Influence Diagnostic in Log-Exponential-Inverse-Exponential {Weibull} Regression Failure Model

ABSTRACT

An exponential-inverse-exponential {Weibull} regression failure model is introduced. Some of its properties like density function, survival function, and hazard function are derived. Maximum likelihood estimates of the parameters of the new model from censored data are obtained. To assess the local influence diagnostic(s) on the parameter estimates, the appropriate matrices are derived. Also, global influence and local influence are used to detect influential observations. Martingale and Deviance residuals are obtained and used to detect outliers and evaluate the model assumptions. A real data is analyzed under Log-Exponential-Inverse-Exponential {Weibull} regression model to show the usefulness of the model. A simulation study is performed to investigate the behavior of the estimates for different sample sizes and censoring percentages.

Keywords: Censored data, *exponential -inverse- exponential {Weibull} distribution, global and local influence,* martingale and Deviance residuals, *survival data.*

1. INTRODUCTION

In this paper, we consider myeloma patient's data. The data set presenting the elapsed time (the event of interest) from diagnosis. The common name of the elapsed time is survival time, the patients under study are called individuals, and the set of data is named survival data. In this kind of the study; if the exact time of the death is known for some patients, the patients or individuals are named observed. On other side, if the exact time of death(failure) is not known individuals are called censored observations. As a matter of fact, the exact time is not known but we only know that it is larger than a certain value called censored time. Here, we consider the censoring times are random variable. In many practical applications, the survival times or lifetimes are affected by regressor variables such as, hemoglobin level, age, sex and other factors. Regression models are widely used to measure this effect. Among these regression models, the log location-scale regression model is considered as an important type of a parametric regression model since it assumes a linear relationship between the log lifetime (regressand variable) and the regressor variables. A location-scale regression model has been commonly used in clinical trials, and has been discussed by many researchers. These include log-Burr XII regression models by [1], a log-extended Weibull regression model by [2], the log-Burr XII regression model for grouped survival data by [3], the log-generalized modified Weibull regression model by [4], the Marshall–Olkin truncated Poisson Weibull regression model by [5], an extended Weibull regression model by [6], the log-beta Weibull regression model by [7], [8] proposed two new regressions, one is parametric and the other is partially linear based on an extended Birnbaum–Saunders distribution, and [9] defined a class of survival models for modeling time-to-event with long-term and obtained some of its structural properties. Also, [10] introduced and study the log-odd log-logistic Weibull (LOLLW) distribution and constructed the LOLLW regression model to investigate the informative censoring mechanism in a type of location-scale regression model, [11] introduced the generalized odd log-logistic flexible Weibull regression model, [12] proposed the heteroscedastic log-odd log-logistic generalized gamma (LOLLGG) regression model for censored data, [13] introduced the logit exponentiated power exponential regression model, [14] proposed the generalized odd log-logistic Maxwell semiparametric regression model which is very flexible for modeling response variable with positive support, [15] introduced a new regression based on the odd log-logistic Marshall–Olkin normal distribution, and [16] proposed two different zero-inflated-right-censored regression models, assuming Weibull and gamma distributions .

The next step after modelling the regression is model assessment to detect the presence of influential and extreme observations. Many authors introduced a framework to detect extreme and influential observations, see for example [17], [18], and [19]. In this paper, we propose a regression failure model based on exponential -inverse- exponential {Weibull} (**EIEW**) distribution. The rest of the paper is organized as follow; In Section 2 we review the **EIEW** distribution. In Section 3, we suggest a log-exponential -inverse-exponential {Weibull} (**LEIEW**) regression failure model of location-scale form. In addition, the maximum likelihood estimators are obtained. We use several diagnostics measures In Section 4. We present two types of residuals In Section 5. A real data set is analyzed in Section 6. Simulation studies are presented in Section 7. Finally, summary and concluding remarks appear in Section 8.

2. EXPONENTIAL -INVERSE- EXPONENTIAL {WEIBULL} DISTRIBUTION

The **EIEW** distribution considered in [20] with parameters α and λ has a density function given by

$$f(t;\alpha,\lambda)=\frac{\alpha\lambda}{t^2}\frac{e^{-\frac{\lambda}{t}}}{1-e^{-\frac{\lambda}{t}}}e^{-\alpha\left(-\ln\left(1-e^{-\frac{\lambda}{t}}\right)\right)},t>0, \tag{1}$$

where $\alpha,\lambda>0$ are shape and scale parameters respectively. Fig. 1 shows the density function defined in (1) for some values of the parameters. Let T a random variable representing the lifetime of an individual or item that follows the **EIEW** density. The survival and hazard functions corresponding to random variable T are given respectively by

$$S(t;\alpha,\lambda)=P(T\geq t)=e^{-\alpha\left(-\ln\left(1-e^{-\frac{\lambda}{t}}\right)\right)}, \tag{2}$$

$$h(t;\alpha,\lambda)=\frac{\alpha\lambda}{t^2}\frac{e^{-\frac{\lambda}{t}}}{1-e^{-\frac{\lambda}{t}}}\ . \tag{3}$$

Fig. 2 and 3 show the survival and hazard functions of T defined in (2) and (3) for some choices of the parameters respectively.

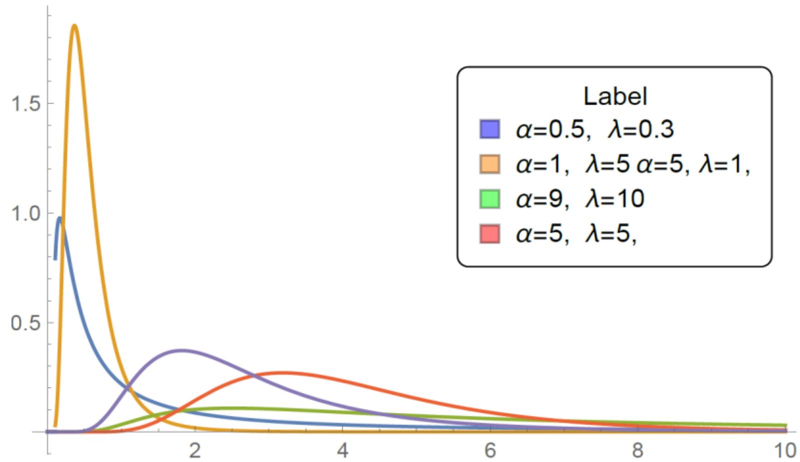


Fig. 1. Plots of **EIEW** distribution for some values of α *and* λ.

