Data Article

Forecasting Currency in Circulation in Nigeria Using Holt-Winters Exponential Smoothing Method.

Abstract

Nigeria is a country located in West Africa. It is the most populous country in Africa, with a population of about 211 million. Nigeria is a federal republic with three tiers of government: federal, state, and local. Nigeria is a diverse country, with over 250 ethnic groups. This study aims to investigate the forecasting of currencies in circulation (CIC) in Nigeria using the Holt-Winters exponential smoothing method (Additive Holt–Winters Model and Multiplicative Holt–Winters Method). The forecasting data are collected from January 1960 to December 2022. The focus of the study is primarily to determine the optimal forecasting approach while considering the pertinent smoothing parameters and determine which of the forecasting method, either the multiplicativeHolt–Winters or additiveHolt–Winters is best in forecasting currencies in circulation (CIC) in Nigeria. Furthermore, the study offers prognostic insights regarding currencies in circulation in Nigeria. Three experiments were conducted with different parameters (α= 0.2, γ= 0.2, δ= 0.5; α= 0.4, γ= 0.3, δ= 0.1; α= 0.1, γ= 0.1, δ= 0.3) that were randomly generated. The experiment with α= 0.4, γ= 0.3, and δ= 0.1 parameters was the best model for forecasting the variable using both the Additive Holt-Winters model and the Multiplicative Holt-Winters method. Based on the comparison of the accuracy measures, this study concluded that the multiplicative Holt-Winters method outperforms the additive Holt-Winters model in all three measures: MAPE, MAD, and MSD. The multiplicative Holt-Winters method has a significantly lower MAPE (5.97894E+00) compared to the additive Holt-Winters model (9.55758E+02). The forecast of currencies in circulation in Nigeria was conducted using the multiplicative Holt-Winters method, and it found that currency in circulation in Nigeria continues to increase as it shows the upper trend.

Keywords: Forecast, Currency in Circulation, Nigeria, Additive Holt–Winters, Multiplicative Holt–Winters.

1. Introduction

Nigeria is a country located in West Africa. It is the most populous country in Africa, with a population of about 211 million (Atoyebi, 2023). Nigeria is a developing country with a growing population. Nigeria is also a resource-rich country with a lot of potential for economic growth. The Central Bank of Nigeria (CBN) is responsible for managing the amount of currency in circulation in Nigeria. The CBN does this by printing money, buying and selling foreign exchange, and setting interest rates. The CBN's goal is to keep the amount of currency in circulation at a level that will promote economic growth without causing inflation. The CBN issues various denominations of notes, which are then distributed to the general population through commercial banks. Presently, there are eight naira note denominations: ₦5, ₦10, ₦20, ₦50, ₦100, ₦200, ₦500, and ₦1000, no Nigeria coin is in circulation at the moment (Awe et al., 2010). The West African Currency Board (WACB) was responsible for overseeing the production of banknotes in Nigeria from 1912 to 1959. Before the establishment of this institution, Nigeria used different forms of currency, such as manilas and cowries (CBN Data Base, 2017). On July 1st, 1959, the WACB notes and coins were phased out when the CBN introduced Nigerian notes and coins. It was on July 1st, 1962, that Nigerian monetary policies were initiated to align with the country's new status (CBN Data Base, 2017).

The term "currency in circulation" (CIC) refers to the total amount of notes and coins circulating within an economy, representing the most liquid form of monetary aggregate (Albert, 2013). Notably, approximately 70% of Nigeria's reserves consist of CIC (Alvan, 2014). The demand for cash from both the general public and the banking system plays a significant role in determining the level of CIC, and changes in this demand serve as important indicators of the economy's monetization and demonetization (Albert, 2014). An annual increase in CIC was reported from

2009 to 2013, although there was a decline in the ratio of CIC to Gross Domestic Product (GDP) during the same period (CBN Annual Statistical Bulletin, 2013). The significance of CIC is evident in its ratio to nominal GDP and its contribution to the overall money supply, highlighting its relevance in any economy (Simwaka, 2006; Stavreski, 1998). Numerous studies have been conducted to investigate various aspects of CIC.

In a study by Ikoku (2014) analyzing the usage of daily, weekly, and monthly bills in Nigeria from January 2000 to December 2010, it was concluded that a combination of mixed models incorporating ARIMA and structural components, along with dummy and seasonal variables, yielded the most accurate results. The study revealed that factors such as exchange rates, interbank rates, seasonality, holidays, and elections significantly influenced the demand for money. Omekara et al. (2015) focused on monthly cash in circulation in Nigeria from January 2010 to December 2014 and determined that the SARIMA (0,1,0)x(0,1,1)12 model best matched the observed patterns in the series. Similarly, Alberto et al. (2012) and Albert et al. (2013) conducted comparable research on currency in circulation at central banks in Ghana and Europe. Bhattacharya and Joshi (2000) reviewed the currency in use in India and concluded that the money demand model and univariate modelling approaches were the most relevant. This current research will focus on CIC in Nigeria using holt–winter exponential smoothing.

Forecasting plays a crucial role in mitigating decision risk by providing insights into potential outcomes when the consequences of an action are significant but uncertain (Jones, 2018). Once the necessary data for a time series is collected, the analyst's next step involves selecting an appropriate forecasting model. Various statistical and graphical tools can aid in this selection process. One recommended approach is to create sequence plots of the time series. Sequence plots visually display the values of the data series over time, with the horizontal axis representing the passage of time and the vertical axis indicating the values (Smith et al., 2020). These plots facilitate the identification of key characteristics, such as trends and seasonality, which are further explored in the report. The presence or absence of these components significantly influences model selection and prediction accuracy.

The foundation of time-series forecasting lies in distinguishing between patterns and random errors within a time series. By identifying trends, long-term changes, and seasonality resulting from factors like variations in usage and demand, the goal is to separate the underlying pattern from the noise. Several time series forecasting techniques are available, including Moving Averages, Linear Regression with Time, and Exponential Smoothing. In this report, the focus is primarily to determine the optimal forecasting approach while considering the pertinent smoothing parameters and determine which of the forecasting method, either the multiplicativeHolt–Winters or additiveHolt–Winters is best in forecasting currencies in circulation (CIC) in Nigeria. Furthermore, this study offers prognostic insights regarding currencies in circulation in Nigeria.

2. Literature Review

The study conducted by the Reserve Bank of New Zealand, as evaluated by Cassino et al. (1997), provides insights into the forecasting of CIC. The researchers explored various variations of the autoregressive integrated moving average (ARIMA) model, including seasonal moving average (SMA) components and seasonal autoregressive (SAR) terms. However, they found that the ARIMA model lacked a solid theoretical economic foundation. Being a univariate model, it relied solely on historical data and had limited responsiveness to external shocks. This limitation resulted from the assumption of a stationary time series process centred around a specific mean.

Consequently, the predictive accuracy of the ARIMA model was compromised. Among the models examined, including the money demand model, ARIMA1 (SMA), and ARIMA2 (SAR), the ARIMA model performed the best, yielding the lowest percentage root mean square errors for out-of-sample forecasts. Nevertheless, the ARIMA model's effectiveness diminishes when applied to high-frequency data.

Ortiz (2015) conducted a study on the precision rate of the Holt-Winters model in forecasting exchange rates by using particle swarm optimization. In particular, an exponential smoothing method, through the use of the Holt-Winters Model, was used for the purpose of forecasting exchange rates. The search for optimal smoothing constants was performed via computer simulation using Particle Swarm Optimization. Results from the experiment showed that Particle Swarm Optimization is capable of calculating reliable smoothing constants, thereby producing forecasts that accurately determine the direction (i.e. fall or rise) of exchange rates. Moreover, the computed Mean Absolute Deviation (MAD) and Residual Standard Error (RSE) from the exchange rate forecasts based on actual observed data suggest that PSO may also be utilized to enhance forecasting precision.

