Original Research Article

The Mechanization Thought and its Characteristics Contained in Newton's Mathematics

Abstract: Understanding the characteristics of Newtonian mathematics is of great value. By analyzing some of Newton's representative mathematical contents, we found that: Newton used the thought of mathematical mechanization in his mathematics.Newton applied the thought of mathematical mechanization to the study not only of calculus but of arithmetic, algebra, polynomials, higher-order equation, geometry, series, differential equation, etc. Newton's application of this thought in traditional mathematics is very similar to that of ancient Chinese mathematicians. While in modern mathematics, his procedures are briefer, faster, and more comprehensive than that of other mathematicians at that time. Newton's wide application of the thought of mathematical mechanization strongly promoted the development of Western mathematics. Like Descartes and Leibniz, Newton was also a pioneer of the development and practical application of the thought of mathematical mechanization in the Western world.

Key words: Newton; Mechanization; Mathematics; Differential; Equation

1. Introduction

Isaac Newton (1643-1727), a famous British astronomer, physicist, mathematician, and chemist, has made great contributions to the progress of human science(Wu, 2023; Rossi, 2023; Wu, 2018; Dampier, 1930). Referring to Newton's mathematics, what people are most familiar with is his invention of calculus and the Newton-Leibniz formula, which widely applied the idea of infinitesimal analysis to mathematics (Bell, 1986; Dunham, 1990; Kline, 1990). As a matter of fact, when Newton studied and applied mathematics, he also paid close attention to the procedurization, automatization, and algorithmization of mathematical calculation, and applied thought of mathematical mechanization. The thought of mathematical mechanization is defined as a standardized idea that every step in the process of mathematical calculation or proof should be carried out in strict accordance with the established steps (Wu, 2006; Wu, 1996). Newton's thought of mathematical mechanization is not only clear but also in some ways similar to China's traditional idea of mathematical mechanization. In other aspects, it is clearer and more concise than the predecessors, therefore, it has more practical value.

2. Newton's mathematics embodies the explicit thought of mathematical mechanization

In 1666, Newton wrote the paper titled *Treatise on Fluxions*, which officially announced the birth of the new discipline, calculus (Boyer, 1959; Li, 2011). In this paper, he gave two methods to find the differentiation. The first method is as follows (Newton, 1736):

PROBLEM Ⅰ: Given the relationship that the flowing quantities have to each other, determine the relationship that exists between the flows.

SOLUTION：Arrange the equation by which the date relation is expressed, next to the dimensions of any of the flowing quantities which it includes, （for example x ）and multiply its terms by any arithmetic progression, and then by $\frac{\dot{x}}{x}$：

Perform this operation separately for any flowing quantity; Then make the aggregate of all these facts equal to the cloud, and you will have the required equation.

EXEMPLE I : The relationship between quantities x & y is explained by the Equation $x^3 - ax^2 + axy - y^3 = 0$. First arrange the terms according to x, then according to y, and multiply them, as you see here done:

$$\mathit{Mult.} \quad x^3 - ax^2 + axy - y^3 \,|\, -y^3 + axy \begin{array}{l} -ax^2 \\ +x^3 \end{array}$$

$$\mathit{Per} \quad \frac{3\dot{x}}{x};\ \frac{2\dot{x}}{x};\ \frac{\dot{x}}{x};\ \ 0 \ \ |\frac{3\dot{y}}{y};\ \frac{\dot{y}}{y};\ \ 0$$

$$\mathit{Whence\ it\ is\ made}:\ 3x^2\dot{x} - 2ax\dot{x} + ay\dot{x} * \quad -3y^2\dot{y} + ax\dot{y} *$$

The collection of products is $3x^2\dot{x} - 2ax\dot{x} + ay\dot{x} - 3y^2\dot{y} + ax\dot{y} = 0$； Which equation shows what relation there is between the fluxions $\dot{x}$, & $\dot{y}$.

From the above, it can be learned that Newton's first method of find the differentiation is:

First of all, arrange polynomials in the equation according to the dimensions of some fluent quantities from high to low;For instance, if the original equation is $x^3 - ax^2 + axy - y^3 = 0$, the operation in this step is to list the following two polynomials (one on x and one on y): $x^3 - ax^2 + axy - y^3$ and $-y^3 + axy - ax^2 + x^3$.