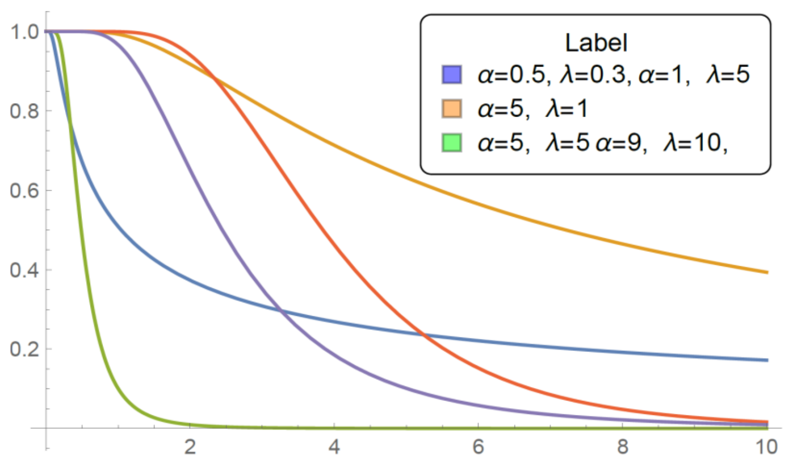


Fig. 2. Plots of the survival function for **EIEW** distribution for some values of α *and* λ.

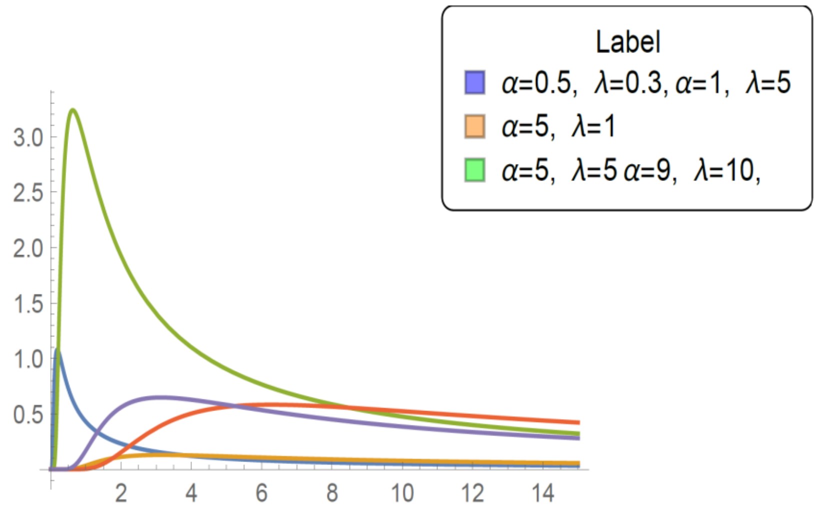


Fig. 3. Plots of the hazard rate function for **EIEW** distribution for some values of $\alpha \; and \; \lambda$.

2.1 Parameters estimation

Let $t_1, t_2, \dots, t_n$ be a random sample of random variate T with **EIEW** distribution. The likelihood function for complete sample is given by

$$L(t;\alpha,\lambda) = \frac{(\alpha\lambda)^n}{\sum_{i=1}^{n} t_i^2} \; \frac{\sum_{i=1}^{n} e^{-\frac{\lambda}{t_i}}}{1-\sum_{i=1}^{n} e^{-\frac{\lambda}{t_i}}} \; \sum_{i=1}^{n} e^{-\alpha\left(-\ln(1-e^{-\frac{\lambda}{t_i}})\right)}.$$

The likelihood function for Type II censored sample is given by

$$L(\text{t};\alpha,\lambda) = \frac{(\alpha\lambda)^r}{\sum_{i=1}^{r} t_i^2} \frac{\sum_{i=1}^{r} e^{-\frac{\lambda}{t_i}}}{\sum_{i=1}^{r}\left(1-e^{-\frac{\lambda}{t_i}}\right)} e^{-\sum_{i=1}^{r}\alpha\left(-\ln(1-e^{-\frac{\lambda}{t_i}})\right)} e^{-\sum_{i=r+1}^{n}\alpha\left(-\ln(1-e^{-\frac{\lambda}{t_i}})\right)},$$

where r is the observed number of failures.

The log likelihood function for censored sample can be written as

$$\ell(\text{t};\alpha,\lambda) = r(log\alpha + log\lambda) - \sum_{i=1}^{r} logt_i^2 - \sum_{i=1}^{r}\frac{\lambda}{t_i} - (1-\alpha)\sum_{i=1}^{r} log\left(1-e^{-\frac{\lambda}{t_i}}\right) + \alpha\sum_{r+1}^{n}\log\left(1-e^{-\frac{\lambda}{t_i}}\right).$$

The maximum likelihood estimates of the parameters are obtained by maximizing $\ell(\text{t};\alpha,\lambda)$ with respect to α and λ either directly or by differentiating $\ell(\text{t};\alpha,\lambda)$ with respect to α and λ and equating the derivative to zero and solving the resulting equations simultaneously.

3. LOG- EXPONENTIAL -INVERSE EXPONENTIAL {WEIBULL} REGRESSION MODEL

3.1 Transformation (Location- Scale) regression model

In many medical applications, life times may be affected by regressor variables such as hemoglobin, age, sex, serum calcium measurement at diagnosis as well as other factors. The regressor variables vector is denoted by $\boldsymbol{x} = (x_1, x_2, \dots, x_p)^T$ which is related to the regressand variable $Y = log(T)$ through a regression model. Considering two reparametrizations $\gamma = exp(-\lambda)$ and $\delta = 1/\alpha$, the density function of Y can be written as

$$f(y) = \frac{1}{\delta}\frac{e^{-(y-\gamma)}e^{-e^{-(y-\gamma)}}}{1-e^{-e^{-(y-\gamma)}}} exp\left[\frac{-1}{\delta}\left\{-ln\left[1-e^{-e^{-(y-\gamma)}}\right]\right\}\right], \tag{4}$$

where $y > 0$, $\delta > 0$, and $-\infty < \gamma < \infty$, with survival function given by

$$S(y) = e^{y}\, e^{\frac{-1}{\delta}\left(-ln\left(1-e^{-e^{-(y-\gamma)}}\right)\right)}. \tag{5}$$

We can write the model (4) as a log-linear model

$$Y = \gamma + \delta Z\,, \tag{6}$$

where Z is a variable with density function

$$f(z) = \mathrm{e}^{-\mathrm{Z}\delta}\,\frac{e^{-\mathrm{e}^{-\mathrm{Z}\delta}}}{1-e^{-\mathrm{e}^{-\mathrm{Z}\delta}}} exp\left[\frac{-1}{\delta}\left\{-ln\left[1-e^{-\mathrm{e}^{-\mathrm{Z}\delta}}\right]\right\}\right], \tag{7}$$

and hazard function

$$h(z) = \frac{\mathrm{e}^{-\gamma}e^{-\mathrm{e}^{-\mathrm{Z}\delta}}}{1-e^{-\mathrm{e}^{-\mathrm{Z}\delta}}}\,,$$

where $\delta > 0$, $-\infty < \gamma < \infty$, and $-\infty < z < \infty$.

