__Original Research Article__

Solitons of the Jimbo- Miwa equation through the sine-Gordon expansion scheme

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ABSTRACT

In this article, by using the sine-Gordon expansion method the Jimbo-Miwa equation provides some soliton of hyperbolic type functions. The obtained solutions have a few arbitrary parameters which are very impactful to explain the nature of the solitary wave. We have explained the nature of the wave profile for different values of the free parameters of the obtained solutions via Matlab software.

Keywords: Nonlinear evolution equations; the Jimbo-Miwa equation; the sine-Gordon method; Solitons.

1. INTRODUCTION

Natural processes and physical phenomena frequently exhibit intricate nonlinear characteristics which lead to different types of non-linear mathematical models. Differential equations (DEs), particularly nonlinear partial differential equations (PDEs) are the key feature of those models. For instance, the Korteweg–de Vries (KdV) equation is proposed to study the surface water wave [1,2], the Kadomtsev–Petviashvili (KP) equation expresses the dynamics of 2D solitary waves [3], the Schrödinger equation expresses the dynamics of electromagnetic wave [4], etc. The Jimbo and Miwa (JM) equation was first developed by Jimbo and Miwa [5] to clarify some intriguing (3 + 1)-dimensional waves [6]. This JM equation is considered the 2nd equation of the KP hierarchy [7]. To get a better knowledge and explanation of different natural phenomena, finding solutions to the respective models is necessary. The JM equation has been the subject of extensive research, and numerous researchers have used various strategies to discover various types of solutions. Multiple-soliton solutions to the JM equation were discovered in [8] using the simplified Hirota's approach [8–10]. Hirota bilinear scheme [11-13] was applied to solve the JM equation. The generalized Riccati equation mapping scheme was also used to find exact solutions for the (3 + 1)-JM equation [14]. Two different forms of variable separation solutions for the (3+1)-dimensional JM equation are found using the multi-linear variable separation method [15]. Moreover, four different lump-type solutions to the (3+1)- dimensional JM equation were obtained using the Hirota bilinear form [7]. To our knowledge, the (3+1)- dimensional JM equation has not yet been solved by using the sine-Gordon expansion scheme. The purpose of this study is to investigate the stated equation through the sine-Gordon expansion method and attain some fresh and broad-ranging solutions. Besides, demonstrate the wave profile of the obtained solutions to understand the nature of the solitons via MATLAB.

2. METHODOLOGY

In this section sine-Gordon expansion method will be briefly explained. A nonlinear evolution equation of variables x,y,z,t is considered as follows:

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}-\frac{1}{m^2}\frac{\partial^2 U}{\partial t^2}=\sin(U). \tag{2.1}$$

where $U = U(x, y, z, t)$ is the wave function, while m denotes any real constant. Now, assuming the transformation below,

$$U(x,y,z,t)=U(\xi), \qquad \xi=\alpha x+\beta y+\gamma z-\omega t, \tag{2.2}$$

where α, β and γ are ratios of the wave's directions and ω is the traveling wave's speed.

Equation (2.1) can be changed by using (2.2), resulting in a nonlinear equation of the following form

$$\frac{d^2U}{d\xi^2}=\frac{1}{\left(\alpha^2+\beta^2+\gamma^2-\frac{\omega^2}{m^2}\right)}\sin(U). \tag{2.3}$$

Equation (2.3) can be restored as bellow

$$\left(\frac{d}{d\xi}\left(\frac{U}{2}\right)\right)^2=\frac{1}{\left(\alpha^2+\beta^2+\gamma^2-\frac{\omega^2}{m^2}\right)}sin^2\left(\frac{U}{2}\right)+k, \tag{2.4}$$

here k is a integration constant.

