

Bayesian Estimation of Exponentiated Rayleigh Distribution under Symmetric and Asymmetric Loss Functions

Abstract

In this paper, we have considered the estimation problem of one-parameter exponentiated Rayleigh distribution. The parameters are estimated using likelihood based inferential procedure. We have computed MLEs and Bayes estimates under informative and non-informative priors along with six different loss functions, the Bayes estimation was obtained “Squared error, Linear exponential, Precautionary, Entropy, De Groot and non-Linear exponential loss functions”. Finding a good estimator of the unidentified shape parameter is the study's main goal. The Bayesian estimates of the parameter of exponentiated Rayleigh distribution are obtained using Markov chain Monte Carlo (MCMC) simulation method. All the computations are performed in OpenBUGS and R software.

Keywords: Bayesian Estimation, MLE, Bayes estimate, Exponentiated Rayleigh Distribution, Loss function, Prior, Posterior,

1. INTRODUCTION

For modelling data, Burr (1942) introduced twelve different forms of cumulative distribution function. Among them Burr Type X and Burr Type XII received the maximum attention, where Surles and Padgett (2001) observed that the Burr Type X distribution (Exponentiated Rayleigh distribution) can be used quite effectively in modeling strength data and also modeling general lifetime data. Several aspects of the ER distribution were studied in literature, see for example Sartawi and Abu-Salih (1991), Jaheen (1995), Ahmed, Fakhry and Jaheen (1997), Karam and Jbur (2014), Feroze and Aslam (2012) and Sindhu and Aslam (2014). The cumulative distribution function (CDF) and the probability density function (pdf) of the ER distribution with shape parameter ($\theta > 0$) are respectively as follows: (1994) Numerous authors have worked on the generalization of Rayleigh distribution; among them are Voda (2007), Kundu and Raqab (2005), Raqab and Madi (2009), Merovci (2013), Dey et al. (2014), Merovci and Elbatal(2015), Ahmad et al. (2015), Saima et al. (2016), Ateeq et al. (2019), Sofi et al. (2019).

$$F(x, \theta) = (1 - e^{-x^2})^\theta ; x > 0; \theta > 0 \quad (1)$$

$$f(x; \theta) = 2\theta x e^{-x^2} (1 - e^{-x^2})^{(\theta-1)} ; x > 0; \theta > 0 \quad (2)$$

The random number has been generated by inverse transformation method, which is for uniform random U:

$$X_i = \left(-\ln(1 - U_i^{1/\theta})\right)^{0.5} ; \quad i = 1, 2, \dots, n \quad (3)$$

Studying a particular phenomenon via the challenge of estimating the unknown parameters in statistical distributions is one of the significant problems that individuals who are interested in applied statistics face. This study examines the estimations of the ER distribution's unknown shape parameter. The ER distribution is a significant distribution in operations research and statistics, and it is used in a number of fields including biology, agriculture, and other sciences. Considering the Bayesian analysis of the unknown parameters with various priors (informative and non-informative) and loss functions for complete samples is the main goal of this.