The second step is to list the factors corresponding to each term of the polynomials obtained in the first step. For the polynomials listed in the first step, the below factors will be obtained after this step:

$$\frac{3\dot{x}}{x};\ \frac{2\dot{x}}{x};\ \frac{\dot{x}}{x};\ \ 0 \ \text{ and } \ \frac{3\dot{y}}{y};\ \frac{\dot{y}}{y};\ \ 0.$$

The third step is to multiply each term of the polynomial with the corresponding factor to obtain the product. For the above polynomials, the operation in this step is to obtain two products: $3x^2\dot{x} - 2ax\dot{x} + ay\dot{x} + 0$ and $-3y^2\dot{y} + ax\dot{y} + 0$.

The fourth step is to add up all the obtained products and make the result equal to 0 to form a new equation which is the differential form of the original equation. For the example above, this step is to add the above two polynomials to form a new equation $3x^2\dot{x} - 2ax\dot{x} + ay\dot{x} - 3y^2\dot{y} + ax\dot{y} = 0$, which is the differential equation sought in this problem.

In this paper, the second method of differential calculation that Newton provided is as follows (Newton, 1736):

If the equation is $x^3 - ax^2 + axy - y^3 = 0$, let $x + \dot{x}o$ & $y + \dot{y}o$ substitute x & y respectively, you will get:

$$\left.\begin{array}{c} x^3 + 3x^2\dot{x}o + 3x\dot{x}o\dot{x}o + \dot{x}^3o^3 \\ -ax^2 - 2ax\dot{x}o - a\dot{x}o\dot{x}o \\ -ax^2 - 2x\dot{x}o - a\dot{x}o\dot{x}o \\ +ax\dot{y}o \\ -y^3 - 3y^2yo - 3y\dot{y}o\dot{y}o - y^3o^3 \end{array}\right\} = 0$$

But, by hypothesis, $x^3 - ax^2 + axy - y^3 = 0$, therefore erase these terms, and divide the rest by o & will remain:

$$3x^2\dot{x} - 2ax\dot{x} + ay\dot{x} + axy - 3y^2\dot{y} + 3x\dot{x}^2o - a\dot{x}^2o + a\dot{x}\dot{y}o - 3y\dot{y}^2o + \dot{x}^3o^2 - y^3o^2 = 0;$$

But when we have invented o, an infinitesimal quantity, so as to be able to explain the quantities of moments, the terms drawn into it can be considered for nothing when compared with others; therefore I neglect them, and we got $3x^2\dot{x} - 2ax\dot{x} + ay\dot{x} + ax\dot{y} - 3y^2\dot{y} = 0$, as we found previously in Example 1.

From the paragraph above, it can be concluded that Newton's second method of finding the differentiation is as following steps:

Step 1: Replacing the quantities x and y in the original equation into $x + \dot{x}o$ and $y + \dot{y}o$ respectively to obtain a new equation.

Step 2: Subtracting the original equation from the newly obtained new equation.

Step 3: Dividing all the terms of both sides of the equation by o.

Step 4: Removing all terms with o in the new equation and the result is the desired equation.

It can be seen that each method given by Newton for finding the differential of an equation (or equation that gives the relation between the fluxions) is a procedure, exactly, a set of operational steps. These steps are not only explicit but also sequential, which means they cannot be interchanged arbitrarily. Therefore, Newton's method of differentiation contains explicit thought of mathematical mechanization.

Additionally, Newton began his tenure as Lucasian Professor of Mathematics at Trinity College, Cambridge in 1669 and left Cambridge in 1689 to become a Fellow of the Royal Society. During this period, he continued to teach mathematics, mainly algebra to be precise, to students at Cambridge (Richard, 2010). In his lectures on algebra, Newton gave the following method for finding the divisors or the factorization of polynomials (Newton, 1720):

Then set also down the arithmetical progressions which run throw the divisors of all the numbers proceeding from the greater terms to the less, in the order that the terms of the progression 3, 2, 0, -1, -2, proceed, and whose terms differ either by unity, or by some proposed. If any progression of this kind occurs, that term of it which stands in the same line with the term o of the first progression, divided by the difference of the terms, will compose the quantity by which you are to attempt the division.