In a similar vein, Bhattachrya and Joshi (2000) conducted a comprehensive investigation of forecasting methods for CIC in an emerging market, focusing on their significance in maintaining monetary stability in the Indian economy. While acknowledging the efficacy of the money demand model and univariate modelling as primary approaches, they observed that these models were ill-suited for high-frequency data, often producing unsatisfactory results compared to quarterly and annual data. The primary reason behind this discrepancy was the absence of income data beyond quarterly frequency in the money demand models. In theory, univariate models should effectively capture historical patterns and account for seasonal events that influence CIC, such as holidays that increase the demand for money. However, due to inadequate intra-month and lag specifications, these models failed to account for these impacts, leading to inaccurate estimates.

Lima, Gonçalves, and Costa (2019) have undertaken a study that examines the prediction of economic time series that display pronounced trends and seasonal patterns. The researchers have proposed an approach that uses Holt-Winters Exponential Smoothing to enhance the likelihood of capturing diverse patterns in the data and, thereby, augment forecasting performance. The study involves a comparative analysis of the accuracy of Holt-Winters models (additive and multiplicative) in forecasting and aims to provide novel insights into the methods employed through this approach. The selection of these methods is based on their capacity to model trends and seasonal fluctuations that are inherent in economic data. The models are fitted to time series of e-commerce retail sales in Portugal, and the study evaluates the accuracy of additive and multiplicative output.

To address the shortcomings associated with lag effects and enhance the model's accuracy, Bhattachrya and Joshi (2000) proposed an intra-month (weekly) univariate model incorporating two seasonal dummy variables. By incorporating day-of-the-month and month-of-the-year effects through these dummy variables, along with additional variables to account for the impact of holidays and festivals, the researchers aimed to improve the empirical specification. The findings of their study demonstrated significant effects at both the month-of-the-year and day-of-the-month levels. Moreover, the inclusion of variables to capture the influence of celebrations and holidays proved beneficial in simulating CIC in India. Bhattachrya and Joshi (2000) highlighted that the coefficients of the months in the model exhibited a significant increase in April, a month associated with major agricultural purchases.

Olayiwola and Atoyebi (2021) conducted an investigation on the Box-Jenkins Approach to Fuel Prices and Currency Strength in the International Market. The study used an ARIMA to predict the fuel prices and the strength of the South African rand in the international market, based on 35 years of monthly data. The fuel prices were observed to demonstrate an upward trend variation. Upon estimating numerous competing models for each variable, ARIMA (3, 1, 1), ARIMA (3, 1, 1), ARIMA (1, 1, 2) and ARIMA (1, 0, 1) were found to be the most suitable models for modelling and predicting the future values of diesel, paraffin, petrol and exchange rate (ZAD-USD), respectively. The corresponding forecasting accuracies were 93.4%, 91.7%, 91.5% and 79.3%. The study concluded that using these models in predicting the future values of fuel prices and the strength of South African currency against the United States of America dollars can help policymakers and all stakeholders make informed decisions in their planning.

In their study on the currency in circulation in the country, Mwale et al. (2004) emphasized the importance of CIC and identified two indicators to assess its significance: the proportion of CIC to the total money supply (CIC/money supply) and its relationship to the GDP. An increase in the percentage of CIC in the money supply indicated reduced funds available for lending, potentially hindering economic growth by limiting the amount of money in banks and deposit institutions. On the other hand, a high level of CIC suggested increased economic transactions, which could contribute to inflationary pressures and indicate an economic boom. Analyzing annual data from 1965 to 2004, Mwale et al. discovered a seasonal trend in the data attributed to seasonal agricultural activity. Mwale et al. (2004) undertook the task of modelling the level of CIC in Nigeria using a traditional multivariate regression model based on the demand for money. Despite incorporating various variables, such as nominal GDP growth rate, interest rates, shadow economy indicators, indicators of small-scale agricultural activities, electronic transactions, and a dummy variable, the model's explanatory power was limited, accounting for only around 60% of the fluctuations in the CIC/money supply. The authors did observe a negative relationship between deposit rates and the amount of money in circulation, suggesting that higher deposit rates led to reduced money supply due to increased saving behaviour. Additionally, the model indicated significant effects of small-scale agricultural production activities and the shadow economy. However, the GDP growth rate and a dummy election variable proved to have marginal usefulness in predicting the movement of the CIC/money supply.

Dheerasinghe (2006) emphasized the importance of estimating the currency in circulation (CIC) for the Sri Lankan Central Bank, highlighting its significance as a leading indicator of economic growth and accounting for over 65% of the total reserve funds. While acknowledging the unsuitability of commonly used approaches for high-frequency data, Dheerasinghe agreed with the Reserve Bank of India's concerns. She also recognized the limitations of the traditional demand for money model, which lacked more frequent income data beyond quarterly periods. Furthermore, she observed that the alternative approach of univariate modelling had shown inefficiency in capturing certain effects and seasonality, possibly due to variations in intra-month and intra-week lags. In response, Dheerasinghe proposed an alternative univariate modelling approach that aimed to independently address trends, seasonal patterns, and cycles in the series, considering the drawbacks of existing methods. Successfully identifying cyclical dynamics in the data, Dheerasinghe and her team incorporated autoregressive and moving average (ARMA) terms into their modelling approach. They employed time and time-squared series to capture the stochastic trend in the data, accounting for both linear and non-linear trends. Based on their findings, all three approaches adequately fit the data and captured various effects and seasonality, displaying satisfactory performance in out-of-sample forecasting for Sri Lanka. The model selection process involved criteria such as the lowest Akaike and Schwartz information criteria, maximization of R-squared, and minimization of mean square error of the forecasts.

Norat (2008) employed a structural time series (STS) technique, following a similar approach to Dheerasinghe, to forecast the CIC in Nigeria. The STS model utilized structural equations to incorporate trends, seasonality, cycles, and other properties of the time series. Norat applied this model to analyze CIC data from the United Kingdom, specifically considering volatile periods such as Christmas and New Year's celebrations, during the years 2005 to 2006. By comparing the outcomes with those of an exponential smoothing model, Norat claimed that the STS model demonstrated superior performance outside the sample. Moreover, Norat (2008) established that smaller out-of-sample forecast errors were obtained when the estimates of the STS forecast were weighted with CIC demand from participants in the British Notes Circulation Scheme.

Furthermore, the study conducted by Riazuddin and Khan (2005) focused on modelling CIC and advocated for the effectiveness of the ARIMA model. They argued that incorporating Islamic calendar effects into the ARIMA model enhanced its performance. By including Islamic event dummy variables, they claimed to improve the prediction of cash movement in circulation in Pakistan during the estimation sample period from July 1972 to June 1999. However, the extent of improvement and the reliability of this approach were not extensively discussed. The out-of-sample forecasts exhibited a mean absolute percent error of 0.504 percent, which may seem low, but the lack of contextual comparison or comprehensive evaluation of other forecasting methods hampers a thorough assessment.