If $k = 0$, $\phi(\xi) = \left(\frac{U}{2}\right)$ and $l^2 = \frac{1}{\left(\alpha^2+\beta^2+\gamma^2-\frac{\omega^2}{m^2}\right)}$, then from (2.4) we have

$$\frac{d\phi}{d\xi}=l\sin(\phi). \tag{2.5}$$

Equation (2.5) changes into the following when $l = 1$

$$\frac{d\phi}{d\xi}=\sin(\phi). \tag{2.6}$$

We develop the following relations by using the variable separation principle

$$\sin(\phi)=\sin(\phi(\xi))=\frac{2f\exp(\xi)}{1+f^2\exp(2\xi)}=\text{sech}(\xi), \qquad \text{for } f=1, \tag{2.7}$$

$$\cos(\phi)=\cos(\phi(\xi))=\frac{-1+f^2\exp(2\xi)}{1+f^2\exp(2\xi)}=\tanh(\xi), \qquad \text{for } f=1, \tag{2.8}$$

here f is a constant of integration.

We now take into consideration a (3+1)-dimensional nonlinear evolution equation with four variables x, y, z and t as follows:

$$\psi\left(U,\frac{\partial U}{\partial x},\frac{\partial U}{\partial y},\frac{\partial U}{\partial z},\frac{\partial U}{\partial t},\frac{\partial^2 U}{\partial x^2},\frac{\partial^2 U}{\partial y^2},\frac{\partial^2 U}{\partial z^2},\frac{\partial^2 U}{\partial t^2}\dots\right)=0, \tag{2.9}$$

where $U = U(x, y, z, t)$ is an undefine wave function, ψ is a polynomial of function U and of its derivatives. Here, the space variables are x, y and z, the temporal variable is t and partial derivatives of the function U with respect to x, y, z and t respectively are $\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}, \frac{\partial U}{\partial t}$ ... etc.

We begin the solution of equation (2.9) as bellows in accordance with the sine-Gordon expansion method

$$U(\xi) = M_0 + \sum_{r=1}^{R} tanh^{r-1}(\xi)\,[N_r \operatorname{sech}(\xi) + M_r \tanh(\xi)]. \tag{2.10}$$

Using the identities (2.7) and (2.8) into the solution of (2.10), we have

$$U(\phi(\xi)) = M_0 + \sum_{r=1}^{R} cos^{r-1}\phi(\xi)\,[N_r \sin(\phi(\xi)) + B_r \cos(\phi(\xi))]. \tag{2.11}$$

Value of R can be calculated from the resulting nonlinear equation using the balancing principle, taking into account term, the greatest power nonlinear and the higher derivative. A algebraic system of equations is produced by leveling the coefficients of $[sin^q(\phi(\xi))\, cos^q(\phi(\xi))]$ to zero. The values of $M_r, N_r, \alpha, \beta, \gamma$ and ω are obtained by solving this system of algebraic equations. At last, the desired solution of the equation (2.9) can be obtained by using the values of $M_r, N_r, \alpha, \beta, \gamma$ and ω into solution (2.10),

3. APPLICATION

In this section, the SGE method is exploited to obtain standard explicit and solitary wave solutions of the Jimbo-Miwa equation in the form

$$u_{xxxy} + 3u_x u_{xy} + 3u_y u_{xx} + 2u_{yt} - 3u_{xz} = 0. \tag{3.1}$$

To investigate the solitons through the MSE method to Eq. (3.1) we apply the wave transformation

$$u(x, y, z, t) = V(\xi),\ \xi = k(x + y + z - \omega t) \tag{3.2}$$

By using Eq. (3.2) in Eq. (3.1) we attain the nonlinear equation as follows

$$k^2 V^{iv} + 6kV'V'' - (2\omega + 3)V'' = 0, \tag{3.3}$$

where $'$ represents the derivatives with regard to ξ. Substituting $V'(\xi) = U(\xi)$ in Eq. (3.3) and integrating then we get the following differential equation:

$$k^2 U'' + 3kU^2 - (2\omega + 3)U = 0. \tag{3.4}$$

Applying the balancing principle between the highest degree of the nonlinear term U^2 and the derivative term U'' and gives the value of $N = 2$. Using the value of N we attain the shape of the solution function of Eq. (3.4) as in solution (2.10) as follows

$$U(\phi) = M_0 + N_1 \sin(\phi) + M_1 \cos(\phi) + N_2 \sin(\phi)\cos(\phi) + M_2 \cos^2(\phi). \tag{3.5}$$

