, for every $u, v \in \tilde{V}'$.

Definition 4.19. If a node adjacent to other nodes of $\mathbb{G}_I^*$ has different total degrees, then $\mathbb{G}_I^*$ is called a Totally Irregular BIAFG. i.e., $D(u) \neq D(v)$, for every $u, v \in \tilde{V}'$.

Theorem 4.20. *Every complete BIAFG $\mathbb{G}_I^*$ is strong but the converse is not true.*

Proof. Consider a BIAFG graph $\mathbb{G}_I^*$ which is complete. we know that a complete BIAFG has atmost one fuzzy bridge. As every fuzzy bridge is strong, $\mathbb{G}_I^*$ is strong but a strong arc need not be a fuzzy bridge, hence the converse is not true □

Definition 4.21. A subset $\mathcal{D} \subset \tilde{V}'$ is known as dominating set of BIAFG $\mathbb{G}_I^*$ if for every vertex $p \in \tilde{V}' - \mathcal{D}$, $\exists q \in \mathcal{D}$ such that p dominates q.

Definition 4.22. The minimal dominating set is a dominating set $\mathcal{D} \subset \tilde{V}'$ if no proper subset of $\mathcal{D}$ is a dominating set of $\mathbb{G}_I^*$. The maximum cardinality of all the minimal dominating set in $\mathbb{G}_I^*$ is the domination number of $\mathbb{G}_I^*$ and is denoted by $\gamma_A(\mathcal{G})$

Theorem 4.23. *Let $\mathbb{G}_I^*$ be a bipolar intuitionistic anti fuzzy graph. A dominating set $\mathcal{D}$ of $\mathbb{G}_I^*$ is a minimal dominating set if and only if for each $u \in \mathcal{D}$, one of the following two conditions holds.*
(i) $N(u) \cap \mathcal{D} = \phi$.
(ii) There is a vertex $v \in \tilde{V}' - \mathcal{D}$ such that $N(v) \cap \mathcal{D} = \{u\}$.

Proof. Let $\mathbb{G}_I^*$ be a bipolar intuitionistic anti fuzzy graph. Let $\mathcal{D}$ be a minimal dominating set of $\mathbb{G}_I^*$ and $u \in \mathcal{D}$. Let $\mathcal{D}_u = \mathcal{D} - \{u\}$. Then $\mathcal{D}_u$ is not a dominating set as $\mathcal{D}$ is a minimal dominating set. Hence there exists $v \in \tilde{V}' - \mathcal{D}_u$ such that v is not dominated by any element of $\mathcal{D}_u$.

Case (i). If $v = u$, then $v = u$ is not dominated by any element of $\mathcal{D}_u$ and hence it is not dominated by any element of $\mathcal{D}$ and hence, $N(u) \cap \mathcal{D} = \phi$.

Case (ii). If $v \neq u$ then u dominates v as $\mathcal{D}$ is a minimal dominating set of $\mathbb{G}_I^*$ and hence, $N(v) \cap \mathcal{D} = \{u\}$.

Conversely, let $\mathcal{D}$ be a dominating set of $\mathbb{G}_I^*$ and for each $u \in \mathcal{D}$, one of the following two conditions holds.
(i) $N(u) \cap \mathcal{D} = \phi$.
(ii) There is a vertex $v \in \tilde{V}' - \mathcal{D}$ such that $N(v) \cap \mathcal{D} = \{u\}$. Suppose if $\mathcal{D}$ is not a minimal dominating set of $\mathbb{G}_I^*$ then $\mathcal{D}_1 \subset \mathcal{D}$ is a dominating set of $\mathbb{G}_I^*$. Consider an element $u \in \mathcal{D}$ and $u \notin \mathcal{D}_1$. Then $u \in V - D_1$ and there exists $w \in \mathcal{D}_1$ such that w dominates u and so $w \in N(u)$. Also $w \in \mathcal{D}_1 \subset \mathcal{D}$ and hence $N(v) \cap \mathcal{D} \neq \phi$.