Griffiths and Higham (2010), while primarily focusing on forecasting pandemic data during the Covid-19 pandemic, introduced the SIR model (Vulnerable, Infected, and Removed) as a system of initial value problems formulated as ordinary differential equations. Although they employed multiplicative Holt-Winters exponential smoothing, the authors acknowledged that this method is suitable only when trend patterns are the sole influencing factor in the data. They also noted that the method fails to capture seasonal trends when the data exhibits both trend and seasonality. To address this limitation, they incorporated an additional parameter into Holt-Winters exponential smoothing. However, the study used the multiplicative methodology of Holt-Winters exponential smoothing without providing a comprehensive evaluation of its effectiveness or comparing it with alternative models.

Riazuddin and Khan, and Norat supported the effectiveness of their respective approaches without thoroughly evaluating the extent of improvement and the robustness of these models. Balli and Elsamadisy claimed the superiority of the ARIMA model over the univariate regression model but lacked comprehensive details in their analysis. Griffiths and Higham introduced the SIR model with multiplicative Holt-Winters exponential smoothing, yet they did not extensively evaluate or compare it with alternative approaches. Thus, further scrutiny and comprehensive evaluation are necessary to ascertain the reliability and effectiveness of these models in accurately forecasting CIC. This current study aims to investigate other methods of forecasting CIC in Nigeria.

In conclusion, the reviewed studies shed some light on the forecasting of CIC using different models. The ARIMA model showed superior performance compared to conventional money demand models in CIC forecasting, but it suffered from limitations due to its reliance on historical data and lack of a theoretical economic foundation. Money demand models and univariate approaches, despite their theoretical promise, failed to accurately capture the impact of high-frequency data. Bhattachrya and Joshi proposed an intra-month univariate model with seasonal dummy variables, which improved empirical specification and accounted for the influence of celebrations and holidays on CIC. These findings highlight the complexities and challenges of accurately forecasting CIC, underscoring the need for further research. The studies under review employed different modelling approaches to forecast CIC. Mwale et al. utilized a traditional multivariate regression model that captured some factors influencing CIC but had limited explanatory power. Dheerasinghe proposed an alternative univariate modelling approach aiming to address trends, seasonal patterns, and cycles, demonstrating satisfactory performance. However, it is crucial to consider the limitations and areas for improvement in these models.

3. **Methodology**

Accurate prediction is crucial for projecting future performance and plays a vital role in comprehensive planning and management processes. Forecasting techniques can be grouped into four categories: time series univariate, multivariate or causal, qualitative or technological, and other quantitative methods. In the study conducted by Guizzi et al. (2015), which focused on modelling temperature behavior in Caserta, Italy, the authors compared exponential smoothing models and autoregressive moving average models. They employed the Holt-Winters method, a statistical technique within the time series univariate approach. Holt (2004) extended the use of exponentially weighted moving averages to account for trend and seasonal variations, while Taylor (2003) utilized the Holt-Winters approach for short-term electricity demand forecasting. DeLurgio (1998) noted that the Winters method and Fourier series analysis are also adaptable techniques that capture the trend, level, and seasonality of time series data. Puah et al. (2016) used the additive Holt-Winters method to analyze rainfall patterns in the Langat River Basin in Malaysia. The Holt-Winters exponential smoothing approach is widely employed for forecasting various time series. This study aims to predict the monthly CIC in Nigeria, fit an additive Holt-Winters exponential smoothing model, and evaluate the accuracy measures.

3.1. Method of smoothing

In the field of time series forecasting, it is common for data to exhibit trend patterns, which can be characterized by consistent increases or decreases in the data. The Holt-Winters exponential smoothing technique is a forecasting method that uses exponential smoothing based on previous forecasting results. This technique incorporates a parameter to effectively handle seasonal patterns in the data (Kalekar, 2004; Pongdatu & Putra, 2018). It is suitable for predicting time-series data that exhibits both seasonal and trend patterns simultaneously. Smoothing, as described by Abd Jalil, Ahmad, and Mohamed (2013), involves calculating the average value over multiple years to estimate the value of a specific year. According to Makridakis et al., (1999), the smoothing approach can be categorized into two types: the exponential smoothing method and the smoothing method. Exponential smoothing is used to forecast data influenced by seasonal or trend patterns by assigning different weights to historical data with exponentially decreasing significance (Makridakis et al., 1999).

3.2. The multiplicative Holt-Winters exponential smoothing method

Charles Holt and Peter Winters developed the Holt-Winters forecasting algorithm, it employs a smoothing technique to forecast data features (Panda, 2020; Makatjane and Moroke, 2016). Exponential smoothing is used to smooth time series data by assigning exponentially decreasing weights to historical data values. This method encompasses three types of exponential smoothing. The first type is single exponential smoothing, which is suitable for univariate data without systematic structures like trends and seasonality (Djauhari, et al., 2020). According to

Panda (2020), single exponential smoothing employs a smoothing factor parameter ranging from 0 to 1. A smaller value indicates slower learning that relies more on historical data, while a larger value indicates faster learning that relies more on recent data. The second type, double exponential smoothing, incorporates two smoothing parameters, α and β, to account for trend changes. It considers both additive and multiplicative trends, and dampening can be applied by reducing the trend size for future forecasts (Bezerra & Santos, 2020). The third type, triple exponential smoothing, is employed when a series exhibits seasonal changes and allows for seasonality. It depends on three parameters, α, β, and γ, which range between zero and one (Bezerra & Santos, 2020). The Holt-Winters triple exponential smoothing technique, named after its creators, Charles Holt and Peter Winters, is particularly effective for detecting shifting levels, trends, and seasons over time using additive or multiplicative seasons. The multiplicative model of Holt-Winters triple exponential smoothing is used to forecast the original data at time t (Kurita, Sugawara, & Ohkusa, 2020; Gupta & Pal, 2020; Makatjane & Moroke, 2016; Jere & Siyanga, 2016; Djakaria, 2019).

$$L_t = \alpha \frac{Y_t}{S_{t-S}} + (1-\alpha)(L_{t-1+T_{t-1}})$$

• Trend patterns smoothing (at the time *t*)

$$T_t = \gamma(L_t - L_{t-1}) + ((1-\gamma)T_{t-1})$$

• Seasonal patterns smoothing (at the time *t*)

$$S_t = \delta \frac{Y_t}{L_t} + (1-\delta)S_{t-S}$$

So, the *p*-period forecasting forward is

$$\hat{Y}_{t+p} = (L_t + pT_t)S_{t-S+p}$$

where ($0 < \alpha, \gamma, \delta < 1$).

3.3. Evaluation of forecasting model

The analysis of data patterns is crucial in selecting an appropriate forecasting technique. The accuracy of a forecasting approach is determined by the prediction error rate (Sumitra and Basri, 2020). It is reasonable to expect that no forecasting approach can predict the future state of the data with perfect accuracy, leading to some degree of inaccuracy. As the forecasting model's error rate decreases, the forecasted outcome becomes closer to the actual outcome. Various statistics, such as mean absolute deviation (MAD), mean squared deviation (MSD), and mean absolute percentage error (MAPE), are used to quantify prediction errors. However, using MAD and MSD as indicators of forecasting accuracy can be problematic because they do not facilitate comparisons across different time series or time intervals. Additionally, the absolute measures MAD and MSD are influenced by the size of the time-series data. Moreover, MAD and MSD may not align with human perception as they involve the square of values. To address these limitations, this study used MAPE as an alternative measure of forecasting accuracy.