The double derivative of the solution (3.5) is shown below

$$U''(\phi) = -N_1\sin^3(\phi) + N_1\cos^2(\phi)\sin(\phi) - 2M_1\sin^2(\phi)\cos(\phi) - 5N_2\,cos(\phi)\sin^3(\phi) + N_2\cos^3(\phi)\sin(\phi) + 2M_2\sin^4(\phi) - 4M_2\cos^2(\phi)\sin^2(\phi). \quad (3.6)$$

Substitute the values in (3.4) and equating the like power of $\sin(\phi)$, $\cos(\phi)$ and constant term, we find the following results of the free parameters:

Set-1: $\omega = -\frac{k^2}{2} - \frac{3}{2}, M_0 = \frac{2k}{3}, M_1 = 0, M_2 = -k, N_1 = 0, N_2 = \pm\sqrt{-k}$

Set-2: $\omega = \frac{k^2}{2} - \frac{3}{2}, M_0 = k, M_1 = 0, M_2 = -k, N_1 = 0, N_2 = \pm\sqrt{-k}$

Putting the value of the free parameters mentioned in the set -1 into the solution (3.5), we attain

$$U_1 = \frac{k(-sinh(\xi) + 2i)}{(3sinh(\xi) + 3i)}; \quad (3.7)$$

where $\xi = k\left(x + y + z + \left(\frac{k^2}{2} + \frac{3}{2}\right)t\right)$.

Putting the value of the free parameters mentioned in the set -2 into the solution (3.5), we attain

$$U_2 = \frac{k(1 + i\sinh(\xi))}{cosh(\xi)^2}; \quad (3.8)$$

where $\xi = k\left(x + y + z - \left(\frac{k^2}{2} - \frac{3}{2}\right)t\right)$.

Furthermore, we have achieved more solutions for the accomplished values of the free parameter but not noted here for repetition.

4. PHYSICAL EXPLANATION

In this section, we will explain the physical significance of the soliton of the obtained solution functions of JM equation. At the beginning, the solution function (3.7) represents a spike type soliton is shown in Fig.-1(a) for the value of $k = \pm 4$.

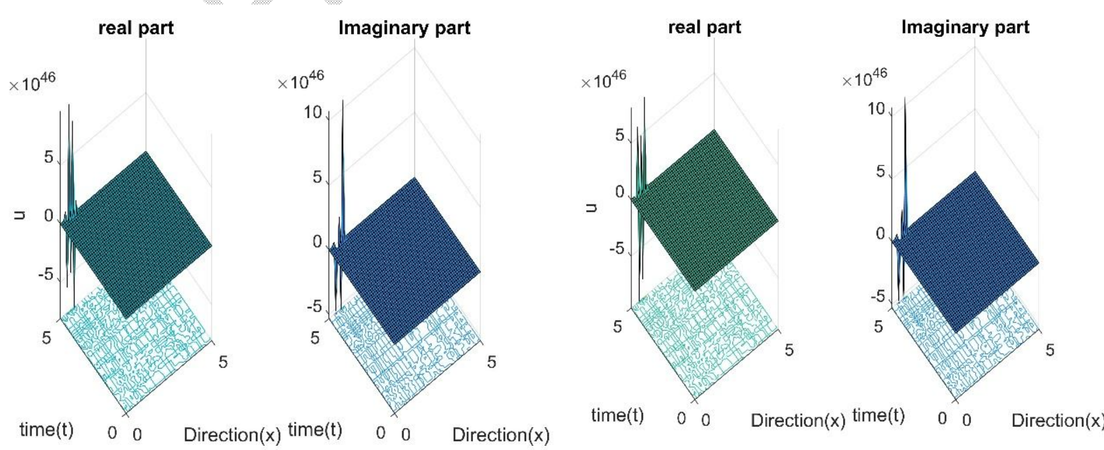

Fig-1(a): wave profile of solution (3.7) for $k = \pm 4$

When we choose the value of the free parameter as $k = \pm 1$, then the same solution provides more spike asserted in fig.-1(b).

