Original Research Article

Numerical simulation of heat transfer in a wavy enclosure with and without the presence of fins

Abstract

This study focuses on numerically simulating heat transfer in an enclosure with a wavy shape, both with and without the presence of fins. The research paper specifically investigates the case of a single vertical fin attached to the lower heated wall of the enclosure. The lower wall is maintained at a constant cold temperature, while the upper wall is kept cold, and the wavy walls are assumed to be constant heated temperature. Additionally, a magnetic field of certain strength is applied parallel to the x-axis. The flow inside the enclosure is characterized by a Prandtl number of 0.71. The researchers conducted a parametric study to examine the impact of the Richardson number on the fluid flow and heat transfer characteristics within the enclosure. The findings indicate that as the fluid flow and heat transfer characteristics within the enclosure. The findings indicate that as the Hartmann number increases (while keeping the Hartmann number constant) the heat transfer rate is enhanced. The results are presented through various graphical representations, such as streamlines, isotherms, velocity and temperature distributions, as well as the local Nusselt number. To validate their findings, the researchers compare their results with previously published works in the field.

Keywords: Mixed convection, Wavy enclosure and Fin.

Nomenclatures			
W	Enclosure height and width	x	horizontal coordinate
A	amplitude	X	dimensionless horizontal coordinate
N	number of oscillations	y	vertical coordinate
B	dimensionless fin thickness	Y	dimensionless vertical coordinate
g	gravitational acceleration		
h	fin position	Greek symbols	
H	dimensionless fin position	α	thermal diffusivity
L	dimensionless fin length	ρ	local density
Nu_L	local Nusselt number	β	coefficient of thermal expansion
Pr	Prandlt number	σ	electrical conductivity
Re	Reynolds number	μ	dynamic viscosity
Ha	Hartmann number	υ	kinematic viscosity
Ri	Richardson number	θ	dimensionless temperature
B_0	Magnetic field strength	Subscripts	

Gr	Grashof number	c	cold
u, v	velocity in *x* and *y* direction respectively	h	hot
U, V	dimensionless velocity in *X* and *Y* direction respectively		

1. Introduction

Computational fluid dynamics (CFD) involves using numerical computations to approximate real-world systems, aiming to enhance the understanding of their performance. Engineering rely on CFD codes that can generate accurate and precise results by discretizing the system into finite grids. CFD encompasses a wide range of applications, from automated engineering design techniques to replacing experimental investigations with detailed solutions of the Navier-Stokes equations for the analysis of intricate fluid flows. Extensive research has been conducted on various cavity shapes in the field of heat transfer. Bilgen [1] numerically investigated natural convection heat transfer inside a differentially heated cavity with a horizontal fin attached to the hot wall. The study revealed that the heat transfer rate was minimized when the fin was positioned at or near the middle of the heated wall. Shi and Khodadi [2] studied steady laminar natural convection heat transfer in a differentially heated square cavity with a thin fin on the hot wall. They discovered that higher Rayleigh numbers resulted in enhanced heat transfer, regardless of the fin's position or length. Ben-Nakhi et al. [3] conducted a numerical investigation on conjugate natural convection in a square enclosure with an inclined thin fin of arbitrary length. M. Fayz-Al-Asad et al. [4] examined natural convection flow in a hexagonal enclosure with a vertical fin, while in another study [5], they explored MHD free convection heat transfer with a vertical fin in a square wavy cavity. Xu et al. [6] studied the effect of fins and their height on the transition of natural convection flow in a cavity, observing a transition from steady to periodic unsteady flow near the finned wall at a critical Rayleigh number dependent on the fin length. Gdhaidh et al. [7] analytically investigated the enhancement of natural convection heat transfer in a closed enclosure using parallel fins, noting that increasing the number of fins led to a decrease in the maximum temperature of the heat source. Elatar et al. [8] performed a numerical study on laminar natural convection inside a square enclosure with an adiabatic horizontal wall and a single horizontal fin attached to different enclosure with an adiabatic horizontal wall and a single horizontal fin attached to different positions with varying lengths on the hot wall. They examined the influence of Rayleigh number, fin lengths, and fin positions on the fluid flow structure and heat transfer characteristics. Finally, Fayz-Al-Asad et al. [9] analyzed the effect of fin length and location on natural convection heat transfer in a wavy cavity. Saha et al. [10] conducted research on mixed convection heat transfer in a lid-driven cavity with a wavy bottom surface.

