Original Research Article

MATHEMATICAL MODEL OF THE INFLUENCE OF THE PARAMETERS OF A SOLAR COOKER WITH SOLAR CONCENTRATOR

ABSTRACT

This work focuses on the influence of the parameters of a solar cooker with solar concentrator. The equations governing the heat transfers in this solar cooker are deduced from the analogy between heat transfers and electrical transfers. These equations are discretised and solved by an implicit finite difference method, using the Gaussian algorithm coupled with an iterative procedure after modelling.

A numerical code simulating the operation of the said solar cooker, using meteorological data from the city of Bangui, was established. This simulation influences the parameters as well as the different components of the cooker such as the materials used and its dimensions. Also, the effect of the solar irradiation gradient on the thermal performance of the cooker was studied. The results show that the mirror is a good reflector. It should be noted that the optimal dimensions (60*50*50cm) of the cooker give a good thermal performance and the thickness of the wall of the pot is 3mm which favours a good thermal conduction.

The numerical work carried out with the SIMUSOL software was aimed at analysing the influence of solar illumination, the angle of opening of the concentrator on the temperature reached at its hearth, and the cooking time of various foods (corn, rice, eggs, meat [8-15]). These results show that the temperature at the focus of the concentrator varies between 150 and 300°C depending on the size of the concentrator. In addition to these, the cooking time depends on the foodstuff and the intensity of the solar flux. The energy efficiency reached 60% with a cooking time of 35 to 120 minutes. These studies led to the mathematical model of our study.

Key words: parameter, concentrator, materials, transfer, thermal.

1. INTRODUCTION

The scarcity of fossil fuels has led the world to turn to renewable energies. One of these is solar energy, which can be used to combat deforestation through its various applications. One of these applications is solar cooking, which has been the subject of several experimental and numerical studies.

Several of these numerical studies carried out on solar cookers aim to predict their thermal behaviour.

Most of this work is based on modelling and simulation of the cooker using MATLAB and TRNSYS.15TM software. The aim is to determine the thermal efficiency, the temperature of the fluid at the outlet of the tube at the concentrator hearth and the overall heat loss coefficient [1-3]. These results show that the outlet temperature of the heat transfer fluid varies from 135 to 185°C depending on the size of the concentrator, while the thermal efficiency is around 60.5%. The overall heat loss coefficient is low with the optical efficiency above 61%. In other works, the analysis of the influence of the absorber tube diameter, solar concentrator and the tilt angle of the parabolic trough on the energy efficiency and the temperature of the fluid at the focus of the parabolic trough has been done [4-7].

This analysis shows that the water temperature can reach values above 106°C. In addition, the energy efficiency and thermal performance decreases significantly with an increase in the angle of inclination of the parabolic trough.

The numerical work carried out with the SIMUSOL software was aimed at analysing the influence of solar illumination, the angle of opening of the concentrator on the temperature reached at its hearth, and the cooking time of various foods (corn, rice, eggs, meat [8-15]). These results show that the temperature at the focus of the concentrator varies between 150 and 300°C depending on the size of the concentrator. In addition to these, the cooking time depends on the foodstuff and the intensity of the solar flux. The energy efficiency reached 60% with a cooking time of 35 to 120 minutes. These studies led to the mathematical model of our study.

2. MATHEMATICAL MODEL

2.1-Physical model

In order to visualise the different heat transfer phenomena, we have shown the solar cooker by means of a physical representation. The solar cooker is shown in figure 1.

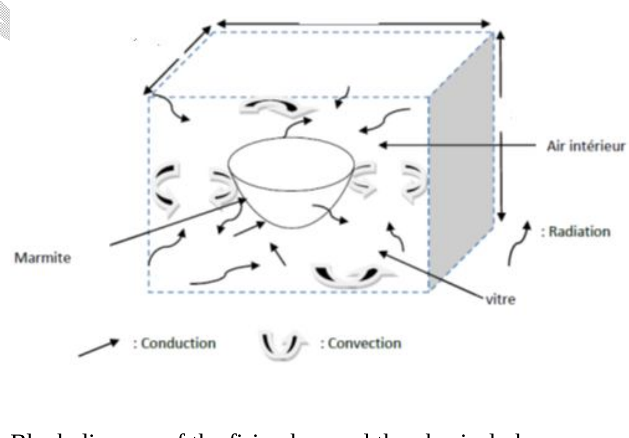

Figure 1: Block diagram of the firing box and the physical phenomena occurring in it

