Original Research Article

ON THE EXISTENCE OF A HOLLOW NEUTRON [STAR]

Abstract:

Based on the Ni's solution of Einstein Field Equations (EFE) we discuss the possibility of formation of a new type Neutron Star in which case the maximum density appears at a distance far from its center (not at the center for normal neutron star case). As the inner core of such Neutron Star is a hollow sphere consideration of vacuum energy at the core offers the possibility of two types neutron star models — one with no exotic phase at the core and another one with a significant portion of it in the new phase. Ni's solutions of EFE provide a new type neutron star model with hollow sphere in the inner core without violating the general relativity conditions. Based on the properties (i.e., findings of Ni's solutions) : a) void inside the center of the neutron star emerges naturally, and b) the solution leads to a mechanism for neutron stars to avoid collapse into black holes but remain regular, this author suggests the existence of a hollow neutron star is possible. If such type neutron stars really exist then it is also proposed that this new type Neutron Star can be detected through observation of electromagnetic counterpart by LIGO and VIRGO. Some problems related to this hollow neutron star are [discussed].

Key Words : Field Equations, Exact solutions, Relativistic stars, Neutron star core, Asymmetric [matter]

1. INTRODUCTION

In 1915 Einstein General theory of Relativity (GTR) introduces the idea that gravitation is not a force, rather it is a consequence of curvature in a four dimensional space-time [1] i.e.,matter tells space how to curve and space tells matter how to move [2].

Thus solutions of Einstein's Field Equations (EFE) of general relativity provide the picture of space-time which relate the Einstein tensor to stress-energy tensor representing the distribution of energy, momentum and stress in the space-time manifold. The fact is that the Einstein's field equations alone are not enough to determine the evolution of a gravitational system in many cases where dependency on the dynamics of matter and energy are relevant i.e. on the gravitational field. Since the Einstein's field equations are a system of coupled non-linear partial differential equations, several effective techniques / approximations offer different exact solutions (Lorentz metrics) which are comfortable to express a physically realistic stress-energy tensors which are in general found in complex, non-physical stress in space. This means that such types of materials that would go into the makeup of such spaces are seldom found in nature (practically impossible to create such situation on the earth), it is frequently desired to understand the geometrical properties of such space of a given mass distribution. So, exact solutions of Einstein's field equations play important role in physical problems[3].

In the year 1916 Schwarzschild [4] first gave the solution (interior and exterior) of the Einstein's field equation with a prediction on the existence of a black-hole. This creates interest on studies of stellar compact objects in the realm of relativistic astrophysics. Searching for the exact solutions and the best of these solutions provide a limit on certain physical parameters (such as mass, radius, etc.) as well as the behavior of gravitational field. For this reason the construction of exact solutions, therefore, can be used for physical modeling and there after to predict the measurable quantities about physical systems of compact objects such as white dwarfs, neutron stars, etc.

The study of neutron stars contributes to our knowledge of fundamental physics, stellar evolution, astrophysics and cosmology. It helps us unravel the mysteries of the universe and provides insights into the nature of matter, gravity, and the extreme physical conditions that exist in our universe. Neutron stars play a crucial role in various astrophysical phenomena, such as pulsars (rapidly rotating neutron stars that emit beams of electromagnetic radiation) , X-ray binaries (systems where a neutron star accretes matter from a companion star) and magnetars (neutron stars possess interior strong magnetic fields). Studying these systems helps us to understand processes like stellar evolution, accretion physics, physics of matter under super strong magnetic field and the dynamics of compact binary systems. Neutron stars also provide insights into the overall structure, evolution, and dynamics of galaxies and the universe as a whole. This work investigates the possible existence of “hollow neutron stars”. Studying hollowneutron stars, or neutron stars with a maximumdensity located at a distance from their center, would be of significant interest and importance because it would challenge our current understanding of the stellar structure and the behavior of matter under extreme conditions.

The remainder of this paper is organized as follows: In sect.2 we describe the Ni’s solutions of Einstein’s Field Equations (EFE) used in this work. The concept of “Vacuum Energy” is presented in Sect.3. The details of Neslušan’s work on Ni’s solutions are discussed in Sect.4. The comparison of normal neutron star and hollow neutron star is presented in Sect.5. Physical significance of the existence of hollow neutron star is described in Sect.6.Finally, we briefly conclude and discussed future prospects of the hollow neutron star in Sect.7.