Based on the literature review conducted and the author's understanding, there is a scarcity of research focusing on wavy enclosures with vertical fins. Furthermore, no existing studies have numerically simulated mixed convection heat transfer in wavy enclosures, both with and without fins. Hence, the aim of this current study is to investigate the enhancement of heat transfer rate in fluid flowing through wavy enclosures, considering the presence of fins as well as without fins.

The upcoming section of this chapter is organized as follows. To begin with section 1 introduces the specific physical configurations that are relevant to the present study. Subsequently, section 2 provides a detailed discussion on the mathematical model chosen for this research, including the governing equations and boundary conditions. The numerical scheme employed in this study is elucidated in section 3.

2. Physical Configuration

Figure 1 illustrates the physical model and boundary conditions of the current problem being investigated. It depicts a two-dimensional wavy enclosure with a side length W. The bottom and to walls of the enclosure is maintained at a constant cold temperature T_c,. The two sided wavy walls are heated temperature T_h(where$T_h > T_c$). Additionally, a heated fin with a length (l) and thickness (b) is attached to the bottom wall at a position (h). The top wall lid moves left to right with constant velocity $u = U_0$ and bottom wall lid moves right to left with constant velocity $u = -U_0$.

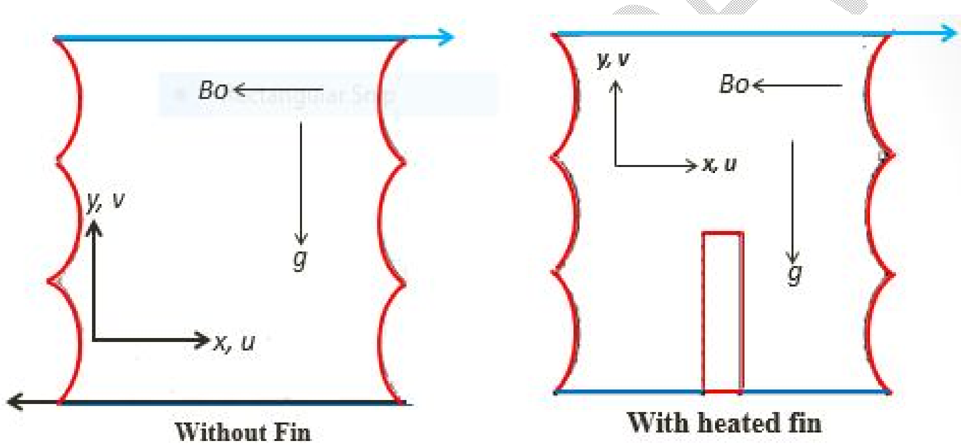


Figure 1: Schematic view of the enclosure with considered in the present work

3. Mathematical Formulations

The governing dimensional differential equations can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \sigma {B_0}^2 v + g\beta(T - T_c) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

The variables used in the mathematical model are as follows: u and v represent the velocity components in the x and y directions, respectively. p denotes the pressure, ρ represents the density, μ is the dynamic viscosity, β is the coefficient of thermal expansion, σ is the electrical conductivity, B_0 signifies the magnetic field, T indicates the temperature, g represents the gravitational force, and α represents the thermal diffusivity.