3. DESCRIPTION OF THE PROPOSED SYSTEM

The solar cooker proposed in this thesis is composed of a solar concentrator and a parallelepiped enclosure, where the pot containing the food to be cooked is placed. The pot is a semi-spherical tank with a black painted outer wall. The walls of the parallelepiped enclosure, except for the one opposite the concentrator, are made of wood measuring 0.65m×0.50m×0.50m, 3 cm thick and with a layer of glass wool 2 cm thick for insulation. A 4mm thick glass pane ensures the transmission of part of the solar flux reflected by the concentrator onto the pot, which is placed in this enclosure. Access to the interior of the enclosure is provided by one of the vertical walls adjacent to the glass wall, designed as a door.

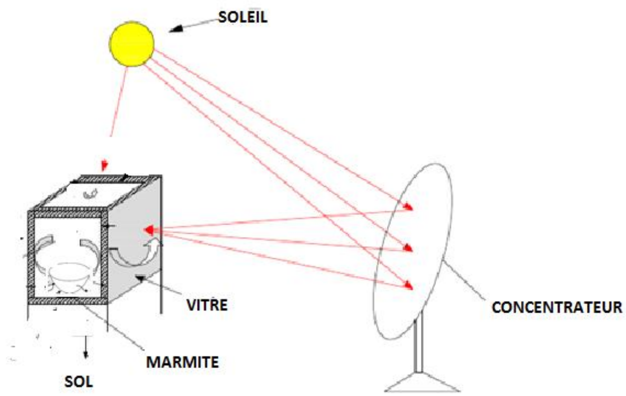

Figure 2 : synoptic diagram of the cooker

4. EQUATION OF TRANSFER

The application of Ohm's law to the electrical analogue given by equation (Eq1) leads to the system of equations of heat and mass exchange coefficients

$$M_i C_{pi} \frac{\partial T_i}{\partial t} = \varphi_{sol} \times S_i + \sum_{k=1}^{m} \sum_{j=1}^{n} (h_{r,j-k} + h_{cv,j-k} + h_{cond,j-k})(T_k - T_j)$$

On both sides of the glass we have heat transfer by radiation and natural convection with the environment, and heat transfer by conduction through the facades

$$m_V C_p \frac{\partial T_{ve}}{\partial t} = I\alpha_v S_v + h_{rv} S_v (T_{ciel} - T_{ve}) + h_{rvsol} S_v (T_{sol} - T_{ve}) + h_{cv} S_v (T_{amb} - T_{ve}) + \frac{\lambda_v S_v}{e}(T_{vi} - T_{ve})$$

$$m_V C_p \frac{\partial T_{vi}}{\partial t} = I\alpha_v S_v + h_{rmv} S_m (T_{pme} - T_{vi}) + h_{cvv} S_v (T_{ai} - T_{vi}) + \frac{\lambda_v S_v}{e_v}(T_{ve} - T_{vi})$$

Inside the enclosure, the speed of air movement is low, the heat transfer is by convection between the glass and the air and then between the glass and the wall of the pot.

$$m_{ai}\, C_{pai} \frac{\partial T_{ai}}{\partial t} + m_{ai} C_{Pai} V_{ai} \frac{\partial T_{ai}}{\partial x} = h_{cvai}\, S_v (T_{vi} - T_{ai}) + h_{cvm}\, S_m (T_{pme} - T_{ai})$$

In this equation, we will neglect the term $\frac{\partial T_{ai}}{\partial x}$- in front of the term $\frac{\partial T_{ai}}{\partial t}$ indeed as the speed of displacement is very weak, we consider that the variation of temperature in a given point due to the exchanges with the various mediums present is considerable in front of the variation of temperature due to the displacement of the air.