Given $\mathcal{D}$ is not a minimal dominating set, then there is a vertex $v \in \tilde{V}' - \mathcal{D}$ such that either v is dominated by more than one vertex of $\mathcal{D}$ or there exist an element $u \in \mathcal{D}$ such that u does not dominate any v for all $v \in \tilde{V}' - \mathcal{D}$.
Case (i). Let $u, w \in \mathcal{D}$ dominates v and $u, w \in N(v)$. Then $N(v) \cap \mathcal{D} = \{u, w\} \neq \{u\}$
Case (ii). Then for this $u \in \mathcal{D}, N(v) \cap \mathcal{D} \neq \{u\}$ for all $v \in \tilde{V}' - \mathcal{D}$. Hence, conditions (i) and (ii) do not hold because of the assumption that $\mathcal{D}$ is not a minimal dominating set of $\mathbb{G}_I^*$. Hence $\mathcal{D}$ is a minimal dominating set of $\mathbb{G}_I^*$. □

Theorem 4.24. *Let $\mathbb{G}_I^*$ be a BIAFG with order r and size s and the minimum degree δ, then $s - r \leq \gamma_A(\mathbb{G}_I^*) \leq r - \delta$.*

Proof. Let $\mathbb{G}_I^*$ be a BIAFG. Consider $\mathcal{D}$ be a dominating set of $\mathbb{G}_I^*$. Let s be the sum of the fuzzy cardinality of all the edges and r be the sum of all vertices. Therefore the difference between the order and size of BIAFG is minimum and the domination number of BIAFG is the minimum cardinality over all the vertices. Hence $|\tilde{V'} - \mathcal{D}| = r - \gamma_A(\mathbb{G}_I^*)$. then there exists at most $deg(\mathbb{G}_I^*)/2$ edges incident from $\tilde{V'} - \mathcal{D}$ to $\mathcal{D}$. Thus, $r - \gamma_A(\mathbb{G}_I^*) \leq s$ This implies that $s - r \leq \gamma_A(\mathbb{G}_I^*)$ (1) Let v be the vertex with minimum degree δ. v must be adjacent to strongly dominate vertices in $\mathbb{G}_I^*$. Hence, $V - N(v)$ is a dominating set. Therefore, $\gamma_A(\mathbb{G}_I^*) \leq r - \delta$ (2). From (1) and (2) we have, $s - r \leq \gamma_A(\mathbb{G}_I^*) \leq r - \delta$ Hence the proof. □

Theorem 4.25. *Let $\mathbb{G}_I^*$ be a complete BIAFG and let $\mathcal{D}$ be the minimal dominating set in $\gamma_A(\mathbb{G}_I^*)$, then $(\tilde{V'} - \mathcal{D})$ is also complete.*

Proof. Let $\mathbb{G}_I^*$ be a complete BIAFG with vertex set $v_i \in \tilde{V'}, i = 1, 2, 3, ..., n$. Let $\mathcal{D}$ be the minimal dominating set in $\mathbb{G}_I^*$. Hence, the resultant vertices in $(\tilde{V'} - \mathcal{D})$ are dominates every other vertex. Therefore, it is complete. Hence the proof. □

Definition 4.26. Let $\mathbb{G}_I^*$ be a BIAFG. A set $\mathcal{D}_d \subset \tilde{V'}$ is a 2-dominating set if for every node in $\tilde{V'} - \mathcal{D}_d$ is dominated by minimum two vertices in $\mathcal{D}_d$. The maximum cardinality of all minimal 2-dominating sets of $\mathbb{G}_I^*$ is said to be 2-domination number of $\mathbb{G}_I^*$ and is denoted as $\gamma_{2d}(\mathbb{G}_I^*)$.