Dimensional boundary conditions:

On the top wall: $u = U_0, v = 0, T = T_c$
On the bottom wall: $u = -U_0, v = 0, T = T_c$
On the left wavy wall: $u = 0, v = 0, T = T_h$
On the right wavy wall: $u = 0, v = 0, T = T_h$

For the Fin surface**:** $0 \leq x \leq l$; $x = h + \frac{b}{2}$ and $x = h - \frac{b}{2}$; $u = v = 0$

By employing the subsequence set of dimensionless parameters, the original governing equations in their dimensional form can be transformed into dimensionless equations.

$$X = \frac{x}{W};\ Y = \frac{y}{W};\ U = \frac{uW}{\alpha};\ V = \frac{vW}{\alpha};\ P = \frac{pW^2}{\rho\alpha^2};\ \theta = \frac{T - T_c}{T_h - T_c};\ H = \frac{h}{W};\ L = \frac{l}{W} \text{ and } B = \frac{b}{W} \quad (5)$$

Incorporating the dimensionless variables into equations (1) to (4), where X and Y represent the coordinates along the horizontal and vertical directions, respectively, U and V denote the dimensionless velocity components in the X and Y directions, θ represents the dimensionless temperature, and P signifies the dimensionless presser, the resulting set of equations in their dimensionless for can be obtained.

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \quad (7)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) - Ha^2 PrV + RiPr\theta \quad (8)$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \quad (9)$$

The transformed dimensionless boundary conditions are as follows:

On the top wall: $U = 1, V = 0, \theta = 0$
On the bottom wall: $U = -1, V = 0, \theta = 0$
On the left vertical wall: $U = 0, V = 0, \theta = 1, 1 - A(1 - \cos(2N\pi X))$

On the right vertical wall: $U = 0, V = 0, \theta = 1,\ A(1 - \cos(2N\pi X))$

For the fin surface**:** $0 \leq X \leq L;\ X = H + \frac{B}{2}$ and $X = H - \frac{B}{2};\ U = V = 0$

Nusselt Number

First, the heat transfer by conduction was equated to the heat transfer by convection

$$h^{*}\Delta T = -k\frac{\partial T}{\partial n} \tag{10}$$

where n is the normal distance to coordinate surface. By introducing the dimensionless variables, defined in equation (5) into equation (11), local Nusselt number is defined as:

$$Nu_L = -\frac{\partial \theta}{\partial N}\bigg|_{Surface} \tag{11}$$

4. Numerical Technique

The nonlinear governing partial differential equations are transferred into a system of integral equations by using the Galerkin weighted residual finite-element method. The integration involved in each term of these equations is performed with the aid of Gauss quadrature method. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations with the aid of Newton's method. Lastly, Triangular factorization method is applied for solving those linear equations.

4.1: Program validation

Comparison between streamlines and isotherms for graphical solution are shown in Figure 2. It is clear from the figures that there is an excellent agreement between two results.