As if the polynomial is $x^3 - x^2 - 10x + 6$, by substituting, one by one, the terms of this progression 1, 0, -1, for x, there will arise the numbers -4, 6, $+14$, which, together with all their divisors, I place right against the terms of the progression 1, 0, -1. The following formula is obtained:

$$\begin{array}{r|r|l|l} 1 & 4 & 1\cdot 2\cdot 4\cdot & +4\cdot \\ 0 & 6 & 1\cdot 2\cdot 3\cdot 6 & +3\cdot \\ -1 & 14 & 1\cdot 2\cdot 7\cdot 14 & +2\cdot \end{array}$$

Then, because the highest term x^3 is divisible by no number but unity, I seek among the divisors a progression whose terms differ by unity, and (proceeding from the highest to the lowest) decrease as the terms of the lateral progression 1, 0, -1. And I find only one progression of this sort, viz. 4, 3, 2, whose term therefore 3 I chose, which stands in the same line with the term o of the first progression 1, 0, -1. and I attempt the division by $x+3$, and find it succeeds, there coming out $x^2 - 4x + 2$.

Again, if the polynomial is $6y^4 - y^3 - 21y^2 + 3y + 20$, for y I substitute successively 2, 1, 0, -1, -2 and the resulting numbers 30, 7, 20, 3, 34, with all their divisors, I place by them as follows:

$$\begin{array}{r|r|l|l} 2 & 30 & 1\cdot 2\cdot 3\cdot 5\cdot 6\cdot 10\cdot 15\cdot 30 & +10\cdot \\ 1 & 7 & 1\cdot 7\cdot & +\ 7\cdot \\ 0 & 20 & 1\cdot 2\cdot 4\cdot 5\cdot 10\cdot 20 & +\ 4\cdot \\ -1 & 31 & 1\cdot 3\cdot & +\ 1\cdot \\ -2 & 34 & 1\cdot 2\cdot 17\cdot 34 & -\ 2\cdot \end{array}$$

And among the divisors I perceive there is this decreasing arithmetical progression $+10$, $+7$, $+4$, $+1$, -2. The difference of the terms of this progression, viz. 3, divides the highest term of the quantity $6y^4$. Wherefore I adjoin to the letter y the $+4$, which stands in the row opposite to term o, divided by the difference of the terms, viz. 3, and I attempt the division by $y + \frac{4}{3}$, or which is the same thing, by $3y+4$, and the business succeeds, there coming out $2y^3 - 3y^2 - 3y + 5$.

Similarly, it can be seen here that Newton implants the explicit mathematical mechanization thought in the method of factoring polynomials.

In fact, in Newton's lectures on algebra, in addition to methods on the factorization of polynomials, there are also methods on the four algebraic operations, exponentiation and extracting roots, methods on how geometric problems are transformed into algebraic equations, and methods for finding roots of higher equations, etc (Newton, 1744). These methods, which Newton gave by steps and procedures, all gave specific examples and their step-by-step diagrams of solutions, and all of which evidently implied the thought of mathematical mechanization.

Around 1692, Newton finished a book about the integration of curves titled *Treatise on the Quadrature of Curves*, detailing his method of finding the integral of a curve. By carefully analyzing his method of finding the integral of a curve, it can be found that it is also given by a procedure (Newton, 1762). This procedure is also highly sequential and not interchangeable, which shows the thought of mathematical mechanization.

3. The thought of the mathematical mechanization of Newton and that contained inancient mathematics in China

It is well known that ancient mathematics in China has an obvious tendency for algorithmization and mechanistic preference (Compilation Group of Brief History of Chinese and Foreign Mathematics, 1986; Guo, 2006; Wu, 2006). Comparing the mathematical mechanization thought of Newton with the mathematical mechanization thought contained inancient mathematics in China, it can be found that they have great similarities, especially when performing elementary mathematical operations, for example, when extracting the roots of the equation.