2. Ni's Solution ofEFE

In 2011 Ni published the result of his solution of the Einstein's Field Equations, based on Tolman-Oppenheimer-Volkoff equations [5,6], for the structure of stable neutron stars [7]. Considering the general static metric for stars (inside neutron stars) with spherical symmetry

$$ds^2 = -e^{\lambda} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2 + e^{\nu} dt^2 \quad (1)$$

and the auxiliary function $u = r (1 - e^{-\lambda})/ 2$ (2)

he obtained the field equations (in c = G = 1 units)as

$$(du / dr) = 4\pi\rho r^2 \quad (3)$$

$$(dp / dr) = -\{ (p + \rho) / r (r - 2u)\} (4\pi p^2 r^3 + u) \quad (4)$$

Where ρ and p are the macroscopic energy density and pressure measured in proper coordinators, respectively. Note that equ. (3) and (4) are the Tolman-Oppenheimer-Volkoffequations.

For neutrons, inside the neutron star, with rest mass m_o obeying the Fermi Dirac statistics, he used parametric form of ρ and p with the parameter 't" related to the Fermi momentum $P_F$as

$$t = 4 \ln \{ P_F / m_o c + [1 + (P_F / m_o c)^2]^{1/2}\} \quad (5)$$

energy density $\rho = K (\sinh t - t)$ (6)

$$p = ⅓ K (\sinh t - 8 \sinh t/2 + 3 t) \quad (7)$$

where $K = \pi m_o^4 c^5 / (4h^3)$ and the density of the particle number $n = N/V = 8\pi P_F^3 / 3 h^3$.

Putting the boundary condition for neutron star with the initial values $u = u_i$, $p = p_i$ at r = 0 to the radius r = r_o where ρ = 0 at the surface boundary of the star he obtained a new type solution i.e. there is a void at r = 0 (i.e. at the centre of the neutron star) though the solutions satisfied the Einstein's equation i.e. from r = 0 to r = r_i , ρ = 0 and p = 0. This means that there is no particles and thus pressure is zero in this void region. Beyond this void, from r = r_i to r =r_o

ρ ≥ 0 and p ≥ 0 the density and pressure increase first from zero at r = r_i . After reaching a maximum,ρandpthendecrease.Attheouterboundaryr=r_o,pandρreachzeroi.e.,

a) after reaching a maximum, p decreases to zero at the outer radius r_o ,

b) the structure parameter 'ρ' increases from zero at the inner radius r_i to a maximum at somewhere and then decreases to zero at the outer radius r = r_o (seefigure1)

c) The outer physical surface of these stars (i.e., neutron stars) is always situated above the corresponding event horizon.

The significant findings of Ni's study can be expressed as :

i) His solutionsfor hollow neutron stars are without maximum mass limit (i.e., no upper mass limit of these objects) which do not depend on the special property of the equation of state for the starmatter.

ii) The solutions with void inside the center of the star emergenaturally,

iii) In the case when the void is very small, even then the particle number 'N' and the mass 'm' of the star do not have a maximumlimit.

iv) His solution leads to a mechanism for neutron stars to avoid collapse into blackholes but remainregular.

3. Vacuum EnergyConcept

The recent observations of gravitational waves (GWs) [8] open a new direction for exploring fundamental physics. It has been speculated earlier that there may be a new phase of nuclear matter at the core of the neutron stars [9 – 11]. If such a phase exists in the interior of the neutron star then

(a) it is expected vacuum energy (VE) at the center of it[12];

(b) the presence of vacuum energy in the inner core of neutron star indicates a new QCD phase at large densities as well as appearance of vacuum energy in the equation of state (EOS) for a newphase.

Not only that, this will make neutron star the only known objects where vacuum energy might make up a non-negligible fraction of the total mass [13].