4. Data Description

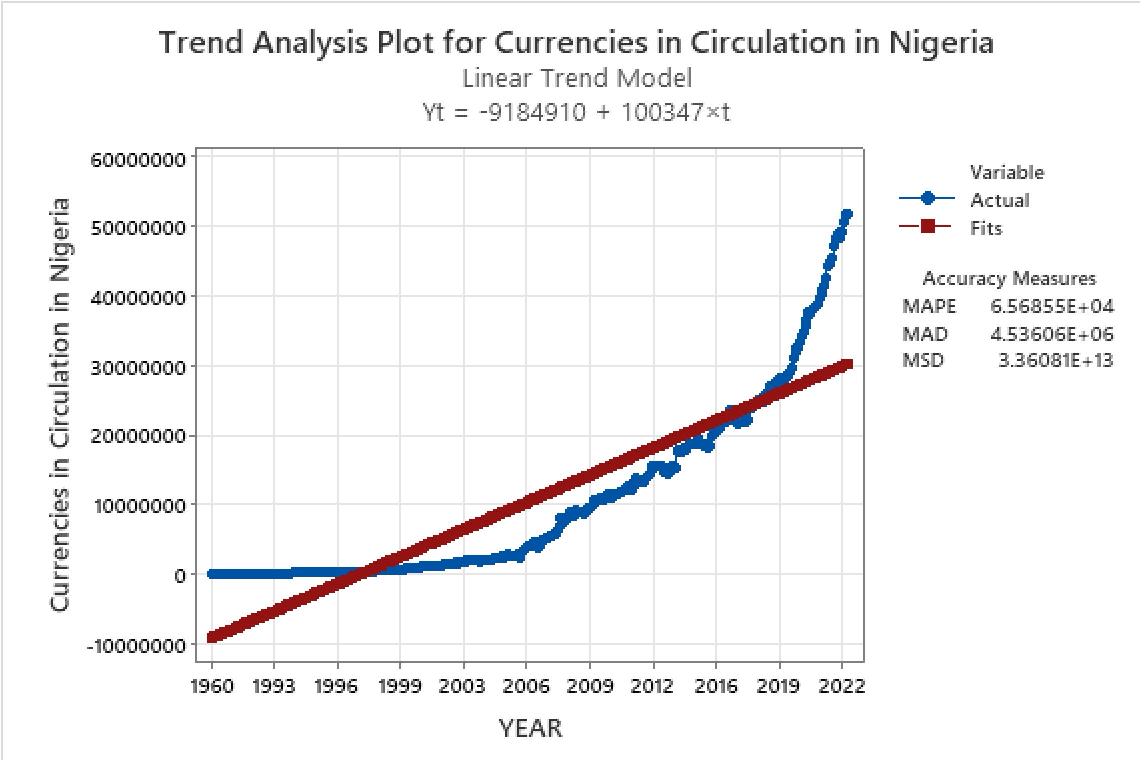

Figure 1: Trend Analysis Plot for Currencies in Circulation in Nigeria

Plotting the time series data of actual CIC in Nigeria from January 1993 to December 2022 is illustrated in Figure 1. The plot effectively demonstrates that the data is subject to both trend and a seasonal pattern. Based on these observations, it appears that the dataset is influenced by both a trend (increasing over time) and a seasonal pattern (repeating fluctuations within each year).

4.1. The determination of the best model for predicting the currencies in circulation in Nigeria:

The Holt-Winters exponential smoothing method uses three parameters - the level parameter (α), trend parameter (γ), and seasonal parameter (δ). Consequently, multiple forecasting models will be derived with varying parameters in this study. Determination of the most suitable model for predicting CIC in Nigeria using this method can be achieved by employing the MAPE value.

Specifically, the chosen model that will be considered better will be the model with the smallest MAPE value. The following models are proposed for predicting CIC in Nigeria using the multiplicative Holt-Winters exponential smoothing method and AdditiveHolt–Winters Model. Three trials and errors were conducted with different parameters that were randomly generated; the results of the random numbers are; α= 0.2, γ= 0.2, δ= 0.5; α= 0.4, γ= 0.3, δ= 0.1; α= 0.1, γ= 0.1, δ= 0.3.

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	A	B	C	D	E
1	**α**	**γ**	**δ**		
2	0.2	0.2	0.5		
3	0.4	0.3	0.1		
4	0.1	0.1	0.3		
5					

Figure 2: Random number for α, γ, δ

Experimental 1:

4.1.1. Multiplicative Holt–Winters Method: α= 0.2, γ= 0.2, δ= 0.5

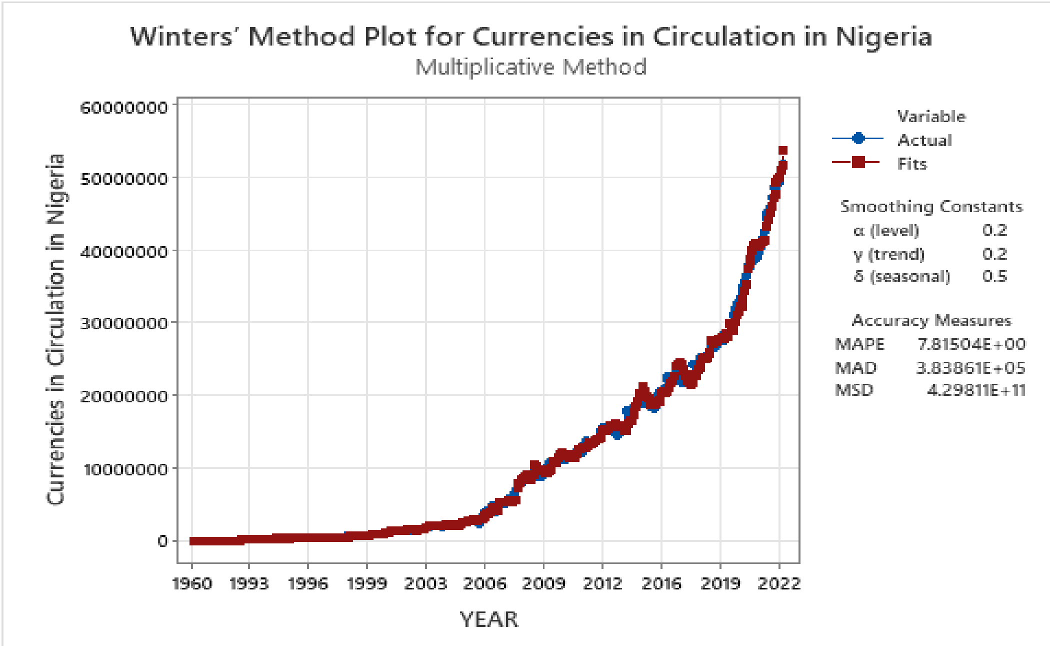

Figure 3: Multiplicative Holt–Winters Method with α= 0.2, γ= 0.2, δ= 0.5

4.1.2. Additive Holt–Winters Model: α= 0.2, γ= 0.2, δ= 0.5

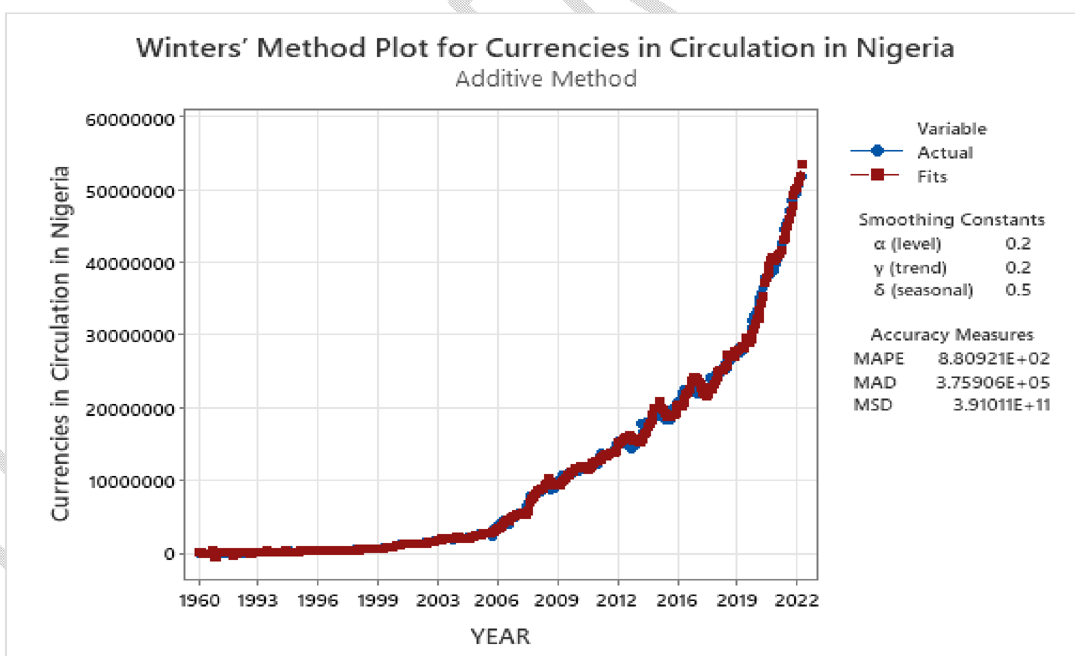

Figure 4: Additive Holt–Winters Model with α= 0.2, γ= 0.2, δ= 0.5

Experimental 2:

4.1.3. Multiplicative Holt–Winters Method: α= 0.4, γ= 0.3, δ= 0.1

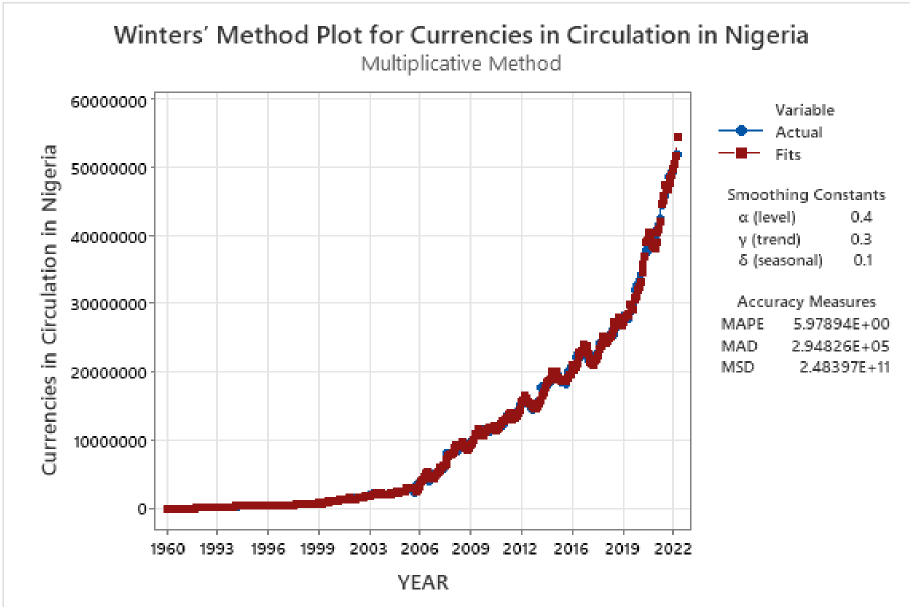

Figure 5: Multiplicative Holt–Winters Method with α= 0.4, γ= 0.3, δ= 0.1

Additive Holt–Winters Model: α= 0.4, γ= 0.3, δ= 0.1

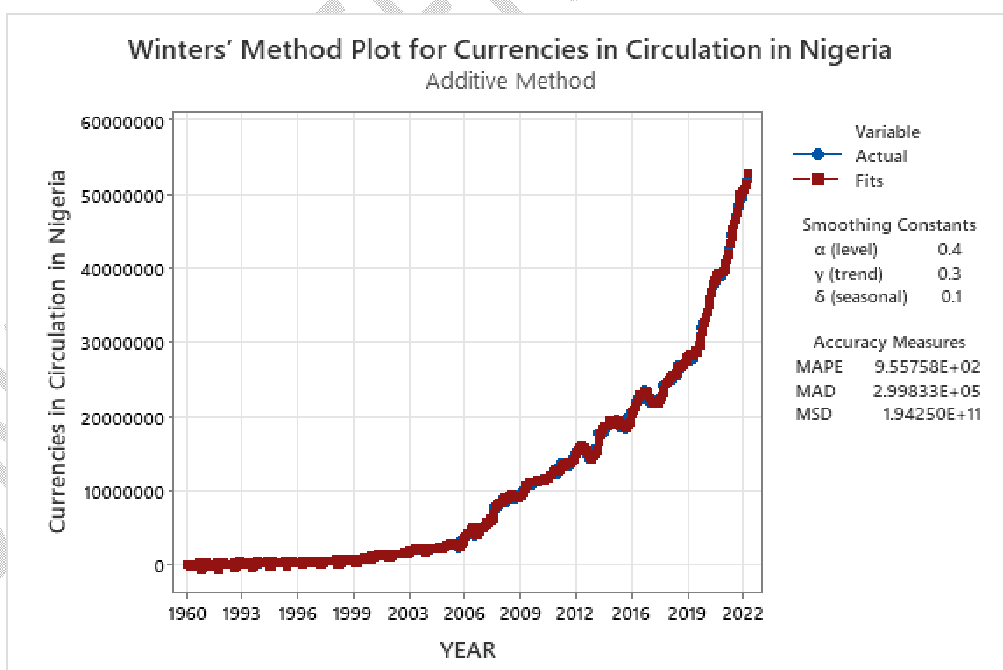

Figure 6: Additive Holt–Winters Model with α= 0.4, γ= 0.3, δ= 0.1

Experimental 3:

4.1.4. Multiplicative Holt–Winters Method: α= 0.1, γ= 0.1, δ= 0.3

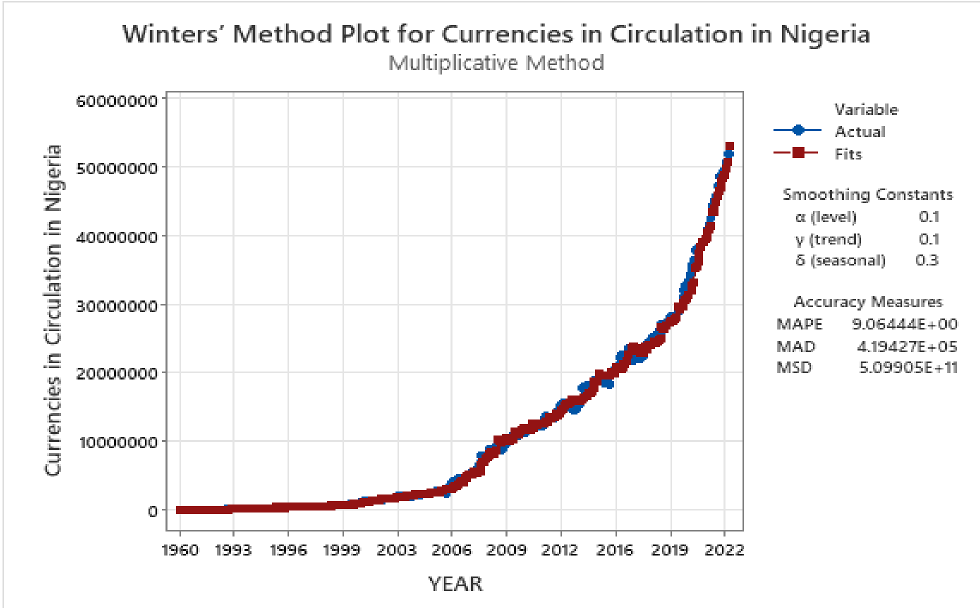

Figure 7: Multiplicative Holt–Winters Method with α= 0.1, γ= 0.1, δ= 0.3

4.1.5. Additive Holt–Winters Model: α= 0.1, γ= 0.1, δ= 0.3

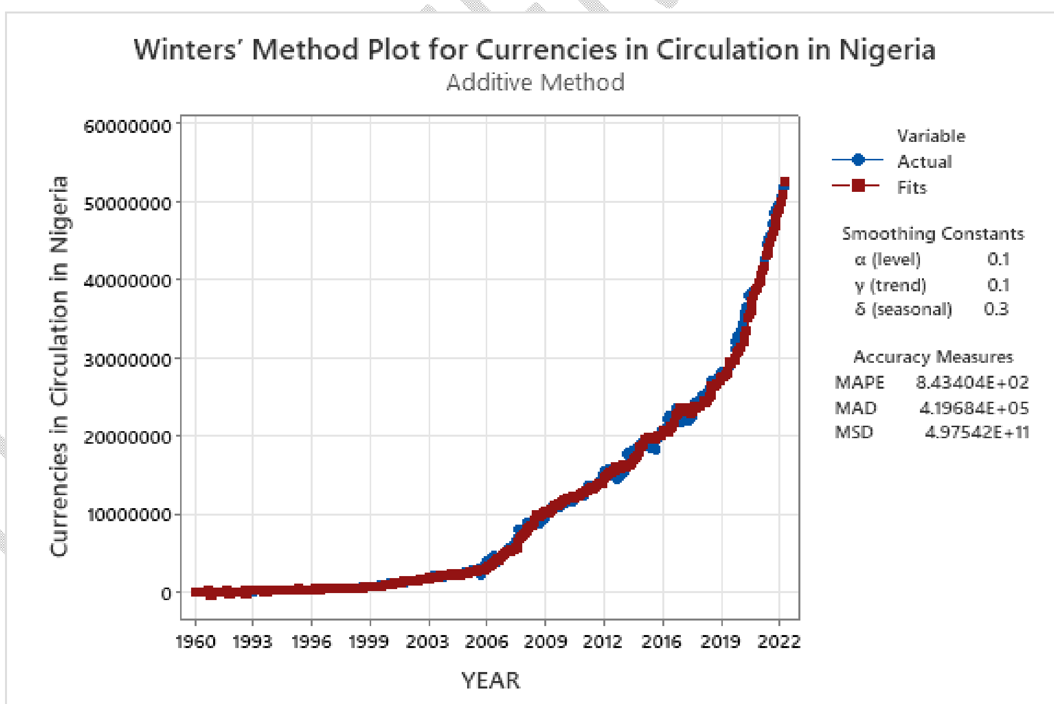

Figure 8: Additive Holt–Winters Model with α= 0.1, γ= 0.1, δ= 0.3

Based on these accuracy measures, it revealed that Experimental 2 has the lowest MAPE (5.97894E+00), indicating a lower average percentage error. Experimental 2 also has the lowest MAD (2.94826E+05), representing a lower mean absolute deviation. Additionally, Experimental 2 has the lowest MSD (2.48397E+11), indicating a lower mean squared deviation. Considering these factors, Experimental 2 performs the best across all three accuracy measures. Therefore, Experimental 2, with α= 0.4, γ= 0.3, and δ= 0.1, is the best model for forecasting the variable using the multiplicative Holt-Winters model. Considering the Additive Holt-Winters model, based on the accuracy measures, it can be inferred that Experimental 2 has the lowest MAPE (9.55758E+02), indicating a lower average percentage error. Experimental 2 also has the lowest MAD (2.99833E+05), representing a lower mean absolute deviation. Additionally, Experimental 2 has the lowest MSD (1.94250E+11), indicating a lower mean squared deviation. Experimental 2 also performs the best across all three accuracy measures. Therefore, Experimental 2, with α= 0.4, γ= 0.3, and δ= 0.1, is still the best model for forecasting the variable using the Additive Holt-Winters model.

This study will further determine which of the forecasting method, either the multiplicativeHolt–Winters or additiveHolt–Winters, is best in forecasting CIC in Nigeria.

4.2. The determination of the best forecasting method for predicting the currencies in circulation in Nigeria:

The Holt-Winters approaches are particularly suitable for time series exhibiting both trend and seasonal characteristics. Two Holt-Winters techniques have been developed for time series with linear trends. The multiplicative Holt-Winters exponential smoothing method and the additive

Holt-Winters model are two popular techniques used for time series forecasting and decomposition. The Multiplicative Holt-Winters method is effective for time series with growing (multiplicative) seasonal variations, while the Additive Holt-Winters method is suitable for time series with constant (additive) seasonal variations.

4.2.1. Holt-Winters Method

The Holt-Winters methodology, initially proposed by Winters, involves estimating three smoothing parameters associated with level, trend, and seasonal variables (Bermudez, Segura, and Vercher, 2006). It is a specialized exponential smoothing method tailored for handling seasonal data.

4.2.2. Additive Holt-Winters Method

The additiveHolt–Winters method is presented in the following equations.

$$y_T = \beta_0 + \beta_1 t + sn_T + \varepsilon_T$$

Estimate of the level at time T

$$l_T = \alpha(y_T - sn_{T-L}) + (1-\alpha)(l_{T-1} + b_{T-1})$$

Estimate of the growth rate (or trend) at time T

$$b_T = \gamma(l_T - l_{T-1}) + (1-\gamma)b_{T-1}, 0 \leq \alpha, \gamma \leq 1$$

Estimate of the seasonal factor at time T

$$sn_T = \delta(y_T - l_T) + (1-\delta)sn_{T-L}, where, 0 \leq \delta \leq 1$$

p- Step ahead forecast made at time T

$\hat{Y}_{T+p}(T) = l_T + pb_T + sn_{T+p-L}$ Where, 1,2,...