In ancient China, chips or bones were used to perform the operation of extracting the square root of a number in the following way (Zhang, 1994):

Step 1 (Grouping the figures and employing 1): Dividing the figures of the initial number extracted into groups from right to left, 2 figures a group. Then the number 1 is taken at the bottom-right to the number. After that, let 1 move to the left, 2 places at a time. This is done until the leftmost end.

Step 2 (Finding the radicand of the first group number): Finding an integral part of the square root of the first group of figures above the leftmost 1 as the first figure of the root.

Step 3 (Making a difference): Making the figure obtained last step to be multiplied by 1 to get A, and multiplied by A to get a product. Then the group of numbers above 1 subtracts this product.

Step 4 (Complementing): Making the integer part of the root to be multiplied by 1, and add the result to A to get B.

Step 5 (Setting 1 back): Moving the number B back one place to the right, and the below number 1 moves back 2 places.

Step 6 (Finding the next digit of the root): Estimating how many times the number above B is B, and this number is used as the next digit of the root.

If the group of numbers in the sixth step is not the last group after the above steps, the third step to the sixth step should be repeated more times.

If the number to be extracted is a fraction, and its denominator is a square number, then square root of the denominator can be extracted directly and the numerator is operated according to the above procedure. If the denominator is not a square number either, the problem should be transformed by using the formula $\sqrt{\frac{a}{b}} = \frac{\sqrt{ab}}{b}$.

The method of extracting the cubic root of a number is stated as follows (Zhang, 1994):

Step 1 (Group the figures and employ 1): Dividing the figures of the initial number extracted into groups from right to left, 3 figures a group. Then the number 1 is taken at the bottom-right to the number. After that, let 1 move to the left, 3 places at a time. This is done until the leftmost end.

Step 2 (Find the cube root of the first group number): Finding an integral part of the cube root of the first group of figures above the leftmost 1 as the first figure of the root.

Step 3 (Make a difference): Making the figure obtained last step to be multiplied by 1 to get A, and then multiplied by A to get a product B, then multiplied by B to get another product C. Then the group of numbers above 1 subtracts C.

Step 4 (Complementing): Making the integer part of the root firstly to be multiplied by 1 and add the result to A to get C. It is next multiplied by C and add the result to B to get D. Lastly, it is multiplied by 1 and then add the product to A to get E.

Step 5 (Setting 1 back): Moving number D back 1 place to the right, E 2 places, and the below number 1 moves back 3 places.

Step 6 (Find the next digit of the root): Estimating how many times the number above E is E, and this number is used as the next digit of the root.

As the method of extracting square root, the third step to the sixth step should be repeated more times, if the group of numbers in step 6 is not the last.

Similarly, if the number to be extracted is a fraction, and its denominator is a cubic number, then the cube root of the denominator can be extracted directly and the numerator is operated according to the above procedure. If the denominator is not a cubic number either, the problem should be transformed using the formula $\sqrt[3]{\frac{a}{b}}=\sqrt[3]{\frac{ab^2}{b^3}}=\frac{\sqrt[3]{ab^2}}{b}$.

While Newton's method of extracting square root is (Newton, 1720):

It is first to be noted with points in every other place, beginning from unity; then you are to write down such a figure for the quotient, or root, whose square shall be equal to, or nearest, less than the figure or figures to the first point. And then subtracting that square, the other figures of the root will be found one by one, by dividing the remainder by the double of the root as far as extracted, and each time taking from that reminder the square of the figure that last came out, and the decuple of the aforesaid divisor augmented by that figure.

Thus to extract the root out of 99856, first point it after this manner, $\dot{9}9\dot{8}5\dot{6}$; then seek a number whose square quotient; and then having subtracted from 9, 3×3, or 9, there will remain 0; to which set down the figures to the next point, viz. 98 for the following operation. Then taking no notice of the last figure 8, say, how many times is the double of 3, or 6, contained in the first figure 9? Answer 1; wherefore having written 1 in the quotient, subtract the product of 1×61, or 61, from 98, and there will remain 37, to which connect the last figures 56, and you'll have the number 3756, in which the work is next to be carried in. Wherefore also neglecting the last figure of this, viz. 6, say, how many times is the double of 31, or 62, contained in 375, (which is to be guessed at from the initial figure 6 and 37, by taking notice how many times 6 is contained in 37?) Answer 6; and writing 6 in the quotient, subtract 6×626, or 3756, and there will remain 0; whence it appears tat the business is done; the root coming out 316. (Newton's method of extracting the square root is shown in Fig. 1)