Using the solutions of the Tolman-Oppenheimer-Volkoff (TOV) equations [5,6] for spherically symmetric and mass distribution of the star (neutron star) and assuming the presence of vacuum energy in the inner most layer of the neutron star Csáki et al [13] recently investigated for finding the contribution of a vacuum energy in exotic high density phases in different EOSs for modeling and the maximum stable mass of neutron stars. In that study their significant findings are:

i. the compactness of the neutron star can be characterized by the radius of a neutron star with a fixed mass ;

ii. For a given mass, there are two possible types of stars — a) one with no exotic phase in the core and another with a significant portion of the star in the new phase.

In other words, introducing the non-zero vacuum energy in the interior of neutron star can create a disconnected branch of stable neutron star solutions that allow the possibility of having two neutron stars of the same mass but with significantly different radii.

4. Contribution of Neslušan's work

In the conventional structure of neutron star mass is distributed from the surface boundary to its center. No void or hollow is at the center of the neutron star whereas Ni's solution corresponds to neutron stars with a void at their centers. In general, the mass density and pressure of a normal neutron star increase more and more as one proceeds from surface to its center i.e., density and pressure both attend the maximum value at the center. The structure of neutron star nuclear matter at and around the centre is not yet known fully because of various factors such as asymptotic freedom, de-confinement of quark matter, effect of strong internal magnetic field, etc. This means that uncertainties remain in the equation of state of nuclear matter inside the core of neutron star, resulting which the maximum mass of neutron star still remains unsettled. Not only that, the different values of the maximum mass of neutron star appear depending on the nature of interaction among the constituents of nuclear matter considered.

In the case of Ni's study (e.g., a void exists inside the neutron star core) the mass density and pressure increase first from zero at the inner radius r_i (e.g., at the boundary of the interior hollow sphere to a peak (somewhere) and then decrease to zero at the outer radius r_o (i.e., outer surface of the hollow neutron star).

Based on the Ni's solutions of EFE Neslušan [14] estimated the distribution of matter insidethe star for constructing the neutron star model. The rest mass M_o (i.e. the sum of the rest masses of all neutrons constituting the object) of the neutron star he estimated using the traditional theory of these objects (i.e. Misner et al [2]) as

$$M_o = 4\pi m_n \int_{R_{in}}^{R_{out}} n\, r^2 / \sqrt{1 - 2u/r}\ .\ dr \qquad (8)$$

where n = n (r) the number density of neutrons given by the relation

$$n = (8\pi / 3 h^3)\,(m_n c)^3 \sinh^3 t/4 \qquad (9)$$

Using the Ni's solutions the significant results obtained by Neslušan are:

I. There is a variety of neutron stars constituted by the same number of neutronsi.e. for a given rest mass a variety of neutron star models can be constructed with the different other characteristics. For example, for the rest mass 0.669 $M_\odot$they obtained two neutron star models with the same distance r_o(see table I)[14]

Table 1. Model description

	NSmodel 1		NS model 2	
r_o (km)	r_{in} (km)	r_{out} (km)	r_{in} (km)	r_{out} (km)
0.001	2.824×10^{-11}	12.119	$8..981 \times 10^{-11}$	6.8558
0.01	2.824×10^{-8}	12.119	8.981×10^{-8}	6.8558
0.1	2.820×10^{-5}	12.119	8.926×10^{-5}	6.8538
0.5	4168×10^{-3}	12.159	1.0301×10^{-2}	6.6126
1.0	4738×10^{-2}	12.391	8.178×10^{-11}	4.7028

II. Neutron star model is a hollow sphere with a cavity in its interior and the gravitational attraction in the cavity is outwardoriented.

III. The models of compact objects are with no singularity other than the Big Bangtype singularity which can exist in the universe i.e. the conditions are realistic.