Where,

Initial level= $l_0 = \beta_0$ intercept

Initial growth rate= $b_0 = \beta_1$ = Slope

$$S_T = y_T - \hat{Y}, \qquad and\ \bar{S}_{[i]} = \frac{1}{L}\sum_{k=i=1}^{\leq n} S_{2k+1}$$

L = No. of seasons in a year

Initial seasonal factors $Sn_{i-L} = \bar{S}_{[i]}$, $i = 1, 2, \ldots, L$

4.2.3. Multiplicative Holt-Winters Method

The multiplicativeHolt–Winters method is presented in the following equations.

$$y_T = (\beta_0 + \beta_1 t) \times sn_T \times \varepsilon_T$$

Estimate of the level at time T

$$l_T = \alpha(y_T / sn_{T-L}) + (1-\alpha)(l_{T-1} + b_{T-1})$$

Estimate of the growth rate (or trend) at time T

$$b_T = \gamma(l_T - l_{T-1}) + (1-\gamma)b_{T-1}, \qquad 0 \leq \alpha, \gamma \leq 1$$

Estimate of the seasonal factor at time T

$$sn_T = \delta(y_T / l_T) + (1-\delta)sn_{T-L}, where, 0 \leq \delta \leq 1$$

p- Step ahead forecast made at time T

$$\hat{Y}_{T+p}(T) = (l_T + pb_T) \times sn_{T+P-L}, p = 1,2, \ldots$$

Where,

Initial level= $l_0 = \beta_0$intercept

Initial growth rate= $b_0 = \beta_1$ =Slope

$S_T = y_T / \hat{Y}_T\ and\ \bar{S}_{[i]} = \frac{1}{L}\sum_{k=i=1}^{\leq n} S_{2k+1}$

L = Number of seasons in a year

Normalize Constant =

$$CF = L / \sum_{i=1}^{L} \bar{S}_{[i]}$$

Initial seasonal factors =

$$sn_{i-L} = \bar{S}_{[i]}[CF]$$

where, I = 1, 2, ..., L

4.2.4. Model Performance

The goodness of fit is a crucial criterion for evaluating the accuracy of a projected model in relation to the actual value. The accuracy of the model's forecasts was assessed according to pre-established standards.

Mean Absolute Deviation: $MAD = \frac{\sum|e_t|}{n}$

Sum Squared Error: $SSE = \sum e_t^2$

Mean Squared Error: $MSE = \frac{\sum e_t^2}{n}$

Root Mean Squared: $RMS = \sqrt{\frac{\sum e_t^2}{n}}$

Mean Absolute Scaled Error:

$$MASE = \frac{1}{n}\sum_{i=1}^{n}\frac{\sum_{i=1}^{n}|y_t - \hat{Y}_t|}{\frac{1}{n-1}\sum_{i=2}^{n}|y_t - y_{t-1}|}$$

$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$$

Percentage Error :

$$PE_t = \frac{(y_t - \hat{Y}_t)}{y_t} \times 100$$

Adjusted R^2 :

$$\bar{R}^2 = 1 - \frac{(n-1)RSS}{(n-k)TSS} = \frac{(n-1)}{(n-k)}R^2$$

Mean Percentage Error :

$$MPE = \frac{\sum PE_t}{n}$$

Mean Absolute Percentage Error :

$$MAPE = \frac{\sum|PE_t|}{n}$$

Using the parameter α= 0.4, γ= 0.3, and δ= 0.1;

4.2.5. Multiplicative Holt–Winters Method: α= 0.4, γ= 0.3, δ= 0.1

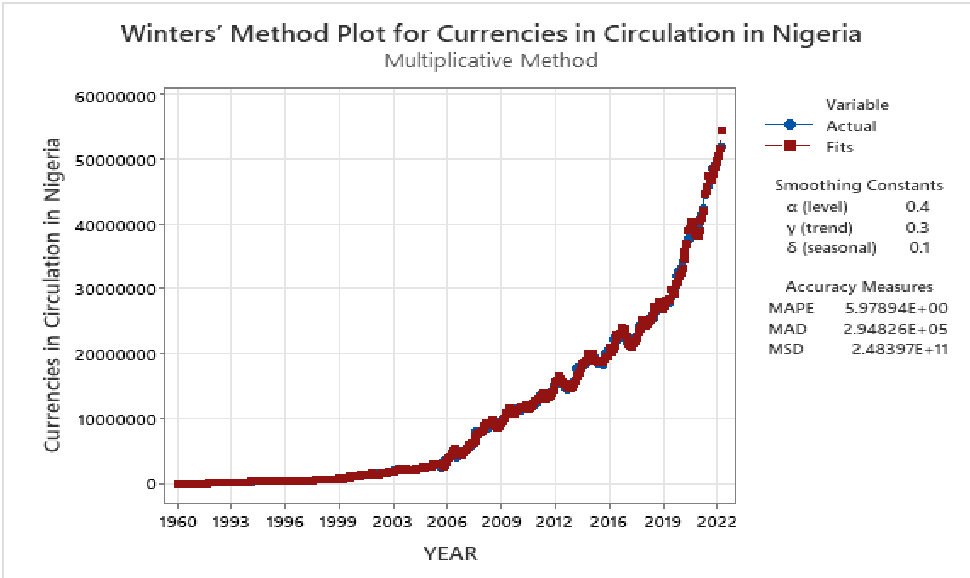

Figure 9: Multiplicative Holt–Winters Method with α= 0.4, γ= 0.3, δ= 0.1

4.2.6. Additive Holt–Winters Model: α= 0.4, γ= 0.3, δ= 0.1

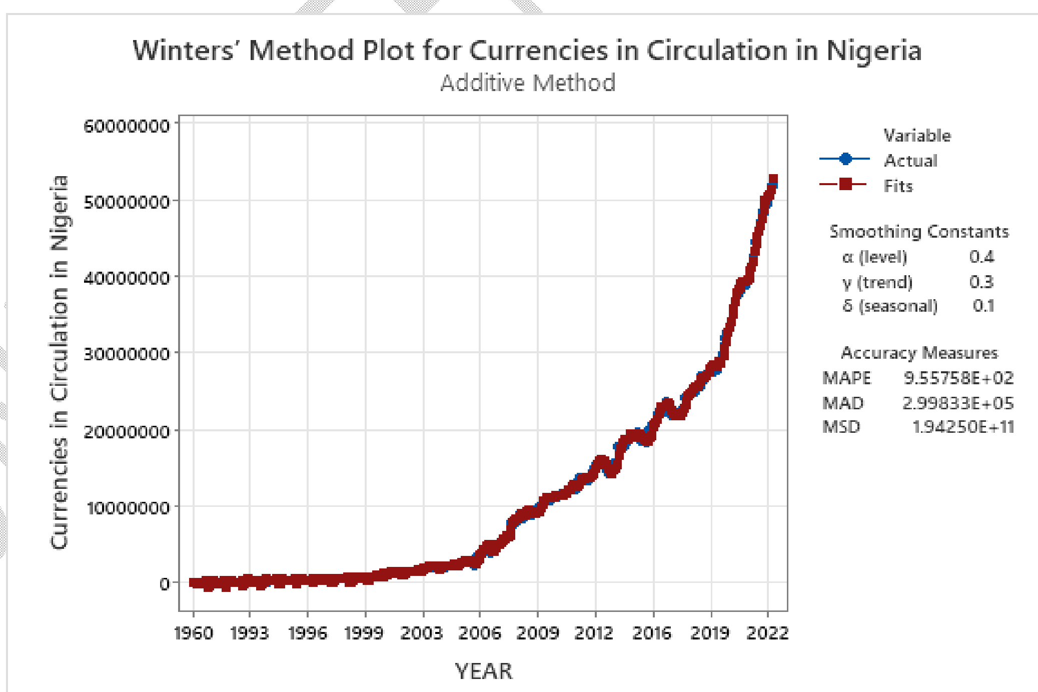

Figure 10: Additive Holt–Winters Model with α= 0.4, γ= 0.3, δ= 0.1

Comparing the accuracy measures, it can be inferred that the multiplicative Holt-Winters method outperforms the additive Holt-Winters model in all three measures: MAPE, MAD, and MSD. The multiplicative Holt-Winters method has a significantly lower MAPE (5.97894E+00) compared to the additive Holt-Winters model (9.55758E+02). Similarly, the multiplicative Holt-Winters method has lower MAD and MSD values. Based on these accuracy measures, it can be concluded that the multiplicative Holt-Winters method is better for forecasting the CIC in Nigeria than the additive Holt-Winters model. Therefore, the multiplicative Holt-Winters method forecasts the monthly CIC in Nigeria.