$\dot{9}9\dot{8}5\dot{6}$ (316
9
6)98
61
62)3756
3756
0

Fig. 1 Newton's method of extracting the square root

Newton's method of extraction of cubic root is as follows (Newton, 1720):

Every third figure beginning from unity is first of all to be pointed, and then such a figure is to be written in the quotient, whose greatest power shall either be equal to the figure or figures before the first point, or next less under them; and then having subtracted that power, the next figure will be found by dividing the remainder augmented by the next figure of the resolved, by the next least power of the quotient, multiplied by the index of the power to be extracted. And having again subtracted the power, the next figure will be found by dividing the remainder augmented by the next figure of the resolved, by the next least power of the quotient, multiplied by the index of the power to be extracted. And having again subtracted the power of the whole quotient from the first resolved, the third figure will be found by dividing that remainder augmented by the next figure of the resolved, by the next least power of the whole quotient, multiplied by the index of the power to be extracted.

Thus to extract the cube root of 13312053, the number is first to be pointed after this manner, viz. $1\dot{3}31\dot{2}05\dot{3}$. Then you are to write the figure 2, whose cube is 8, in the first place of the quotient, as which is the next least cube to the figures 13, which is not a perfect cube number or to the first point; and having subtracted that cube, there will remain 5; which being augmented by the next figure of the resolved 3, and divided by the triple square of the quotient 2, by seeking how many times 3×4 or 12, is contained in 53, it gives 4 for the second figure of the quotient. But since the cube of the quotient 24, *viz.* 13824 would come out too great to be subtracted from the figures 13312 that proceed the second point, there must only 3 be written in the quotient: then the quotient gives the square 529, which again multiplied by 23 gives the square 529, which again multiplied by 23 gives the cube 12167, and this taken from 13312, will leave 1145; which augmented by the next figure of the resolved 0, and divided by the triple square of the quotient contained in 11450, it gives 7 for the figure of the quotient. Then the quotient 237, multiplied by 237, gives the square 56169, which again multiplied by 237 gives the cube 13312053, and this taken from the resolved leaves 0. whence it is evident that the root sought is 237. (Newton's method of extracting the cube root is shown in Fig. 2.)

$$1\dot{3}312\dot{0}53\dot{(}237$$
$$\textit{Subtract the Cube } 8$$
$$12)\ \textit{rem.}\ 53(4\ \textit{or}\ 3$$
$$\textit{Subtract Cube } 12167$$
$$1587)\ \textit{rem.}\ 11450(7$$
$$\textit{Subtract Cube } 13312053$$
$$\text{Re}\,\textit{mains}\ 0$$

Fig. 2 Newton's method of extracting the cube root

Comparing the method of extraction in ancient China with that of Newton's, it is obvious to see that despite their different details, both of them have clear procedures which are highly coherent and sequential; both of them have circular iterations so that the correct result can be attained smoothly according to the fixed order; and both of them do not have any description or proof of the rationality of the process. Therefore, there is a great similarity between Newton's thought of mathematical mechanization and the mathematical mechanization thought contained inancient mathematics in China.They have the same connotation of ideas from the point of view of solving procedures.

4. Newton's thought of the mathematical mechanization and that used by Descartes

René Descartes (1596-1650) is credited with pioneering the mechanization of modern mathematics (Kline, 1990), due to the methodology given in his *Geometry* on the transformation of any geometric problem into an equation problem. It is as follows:

The first step is to arithmetizegeometry, i.e., to define the rules of arithmetic operations for geometric elements, such as addition, subtraction, multiplication, and division of lines.

In the second step, the geometric problem is reduced to an algebraic equation. The method is to find two different representations of a quantity and then make them equal to form an equation. When there are more than one equations, they are reduced to one equation by elimination.

The third step is to judge the category of the final equation and solve it with the fixed method of that category.