Theorem 4.27. *Let $\mathcal{D}_d$ be a 2-dominating set of BIAFG $\mathbb{G}_I^*$.If there exists more than one vertex in $\tilde{V'} - \mathcal{D}_d$, then every 2-dominating sets of BIAFG $\mathbb{G}_I^*$ is a split dominating set*

Proof. Let $\mathcal{D}_d$ represents a 2-dominating set.
Case(i): Suppose $\tilde{V'} - \mathcal{D}_d$ contains a single vertex then there is no need to observe connected or disconnected.
Case(ii): Suppose there is more than one vertex in $\tilde{V'} - \mathcal{D}_d$. Let u and v be the any two vertices in $tildeV'$. If u and v are strong neighbors, then u occur in $\tilde{V'} - \mathcal{D}_d$ and v occur in $\mathcal{D}_d$. Therefore, any vertex in $\tilde{V'} - \mathcal{D}_d$ including u will be disconnected. Further it implies $\mathcal{D}_d$ is a split dominating set. Suppose u and v are not strong neighbor, then they may be in one set of $\tilde{V'} - \mathcal{D}_d$ and $\tilde{V'} - \mathcal{D}_d$ is disconnectedby having a vertex set includes u and v and remaining vertices occur in another set. This implies 2-dominating set $\mathcal{D}_d$ is a split dominating set of BIAFG $\mathbb{G}_I^*$. □

Theorem 4.28. *For any BIAFG $\mathbb{G}_I^*$, $\gamma(\mathbb{G}_I^*) \leq \gamma_{2d}(\mathbb{G}_I^*)$*

Proof. Let $\mathbb{G}_I^*$ be any BIAFG. Let $\mathcal{D} \subseteq \mathcal{M}$ be a dominating set and $\mathcal{D}_d \subseteq \mathcal{M}$ be a 2-dominating set of $\mathbb{G}_I^*$. If $\mathcal{D} = \mathcal{D}_d$, then $\gamma(\mathbb{G}_I^*) = \gamma_{2d}(\mathbb{G}_I^*)$. If $\mathcal{D} \neq \mathcal{D}_d$, then $\mathcal{D}_d$ has atleast one vertices more than $\mathcal{D}$ and hence, $\gamma(\mathbb{G}_I^*) < \gamma_{2d}(\mathbb{G}_I^*)$ Hence, $\gamma(\mathbb{G}_I^*) \leq \gamma_{2d}(\mathbb{G}_I^*)$ □

Theorem 4.29. *The 2-dominating set of an BIAFG exists only if every vertex in $\tilde{V'} - \mathcal{D}_d$ contains at least two other vertices as strong neighbors.*

Proof. Let $\mathcal{D}_d$ is a 2-dominating set of an BIAFG. If there exists a vertex in $\tilde{V}' - \mathcal{D}_d$ with a single strong neighbor, let it be u and its strong neighbor is v.

Case 1. If $v \in \tilde{V}' - \mathcal{D}_d$, then $u \in \tilde{V}' - \mathcal{D}_d$ has no strong neighbor in $\mathcal{D}_d$, this implies that $\mathcal{D}_d$ cannot be a 2-dominating set.

Case 2. If $v \in \mathcal{D}_d$ such that $u \in \tilde{V}' - \mathcal{D}_d$ has exactly one strong neighbor, again, this implies that $\mathcal{D}_d$ cannot be a 2-dominating set.

We obtain contradiction in both the cases. Hence, there exist at least two strong neighbors for every vertex in $\tilde{V}' - \mathcal{D}_d$. □

5. Secure Edge Domination in Bipolar Intuitionistic Anti Fuzzy Graph

Definition 5.1. Let $\mathbb{G}_I^* = (\tilde{V}', \tilde{E}')$ be a Bipolar Intuitionistic Anti Fuzzy Graph. A set $\mathcal{S} \subseteq \tilde{E}'$ is said to be an edge dominating set of $\mathbb{G}_I^*$ if every edge not in $\mathcal{S}$ is incident to some edge in $\mathcal{S}$. It is said to be minimal if no proper subset of $\mathcal{S}$ is an edge dominating set. The edge domination number is denoted as $\gamma_e(\mathbb{G}_I^*)$

Example 5.2.