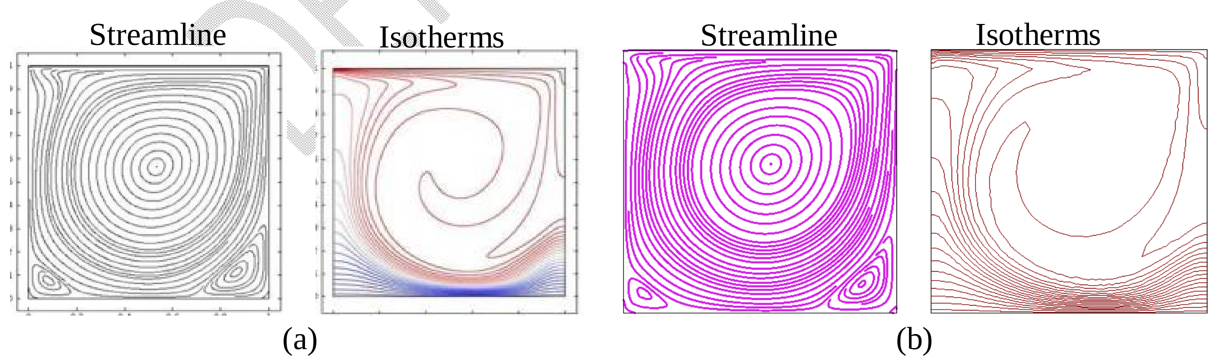
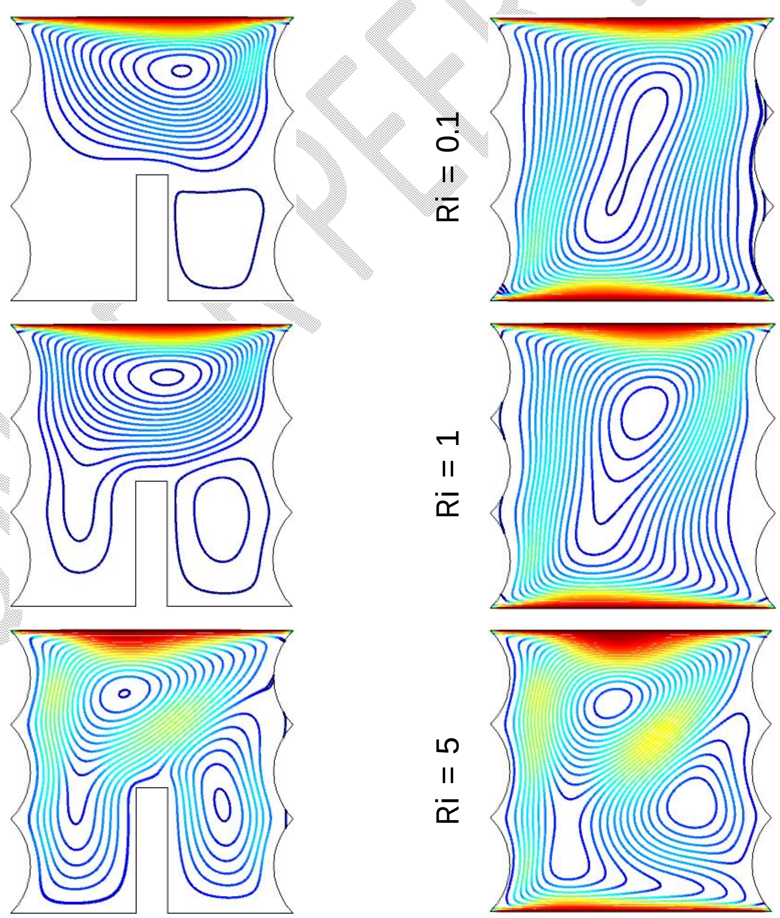


Figure 2: Comparison between isotherms and streamlines for numerical solution of (a) Shaha et al. [10] $Ra = 10^5$, $Re = 100$, $Ha = 50$ and (b) Present Study.

5. RESULTS AND DISCUSSION

This section presents the numerical simulation results of mixed convection heat transfer in a wavy enclosure, both without a fin and with a fin. The results are obtained for various Richardson numbers, considering the presence and absence of a fin. The findings of this parametric study are illustrated in Figures 3 to 7.

Figure 3 depicts the influence of the Richardson number $(Ri = 0.1, 1, 5, 10)$ on the streamlines while Figure 4 demonstrates the effects on the isotherms in the present numerical investigation. For the given parameters $Ha = 50$ and $Pr = 0.71$, the Richardson number quantifies the relative importance of combined forced and free convection. Analyzing Figure 3, it is observed that when $Ri = 0.1$, the buoyancy force inside the enclosure is considerably strong. This results in the formation of a single vortex generated by the movement of the lid wall when a fin is present, while without a fin, the streamlines become intensified near the upper and lower walls, forming a vortex in the middle region of the enclosure. Furthermore, for a Richardson number of $Ri = 1$, the flow structure exhibits similarities to the case of $Ri = 0.1$, with the presence of two vortices inside the enclosure. One vortex is dominant, while the other is a minor vortex. A the Richardson number increases further $(Ri = 5$ and $Ri = 10)$, the buoyancy force becomes even more pronounced. In this scenario, two vortices are observed, with one moving along the right half and the other along the left half of the enclosure.