The example provided by Descartes is as follows (Descartes, 1992):

Let GL be a ruler, and GL can rotate around the G point (as shown in the Fig. 3). It intersects the fixed point L with the figure $CNKL$ (a triangle) that can slide along the KA. The intersection of GL and CNK is point C. When point L moves, the trajectory of C will be EC. The equation is discussed below.

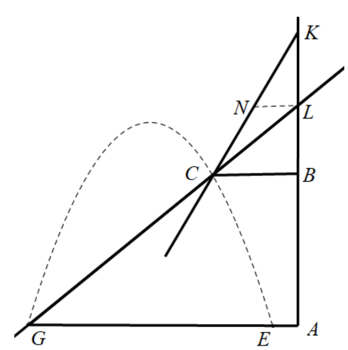

Fig. 3 Cartesian geometric method

Then a line CB is made through point C which is parallel to GA. Let CB and BA be y and x. Let the three fixed quantities of GA, KL, NL be a, b and c.

Because $NL : LK = CB : BK$, then $BK = \frac{b}{c} y$.

Therefore, $BL = \frac{b}{c} y - b$, $AL = x + \frac{b}{c} y - b$.

Because $CB : LB = AG : LA$, we can get an equation $\frac{ab}{c} y - ab = xy + \frac{b}{c} y^2 - by$.

So the desired equation is $y^2 = cy - \frac{cx}{b} y + ay - ac$.

The transformation of geometric problems into equation problemsalso appears in Newton's mathematics (Newton, 1720):

The right line AB be to be cut or divided in mean and extreme proportion in C, that is, so that, BE, the square of the greatest part, shall be equal to the rectangle BD contained under the whole, and the least part; having put $AB = a$, and $BC = x$, then will $AC = a - x$, and $x^2 = a$ into $a - x$; an equation which by reduction gives $x = -\frac{1}{2} a + \sqrt{\frac{5}{4} aa}$.

There is another example. As if the question be of an isosceles CBD inscribed in a circle, whose sides BC, BD, and base CD, are to be compared with the diameter of the circle AB. This may either be proposed of the investigation of the diameter from the given sides and base, or of the investigation of the basis from the given sides and diameter; or lastly, of the investigation of the sides from the given base and diameter; but however it be proposed, it will be reduced to an equation by the same series of an analysis. viz. If the diameter be sought, I put $AB = x$, $CD = a$, and BC or $BD = b$. Then (having drawn AC) by reason of the similar triangles ABC, and CBE, AB will be to $BC :: BC : BE$, or $x : b :: b : BE$. Wherefore, $BE = \frac{bb}{x}$. Moreover, CE is $= \frac{1}{2} CD$, or to $\frac{1}{2} a$; and by reason of the right angle CEB, $CEq + BEq = BCq$, that is, $\frac{1}{4} a^2 + \frac{b^4}{x^2} = b^2$. Which equation, by reduction, will give the quantity x sought.

But if the side BC or BD be sought, put $AB = c$, $CD = x$, and BC or $BD = b$. Then (AC being drawn) because of the similar triangles ABC and CBE, there is $AB : BC :: BC : BE$, or $c : x :: x : BE$. Wherefore $BE = \frac{bb}{c}$; and also $CE = \frac{1}{2} CD$, or $\frac{1}{2} x$. And because the angle CBE is right, $CEq + BEq = BCq$, that is, $\frac{1}{4} x^2 + \frac{b^4}{c^2} = b^2$; an equation which will give by reduction the sought quantity x.

But if the side BC or BD be sought, put $AB = c$, $CD = a$, and BC or $BD = x$. And (AC being drawn as before) by reason of the similar triangles ABC and CBE, AB is to $BC :: BC : BE$, or $c : x :: x : BE$. Wherefore $BE = \frac{xx}{c}$. Moreover, $CE = \frac{1}{2} CD$, or $\frac{1}{2} a$;

and by reason of the right angle CEB, $CEq + BEq = BCq$, that is, $\frac{1}{4}a^2 + \frac{b^4}{c^2} = x^2$; and the equation, by reduction, will give the quantity sought, viz. x.

Comparing Newton's and Descartes' methods of transforming geometric problems into equation problems, it is clear that Newton's method is no less strategic, procedural, and sequential than Descartes' method. Notably, Newton's method is significantly more concise and fast.