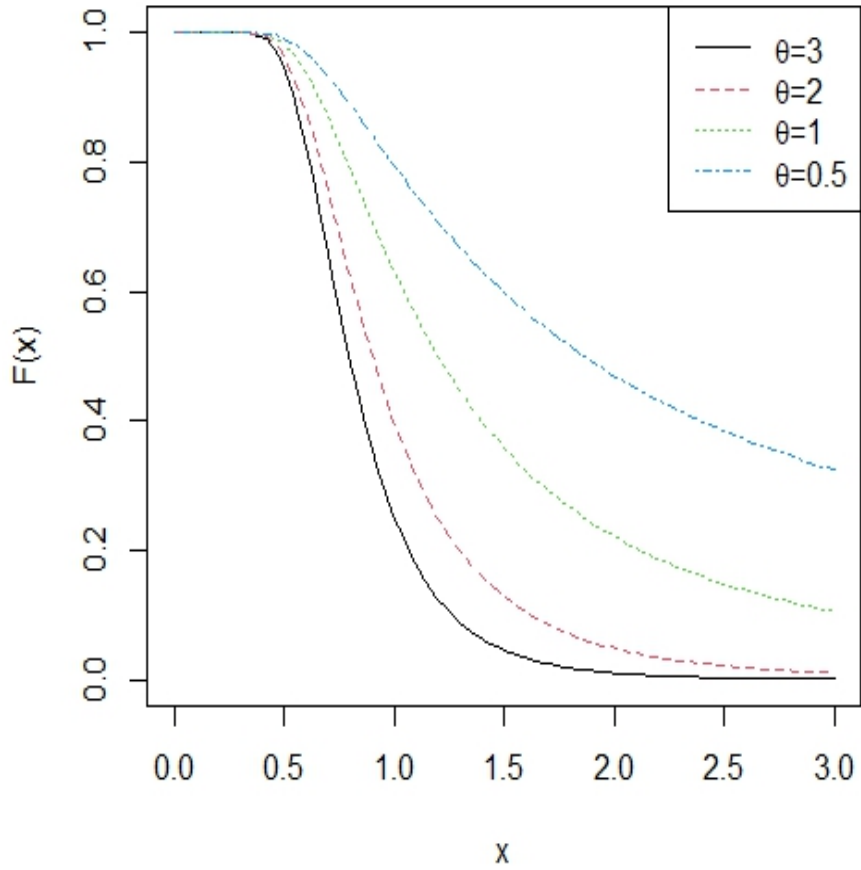


Figure 1. plot of cumulative density function at different value of parameter

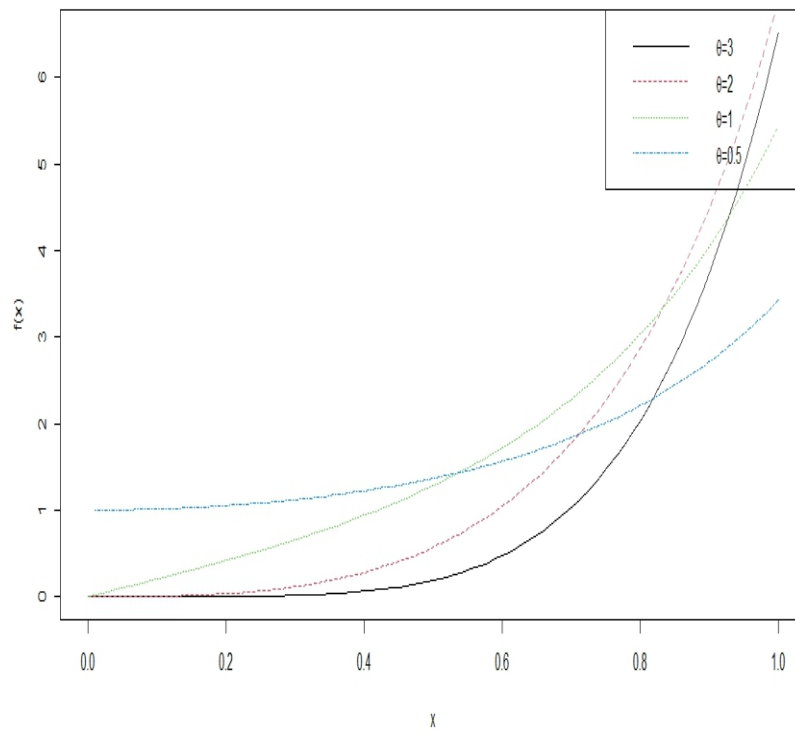


Figure 2. plot of probability density function at different value of parameter

2. LIKLEHOOD FUNCTION

Let X_1, X_2, \dots, X_n be a random sample from ER distribution with shape parameter $\theta > 0$. Therefore, the likelihood function of θ , from (1), as follows as:

$$L(\theta|data) = \prod_{i=1}^n f(x_i, \theta) \quad (4)$$

$$L(\theta|data) = \prod_{i=1}^n \left(2\theta x e^{-x_i^2 + \ln(1-x_i^2)^{-1}} e^{-\theta \ln(1-e^{-x_i^2})^{-1}} \right) \quad (5)$$

$$L(\theta|data) = q_1 \theta^n e^{-\theta [\sum_{i=1}^n \ln(1-e^{-x_i^2})^{-1}]} \quad (6)$$