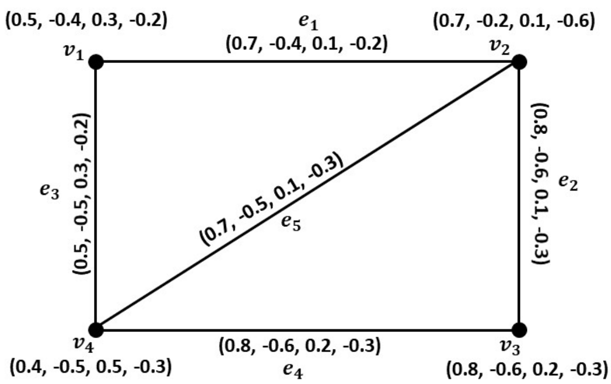

Figure 3. Edge domination in BIAFG

In the above figure 3, $\{e_1, e_2, e_4, e_5\}$,$\{e_2, e_3, e_5\}$,$\{e_1, e_3, e_4\}$ are edge dominating sets of $\mathbb{G}_I^*$.$\{e_1, e_4\}$ $\{e_3, e_2\}$,$\{e_5\}$ are minimal edge dominating sets of $\mathbb{G}_I^*$. Among all the minimal dominating sets, $\{e_5\}$ has minimum cardinality and edge domination number $\gamma_e(\mathbb{G}_I^*) = 1.06$.

Definition 5.3. Let $\mathbb{G}_I^* = (\tilde{V}', \tilde{E}')$ be a BIAFG. Let $\mathcal{S}$ be a minimum edge set of $\mathbb{G}_I^*$. If $\tilde{E}' - \mathcal{S}$ contains an edge dominating set $\mathcal{S}'$ of $\mathbb{G}_I^*$, then $\mathcal{S}'$ is said to be inverse edge dominating set of $\mathbb{G}_I^*$. The minimum cardinality out of all minmal inverse edge dominating sets is said to be inverse edge domination number and is denoted as $\gamma_e^{-1}(\mathbb{G}_I^*)$.

Definition 5.4. Let $\mathbb{G}_I^* = (\tilde{V}', \tilde{E}')$ be a BIAFG. An edge dominating set $\mathtt{F}$ of $\tilde{E}'$ is a secure edge dominating set if for every edge $e \in \tilde{E}' - \mathtt{F}$, there exists an edge $f \in \mathtt{F}$, which is adjacent to e such that $\{(\mathtt{F} - \{f\}) \cup \{e\}\}$ is an edge dominating set.

Definition 5.5. Let $\mathtt{F}$ be a secure edge dominating set of a BIAFG. Then it is said to be minimal secure edge dominating set if no proper subset of $\mathtt{F}$ is a secure edge dominating set. Minimum cardinality among all the minimal secure edge dominating sets is called secure edge domination number of $\mathbb{G}_I^*$ and is denoted by $\gamma_{se}^*(\mathbb{G}_I^*)$