IV. The concept of hollow sphere, as already claimed by Ni, is able to a model of stable compact object of whatever large mass i.e. this model is valid not only for the fullsphere concept of neutron stars but also valid for in the astrophysics of realobjects

5. Normal Neutron Star Vs. Central Hollow Sphered Neutron Star

In 1967 the discovery of Pulsars [15, 16] provided the first evidence of the existence of neutron star. Pulsars are neutron stars neutron stars that pulses of radiation once per rotation. Neutron stars are a class of extremely dense, compact stars, though to be composed primarily of neutrons. Normal neutron stars are typically about 20Km (12 miles) in diameter having mass ranges between 1.8 $M_{\odot}$ and 1.97$M_{\odot}$but most are 1.35$M_{\odot}$. The average densities of these stars are extremely high — about 10^{14} g.cm^{-3}. The boundary surface of this star is solid, intermediate layers are mostly neutrons and are probably in the super-fluid, super-conductor states [17, 18]. It is not yet known definitely what is at the centre of the neutron star [19]. Theoretical studies suggest [20 – 22] hyperons, kaons, pions, quark matter are at the inner core of the normal neutronstar.

In 2011 Jun Ni [7] published the solution of Tolmann- Oppheneimer- Volkoff equations and suggested this new kind stable neutron star which has a hollow sphere with the inner physical surface and cavity inside. Ni's solution based compact stars are neutron stars having a hollow sphere at the inner core this star. This type compact stars are unknown today but observations of LIGO and VIRGO as well as space based NICER [23,24] will definitely detect this kind of compact stars and theoretical studies will increase our knowledge of understanding such type neutron stars. A comparison of physical properties of the normal neutron star and hollow sphere neutron star are given below :

Table 2. Comparative analysis between Normal and Hollow Neutron Star

Normal Neutron Star	Hollow Neutron Star
1) Normal massive neutron stars are formed from supernovaexplosions. 2) Solutions of EFE correspond to neutron stars without a void at their centers and offer maximum density in the core. 3) There is uncertainty in maximum mass limit ofneutron stars, exceeding which it would collapse into blackholes and disappear. As per observation the maximum mass of neutron star obtained till date is 2.14 $M_\odot$(PSR J0740+6620) [25,26]. On the otherhand, observations by LIGO and VIRGO claims the mass limit for the compact object detected in the binary merger GW190814 is in the range 2.5 – 2.67 $M_\odot$[24,27]. But the detected object is either the most massive neutron star or the least massive blackhole. Theoretical studies [28] extend the mass limit up to 3.2$M_\odot$. 4) The maximum mass limit of the neutron star depends on the choice Equation of States (EoS) of the starmatter.	1) Neutron stars with Hollow sphere are formed possibly from binary neutron starsmerger 2) Ni's solutions [7] of EFE correspond to neutron stars with a void at their centers and maximum density occurs beyond the core. 3) Ni's solutions show without maximum mass limit for TOV equations i.e. there is no upper mass limit of these objects. These compact objects (Neutron stars) can accommodate infinite number of neutron particles. Even in the case that the void is very small, the particle number N and the mass 'm' of the star do not have a maximumlimit. 4) Solutions without maximum mass limit do not depend on the special property of EoS of the starmatter.

5) In the case of normal neutron star matter density gradually increases from the surface boundary of the star and then reaching maximum at the centre of the star. 6) Asymptotic freedom behavior appears in the core of the star[29,30] 7) Exceeding the maximum mass limit normal neutron star collapses into black-hole anddisappear.	5) In this case the stars do not have a maximum mass limit which leads to a mechanism for neutron stars to avoid collapse into black holes and remain regular. 6) No appearance of asymptotic freedom phase in the interior of this neutronstar. 7) Matter density starts increasing from the boundary surface , after a few km away maximum density arises and then again decreases gradually upto the inner radius of the hollow sphere/ core.

6. Physical significance of the existence of hollow neutron star

Neutron stars are incredibly dense objects formed from the remnants of massive stars that have undergone gravitational collapse. The core of a neutron star is thought to consist predominantly of densely packed neutrons, hence the name. The density tends to be highest at the center of the star, gradually decreasing as you move outward. However, the exact distribution of density within a neutron star is still an active are of research, and variations and complexities may not yet be fully understood.

The first model of neutron star was worked out for the spherically symmetric, non-rotating compact object by Oppenheimer and Volkoff [5]. Their model includes the equation of states (published by Chandrasekhar [31]) for a pure neutron, extremely cold and degenerate gas. The main facts were:

a) There is no solution for a stable configuration of dead star which has no internal source of energy;

b) If the mass of the star exceeds a certain critical limit then the star has to collapse to its event horizon i.e., the non-existence of the stable configuration of very massive stars and their collapse to black hole is inevitable.