Such that $q_1 = 2^n e^{\left(\sum_{i=1}^n \ln x_i - \sum_{i=1}^n x_i^2 + \sum_{i=1}^n \ln(1-e^{-x_i^2})^{-1} \right)}$

3. Bayesian Estimators using Different Prior and Loss Functions

In this section Bayesian Estimators of the shape parameter for four different prior functions and under six different loss functions has been determined.

3.1 Types of loss functions: -

If $\hat{\theta}$ represent of estimator for the shape parameter θ , then for:

1. Squared error loss function (SELF): SELF defined as:(2011)

$$L(\hat{\theta}, \theta) = c(\hat{\theta} - \theta)^2; \hat{\theta}_{self} = E(\theta) \quad (7)$$

2. Linear exponential loss function: the loss function defined as: (2011)

$$L(\hat{\theta}, \theta) = \left(e^{c(\hat{\theta}-\theta)} - c(\hat{\theta} - \theta) - 1 \right); \hat{\theta}_{linex} = -\frac{1}{c} \ln E(e^{-c\theta}) \quad (8)$$

3. Precautionary loss function: the Precautionary loss function defined as: (2012)

$$L(\hat{\theta}, \theta) = \frac{(\hat{\theta} - \theta)^2}{\hat{\theta}}; \hat{\theta}_{plf} = \sqrt{E(\theta^2)} \quad (9)$$

4. Entropy loss function (ELF): ELF defined as: (2011)

$$L(\hat{\theta}, \theta) = \left(\frac{\hat{\theta}}{\theta} \right)^t - t \ln \left(\frac{\hat{\theta}}{\theta} \right) - 1; \hat{\theta}_{elf} = \left(E(\theta^{-t}) \right)^{-\frac{1}{t}} \quad (10)$$

5. De Groot loss function (DLF): DLF defined as: (2011)

$$L(\hat{\theta}, \theta) = \left(\frac{(\hat{\theta} - \theta)}{\hat{\theta}} \right)^2; \hat{\theta}_{dlf} = \frac{E(\theta^2)}{E(\theta)} \quad (11)$$

6. Non- Linear exponential loss functions is defined as:

$$L(\hat{\theta}, \theta) = \left(e^{c(\hat{\theta}-\theta)} + c(\hat{\theta} - \theta)^2 - c(\hat{\theta} - \theta) - 1 \right); \hat{\theta}_{Nlinex} = \frac{1}{c+1} \ln E(e^{-c\theta}) - 2E(\theta) \quad (12)$$

3.2 The Posterior distributions with different priors

For the random variable X, the posterior density function of the shape parameter θ is:

$$P(\theta|\underline{x}) = \frac{L(\theta|\underline{x}) \cdot p(\theta)}{\int_0^\infty L(\theta|\underline{x}) \cdot p(\theta) d\theta} \quad (13)$$

For ‘‘Bayesian estimation’’, we describe two distinct posterior distributions under complete samples, and two distinct prior distributions for the shape parameter.