The pot is considered as an absorber, it has two faces: external and internal. We write the differential equations on each side where the heat transfer is done by convection, radiation and conduction.

$$m_m\, C_{pm} \frac{\partial T_{pme}}{\partial t} = I\tau_v \alpha_m + h_{cvm}\, S_m (T_{ai} - T_{pme}) + h_r S_m (T_{vi} - T_{pme}) + 2\pi\lambda_m \left(\frac{R_2 - R_1}{R_2 . R_1}\right)(T_{pmi} - T_{pme})$$

$$m_m C_{pm} \frac{\partial T_{pmi}}{\partial t} = 2\pi\lambda_m \left(\frac{R_2 - R_1}{R_2 . R_1}\right)(T_{pme} - T_{pmi}) + h_f S_m (T_f - T_{pmi})$$

Equation (7) justifies the evolution of the temperature of the fluid in the kettle

$$m_f C_{Pf} \frac{\partial T_f}{\partial t} + m_f C_{Pf} V_f \frac{\partial T_f}{\partial x} = h_f S_m (T_{pmi} - T_f)$$

Heat is exchanged by the three modes of heat transfer: conduction, convection and radiation.

5. RESULTS AND DISCUSSION

5.1-INFLUENCE OF THE NATURE OF THE CONCENTRATOR COATING MATERIALS

In this section we present two types of concentrator coated with different materials whose reflectivity and emissivity properties are shown in the table below.

Table 1: Radiative coefficients of concentrator materials

Materials	Reflectivity	Emissivity
Aluminium foil	0,65	0.09
Mirror	0,99	0,97

Figures 3-4 show the evolution of the temperature at the focus of the concentrator as a function of solar irradiance. As shown in figures 3 and 4, the temperature at the focus of the concentrator with rectangular mirrors on the wall is higher than that of the concentrator with aluminium foil on the wall. Thus, for an opening diameter of 65 cm, the temperature at the focus of the concentrator covered with rectangular mirrors is 160°C and the thermal efficiency is 80%. For a solar irradiance equal to 900W/m² and an opening diameter of the concentrator of 65 cm, the temperature at the focus of the concentrator covered with aluminium film reaches 118°C with a thermal efficiency of 60%. From these results, we retain the concentrator with rectangular mirrors on the wall for further calculations. This confirms the superior optical properties of the mirror as shown in Table 1.

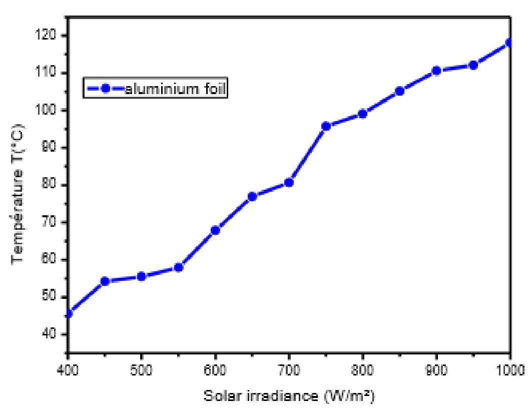

Figure 3: Evolution of the temperature at the focus of the concentrator as a function of solar irradiance. Concentrator wall covered with aluminium foil

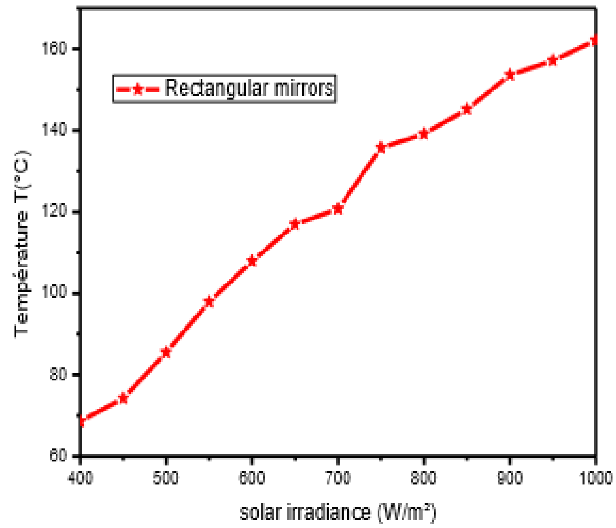

Figure 4: Evolution of the temperature at the focus of the concentrator as a function of solar irradiance. Concentrator wall covered with rectangular mirrors

6. INFLUENCE OF THE DIMENSIONS OF THE COOKER

In this paragraph, we look for the optimal dimensions of the cooker, more precisely the values of the length, height and width of the parallelepipedic enclosure in which the kettle is placed, for which the air temperature reached is maximum under the operating conditions retained previously. Thus, our calculations were made considering a solar irradiance equal to 900 W m-2 and an opening diameter of the concentrator of 0.65 m.