Now, suppose that the scale parameter λ of the **EIEW** distribution depends on the matrix of explanatory variables X, as:

$$\lambda_i = \boldsymbol{x}_i^T\beta\,,$$

Also, suppose that the regression model based on **EIEW** distribution given in (6) relates the response variable Y and the covariate vector $\mathbf{x}$. Hence, the conditional dependence of $y|x$ can be expressed as

$$Y_i = \boldsymbol{x}_i^T\beta + \delta Z_i, \qquad i = 1, \dots, n\,, \tag{8}$$

where $\boldsymbol{x}_i^T = (x_{i1}, x_{i2}, \dots, x_{ip})$ is the explanatory vector, $\beta = \left(\beta_1, \beta_2, \dots, \beta_p\right)^T$, $\delta > 0$ are unknown parameters and Z is a variable with the distribution function in (7).
Using (5), the survival function of $Y|\boldsymbol{x}$ is given by

$$s(y|x) = e^{y}\, e^{\frac{-1}{\delta}\left[-\ln\left[1-e^{-e^{-(y-X_i^T\beta)}}\right]\right]}.$$

3.2 Estimation of the regression model

Assume we have a sample $(y_1, x_1), (y_2, x_2), \dots, (y_n, x_n)$ with distribution (4), where y_i is the logarithm of the survival time and $\boldsymbol{x}_i$ is the covariate vector. The log likelihood function can be written as

$$l(\theta) = \sum_{i\in\mathcal{F}} logf(y_i;\theta) + \sum_{i\in\mathcal{C}} logS(y_i;\theta)$$
$$= rlog[\delta] - \delta\sum_{i\in\mathcal{F}} Z_i - \sum_{i\in\mathcal{F}} \mathrm{e}^{-\delta Z_i} - \left(1-\frac{1}{\delta}\right)\sum_{i\in\mathcal{F}} \log\left[1-e^{-\mathrm{e}^{-\delta Z_i}}\right] + \sum_{i\in\mathcal{C}}(\delta z_i + \gamma_i) + \frac{1}{\delta}\sum_{i\in\mathcal{C}} \log\left[1-e^{-\mathrm{e}^{-\delta Z_i}}\right], \tag{9}$$

where $\theta = (\delta, \beta^T)^T$ is the parameter vector, r is the number of failures, $\mathcal{F}$ represents the set of observed failures, $\mathcal{C}$ represents the set of censored observations, and $Z_i = \frac{y_i - X_i^T \beta}{\delta}$. To obtain the maximum likelihood estimates for the parameter vector $\theta = (\delta, \beta^T)^T$ we will maximize the likelihood function.
The corresponding likelihood equations are

$$\frac{dL(\theta)}{d\delta} = \frac{-r}{\delta} - \frac{1}{\delta^2}\sum_{i=1}^{r} log\left[1 - \mathrm{e}^{-e^{-(y_i - x_i\beta)}}\right] + \frac{1}{\delta^2}\sum_{i=r+1}^{n} log\left[1 - \mathrm{e}^{-e^{-(y_i - x_i\beta)}}\right]$$

$$\frac{dL(\theta)}{d\beta} = \sum_{i=1}^{r} x_i - \sum_{i=1}^{r} x_i e^{-(y_i - x_i\beta)} - (1 + \frac{1}{\delta})\sum_{i=1}^{r} \frac{x_i e^{-(y_i - x_i\beta)} e^{-e^{-(y_i - x_i\beta)}}}{1 - e^{-e^{-(y_i - x_i\beta)}}} + \sum_{i=r+1}^{n} \frac{x_i e^{-(y_i - x_i\beta)} e^{-e^{-(y_i - x_i\beta)}}}{\delta\left[1 - e^{-e^{-(y_i - x_i\beta)}}\right]}$$

$$\frac{d^2L(\theta)}{d\delta d\beta} = -\sum_{i=1}^{r} \frac{x_i e^{-(y_i - x_i\beta)} e^{-e^{-(y_i - x_i\beta)}}}{\delta^2\left(1 - e^{-e^{-(y_i - x_i\beta)}}\right)} - \sum_{i=r+1}^{n} \frac{x_i e^{-(y_i - x_i\beta)} e^{-e^{-(y_i - x_i\beta)}}}{\delta^2\left(1 - e^{-e^{-(y_i - x_i\beta)}}\right)}$$

$$\frac{d^2L(\theta)}{d\beta^2} = -\sum_{i=1}^{r} x_i^2 e^{-(y_i - x_i\beta)} - (1 + \frac{1}{\delta})\sum_{i=1}^{r} \frac{\left[x_i^2 e^{-(y_i - x_i\beta)} e^{-e^{-(y_i - x_i\beta)}}\right]\left[\left(1 - e^{-(y_i - x_i\beta)}\right)\left(1 - e^{-e^{-(y_i - x_i\beta)}}\right) - \left(e^{-(y_i - x_i\beta)} e^{-e^{-(y_i - x_i\beta)}}\right)\right]}{\left[1 - e^{-e^{-(y_i - x_i\beta)}}\right]^2} + \sum_{i=r+1}^{n} \frac{\left[x_i^2 e^{-(y_i - x_i\beta)} e^{-e^{-(y_i - x_i\beta)}}\right]\left[\left(1 - e^{-(y_i - x_i\beta)}\right)\left(1 - e^{-e^{-(y_i - x_i\beta)}}\right) - \left(e^{-(y_i - x_i\beta)} e^{-e^{-(y_i - x_i\beta)}}\right)\right]}{\left[1 - e^{-e^{-(y_i - x_i\beta)}}\right]^2}$$

$$\frac{d^2L(\theta)}{d\delta^2} = \frac{r}{\delta^2} - \frac{2}{\delta^3}\sum_{i=1}^{r} log\left(1 - e^{-e^{-(y_i - x_i\beta)}}\right) + \frac{2}{\delta^3}\sum_{i=r+1}^{n} log\left(1 - e^{-e^{-(y_i - x_i\beta)}}\right)$$

In this paper we will use Mathematica14 to compute the ML estimates. Now we have the variance-covariance matrix say $\ddot{L}(\theta) = \frac{\partial^2 L(\theta)}{\partial\theta^2}|_{\theta=\hat{\theta}}$

$$\ddot{L}(\theta) = \begin{pmatrix} L_{\beta\beta} & L_{\beta\delta} \\ L_{\delta\beta} & L_{\delta\delta} \end{pmatrix}$$

Let $I(\theta) = E[\ddot{L}(\theta)]$ is the expected observed Fisher information matrix and the asymptotic covariance matrix $I^{-1}(\theta)$ of $\hat{\theta}$ can be approximated by the inverse of the $\ddot{L}(\hat{\theta})$.