In order to deal with the above problem on the stable configuration of very massive stars Ni [7] considered in his model “ a finite star-centric distance” instead of “the integration in the center of the star" used in Oppenheimer-Volkoff model, and also the outer physical surface of the star which is always situated above the corresponding event horizon. The significant outcomes of the Ni's model are :

i) There is no upper mass limit (i.e., maximum) of the hollow neutron stars i.e., with the change of the void boundary the mass and particle number of this star can approach infinity.

ii) As the star accommodates infinite number of particles, thus the hollow neutrons stars with large masses can remain stable, resulting which these stars do not collapse into black holes.

iii) As the star is a hollow sphere i.e. cavity inside, the inner physical surface exists.

iv) The distribution of matter is spherically symmetric when observed by the observer located at the center of the star. But the same is no longer observed as spherically symmetric by that observed who is aside the center of the star in a curved space time of general relativity.

v) The energy density and pressure are positive i.e. non-zero and outward oriented gravitational attraction are effective in the upper layers of this star.

vi) It is argued (by Neslusan [14,32]) that the concept of hollow sphere should applicable to the models of real compact objects. This means that existence of hollow sphered neutron star is physically possible.

7. Conclusion

Regarding compact objects (such as white dwarf, neutron star) we generally understand that its internal density increases from the star's boundary surface towards the centre of the object and density is maximum at the core due strong gravitational attraction. Ni's solutions of EFE [7] provide a breakthrough idea in understanding the compact objects with hollow sphere in its interior at the core. Introducing the presence of vacuum energy in the inner cores of neutron stars Csáki etal [13] stress the existence of the new phase, due QCD phase transition , which requires a modification of the traditional concept of compact object i.e. a non-zero vacuum energy can create a disconnected branch of stable neutron star solutions which allow the possibility of having two neutron star models for a given masswith

significantly different radii —

a) Neutron star with no exotic phase in the core ,and

b) Another one with a significant portion of it in the newphase.

Neslušan's study [14], based on Ni's solutions, points out that the distribution of matter which is observed as spherically symmetric when the observer is in its center, is no longer observed as spherically symmetric by the observer aside the center in a curved space-time of general relativity. His analysis suggest that in the interior of such type hollow sphere neutron stars there is a critical distance in which the partial gravitational actions of both upper and lower layers equal each other but in the stable configuration this gravity is again balanced by the gradient of pressure which is always oriented against the gravity. Since the concept of hollow sphere model is applicable for stable compact object of whatever large mass, this is also valid only for full-sphere concept of neutron star, but also valid in the astrophysics of real objects [33] such as pulsars with extraordinarylargemasse.g.J1748-2021B,J1311-3430,B1957+20withmasses(2.5–2.9)$M_{\odot}$,(2.1–2.8)$M_{\odot}$and (2.1 – 2.7)$M_{\odot}$, respectively [34] ; ultra-luminous objects like quasars etc.

Detection of Hollow Neutron Star

Now question is detection of such hollow sphere neutron stars. As neutron star mergers chirp about vacuum energy (as suggested by Csáki etal [13]) so observations of electromagnetic counterparts from neutron star mergers through LIGO and VIRGO offer the access to detect such type objects. Numericalsimulations of neutron star mergers can provide the required information in this matter [35- 37]. Besides observation, theoretical studies of hollow sphere neutron stars can also increase our knowledge of this new type unknown relativistic compact object (neutron star) inastrophysics.

The significance of this central hollow neutron star is ----

a) for equilibrium mass distribution it has no upper limit mass i.e., without a maximum mass limit.

ii) These stars are with a void inside their centers. But one of the most important predictions of general relativity is the existence of maximum mass for any static matter configuration.