Figures 5-8 show the evolution of the air temperature inside the enclosure as a function of the dimensions of the cooker.

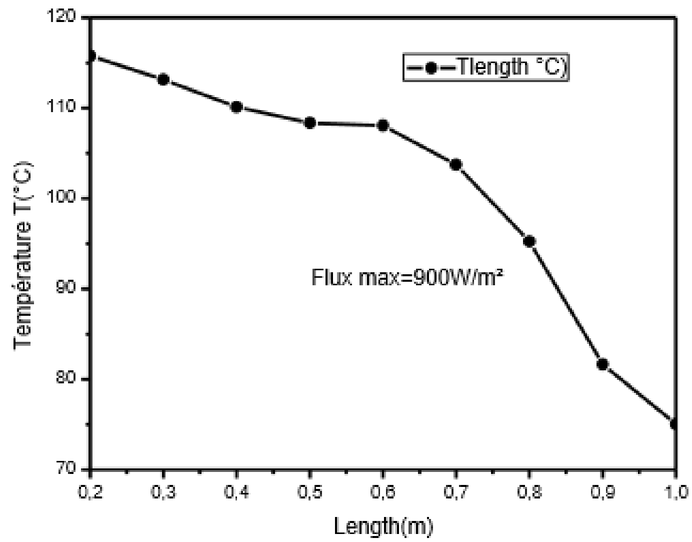

Figure 5:Evolution of the air temperature in the cooker chamber as a function of width

Note that in Figure 5, the air temperature decreases as the length of the cooker enclosure increases.

Figures 6 and 7 illustrate that the air temperature in the cooker chamber increases as the other dimensions (height, width) increase.

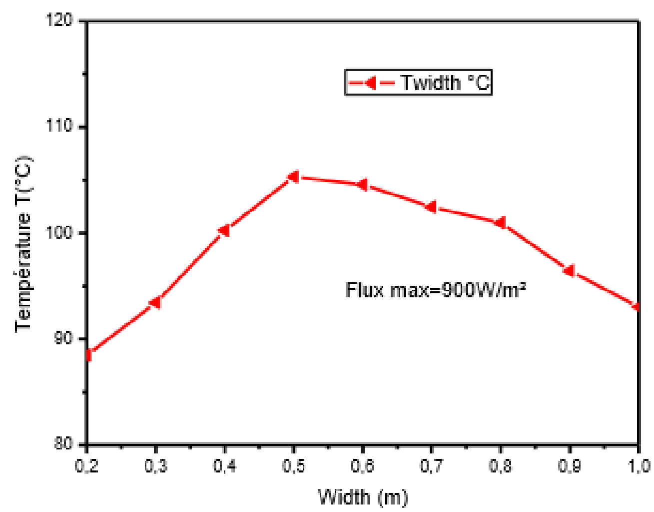

Figure6:Evolution of the air temperature in the cooker chamber as a function of width

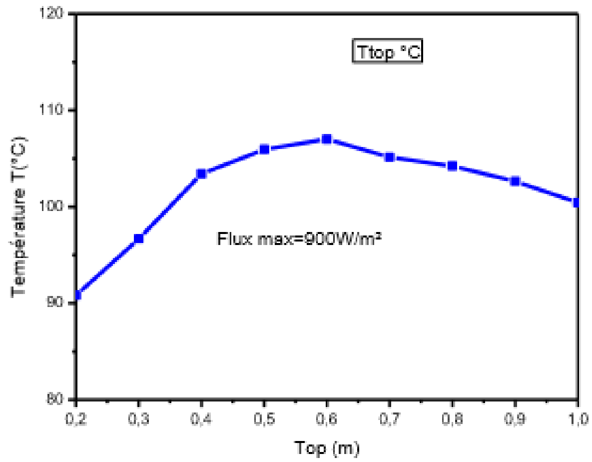

Figure7:Evolution of the air temperature in the cooker chamber as a function of height

In order to define the optimal dimensions of this cooker, Figure 5 illustrates the evolution of the temperature in the cooker chamber as a function of its dimensions (length, width and height). We notice that the optimal dimensions are located in the ABC zone where the temperature is maximum. Therefore, the optimal height and width are 0.50m and the optimal length is 0.65m.