4. SENSITIVITY ANALYSIS

After modelling, it is important to perform a robustness study to detect extreme observations. Extreme observations may be influential and cause distortions in regression analysis. Many authors discussed influential observations detection and proposed many approaches to detect and how to treatment those influential observations. Among them [17], [21], [18], and [22]. Case deletion is one of these approaches that can tell us if that observation is influential or not. This approach deletes one observation – one at a time- from the data to saw if the regression model has any influence or not.

(4.1) Global influence

One method of studying a global influence is case-deletion approach. The approach based on studying the effect of dropping the i[th] case from the data set. Let

$$Y_l = x_l^T\beta + \delta Z_l \quad (10)$$

be the case-deletion model for model (8), where $l = 1,2,\dots,n.,l \neq i$. From now we will use the subscript "(i)" to refer to the original quantity with the i^{th} case deleted. The log likelihood function of model (10) is denoted by $L_{(i)}(\theta)$ and let $\hat{\theta}_{(i)} = (\hat{\delta}_i,\hat{\beta}_i)^T$ be the maximum likelihood estimator of θ from $L_{(i)}(\theta)$, where $\theta_{(i)} = (\delta_i,\beta_i^T)^T$be the parameters vector of $L_{(i)}(\theta)$. To assess the influence of the i^{th} item on $\hat{\theta}$ we use the likelihood distance which proposed by [10] as follow

$$LD_i(\theta) = 2\{L(\hat{\theta}) - L(\hat{\theta}_{(i)})\} \cong \chi^2_{(m)},$$

where m is the dimension of θ.

Another approach of the global influence is known as generalized Cook distance and take the form

$$GD_i(\theta) = (\hat{\theta}_{(i)} - \hat{\theta})^T [\ddot{L}(\theta)]^{-1} (\hat{\theta}_{(i)} - \hat{\theta}) \cong F_{(k,n-k,1-a)}\,.$$

Two measures above assess the influence of the i^{th} case by comparing the difference between the ML estimators for the case-deletion model and the ML estimator for the supposed(basic) model. If the deletion item made a real influence on the estimates, that mean there is need to take an action to that item.

(4.2) Local influence

local influence is one methods of sensitivity analysis. In here, we will describe it for log-exponential -inverse-exponential Weibull (**LEIEW**) regression model with censored data. Calculation of local influence for model (8) can be performed as follow. Let $\hat{\theta}_\omega$ denotes the ML estimator under the perturbed model. Then, the likelihood displacement is

$$LD(w) = 2\{L(\hat{\theta}) - L(\hat{\theta}_w)\}.$$

Let Δ be a $(p+1)\times n$ matrix depends on the perturbation scheme and whose elements are given by $\Delta_{\mathrm{ji}} = \frac{\partial^2 l(\theta\backslash w)}{\partial\theta_j \partial w}|_{\theta=\hat{\theta}\,\&\, w=w_0}$. The normal curvature for θ at direction d, where $||d|| = 1$ is given by

$$C_d(\theta) = 2|d^T\Delta^T\ddot{L}(\theta)^{-1}\Delta d|$$

Also, normal curvatures $C_\delta(\theta)$ and $C_\beta(\theta)$ can be calculated to made various index plots. For example, the index plot of dmax the eigenvector corresponding to $c_{d_{max}}$, the largest eigen value of the matrix $B = \Delta^{\mathrm{T}}\ddot{L}(\theta)^{-1}\Delta$ and the index plots of $C_{d_i}(\delta)$ and $C_{d_i}(\delta)$ named total local influence, where, d_i is a vector of zeros with $n\times 1$ with one at the i^{th} position.

Let, Δ_i^T denotes the i^{th} row of Δ ,then , the curvature at direction d_i can take the form

$$C_i = 2|\Delta_i^T\ddot{L}(\theta)^{-1}\Delta_{\mathrm{i}}|.$$

(4.3) Curvature Calculations

Consider the model (8), and its log likelihood function (9), Now, we will calculate three perturbation schemes. let $\boldsymbol{w} = (w_1,w_2,\dots,w_n)^T$ be the vector of weights and Let us denote $\Delta = (\Delta_1,\dots,\Delta_{\mathrm{p+1}})^T$ as $(p+1)\times n$ matrix with elements $\Delta_{\mathrm{ji}} = \frac{\partial^2 L(\theta\backslash w)}{\partial\theta_i \partial w_i}$, evaluated at $\theta = \hat{\theta}$ and $w = w_0, j = 1,\dots,p+1$ and $i = 1,2,\dots,n$.

<u>(4.3.1) Case- weight perturbation</u>

In this case, the log likelihood function of the case-weights perturbation scheme for postulated model takes the form

$$l(\theta\backslash w) = \sum_{i=1}^{r} w_i log f(y_i;\theta) + \sum_{i=r+1}^{n} w_i log S(y_i;\theta)$$

Then, for **LEIEW** regression model with censored data is

$$L(\theta\backslash w) = -rlog[\delta]\sum_{i=1}^{r} w_i + \sum_{i=1}^{r} w_i(y_i - x_i\beta) + \sum_{i=1}^{r} w_i \mathrm{e}^{-(y_i - x_i\beta)} - (1+\frac{1}{\delta})\sum_{i=1}^{r} w_i \log\left[1 - e^{-\mathrm{e}^{-(y_i - x_i\beta)}}\right] + \sum_{i=r+1}^{n} w_i\, y_i + \frac{1}{\delta}\sum_{i=r+1}^{n} w_i \log\left[1 - e^{-\mathrm{e}^{-(y_i - x_i\beta)}}\right]$$

Where $0 \le w_i \le 1$ and $w = (1, \dots, 1)^T$.

Then the elements of vector Δ_1 can be expressed as

$$\Delta_{1i} = \begin{cases} \frac{-r}{\hat{\delta}} - \frac{1}{\hat{\delta}^2}\sum_{i=1}^{r} \log\left(1 - e^{-e^{-\hat{\delta}\hat{z}_i}}\right) & if\ i \in F \\ \frac{1}{\hat{\delta}^2}\sum_{i=r+1}^{n} \log\left(1 - e^{-e^{-\hat{\delta}\hat{z}_i}}\right) & if\ i \in C \end{cases}$$

The elements of vector Δ_j for $j = 2, \dots, p+1$, take the form

$$\Delta_{ji} = \begin{cases} \sum_{i=1}^{r} x_{ij} - \sum_{i=1}^{r} x_{ij} e^{-\hat{\delta}\hat{z}_i} - (1+\frac{1}{\hat{\delta}})\sum_{i=1}^{r} \frac{x_{ij} e^{-\hat{\delta}\hat{z}_i} e^{-e^{-\hat{\delta}\hat{z}_i}}}{1 - e^{-e^{-\hat{\delta}\hat{z}_i}}} & if\ i \in F \\ \sum_{i=r+1}^{n} x_{ij} + \sum_{i=r+1}^{n} \frac{x_{ij} e^{-\hat{\delta}\hat{z}_i} e^{-e^{-\hat{\delta}\hat{z}_i}}}{\hat{\delta}(1 - e^{-e^{-\hat{\delta}\hat{z}_i}})} & if\ i \in C \end{cases}$$

where $\hat{z}_i = \left(\frac{y_i - x_i\hat{\beta}}{\hat{\delta}}\right)$.