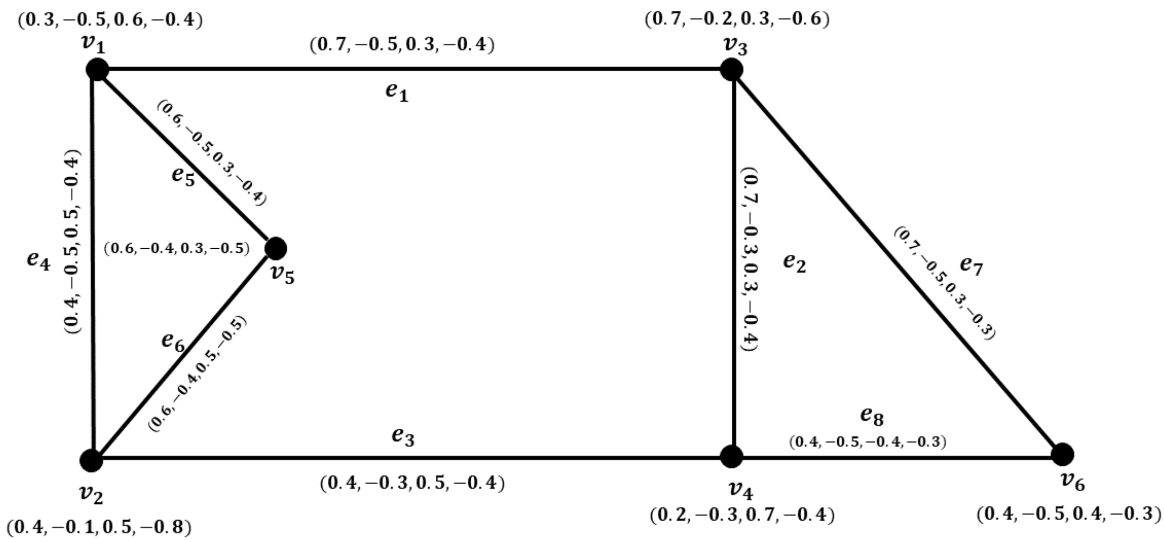

FIGURE 4. Secure Edge Domination in BIAFG

Example 5.6. In Figure 4, $\{e_1, e_2, e_6\}$,$\{e_1, e_2, e_4\}$,$\{e_1, e_3, e_5, e_7\}$, $\{e_2, e_4, e_6, e_8\}$ are some secure edge dominating sets and the secure edge domination number, $\gamma_{se}^*(\mathbb{G}_I^*) = 1.82$

Definition 5.7. Secure total edge dominating set of a BIAFG $\mathbb{G}_I^* = (\tilde{V}', \tilde{E}')$ is defined as a total edge dominating set $\mathtt{T} \subseteq \tilde{E}'$ such that for every $e \in \tilde{E}'/\mathtt{T}$ $\exists$ an edge $t \in \mathtt{T}$, such that e and t are adjacent and $\{(\mathtt{T} - \{t\}) \cup \{e\}\}$.

Definition 5.8. Let $\mathtt{T}$ be a secure total edge dominating set, if no proper subset of $\mathtt{T}$ is a secure total edge dominating set then it is a minimal secure total edge dominating set. Minimum cardinality from all the minimal secure total edge dominating sets is said to be secure total edge domination number and is denoted by $\gamma_{ste}^*(\mathbb{G}_I^*)$

Theorem 5.9. *Prove that* $\mathtt{S}$ *is a secure edge dominating set of a Bipolar Intuitionistic Anti Fuzzy Graph* $\mathbb{G}_I^*$ *where* $\mathtt{S}$ *is a secure total edge dominating set of* $\mathbb{G}_I^*$

Proof. Given a Bipolar Intuitionistic Anti Fuzzy Graph $\mathbb{G}_I^*$. Let $\mathtt{S}$ be a secure total edge dominating set of $\mathbb{G}_I^*$ then $\mathtt{S}$ is a total edge dominating set of $\mathbb{G}_I^*$. Hence $\mathtt{S}$ is an edge dominating set of $\mathbb{G}_I^*$. Let $e \in \tilde{E}'/\mathtt{S}$, then $\exists s \in \mathtt{S}$ such that e and s are two adjacent edges implies e is a strong edge of $\mathbb{G}_I^*$ and $(\mathtt{S} - \{s\}) \cup \{e\}$ is an edge dominating set of $\mathbb{G}_I^*$. Therefore $\mathtt{S}$ is a secure edge dominating set of $\mathbb{G}_I^*$. □

Theorem 5.10. *Let* $\mathbb{G}_I^*$ *be a Bipolar Intuitionistic Anti Fuzzy Graph with no isolated edges. then for every minimal edge dominating set* $\mathtt{F}$*, Prove that* $\tilde{E}' - \mathtt{F}$ *is an edge dominating set.*

Proof. Given a Bipolar Intuitionistic Anti Fuzzy Graph $\mathbb{G}_I^*$, let f be an edge in $\mathtt{F}$. Since $\mathbb{G}_I^*$ has no isolated edges, $\exists$ an edge $e \in N(f)$ which implies $e \in \tilde{E}' - \mathtt{F}$. Hence, Every element of $\mathtt{F}$ is dominated by some element of $\tilde{E}' - \mathtt{F}$. Thus, $\tilde{E}' - \mathtt{F}$ is an edge dominating set. □