Therefore, several problems arise regarding hollow neutron stars :

i) As these neutron stars have no upper mass limit these cover up the mass gap region (2-5)$M_{\odot}$although masses of heaviest neutron stars, hybrid stars and high mass strange quark stars cover up to 2.6$M_{\odot}$[27] and Triaxial type [38,39] quark star up to 3.3$M_{\odot}$[40] provided the secondary compact star in the binary merger GW190814 be the heaviest neutron star. The remnant of GW190814 remains debated whether it is a heaviest neutron star or lightest black hole

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ii) A new evolutionary theory is required to explain the formation of black holes from massive neutron stars.

iii) The central part of the hollow neutron star is vacuum theoretically, but in realistic case may contain which type deconfined quark matter not yet certain.

iv) How to distinguish between the normal neutron star and hollow neutron star through their properties ?

v) Is the hollow neutron star be a good source of gravitational waves ?

vi) The shapes of the inside beam pattern, the outside beam pattern and the combined beam pattern of hollow accretion column depend on different values of mass and radius of the slowly rotating neutron star [41]. Do the shapes will be changed in the case of hollow neutron star ?

Ni studied the general relativistic field equations for neutron stars. It is found that there are solutions for the equilibrium mass distribution without an upper mass limit. The solutions correspond to stars with a void in the center. As the vacuum boundary changes, the star's mass and number of particles can approach infinity.If hollow neutron stars are detected, then it will be a new milestone in the compact object world.

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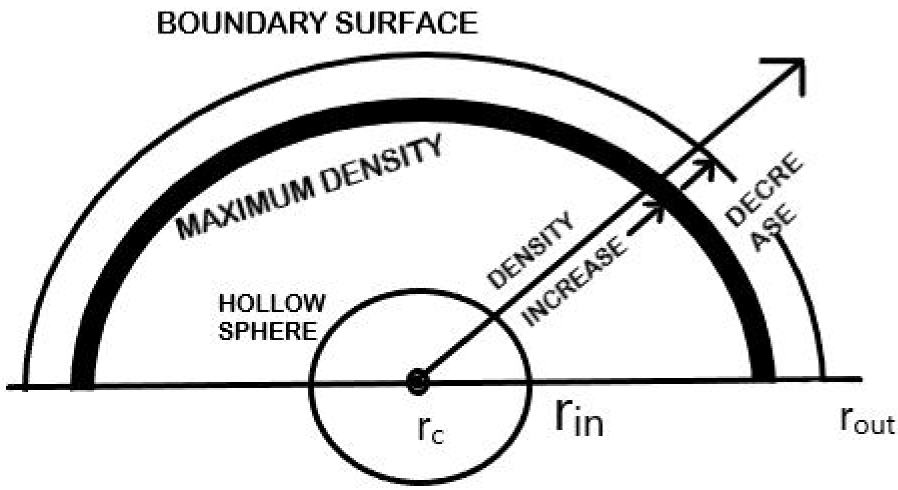
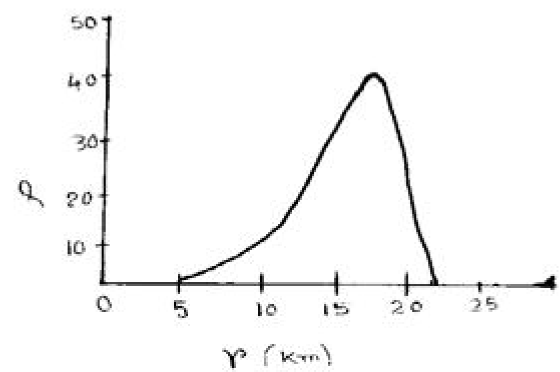


(A)



(B)

Fig.1 : Schematic diagram showing (A) the structure of a hollow sphere neutron star (not scaled). $\mathbf{r_c}$ →Center of the hollow neutron star. $\mathbf{r_{in}}$ → radius of the inner surface boundary and $\mathbf{r_{out}}$ → radius of the boundary surface of the hollow Neutron Star,

respectively. Dark black shaded area marked “maximum density” indicates mass density and pressure are maximum whose location is somewhere between inner and outer surface boundaries of the hollow neutron star.

(B) density ‘ρ’ – radius ‘r’ curve as per Ni’s solution which clearly indicates mass density and pressure with initial value zero at the inner surface boundary, then reach maximum value somewhere and again reach to zero at the boundary surface of the hollow neutron star as per Ni’s solution.