(4.3.2) Response perturbation

Consider the regression model (8) by assuming that each y_i is perturbed as $y_i + \sigma_y w_i = y_i^*$; $i = 1, \dots, n$; $w_i \in \mathbb{R}$, where σ_y is a scale parameter that may be the estimated standard deviation of Y.

The perturbed log likelihood function becomes as follow

$$L(\theta\backslash w) = -rlog[\delta] + \sum_{i=1}^{r}(y_i^* - x_i\beta) + \sum_{i=1}^{r} \mathrm{e}^{-(y_i^* - x_i\beta)} - (1+\frac{1}{\delta})\sum_{i=1}^{r} \log\left[1 - e^{-\mathrm{e}^{-(y_i^* - x_i\beta)}}\right] + \sum_{i=r+1}^{n} y_i^* + \frac{1}{\delta}\sum_{i=r+1}^{n} \log\left[1 - e^{-\mathrm{e}^{-(y_i^* - x_i\beta)}}\right]$$

Then, the elements of vector Δ_{1i} take the form

$$\Delta_{1i} = \begin{cases} -\frac{\hat{\sigma}\hat{h}_i e^{-\hat{h}_i}}{\hat{\delta}^2(1 - e^{-\hat{h}_i})} & if\ i \in F \\ \frac{\hat{h}_i e^{-\hat{h}_i}}{\hat{\delta}^2(1 - e^{-\hat{h}_i})} & if\ i \in C \end{cases}$$

On the other hand, the elements of vector Δ_j for $j = 2, \dots, p+1$, can be expressed as

$$\Delta_{ji} = \begin{cases} -\hat{\sigma}x_{ij}\hat{h}_i - \hat{\sigma}(1+\frac{1}{\hat{\delta}})\frac{x_{ij}(2\hat{h}_i)e^{-2\hat{h}_i}}{\left(1 - e^{-\hat{h}_i}\right)^2} - \hat{\sigma}^2(1+\frac{1}{\hat{\delta}})\frac{x_{ij}\hat{h}_i^{\,2} e^{-\hat{h}_i}}{(1 - e^{-\hat{h}_i})} & if\ i \in F \\ \frac{\hat{\sigma}x_{ij}(2\hat{h}_i)e^{-2\hat{h}_i}}{\left(1 - e^{-\hat{h}_i}\right)^2} + \frac{(-\hat{\sigma} + \hat{\sigma}\hat{h}_i)x_{ij}\hat{h}_i e^{-\hat{h}_i}}{\hat{\delta}(1 - e^{-\hat{h}_i})} & if\ i \in C \end{cases}$$

Where, $\hat{h}_i = e^{-(y_i + w_i\hat{\sigma}_y - x_i\hat{\beta})}$

4.3.3 Explanatory variable perturbation

Again, consider the regression model (8) and consider an additive perturbation on a particular x_i , namely x_t, by making perturbed as $x_{it} + w_i\sigma_t = x_{itw}$; $i = 1, \dots, n$; $w_i \in \mathbb{R}$, where σ_t is a scaled factor.
The perturbed log likelihood function in this case becomes as follow

$$L(\theta \backslash w) = -rlog[\delta] + \sum_{i=1}^{r}(y_i - x_i^{*T}\beta) + \sum_{i=1}^{r} \mathrm{e}^{-(y_i - x_i^{*T}\beta)} - (1 + \frac{1}{\delta}) \sum_{i=1}^{r} \log\,[1 - e^{-\mathrm{e}^{-(y_i - x_i^{*T}\beta)}}] + \sum_{i=r+1}^{n} y_i + \frac{1}{\delta} \sum_{i=r+1}^{n} \log\,[1 - e^{-\mathrm{e}^{-(y_i - x_i^{*T}\beta)}}]$$

where, $x_i^{*T} = \beta_1 + \beta_2 x_{i2} + \cdots + \beta_t(x_{it} + w\sigma_t) + \cdots + \beta_p x_{ip.}$

Then, the elements of vector $\Delta_{1\mathrm{i}}$ take the form

$$\Delta_{1\mathrm{i}} = \begin{cases} -\dfrac{\hat{\sigma}_x \hat{\beta}_t \hat{k}_i e^{-\hat{k}_i}}{\hat{\delta}^2(1 - e^{-\hat{k}_i})} & if\ i \in F \\ \dfrac{\hat{\sigma}_x \hat{\beta}_t \hat{k}_i e^{-\hat{k}_i}}{\hat{\delta}^2(1 - e^{-\hat{k}_i})} & if\ i \in C \end{cases}$$

On the other hand, the elements of vector Δ_j for $j = 2, \dots, p+1$ and $j \neq t$ can be expressed as

$$\Delta_{\mathrm{ji}} = \begin{cases} x_i - x_i \hat{k}_i - (1 + \frac{1}{\hat{\delta}}) \dfrac{\hat{k}_i e^{-\hat{k}_i}}{1 - e^{-\hat{k}_i}} & if\ i \in F \\ \dfrac{x_i \hat{k}_i e^{-\hat{k}_i}}{\hat{\delta}\left[1 - e^{-\hat{k}_i}\right]} & if\ i \in C \end{cases}$$

and the elements of vector Δ_t are given by

$$\Delta_{\mathrm{ti}} = \begin{cases} \hat{\sigma}_x - (\hat{\xi}_i \hat{k}_i \hat{\rho}_i + \hat{k}_i \hat{\sigma}_x) - \dfrac{(1 - e^{-\hat{k}_i})\{\hat{k}_i^2 \hat{\xi}_i \hat{\sigma}_x \hat{\beta} + e^{-\hat{k}_i}(\hat{\rho}_i \hat{\xi}_i \hat{k}_i + \hat{k}_i \hat{\sigma}_x)\} - \hat{\sigma}_x \hat{\beta} \hat{\xi}_i \hat{k}_i^2 e^{-2\hat{k}_i}}{(1 - e^{-\hat{k}_i})^2} & if\ i \in F \\ \dfrac{(1 - e^{-\hat{k}_i})(x_i \hat{k}_i \hat{\sigma}_x \hat{\beta}) - (2\hat{k}_i) x_i e^{-\hat{k}_i} \hat{\sigma}_x \hat{\beta}}{(1 - e^{-\hat{k}_i})^2} & if\ i \in C, \end{cases}$$

where, $\hat{k}_i = e^{-(y_i - (x_i + w_i \hat{\sigma}_x)\hat{\beta})}$, $\hat{\xi}_i = x_{it} + w_i \hat{\sigma}_x$, and $\hat{\rho}_i = x_{it} + \hat{\sigma}_x \hat{\beta}$.