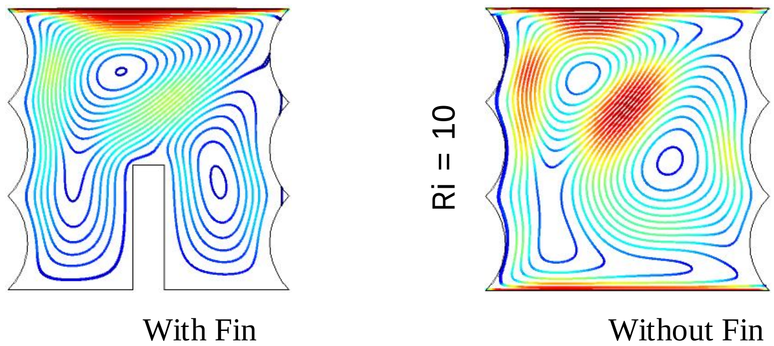
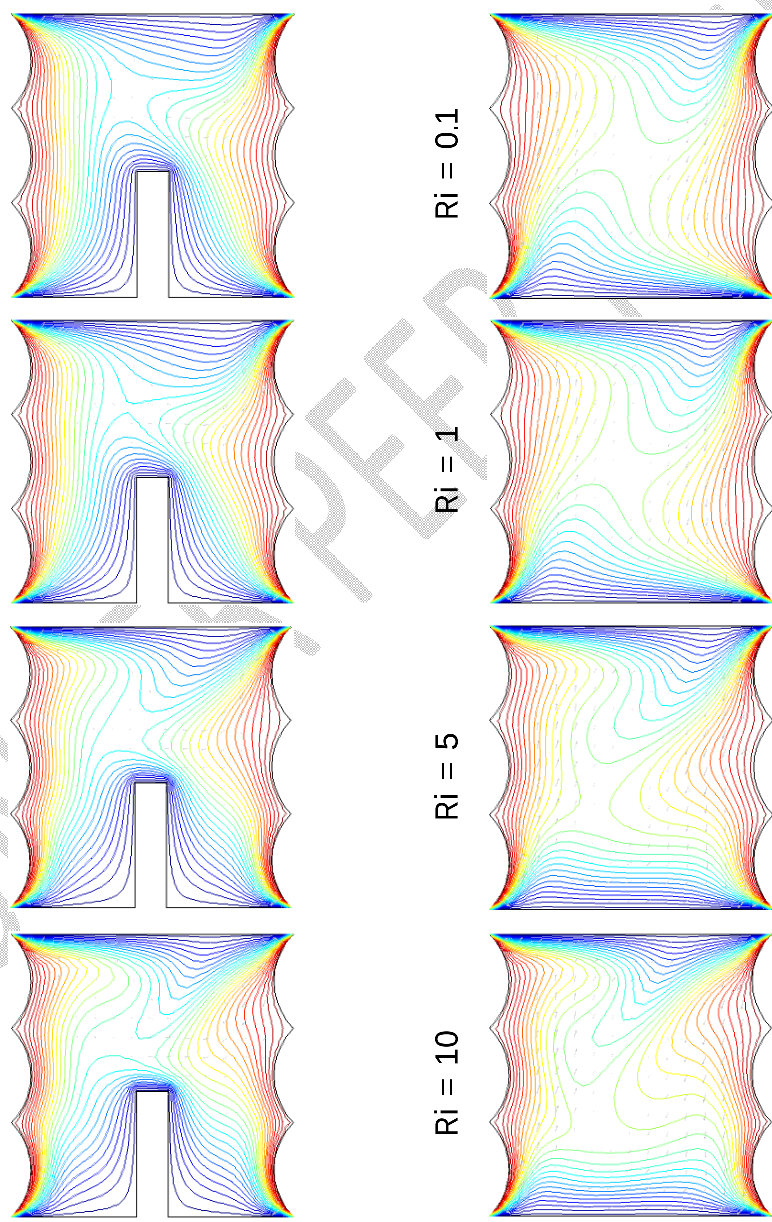