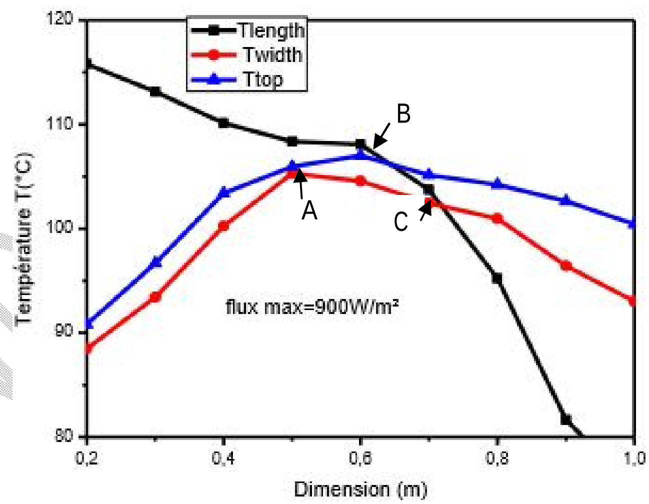

Figure 8: Evolution of the air temperature in the cooker chamber as a function of its dimensions

The result shows that for dimensions between [50cm-65cm], the thermal efficiency of the cooker is in the range of 42-45%. Thus the amount of heat required to increase the temperature of the fluid in the pot is higher the larger its volume. Thus, for a solar flux reflected by the concentrator, the increase in temperature of the fluid in the pot is all the higher as its volume is lower. From this curve, we deduce that the dimensions

0.65×0.50×0.50m for which the energy yield reaches its maximum value are the optimal dimensions of the cooker. In the rest of our calculations, we will use these dimensions.

7. INFLUENCE OF THE DIFFERENT MATERIALS OF THE POT WALLS

figure 9 shows the hourly evolution of the temperature of the external face of the kettle. We analyse the influence of the nature of the materials it is made of.

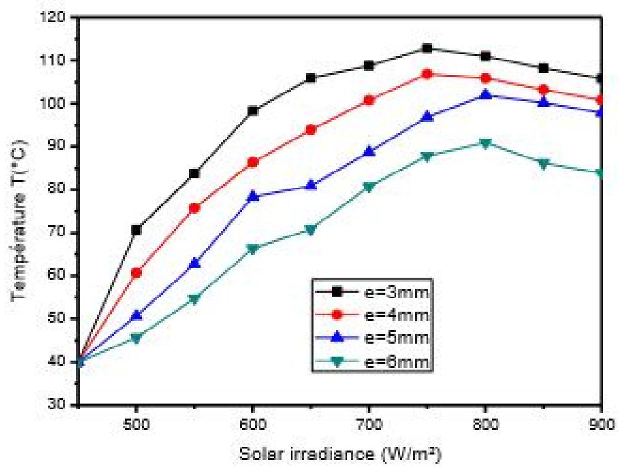

Figure 9: Hourly temperature trends on the outer surface of the wall showing the influence of the nature of the material constituting the wall of the kettle

The values of the thermo-physical properties of these materials are given in Table 2

Table 2: Thermo-physical properties of the materials of which the pot wall is made

Materials	Thermal conductivity (W/m.K)	Specific heat (J/kg.K)	Density Kg/m3
Aluminium	237	896	2700
Steel	50	450	7800
Copper	401	386	8940

The nature of the materials of which the kettle is made plays a very important role in the value of the temperature of the fluid it contains.

We observe that the hourly changes in the highest temperature of the fluid it contains are similar throughout the day. However, the highest values are obtained for the pot with a copper wall, the material with the highest thermal conductivity among those used in our calculations.

The analysis of the influence of the thickness of the pot wall shows that the lower the thickness of the pot, the higher the temperature of the inner side of the pot (Figure 9).