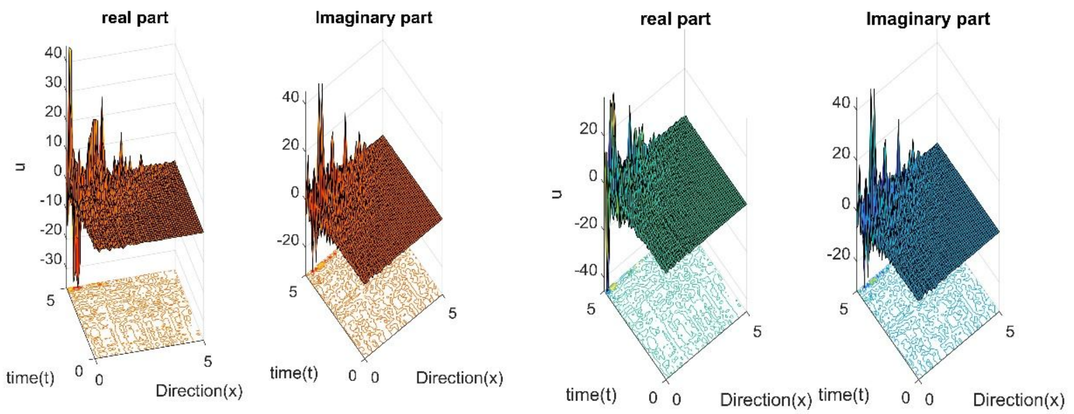

Fig-1(b): wave profile of solution (3.7) for $k = \pm 1$

After selecting the value of the parameter as more small like $k = \pm 0.25$ the solution $U_1(x, y, z, t)$ represents a symmetric type wave marked in fig.-1(c).

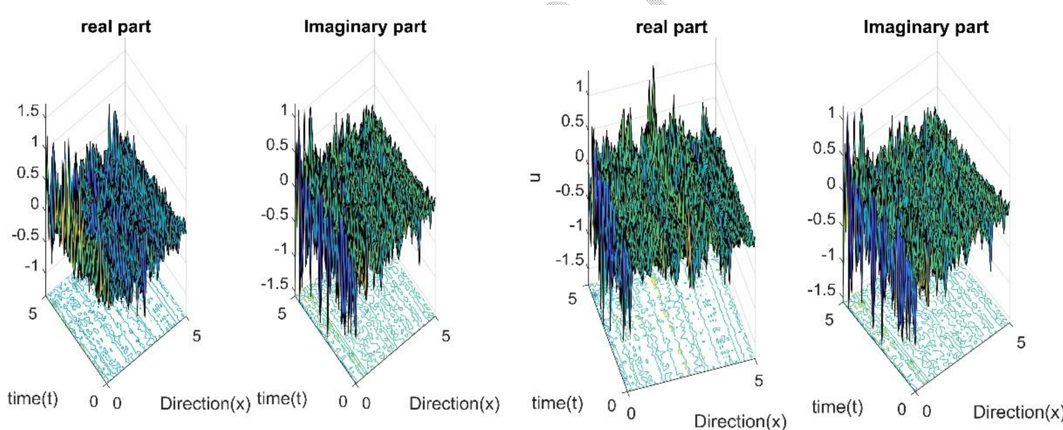

Fig-1(c): wave profile of solution (3.7) for $k = \pm 0.25$

Furthermore, the solution function (3.8) represents the unequal periodic wave shown in Fig.-2(a) and Fig.-2(b) for the value of $k = \pm 0.25$ and $k = \pm 1$ respectively.