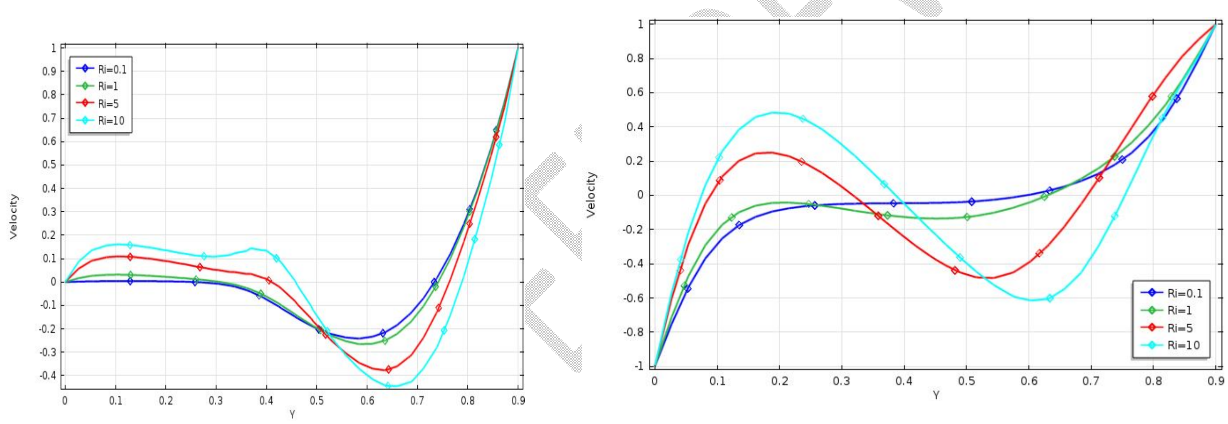
Figure 3: Streamlines with fin for Richardson number $Ri = 0.1,1,5,10$ while $Ha = 50$ and $Pr = 0.71$.



With Fin   Without Fin

Figure 4: Isotherms with fin for Richardson number $Ri = 0.1, 1, 5, 10$ while $Ha = 50$ and $Pr = 0.71$.

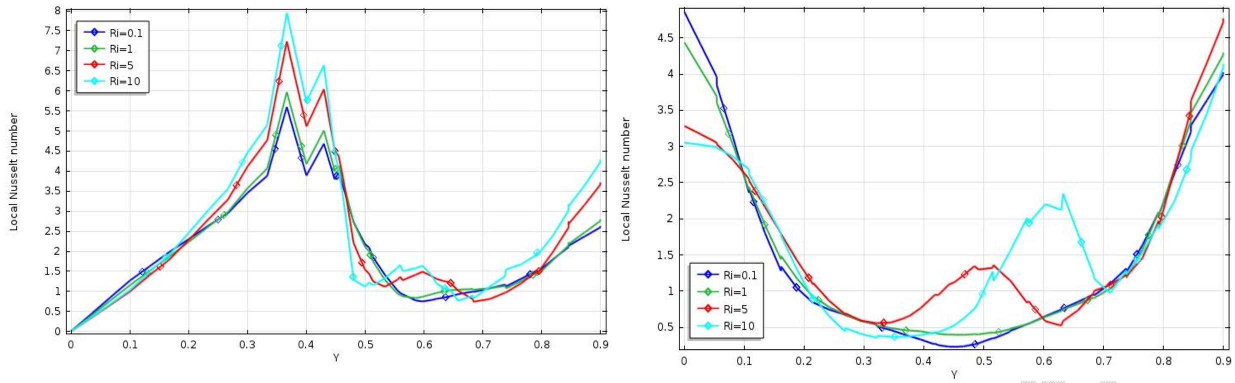
The observed phenomenon can be attributed to the significant influence of the Richardson number on the buoyancy force, which plays a crucial role in shaping the flow field. As indicated by the isotherms shown in Figure 4, the presence of a thick thermal boundary layer is evident near the heated walls (top and bottom walls) and the surface of the fin when the Richardson number is low ($Ri = 0.1$). However, as the Richardson number increases ($Ri = 10$), these thermal boundary layers becomes thinner. Additionally, the curvature of the isotherms intensifies with higher Richardson numbers, causing the heat lines to converge towards the vertical wavy walls and the fin surface. This behavior signifies an enhancement in heat transfer through convection mechanisms.