5. RESIDUAL ANALYSIS

In order to study deflections from the error assumptions, like if there are outliers or not in our regression model, the statisticians proposed many types of residuals such as; Cox-Snell, score, martingale, and Deviance residuals. Martingale and Deviance residuals based on log-exponential -inverse exponential Weibull **LEIEW** regression model will be considered.

5.1 Martingale residuals

According to [23], the martingale residuals take the form

$$r_{\mathcal{M}_i} = \varphi_i + \log\left[S\left(y_i; \hat{\theta}\right)\right], \tag{11}$$

where, φ_i Takes 0 in case of censored observations and takes 1 in case of uncensored observations, and $S(y_i; \hat{\theta})$ is defined as the survival function of **LEIEW** distribution. Due to the skewness distributional form of the martingale residuals, the minimum value of $r_{\mathcal{M}_i}$ is $-\infty$ and the maximum value of $r_{\mathcal{M}_i}$ is +1.

5.2 Deviance residuals

Deviance residuals has a large applied in generalized linear models see for instant [24] and [25]. According to [26], the Deviance residuals take the form

$$r_{\mathcal{D}_i} = sgn(r_{\mathcal{M}_i})\sqrt{-2\{r_{\mathcal{M}_i} + \varphi_i \log[\varphi_i - r_{\mathcal{M}_i}]\}}, \quad (12)$$

where, $r_{\mathcal{M}_i}$ is the martingale residuals, defined in (11), for the i^{th} individual and $sgn(.)$ is the sign function.

6. AN APPLICATION

The multiple myeloma patient's data displayed in [27] is used to illustrate the performance of the **LEIEW** regression model. This data represents the survival time of 65 patients. The explanatory variables for each patient consist of a blood urea nitrogen measurement at diagnosis, hemoglobin at diagnosis, age at diagnosis, Serum calcium measurement at diagnosis and two binary variables censoring indicator and sex. The following variables in the study are:

t_i: Survival times (in months).
y_i: log survival time (in months).
$stus_i$: Censoring indicator (0=alive(censoring), 1=dead).
x_{i0}: Constant=1.
x_{i1}: Logarithm of a blood urea nitrogen measurement at diagnosis.
x_{i2}: Hemoglobin at diagnosis.
x_{i3}: Age at diagnosis (in years).
x_{i4}: Sex (0 male 1 female).
x_{i5}: Serum calcium measurement at diagnosis.

6.1 Maximum likelihood estimation

Maximum likelihood is used to estimate model parameters using Mathematica program. SE, CI 95% and P -values for each parameter are computed. The results are shown in the Table 1. It can be noted that the explanatory variable x_2 is significant for the model at the significance level 5% but x_1, x_3 and x_4 are not significant for this model at level of 5%.

Table 1: The ML Parameter Estimates of the LEIEW regression model – Final Model.

Parameter	Estimate	SE	P -values	CI 95%
δ	0.971			
β0	4.691	0.0482	0.10	(3.928, 5.453)
β1	−1.568	0.1608	0.19	(-4.110, 0.974)
β2	0.154	0.0156	0.00	(-0.092, 0.400)
β3	0.003	0.0042	0.47	(-0.063, 0.069)
β4	0.255	0.0860	0.17	(-1.105, 1.615)
β5	−0.166	0.0243	0.23	(-0.550, 0.218)

Therefore, the final model fitted is:

$$y_i = \beta_2 x_{i2} + \sigma z_i \quad , i = 1,2, \ldots ,65, \quad (13)$$

where y_i follows the **LEIEW** given in (4). Table 5 represents the ML estimates of the parameters in the final model (13). It can be concluded that the log survival time for myeloma patients depend on Hemoglobin, that when the Hemoglobin increases for myeloma patients, the log survival time increases.

6.2 Global influence

The results of influence measure index plots using myeloma data for **LEIEW** regression models are shown in Figure (4) and (5). It is clear that, the Figure of likelihood distance shows that, the observations from 51 to 56 and from 58 to 65 are possible influential observations in **LEIEW** regression model. On the other hand, the Figure of generalized Cook distance, shows that only the first observation may be an influential observation.

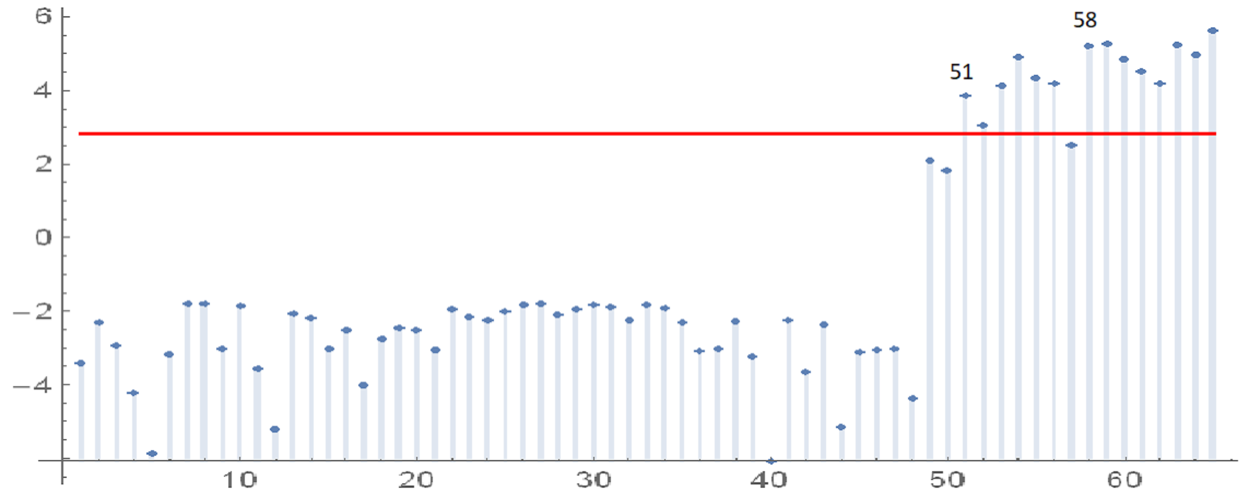


Figure (4) plot index of the Likelihood Distance for **LEIEW** regression model.