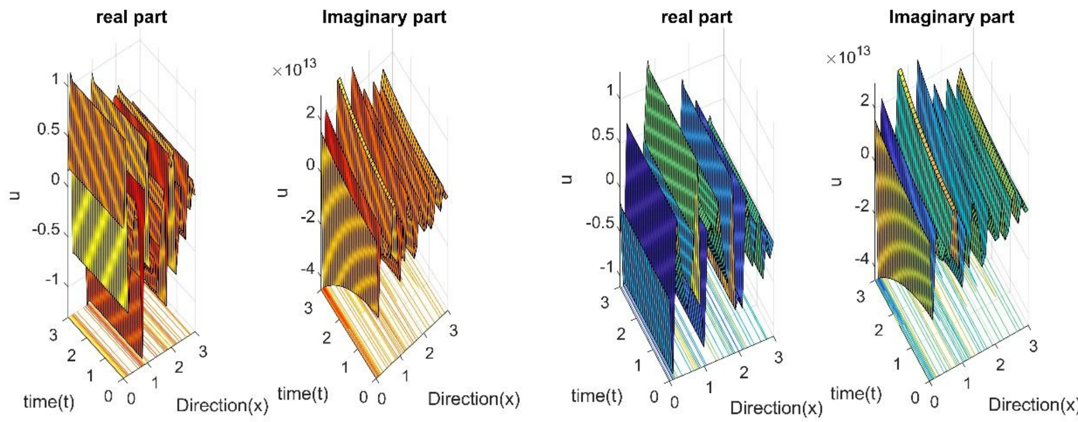

Fig-2(a): wave profile of solution (3.8) for $k = \pm 0.25$

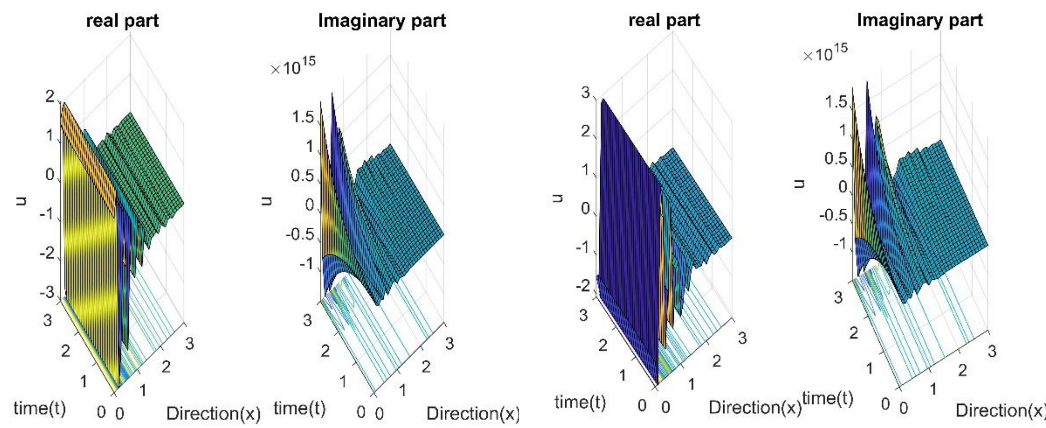

Fig-2(b): wave profile of solution (3.8) for $k = \pm 1$

Besides, we have obtained the same wave profile for the different values of the free parameter of the attained solution functions. We have not documented here for sagacity.

5. CONCLUTION

In this study we have investigated the Jimbo-Miwa equation through the sine-Gordon expansion method and attained some fresh and broad ranging traveling wave solution. By choosing analogous values of the free parameters of the obtained solution function represent different types of waves which has a very important and significance role to describe the reason of the real phenomena. Moreover, the implemented method is straightforward and easy to apply. This process will be very helpful to explain and investigate any nonlinear evolution equation in future research.

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