3.3 The Jeffery’s prior, for the parameter (θ) is:

$$p(\theta) = \frac{1}{\theta}; \theta > 0 \quad (14)$$

Then under the assumption of this prior distribution and by equation (13), the posterior distribution will be as:

$$p_j(\theta|\underline{x}) = \frac{q_1 \theta^n e^{-\theta [\sum_{i=1}^n \ln(1-e^{-x_i^2})^{-1}]} \frac{1}{\theta}}{\int_0^\infty q_1 \theta^n e^{-\theta [\sum_{i=1}^n \ln(1-e^{-x_i^2})^{-1}]} \frac{1}{\theta} d\theta} \quad (15)$$

$$p_j(\theta|\underline{x}) = \frac{\left(\sum_{i=1}^n \ln(1 - e^{-x_i^2})\right)^n}{\Gamma(n)} \theta^{n-1} e^{-\theta\left(\sum_{i=1}^n \ln(1 - e^{-x_i^2})\right)^{-1}} \quad \theta > 0 \quad (16)$$

3.4 The Gamma prior, considered to be:

$$p(\theta) = \frac{b^a \theta^{a-1}}{\Gamma(a)} e^{-\theta b}, \quad \theta > 0, \quad a, b > 0 \quad (17)$$

By equation (13) the posterior distribution under the assumption Gamma prior is:

$$p_j(\theta|\underline{x}) = \frac{q_1 \theta^n e^{-\theta[\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1}]} \frac{b^a \theta^{a-1}}{\Gamma(a)} e^{-\theta b}}{\int_0^\infty q_1 \theta^n e^{-\theta[\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1}]} \frac{b^a \theta^{a-1}}{\Gamma(a)} e^{-\theta b} d\theta} \quad (18)$$

$$p_j(\theta|\underline{x}) = \frac{\left(\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1} + b\right)^{n+a}}{\Gamma(n+a)} \theta^{n+a-1} e^{-\theta[\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1} + b]}; \quad \theta > 0 \quad (19)$$

3.5 Bayesian Estimators under Jeffrey Prior using the six different loss functions:

$$\hat{\theta}_{SELF} = \frac{n}{\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1}} \quad (20)$$

$$\hat{\theta}_{linex} = \frac{1}{c} \ln \left(\frac{\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1}}{\ln(1 - e^{-x_i^2})^{-1} + c} \right) \quad (21)$$

$$\hat{\theta}_{plf} = \sqrt{\frac{n(n+1)}{(\ln(1 - e^{-x_i^2})^{-1})^2}} \quad (22)$$

$$\hat{\theta}_{elf} = \frac{\Gamma(n)^{1/t}}{\Gamma(n-t)} \frac{1}{\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1}} \quad (23)$$

$$\hat{\theta}_{DLF} = \frac{(n+1)}{\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1}} \quad (24)$$

$$\hat{\theta}_{Nlinex} = \frac{\left(\ln \left(\frac{\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1}}{\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1} + c} \right)^n - 2 \frac{n}{\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1}} \right)}{c+2} \quad (25)$$

3.6 Bayesian Estimators under Gamma Prior using the six different loss functions:

$$\hat{\theta}_{SELF} = \frac{n+a}{\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1} + b} \quad (26)$$

$$\hat{\theta}_{linex} = \frac{1}{c} \ln \left(\frac{\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1} + b}{\ln(1 - e^{-x_i^2})^{-1} + c + b} \right)^{(n+a)} \quad (27)$$

$$\hat{\theta}_{plf} = \sqrt{\frac{(n+a)(n+a+1)}{(\ln(1 - e^{-x_i^2})^{-1} + b)^2}}$$

$$\hat{\theta}_{elf} = \frac{\Gamma(n+a)^{1/t}}{\Gamma(n+a-t)} \frac{1}{\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1} + b} \quad (28)$$

$$\hat{\theta}_{DLF} = \frac{(n+a+1)}{\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1} + b} \quad (29)$$

$$\hat{\theta}_{N\text{Linex}} = \frac{\left(\ln \left(\frac{\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1} + b}{\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1} + b + c} \right)^{n+a} - 2 \frac{n}{\sum_{i=1}^n \ln(1 - e^{-x_i^2})^{-1} + b} \right)}{c + 2} \quad (30)$$

4. SIMULATION STUDY

In this section we mainly perform some simulation experiments to observe the behaviour of the different Bayes estimators for the shape parameter proposed in the previous section (3), using different sample sizes “n= 10, 20, 30, 50, 75 and 100” and for two distinct parameters values “ $\theta = 1.5$ and 3”, by applying the Monte Carlo simulation to compare the performance of these estimators using the mean squared error (MSE). All results are computed by R program based on replications and for two different put “a = 3, b= 0.8, c= 1; a = 4, b= 1.2, c= 1”.