Definition 5.11. Let I be a minimal edge dominating set which has minimum cardinality. Then $I' \subseteq \tilde{E}'/I$ is said to be an inverse edge dominating set of $\mathbb{G}_I^*$ with respect to I if I' is an edge dominating set. The minimum cardinality of all inverse edge dominating set of I' of $\mathbb{G}_I^*$ is called inverse edge domination number and is denoted by $\gamma_e^{-1}(\mathbb{G}_I^*)$

Definition 5.12. Let $\mathbb{G}_I^* = (\mathbb{V}, \tilde{E}')$ be a BIAFG. Let I be a minimal secure edge dominating set which has the minimum cardinality. Then $I' \subseteq \tilde{E}' - I$ is said to be an inverse secure edge dominating set of $\mathbb{G}_I^*$ with respect to I if I' is a secure edge dominating set. The inverse secure edge domination number $\gamma_e^{-1}(\mathbb{G}_I^*)$ is the minimum cardinality of an inverse secure edge dominating set of $\mathbb{G}_I^*$.

Definition 5.13. Let $\mathbb{G}_I^* = (\tilde{V}', \tilde{E}')$ be a BIAFG. Let I be a minimal secure total edge dominating set which has the minimum cardinality. Then $I' \subseteq \tilde{E}' - I$ is said to be an inverse secure total edge dominating set of $\mathbb{G}_I^*$ with respect to I if I' is a secure total edge dominating set. The inverse secure total edge domination number $\gamma_{ste}^{-1}(\mathbb{G}_I^*)$ is the minimum cardinality of an inverse secure total edge dominating set of $\mathbb{G}_I^*$

Remark 5.14. Inverse secure total edge dominating set may not exist in all Bipolar Intuitionistic Anti Fuzzy Graphs.

Theorem 5.15. *Let* $\mathbb{G}_I^*$ *be a Bipolar Intuitionistic Anti Fuzzy Graph, If* $\gamma_{ste}^{-1}(\mathbb{G}_I^*)$*-set exists in* $\mathbb{G}_I^*$*, then prove that*

$$\gamma_{se}^*(\mathbb{G}_I^*) \leq \gamma_{ste}^{-1}(\mathbb{G}_I^*)$$

Proof. In a Bipolar Intuitionistic Anti Fuzzy Graph $\mathbb{G}_I^*$, We know that every inverse secure total edge is a secure total edge dominating set. Hence $\gamma_{se}^*(\mathbb{G}_I^*) \leq \gamma_{ste}^{-1}(\mathbb{G}_I^*)$ □

Theorem 5.16. *Let $\mathbb{G}_I^*$ be a BIAFG, If $\gamma_{ste}^{-1}(\mathbb{G}_I^*)$-set exists in $\mathbb{G}_I^*$ then, then prove that*

$$\gamma_{se}^*(\mathbb{G}_I^*) + \gamma_{se}^{-1}(\mathbb{G}_I^*) = |S|$$

Proof. The proof is obvious. □

Theorem 5.17. *A strong BIAFG $\mathbb{G}_I^*$ contains an inverse edge dominating set.*

Proof. Given a Bipolar Intuitionistic Anti Fuzzy Graph $\mathbb{G}_I^*$ with strong edges. Let $\mathtt{F}$ be a minimal edge dominating set of $\mathbb{G}_I^*$. Then by known theorem If $\mathtt{F}$ is a minimal edge dominating set, then $\tilde{E}' - \mathtt{F}$ is also an edge dominating set of $\mathbb{G}_I^*$. Hence $\tilde{E}' - \mathtt{F}$ is an edge dominating set. Thus every strong Bipolar Intuitionistic Anti Fuzzy Graph $\mathbb{G}_I^*$ contains an inverse edge dominating set. □

4. Conclusion We have discussed about different types of domination in bipolar intutionistic fuzzy graphs, The idea of double domination on bipolar Intuitionistic Anti fuzzy graph was presented in this article and discussed some of its properties as well as some results related to Secure dominations in BIAFG. We can extend our research work to total domination other various types of bipolar intuitionistic fuzzy graphs.