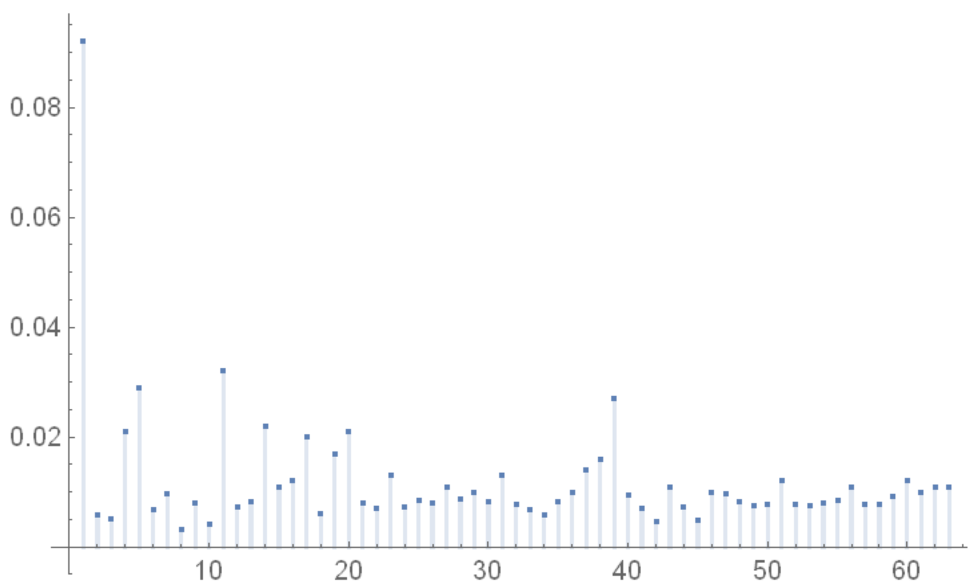
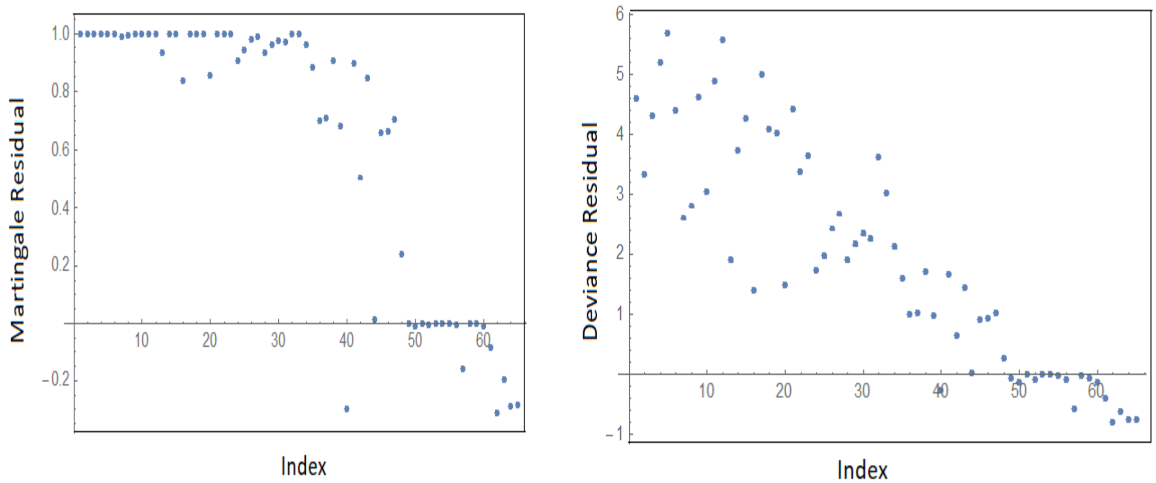


Figure (5) plot index of the generalized Cook distance for **LEIEW** regression model.

6.3. Analysis of residual

We present Fig 6, to detect extreme observations from **LEIEW** regression model. As expected, the graph in Fig.6 (a) indicates that, martingale residuals between -0.3 and 1. The graph in Fig.6 (b) shows the deviance residual (12) for the **LEIEW** regression model and it indicates that no observations appear as possible outlier.



(a) (b)

Fig.6 Index plot of residuals on fitting the **LEIEW** regression model for myeloma patient's data (a) martingale residual and (b) Deviance residuals.

7. Comparison between LEIEW regression model and some regression models

[28] introduced an Exponentiated- Weibull regression model, [1] introduced a log-BurrXII regression model, and [2] introduced a log-extended Weibull regression model. We conduct a comparison between these regression models and LEIEW regression model based on AIC and BIC criteria. Table 2 displays the results of these criteria which show that the **LEIEW** regression model is more appropriate model compared to these regression models with smaller values of AIC and BIC.

Table 2: Statistics AIC and BIC for Comparing the mentioned regression models above and the LEIEW regression model.

Model	AIC	BIC
LEIEW	81	96
Log-extended- Weibull	3372	3390
Log-BurrXII	261	278
Exponentiated- Weibull	249	266

Based on this analysis, we conclude that the **LEIEW** regression model is more appropriate for fitting these data compared to the above regression models.

8. Simulation studies

We perform a simulation study for different values of sample size (n) and censoring percentages $\vartheta\%$ to investigate the accuracy of the MLEs in the **LEIEW** regression model. The lifetimes $T_1, \dots, T_n$ are generated from the **EIEW** distribution (1) under the following re-parametrization variable $Y = log(T)$, $\gamma = log(\lambda)$ and $\delta = 1/\alpha$. Then, we take $y_i = \beta_0 + \beta_1 x_i$, where, x_i is generated from a uniform distribution on the interval $(0, 1)$. Also, the censoring times $\mathcal{C}_1, \dots, \mathcal{C}_n$ are generated from a uniform distribution $(0, \vartheta)$, where ϑ is adjusted until the censoring percentages of $0, 0.10$ or 0.30 are reached. For each combination of $n, \gamma, \delta, \beta_0, \beta_1$ and $\vartheta\%$, 1000 samples are generated, and the **LEIEW** regression model (8) is fitted to each replicated data set. The relative bias (RBias) and the relative mean squared errors (RMSEs) for different sample sizes and censoring percentages are listed in Tables (3). In general, we observe that:

- The RBiase and RMSE values of ML the estimates decrease as the sample size increases.
- The RBiase and RMSE values of ML the estimates decrease when proportions of censoring decrease.
- The RBiase and RMSE values of ML the ML estimates at $\gamma = 2,\ \delta = 0.5,\ \beta_0 = 1,\ \beta_1 = 0.5$ has the smallest values compared to the RBiase and RMSE of ML estimates for the corresponding other sets of parameters.