The mean and MSE values for the Bayesian estimator, are recorded in tables (1), (2), (3) and (4).

Table (1): The mean value of $\hat{\theta}$, when $\theta = 1.5$

| n | Jeffery prior | | | | | |
|------------|-------------------------|------------------------|------------------------|--------------------------|------------------------|---------------------------|
| | B_{SELF} | B_{PLF} | B_{DLF} | B_{LINEX} | B_{ELF} | B_{NLINEX} |
| 10 | 1.6566 | 1.7356 | 1.8124 | 1.5118 | 1.4047 | 1.6125 |
| 20 | 1.5573 | 1.5860 | 1.6253 | 1.4871 | 1.4301 | 1.5464 |
| 30 | 1.5502 | 1.5657 | 1.6017 | 1.5105 | 1.4622 | 1.5345 |
| 50 | 1.5264 | 1.5312 | 1.5462 | 1.5022 | 1.4713 | 1.5144 |
| 75 | 1.5219 | 1.5217 | 1.5320 | 1.5043 | 1.4811 | 1.5132 |
| 100 | 1.5191 | 1.5262 | 1.5245 | 1.5032 | 1.4761 | 1.5112 |

| n | Gamma prior | | | | | |
|------------|-------------------------|------------------------|------------------------|--------------------------|------------------------|---------------------------|
| | B_{SELF} | B_{PLF} | B_{DLF} | B_{LINEX} | B_{ELF} | B_{NLINEX} |
| 10 | 1.8812 | 1.9512 | 2.0247 | 1.7456 | 1.6615 | 1.8357 |
| 20 | 1.6832 | 1.7274 | 1.7453 | 1.6231 | 1.5634 | 1.6606 |
| 30 | 1.6452 | 1.6597 | 1.67082 | 1.5843 | 1.5407 | 1.6328 |
| 50 | 1.5782 | 1.5832 | 1.6182 | 1.5523 | 1.5376 | 1.5632 |
| 75 | 1.5472 | 1.5638 | 1.5679 | 1.5413 | 1.5246 | 1.5539 |
| 100 | 1.5452 | 1.5532 | 1.5509 | 1.5234 | 1.5232 | 1.5423 |

| n | Gamma prior | | | | | |
|------------|-------------------------|------------------------|------------------------|--------------------------|------------------------|---------------------------|
| | B_{SELF} | B_{PLF} | B_{DLF} | B_{LINEX} | B_{ELF} | B_{NLINEX} |
| 10 | 1.9054 | 1.6345 | 2.1420 | 1.7657 | 1.7032 | 1.8632 |
| 20 | 1.7034 | 1.5604 | 1.7676 | 1.6345 | 1.6245 | 1.7682 |
| 30 | 1.6401 | 1.5456 | 1.6875 | 1.6208 | 1.5934 | 1.6456 |
| 50 | 1.5789 | 1.5345 | 1.6556 | 1.5753 | 1.5443 | 1.5965 |
| 75 | 1.5556 | 1.5245 | 1.5236 | 1.5532 | 1.5265 | 1.5534 |
| 100 | 1.5434 | 1.5156 | 1.5123 | 1.5256 | 1.5134 | 1.5431 |