(a) With Fin   (b) Without Fin

Figure 5: Variation of the velocity components along the horizontal line for different Richardson numbers $Ri = 0.1, 1, 5, 10$, while $Re = 100$, $Gr = 10000$, $Ha = 50$ and $Pr = 0.71$.

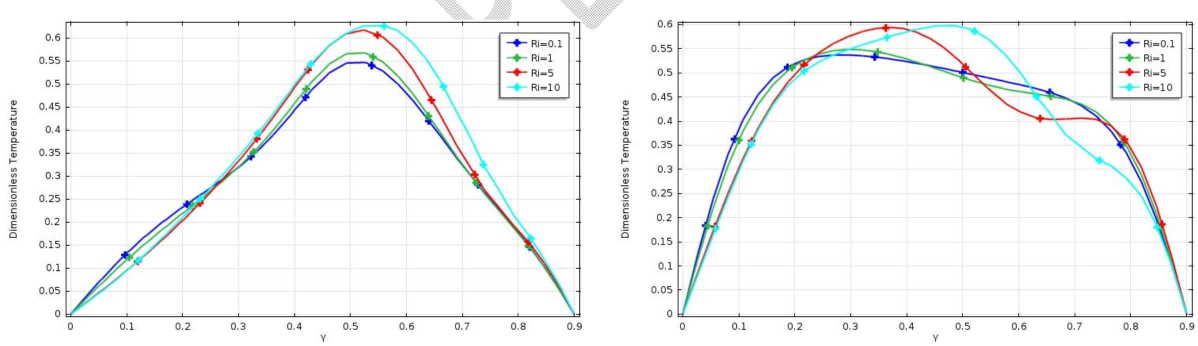
The impact of the Richardson number on the vertical component of velocity profiles along the horizontal center line of the enclosure is illustrated in Figure 5 for , $Ha = 50$ and $Pr = 0.71$. From the figure, it is observed that lower values of the Richardson number result in smaller changes in the velocity profiles. However, as the Richardson number increases, the velocity profiles undergo larger variations. Furthermore, with an increase in the Richardson number, the absolute values of both the maximum and minimum velocity also increase.



(a) With Fin (b) Without Fin

Figure 6: Variation of the local Nusselt number along the horizontal line for different Richardson numbers $Ri = 0.1, 1, 5, 10$, while $Ha = 50$ and $Pr = 0.71$.

Figure 6(a) displays the variation of the local Nusselt number along the horizontal wall for different Richardson numbers in the presence of a fin. The figure reveals that while the changing rate of the local Nusselt number is similar for each Richardson number, it differs from one Richardson number to another. Additionally, the local Nusselt number exhibits two concave-up regions and one concave-down region for every Richardson number. In Figure 6(b), the distribution of the local Nusselt number along the horizontal wall of the enclosure for varying Richardson numbers is shown. It is evident from the figure that the local Nusselt number increases as the Richardson number increases across a significant portion of the bottom heated wall. However, at the midile section of the bottom wall, the local Nusselt number remains zero and does not vary significantly with changes in the Richardson number.



(a) With Fin (b) Without Fin

Figure 7: Variation of the dimensionless temperature along the horizontal line for different Richardson numbers $Ri = 0.1, 1, 5, 10$, while $Re = 100$, $Gr = 10000$, $Ha = 50$ and $Pr = 0.71$.

Figure 7(a) illustrates the variations of the dimensionless temperature along the horizontal wall for different Richardson numbers in the presence of a fin, while Figure 7(b) represents the same for the case without a fin. It can be observed from both figures that for all Richardson numbers, the dimensionless temperature profiles exhibit a concave-up shape.

6. Conclusion

Numerical simulation of heat transfer in a wavy enclosure with and without the presence of fins. Very good agreements were observed with and without fin. Based on the findings, it can be concluded that:

- Velocity, temperature as well as heat transfer rate increased with the increasing Richardson number.
- For all cases considered, two counter rotating eddies were formed inside the enclosure regardless the Richardson number.
- Heat transfer is significantly enhanced when fins are heated compared to when they are not present.

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