Table 3: Values of thermal efficiency and heat exchange coefficient

Wall thickness	Thermal efficiency %
e=3mm	61,38
e=4mm	58,40
e=5mm	44,62
e=6mm	36,91

This result is consistent with the fact that the thermal resistance between the outer and inner surfaces is greater the greater the thickness of the wall. The wall thickness is of the order of a few millimetres. Thus, the thermal efficiency by conduction through the wall of the kettle becomes important when the thickness is low. This result is consistent with the hourly temperature evolution of the fluid in the kettle.

9. CONCLUSION

In the framework of this work, we have carried out a numerical study of the solar cooker.

After a brief presentation on solar energy and its duration of sunshine in Bangui, we drew up a state of the art of solar cookers. A model of heat transfer in this cooker was then proposed. The equations governing the heat transfers are obtained from the thermal balance established on each component of the cooker and based on the nodal method. These equations are discretised by the implicit finite difference method, coupled with an iterative Gaussian algorithm procedure.

The simulation shows that the two wall curves (inner and outer) of the kettle are superimposed due to the negligible wall thickness.

It is clear that the parameters of the cooker play a major role, as they influence the determination of the dimensions of the cooker. In addition, the increase in the temperature of the kettle wall favours the improvement of the thermal efficiency of the solar cooker studied.

ANNEX

Through the pane of thickness ev and thermal conductivity λ_v the conductive transfer coefficient is by definition equal to :

Conductive heat exchange

The coefficient of heat transfer by conduction through the wall of the pot, h_cpmassimilated to a half-sphere, verifies the following expression:

$$h_{cpm} = \frac{2 \times \pi \times \lambda_m \times (R_2 - R_1)}{S_m \times R_2 \times R_1}$$

By convection

Between the external face of the glass and the ambient air

$$h_{cv-a} = 5{,}67 + 3{,}86V_v$$

Between the inner face of the glass and the air inside the parallelepiped enclosure

$$h_{cvvai} = \frac{\lambda_v \times N_{ust}}{H_v}$$

Between the outside of the absorber wall and the air inside

Between the fluid and the inside of the absorber wall

$$h_{cvpmai} = \frac{\lambda_m \times N_{us}}{H_m}$$

By radiation

Between the outer face of the glass and the sky

$$h_{rciel} = [\sigma.S(T_{ve}^2 + T_{ciel}^2)(T_{ve} + T_{ciel})] \times \frac{(1+\cos\alpha)}{2}$$

Where α is the angle of inclination of the pane to the horizontal.

Between the outer face of the pane and the ground

$$h_{rsol} = [\sigma.S(T_{ve}^2 + T_{sol}^2)(T_{ve} + T_{sol})] \times \frac{(1-\cos\alpha)}{2}$$

Between the absorber and the inside of the glass

$$h_{rv-m} = \frac{\sigma.(T_{vi}+T_{pme})(T_{vi}^2+T_{pme}^2)}{\frac{1}{Fr_{vm}}+\frac{1}{\varepsilon_m}+\frac{1}{\varepsilon_v}-1}$$

The Nusselt number has the expression :

$$N_{us} = [0{,}825 + \frac{0{,}387 \times R_a^{\frac{1}{6}}}{[1 + (\frac{0{,}492}{P_r})^{\frac{9}{16}}]^{\frac{8}{27}}}]^2$$

$$3.10^5 < R_a < 3.10^{10}$$

$$N_{ust} = 0{,}27 \times R_a^{\frac{1}{4}}$$

The number of Grashof

$$G_r = \frac{g\beta\Delta T\rho^2 L^3}{\mu^2}$$

The Rayleigh number

$$R_a = \frac{g\beta(T_p - T_\infty)L^3}{a\upsilon^2}$$

The sky temperature is deduced by the correlation of :

$$T_{ciel} = 0{,}0552T_a^{1,5}$$

The room temperature is calculated using the expression:

$$Ta = \overline{Ta} + \Delta Ta \times \sin\left\{\frac{\pi}{12} \times [TL - \left(TLo + \frac{3}{2}\right)]\right\}$$

TL: Local time (h)

TLo: Sunrise time (h)

$\overline{Ta} = \frac{Ta,max+Ta,min}{2}$ et $\Delta Ta = \frac{Ta,ma-Ta,min}{2}$

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