Table (2): The mean value of $\hat{\theta}$, when $\theta = 3$

| n | Jeffery prior | | | | | |
|------------|-------------------------|------------------------|------------------------|--------------------------|------------------------|---------------------------|
| | B_{SELF} | B_{PLF} | B_{DLF} | B_{LINEX} | B_{ELF} | B_{NLINEX} |
| 10 | 3.3243 | 3.4652 | 3.5247 | 2.8184 | 2.8125 | 3.1517 |
| 20 | 3.1145 | 3.1924 | 3.3458 | 2.7857 | 2.7823 | 3.1386 |
| 30 | 3.0764 | 3.1778 | 3.2764 | 2.6219 | 2.7225 | 3.0261 |
| 50 | 3.0621 | 3.1231 | 3.1289 | 2.6708 | 2.7706 | 3.0226 |
| 75 | 3.0423 | 3.0645 | 3.0643 | 2.5823 | 2.5669 | 3.0227 |
| 100 | 3.0351 | 3.0545 | 3.0565 | 2.4596 | 2.5636 | 3.0200 |

| n | Gamma prior (a= 3, b = 0.8) | | | | | |
|------------|------------------------------------|------------------------|------------------------|--------------------------|------------------------|---------------------------|
| | B_{SELF} | B_{PLF} | B_{DLF} | B_{LINEX} | B_{ELF} | B_{NLINEX} |
| 10 | 3.3456 | 2.8732 | 3.4729 | 2.9598 | 2.9556 | 3.2162 |
| 20 | 3.2593 | 2.8978 | 3.3543 | 2.9372 | 2.9618 | 3.1032 |
| 30 | 3.1208 | 2.9343 | 3.2467 | 2.9784 | 2.9738 | 3.0432 |
| 50 | 3.0878 | 2.9991 | 3.1343 | 3.0023 | 3.0056 | 3.0634 |
| 75 | 3.0695 | 2.9922 | 3.0881 | 3.0092 | 3.0084 | 3.0457 |
| 100 | 3.0414 | 2.9821 | 3.0652 | 3.0076 | 3.0073 | 3.0465 |

| n | Gamma prior (a= 4, b = 1.2) | | | | | |
|------------|------------------------------------|------------------------|------------------------|--------------------------|------------------------|---------------------------|
| | B_{SELF} | B_{PLF} | B_{DLF} | B_{LINEX} | B_{ELF} | B_{NLINEX} |
| 10 | 2.9675 | 2.9013 | 2.9292 | 2.9598 | 2.9556 | 3.2162 |
| 20 | 3.1278 | 2.8323 | 3.2567 | 2.9372 | 2.9618 | 3.1032 |
| 30 | 3.1149 | 2.9324 | 3.2076 | 2.9784 | 2.9738 | 3.0432 |
| 50 | 3.0643 | 2.9435 | 3.1465 | 3.0023 | 3.0056 | 3.0634 |
| 75 | 3.0351 | 2.9772 | 3.0881 | 3.0923 | 3.0084 | 3.0457 |
| 100 | 3.0484 | 2.9822 | 3.0652 | 3.0772 | 3.0073 | 3.0465 |