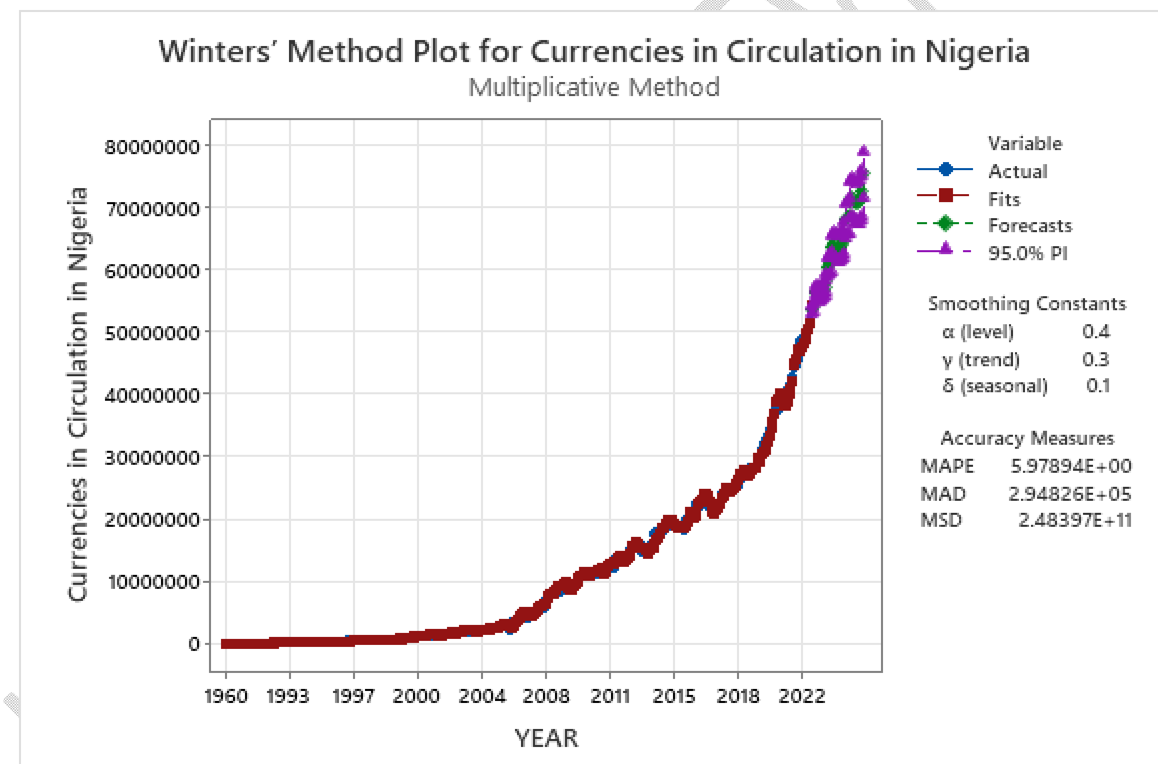

Figure 11: Forcast of Currencies in Circulation in Nigeria using Multiplicative Method

Table 1: Monthly Forecasted Currencies in Circulation in Nigeria using the multiplicative Holt-Winters method

Year	Month	Forecasted Currencies in Circulation
2023	January	53653768
2023	February	54073356
2023	March	56194283
2023	April	56725871
2023	May	56563651
2023	June	56464686
2023	July	56187244
2023	August	56411693
2023	September	56773908
2023	October	57365332
2023	November	58195583
2023	December	60655044
2024	January	61109084
2024	February	61500967
2024	March	63825871
2024	April	64343442
2024	May	64075377
2024	June	63881193
2024	July	63487405
2024	August	63662511
2024	September	63993947
2024	October	64584082
2024	November	65442812
2024	December	68130972
2025	January	68564400
2025	February	68928579
2025	March	71457460
2025	April	71961014
2025	May	71587104
2025	June	71297700
2025	July	70787566
2025	August	70913328
2025	September	71213987
2025	October	71802832
2025	November	72690040
2025	December	75606900

5. Conclusion, Summary and Recommendation

Based on an analysis and discussion of CIC in Nigeria forecasting from January 1960 to December 2022 (using Money Supply as a proxy), it can be concluded that the Holt-Winters exponential smoothing forecasting model is best suited for forecasting Nigeria CIC, with smoothing parameters for level $\alpha= 0.4$, $\gamma= 0.3$, and $\delta= 0.1$. It was found that the multiplicative model yielded better results than the additive model for all series. The trend of CIC in Nigeria, using the Holt-Winters exponential smoothing forecasting data model, shows that CIC in Nigeria will continue to increase (Table 1) because of the rapid population growth in Nigeria and as more people enter the workforce and engage in economic activities, there is an increased demand for the currency to facilitate transactions. Also, the inflation rate has continued to go higher in Nigeria. The high rate of inflation in Nigeria is part of the reason for the increase in the amount of currency in circulation. As prices rise, people need more money to purchase goods and services, which can result in a higher demand for physical currency. It is also pertinent to note that Nigeria has a significant informal sector, where economic activities are conducted outside the formal banking system. This sector often relies heavily on cash transactions, leading to a higher demand for physical currency. And the fact that Nigeria is presently a cash-based society also leads to an increase in CIC in Nigeria. Many individuals and businesses in Nigeria still prefer using physical currency for various reasons, including limited access to banking services, concerns about the security of digital transactions, and a cultural preference for cash. The Nigerian government also plays a role in increasing the amount of currency in circulation. This is because the government prints money to finance its expenditures. It is important to note that an increase in the amount of currency in circulation does not necessarily mean that the Nigerian economy is healthy. There are a number of things that can be done to reduce currency circulation in Nigeria. Some of the most effective measures include:

Promoting the use of electronic payments without inflicting pain on the citizens: The use of electronic payments can help to reduce the amount of cash in circulation. This is because people who use electronic payments do not need to carry cash with them, which means that they are less likely to spend it.

Encouraging people to save their money: When people save their money, it is taken out of circulation. This is because money that is saved is not being used to buy goods and services.

Introducing a cashless society: A cashless society is one in which people do not use cash for transactions. This can be achieved by promoting the use of electronic payments and by making it difficult to use cash.

Enhance financial literacy: Educating the population about the benefits of using formal banking services and digital payment methods can help change the perception and behavior towards cashless transactions. Financial literacy programs can empower individuals to understand and utilize the available financial services effectively.

Improve the efficiency of the banking system: The banking system in Nigeria is often inefficient, which makes it difficult for people to access financial services. This can lead to people using cash for transactions, even when they would prefer to use electronic payments. The government can improve the efficiency of the banking system by providing financial literacy training to the public and by making it easier for people to open bank accounts.

Promote financial inclusion: Financial inclusion is the process of ensuring that everyone has access to basic financial services, such as savings accounts, loans, and insurance. When more people have access to financial services, it reduces the need for cash. The government can promote financial inclusion by providing financial literacy training to the public and by making it easier for people to open bank accounts.

Combat inflation: Implementing effective monetary policies to control inflation can help stabilize prices and reduce the demand for physical currency. The Central Bank of Nigeria plays a crucial role in managing inflation by employing tools such as interest rate adjustments, open market operations, and reserve requirements.

Upgrade currency security features: Enhancing the security features of banknotes can help combat counterfeiting, thereby reducing the circulation of counterfeit currency. Introducing new designs and advanced security technologies can make it more difficult to produce and circulate fake notes.

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