Moreover, Newton had the practice of transforming not only geometric problems into equation problems but also general application problems into equation problems. Regarding a practical problem, Newton's solving strategy is as follows (Newton, 1720):

We must first consider whether the propositions or sentences in which it is expressed, be all of them fit to be denoted in algebraic terms. And if so, then we give names to both known and unknown quantities, as far as occasion requires. And the conditions thus translated to algebraic terms will give as many equations as are necessary to solve it.

As if there are required three numbers in continual proportion whose sum is 20, and the sum of their squares 140; putting *x*, *y*, and *z* for the names of the three numbers sought, the question will be translated out of the verbal to the symbolical expression, as follows (as shown in Table 1):

Table 1 Newton's transformation of an algebraic problem into equation problem

The question in words	The same in symbols
There are sought three numbers on these conditions:	x, y, z?
That they shall be continually proportional.	$x : y :: y : z$ ，或 $xz = y^2$
That the sum shall be 20.	$x + y + z = 20$
And the sum of their squares 140.	$x^2 + y^2 + z^2 = 140$

Take another example. A certain merchant increases his estate yearly by a third part, abating 100*l* which he spends yearly in his family; and after three years he finds his estate doubled. Query, what he is worth?

The equation procedure given by Newton is shown in Table 2.

It can be seen that Newton's thought of mathematical mechanization is more concise, convenient and clear than Descartes', which is more convenient for readers to learn and master.

Table 2 Newton's transformation of another algebraic problem into equation problem

In English.	Algebraically.
A merchant has an estate.	x.
Out of which the first year he expends 100*l*.	$x - 100$.
And augments the rest by one third.	$x - 100 + \frac{x-100}{3}$ or $\frac{4x-400}{3}$.

And so the second year expends 100*l*.	$\frac{4x-400}{3}-100$ or $\frac{4x-700}{3}$.
And augments the rest by a third.	$\frac{4x-700}{3}+\frac{4x-700}{9}$ or $\frac{16x-2800}{9}$.
And so the third year expends 100*l*.	$\frac{16x-2800}{9}-100$ or $\frac{16x-3700}{9}$.
And by the rest gains likewise one third part.	$\frac{16x-3700}{9}+\frac{16x-3700}{27}$ or $\frac{64x-14800}{27}$.
And he becomes at length twice as rich as at first.	$\frac{64x-14800}{27}=2x$.

5. Conclusions

From the beginning of mathematics appearing, there have been two major ideas that have profoundly affected its development direction and process, one is axiomatic thought, and the other is mathematical mechanization thought (Gao, 2001). In ancient China, the latter has always occupied a predominant position. While in the western world, it is axiomatic thought dominated the process of almost all branches of mathematics, especially after ancient Greece. However, starting form the period of Renaissance, some aspects of western mathematics gradually shifted towards mechanization (Wu, 2008). In this process, the renowned French mathematician Descartes and the German mathematician Leibniz (1646-1716) are recognized as the pioneers of mathematical mechanization in the West (Ji, 2001; Hu, 1999). Actually, Newton also made a significant contribution to facilitating the trend of mathematical mechanization. Newton did a lot of mathematical research in his life including not only about calculusbut also arithmetic, algebra, geometry, differential equations, and series, etc.. After careful analysis of the methods and conclusions given by Newton in these mathematical research, many of them are given by using simple and correct procedures and steps with strong order, such as the fluxion method, the method of extracting the root, and the method of finding the factors of algebraic formula, etc. Some of the procedures and steps given are different from that of ancient Chinese mathematics but they share the same thought. Compared with the procedures and steps of other mathematicians at that time, for example, the method of transforming geometric problems into algebraic equations given by Descartes, Newton's algorithms are obviously more concise and faster, and more acceptable and applicable to readers. Therefore, it can be concluded that Newton's mathematical mechanization thought should be exceedingly clear, advanced, and practical at that time. Newton should be one of the most powerful promoters of the tendency of some branches of western mathematics to turn to mechanization tradition in the late Renaissance. He was also a forerunner in the creation and application of western mathematical mechanization thought at the time, with Descartes and Leibniz.

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