Table (3): The MSE value of $\hat{\theta}$, when $\theta = 1.5$

| n | Jeffrey's prior | | | | | |
|-----|-----------------------------|------------------|------------------|--------------------|------------------|---------------------|
| | B _{SELF} | B _{PLF} | B _{DLF} | B _{LINEX} | B _{EFL} | B _{NLINEX} |
| 10 | 0.3343 | 0.2302 | 0.4765 | 0.2187 | 0.2343 | 0.2934 |
| 20 | 0.1345 | 0.1153 | 0.1627 | 0.10834 | 0.1134 | 0.1243 |
| 30 | 0.0807 | 0.0786 | 0.1041 | 0.0797 | 0.0792 | 0.0831 |
| 50 | 0.0486 | 0.5674 | 0.0534 | 0.0468 | 0.0245 | 0.0533 |
| 75 | 0.0323 | 0.3493 | 0.3391 | 0.0310 | 0.0302 | 0.0332 |
| 100 | 0.0241 | 0.2321 | 0.2383 | 0.2354 | 0.0232 | 0.0245 |
| n | Gamma prior (a= 4, b = 1.2) | | | | | |
| | B _{SELF} | B _{PLF} | B _{DLF} | B _{LINEX} | B _{EFL} | B _{NLINEX} |
| 10 | 0.4323 | 0.2141 | 0.6014 | 0.2808 | 0.2543 | 0.3754 |
| 20 | 0.1621 | 0.1054 | 0.2062 | 0.1243 | 0.1162 | 0.1489 |
| 30 | 0.1204 | 0.0802 | 0.1323 | 0.0932 | 0.0853 | 0.1032 |
| 50 | 0.0573 | 0.0453 | 0.0637 | 0.0549 | 0.0483 | 0.0549 |
| 75 | 0.0352 | 0.0304 | 0.0365 | 0.0343 | 0.0323 | 0.0421 |
| 100 | 0.0244 | 0.0234 | 0.0275 | 0.0256 | 0.0231 | 0.0245 |

| n | Gamma prior (a= 4, b = 1.2) | | | | | |
|-----|-----------------------------|------------------|------------------|--------------------|------------------|---------------------|
| | B _{SELF} | B _{PLF} | B _{DLF} | B _{LINEX} | B _{EFL} | B _{NLINEX} |
| 10 | 0.4327 | 0.2145 | 0.6013 | 0.2918 | 0.2532 | 0.3767 |
| 20 | 0.1623 | 0.1052 | 0.2067 | 0.1242 | 0.1160 | 0.1476 |
| 30 | 0.1101 | 0.0801 | 0.1322 | 0.0912 | 0.0857 | 0.1034 |
| 50 | 0.0574 | 0.0463 | 0.0651 | 0.0507 | 0.0478 | 0.0541 |
| 75 | 0.0351 | 0.0306 | 0.0386 | 0.0334 | 0.0312 | 0.0321 |
| 100 | 0.0251 | 0.0224 | 0.0275 | 0.0297 | 0.0251 | 0.0242 |

Table (4): The MSE value of $\hat{\theta}$, when $\theta = 3$

| n | Jeffrey's prior | | | | | |
|-----|-----------------------------|------------------|------------------|--------------------|------------------|---------------------|
| | B _{SELF} | B _{PLF} | B _{DLF} | B _{LINEX} | B _{EFL} | B _{NLINEX} |
| 10 | 1.3456 | 1.6709 | 1.9305 | 0.6836 | 0.9347 | 1.0532 |
| 20 | 0.5326 | 0.5867 | 0.6467 | 0.3875 | 0.4587 | 0.4357 |
| 30 | 0.3189 | 0.3343 | 0.3568 | 0.2557 | 0.2861 | 0.2856 |
| 50 | 0.2018 | 0.2096 | 0.2204 | 0.1743 | 0.1865 | 0.1904 |
| 75 | 0.1306 | 0.1388 | 0.1395 | 0.1183 | 0.1228 | 0.1256 |
| 100 | 0.0972 | 0.0976 | 0.0971 | 0.0876 | 0.0912 | 0.0912 |
| n | Gamma prior (a= 3, b = 0.8) | | | | | |
| | B _{SELF} | B _{PLF} | B _{DLF} | B _{LINEX} | B _{EFL} | B _{NLINEX} |
| 10 | 0.8503 | 1.0097 | 1.2108 | 0.4493 | 0.5734 | 0.6532 |
| 20 | 0.4245 | 0.4609 | 0.5213 | 0.2956 | 0.3453 | 0.3657 |
| 30 | 0.2867 | 0.3078 | 0.3284 | 0.2205 | 0.2423 | 0.2556 |

| | | | | | | |
|------------|--------|--------|--------|--------|--------|--------|
| 50 | 0.1987 | 0.1997 | 0.2121 | 0.1612 | 0.1715 | 0.1784 |
| 75 | 0.1346 | 0.1308 | 0.1353 | 0.1122 | 0.1176 | 0.1216 |
| 100 | 0.0932 | 0.0961 | 0.0976 | 0.0845 | 0.0861 | 0.0812 |

According to tables (3) and (4), as sample size rises, the rate at which estimates approach the true value of the shape parameter increases. Based on the best estimates from experiments 3 and 4, it can be concluded that the performance of the Bayes estimator with the Gamma prior and the linear exponential loss function is preferable to that of other priors with other priors and for various sample sizes.