Table 3. RBiase and RMSEs of the estimates of the LEIEW model for different sample sizes and different censoring proportions, at ($\gamma = 1.25,\ \delta = 1, \beta 0 = 0.5$ and $\beta 1 = 0.5$)

n	$\vartheta\%$	δ		β_0		β_1	
		RBiase	RMES	RBiase	RMES	RBiase	RMES
20	0	0.81629	0.66634	0.76590	0.58661	0.83854	0.35157
	0.10	0.71224	0.50728	0.64437	0.41521	0.92825	0.43083
	0.30	0.88535	0.78385	1.14355	1.30772	1.14662	0.65737
50	0	0.86474	0.74777	0.82019	0.67272	0.83710	0.35037
	0.10	0.80730	0.65174	0.90407	0.81735	0.73203	0.26793
	0.30	0.80684	0.65099	1.03200	1.0653	0.95264	0.45377
100	0	0.73527	0.54063	0.79165	0.62672	0.52746	0.13911
	0.10	0.77914	0.60706	0.64151	0.41154	0.77357	0.29920
	0.30	0.75480	0.56972	0.8505	0.72335	0.77631	0.30133
200	0	0.64125	0.41120	0.58390	0.34094	0.63689	0.20282
	0.10	0.65200	0.42510	0.60973	0.37177	0.72479	0.26266
	0.30	0.63142	0.39870	0.57206	0.32725	0.77939	0.30373

Table 4. RBiase and RMSEs of the estimates of the LEIEW model for different sample sizes and different censoring proportions, at ($\gamma = 0.5,\ \sigma = 0.7, \beta 0 = 0.4$ and $\beta 1 = 0.7$)

n	$\vartheta\%$	δ		β_0		β_1	
		RBiase	RMES	RBiase	RMES	RBiase	RMES
20	0	0.93868	0.88112	0.70292	0.49410	0.16033	0.01285
	0.10	0.95804	0.91784	0.70881	0.50242	0.14523	0.01054
	0.30	0.95666	0.91519	0.64649	0.41794	0.17360	0.01506
50	0	0.96435	0.92998	0.54001	0.29161	0.25226	0.03181
	0.10	0.95273	0.90770	0.50932	0.25941	0.27807	0.03866
	0.30	0.93384	0.87207	0.47603	0.22661	0.29185	0.04258
100	0	0.94845	0.89956	0.52612	0.58203	0.16407	0.01345
	0.10	0.93139	0.86749	0.41942	0.11634	0.00676	0.11634
	0.30	0.97526	0.95114	0.6869	0.47184	0.15691	0.01231
200	0	0.89930	0.80874	0.17422	0.03035	0.08079	0.00326
	0.10	0.94474	0.89254	0.52859	0.27941	0.18859	0.01778
	0.30	0.87534	0.76622	0.26526	0.07036	0.15467	0.01196

Table 5. RBiase and RMSEs of the estimates of the LEIEW model for different sample sizes and different censoring proportions, at ($\gamma = 1.25,\ \delta = 0.2,\ \beta 0 = 1.5$ and $\beta 1 = 0.8$)

n	$\vartheta\%$	δ		β_0		β_1	
		RBiase	RMES	RBiase	RMES	RBiase	RMES
20	0	0.77262	0.11939	0.11701	0.02054	0.14118	0.01594
	0.10	0.80426	0.12936	0.16696	0.04181	0.23953	0.04590
	0.30	0.87157	0.15193	0.17296	0.04487	0.12462	0.01242
50	0	0.85470	0.14610	0.11514	0.01988	0.22021	0.03879
	0.10	0.86925	0.15112	0.13650	0.02795	0.16982	0.02307
	0.30	0.87869	0.15442	0.13644	0.02792	0.16302	0.02126
100	0	0.85090	0.14480	0.11607	0.02020	0.33327	0.08885
	0.10	0.86529	0.14974	0.16829	0.04248	0.35600	0.10139
	0.30	0.84761	0.14369	0.13345	0.02671	0.36628	0.10733
200	0	0.85320	0.14559	0.09796	0.01439	0.26167	0.05477
	0.10	0.85402	0.14587	0.13757	0.02838	0.39386	0.12410
	0.30	0.86363	0.14917	0.09075	0.01235	0.18433	0.02718

Table 6. RBiase and RMSEs of the estimates of the LEIEW model for different sample sizes and different censoring proportions, at ($\gamma = 0.75,\ \delta = 1, \beta 0 = 0.5$ and $\beta 1 = 0.5$)

n	$\vartheta\%$	δ		β_0		β_1	
		RBiase	**RMES**	**RBiase**	**RMES**	**RBiase**	**RMES**
20	0	0.81409	0.66275	0.41162	0.16943	0.67210	0.22586
	0.10	0.84885	0.72055	0.57100	0.32604	0.53896	0.14524
	0.30	0.86487	0.74801	0.75700	0.57305	0.54932	0.15088
50	0	0.78687	0.61917	0.25284	0.06392	0.23001	0.02645
	0.10	0.77688	0.60355	0.34137	0.11653	0.52876	0.13979
	0.30	0.81077	0.65734	0.63083	0.39795	0.75191	0.28268
100	0	0.76959	0.59228	0.28843	0.08319	0.11946	0.00713
	0.10	0.70263	0.49369	0.24494	0.05999	0.32602	0.05314
	0.30	0.76089	0.57896	0.64685	0.41842	0.61879	0.19145
200	0	0.72288	0.52256	0.00906	0.00008	0.32643	0.05328
	0.10	0.75394	0.56843	0.29221	0.08538	0.23823	0.02837
	0.30	0.81204	0.65940	0.66976	0.44858	0.41840	0.08753

Table (3.5). RBiase and RMSEs of the estimates of the **LEIEW** model for different sample sizes and different censoring proportions, at ($\gamma = 2,\ \delta = 0.5, \beta 0 = 1$ and $\beta 1 = 0.5$)

n	$\vartheta\%$	δ		β_0		β_1	
		RBiase	RMES	RBiase	RMES	RBiase	RMES
20	0	0.13566	0.00368	0.46731	0.32756	0.07170	0.00411
	0.10	0.27587	0.01522	0.46587	0.32555	0.05884	0.00276
	0.30	0.17666	0.00624	0.57439	0.49489	0.03753	0.00112
50	0	0.29799	0.01776	0.26897	0.10852	0.06251	0.00312
	0.10	0.19480	0.00758	0.35505	0.18909	0.02652	0.00056
	0.30	0.06840	0.00093	0.41462	0.25786	0.08788	0.00617
100	0	0.12426	0.00308	0.38763	0.22539	0.19599	0.03073
	0.10	0.06864	0.00094	0.27730	0.11534	0.06389	0.00326
	0.30	0.41206	0.03395	0.47925	0.34452	0.01440	0.00016
200	0	0.84878	0.14408	0.04727	0.00335	0.10689	0.00914
	0.10	0.01717	0.00005	1.44719	3.14154	0.23545	0.04435
	0.30	0.01619	0.00005	0.30758	0.14191	0.01871	0.00028

9. Summary and Concluding remarks

We concerned only with parametric forms, so a location-scale regression model based on the **EIEW** distribution is proposed for modeling data. In this article, the **LEIEW** regression model with right censored lifetime data is introduced. ML method was used to estimate model parameters. In addition, the robustness features of the ML estimator from the fitted the **LEIEW** regression model are discussed through residuals and sensitivity analysis. The multiple myeloma patient's data is used to illustrate the performance of the **LEIEW** regression model. Moreover, the results of analysis showed that the proposed model provided more flexible and appropriate fit for the multiple myeloma patient's data compared with some regression models using AIC and BIC criteria. Finally, a simulation study is performed to investigate the behavior of the estimators.

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