References

[1] Akram, M., Bipolar fuzzy graphs, Inform. Sci. 181, (2002),5548-5564 .

[2] Akram, M., Bipolar fuzzy graphs with applications, Knowledge Based Systems 39 (2013),1-8.

[3] Akram, M. and Wieslaw, A.,Regular Bipolar fuzzy graphs, Neural Comput. and Applications., 21 (2012),S197-S205.

[4] Cockayne E.J., Favaron, O and Mynhardt, C.M., Secure domination, weak roman domination and forbidden subgraphs, Bulletin of the Institute of Combinatorics and its Applications 39(2003), 87-100.

[5] Karunambigai, M.G., Akram, M.,Palanivel, K., Domination in bipolar fuzzy graphs, International conference on fuzzy systems, (2013), 7-10.

[6] Karunambigai, M.G., Sivasankar, S., Palanivel, K., Secure domination in fuzzy graphs and intuitionistic fuzzy graphs, Annals of Fuzzy Mathematics and Informatics 14 (2017), 419-431.

[7] Merouane H.B and Chellali, M., On secure domination in graphs, Information Processing Letters 115(2015), 786-790

[8] Somasundaram, A., Domination in fuzzy graphs-II, Journal of Fuzzy Mathematics 20 (2000), 281 - 289.

[9] Zadeh, L.A., Fuzzy Sets, Information and Control 8, (1965) 195-204.

[10] Zhang, W.R., Bipolar fuzzy sets and relations,A computational framework for cognitive modeling and multi agent decision analysis, Proc. of IEEE. Conf., (1994) 305-309.

[11] Zhang, W.R., Bipolar fuzzy sets, Proc. of FUZZ-IEEE, (1998).

[12] Jahir Hussain R, Mujeeburahman TC, Dhamodharan D. Secure domination in bipolar fuzzy graphs. Advances and Applications in Mathematical Sciences.2022;22:1-11.

[13] Jahir Hussain R, Mujeeburahman TC, Dhamodharan D. Some edge domination parameters in bipolar hesitancy fuzzy graph. Ratio Mathematica. 2022;42:157-166.

[14] Ezhilmaran D and Sankar K, Morphism of bipolar intuitionistic fuzzy graphs, Journal of Discrete Mathematical Sciences and Cryptography 18(5), 605-621, (2015).

[15] Sankar K and Ezhilmaran D, Bipolar Intuitionistic Fuzzy Graphs with Applications, International Journal of Research and Scientific Innovation 3, $44 - 52$, (2016).

[16] Kanimozhi A, Rani P and Muthuraj R, Domination in Bipolar Anti Fuzzy Graphs, Advances and Applications in Mathematical Sciences, $2053 - 68, (2022)$.

[17] Atanassov K T, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1), 87-96, (1986).

[18] Jabarulla M.M and Banu K.A., Anti Intuitionistic Fuzzy Graph, Advances and Applications in Mathematical Sciences 20(4), 597-604, (2021).

[19] Nagoor Gani A, Anupriya S, Vankatesh S, Double Domination on Antifuzzy Graphs, Design Engineering, 7, 9794-9805, (2021)

[20] Muthuraj R and Sasireka A, Domination on Anti-Fuzzy Graph, International Journal of Mathematical Archive 9(5), 82-92, (2018).

[21] Muthuraj R, Vijesh V.V and Sujith S, Domination on Intuitionistic anti Fuzzy Graph, Advances in Mathematics: Scientific Journal 8, 484-492, (2019).

[22] Parvathi R and Karunambigai M, Intuitionistic fuzzy graphs, Computational Intelligence Theory Applications 38,139-150, (2006) 5(2), 210-214, (2012).

[23] Mahioub M. Q. Shubatah and Hameed Al-Kohily, Connected Perfect Domination in Bipolar fuzzy graphs, IJRDO - Journal of Mathematics, 7(10),(2021)

[24] Muthuraj R and Kanimozhi A, Double domination on bipolar fuzzy graphs with strong edge and its properties, Malaya Journal of Matematik, 8(4), 1583-1586, (2020)