5 Data Analysis

This section is devoted to illustrate the practical applications of the proposed exponentiated Rayleigh distribution. In order to assess the flexibility of the new model, we analyze one real life data sets taken from literature and the numerical results of exponentiated Rayleigh distribution are compared with its sub-models, namely Rayleigh distribution (RD), Weibull distribution (WD), exponential distribution (ED) and generalized exponential distribution (GED). The model selection is carried out by using different model selection criteria including the negative log-likelihood, Akaike information criteria (AIC) (Akaike 1974), Schwarz Information Criteria (SIC) (Schwarz 1978), Corrected Akaike information criteria (AICC) (Bazdogan 1987). Also, Kolmogorov-Simonov test statistics along with corresponding p-value has been calculated.

Data set 1: The data set due to Smith and Naylor (1987) consists of 63 observations of the strengths of 1.5 cm glass fibres, originally obtained by workers at the UK National Physical Laboratory. This data set was also analysed by Oguntunde et al. (2015) to demonstrate the applicability of Weibull-Exponential distribution.

The data are: "0.55, 0.74, 0.77, 0.81, 0.84, 0.93, 1.04, 1.11, 1.13, 1.24, 1.25, 1.27, 1.28, 1.29, 1.30, 1.36, 1.39, 1.42, 1.48, 1.48, 1.49, 1.49, 1.50, 1.50, 1.51, 1.52, 1.53, 1.54, 1.55, 1.55, 1.58, 1.59, 1.60, 1.61, 1.61, 1.61, 1.61, 1.62, 1.62, 1.63, 1.64, 1.66, 1.66, 1.66, 1.67, 1.68, 1.68, 1.69, 1.70, 1.70, 1.73, 1.76, 1.76, 1.77, 1.78, 1.81, 1.82, 1.84, 1.84, 1.89, 2.00, 2.01, 2.24".

Table: 5. ML estimate and the statistic -2l, AIC, SIC, AICC using data set 1

| Model | ML estimates | -2l | AIC | SIC | AICC |
|--------------|---------------------|------------|------------|------------|-------------|
| ERD | 1.4997 | 172.22 | 178.43 | 185.23 | 179.49 |
| RD | 1.635 | 180.35 | 184.23 | 189.35 | 185.06 |
| WD | 2.543 | 196.23 | 198.67 | 200.50 | 198.58 |
| GED | 5.783 | 194.67 | 202.34 | 213.67 | 185.76 |

6 CONCLUSIONS

The methods described to build a full framework to accommodate academic research and engineering applications seeking to implement modern computational based classical as well as Bayesian approaches related to exponentiated Rayleigh distribution, especially in the area of reliability. We have proposed an integrated procedure for Bayesian inference using Markov chain Monte Carlo methods. For the sake of comparison, we have discussed the maximum likelihood estimation. The above study makes the suggestion that when $\alpha=3$, the performance of the Bayes estimator under the Gamma prior with linear loss function, records full appearance "for all sample sizes," as the best prior distribution, and utilising various loss functions for the entire data set.

A simulation study is carried out to investigate the behaviour of ML estimates for finite sample size. The estimation of parameters is approached by the method of maximum likelihood estimation. The applications of the power exponentiated Rayleigh distribution to real data are provided which show that the new distribution can be used quite effectively to provide better fits than the other competing distributions. We prospect that the proposed model will draw wider applications in statistics

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