Fixed Point Theorems Using Soft Multiplicative Generalized Weak Contractive Mappings

Abstract: In the present paper, we establish fixed point theorems for mappings satisfying generalized weak contractive conditions in soft multiplicative metric space.

Keywords: Soft metric space, Soft multiplicative metric space, Soft multiplicative weak contractive mapping, Fixed point.

MSC: 47H10, 54H25

1. Introduction

In 1874, Cantor defined set theory as a branch of mathematics. This theory deals with the problem that contain certain results. But in real life situations, there are various uncertain problems which have imprecise results. To deal with these problems, Molodtsov[4], in 1999, introduced soft set theory and applied this theory in various fields like game theory, operations research etc. In 2002, Maji et al. [11][12] "worked on soft set theory and its applications in decision making problems". In 2011, Ali et al. [8] "defined various operations in this theory". In 2013, Wardowski [3] worked on soft mappings with the fixed point theorems.

As metric space is one of the prominent branches of mathematics, thus to explore this idea using soft sets, Das and Samanta [15][16] investigated the properties of soft real numbers in 2012 and in 2013, they introduced the concept of soft metric space. After this, several researchers worked on soft metric spaces and their properties. In 2008, Bashirov et al. [1] defined multiplicative metric space. Then various authors worked on this space [6][9][14][19][20][21]. Rathee at al. [17] combined "soft metric space and multiplicative metric space and generated a new space called soft multiplicative metric space".

Fixed point theory plays a vital role in various fields of mathematics. In 2016, Wadkar et al. [2] proved fixed point results related to soft sets and in the same year, Yazar et al. [10] proved some fixed point theorems of soft contractive mappings. In 2017, Hosseinzadeh [5] proved fixed point theorems in soft metric space. Then, Abbas et al. [7] introduced various results on fixed point theorems in soft metric spaces. After this, in 2021, Bhardwaj et al. [13] investigated some new fixed point results in soft metric space. Rathee et al. [17] derived some fixed point theorems in soft multiplicative metric space. In 2017, Solankki et al. [18] "generalize the concept of soft weak contractive mapping and proved various fixed point theorems in soft metric space". Extending the work of Solankki et al. [18] and Rathee et al. [17], we generate some new fixed point theorems using generalized multiplicative weak contraction mapping in soft multiplicative metric space.

2. Preliminaries

This section contains some basic definitions and results which are useful for our research work.

Definition 2.1[15]. "Let I be an initial universal set and Ω be the non-empty parameter set. Then, a pair (T,Ω) is called a soft set over I if T is a set valued mapping on Ω taking values in 2^I i.e., $T:\Omega \rightarrow 2^I$."

Definition 2.2[15]. "A soft set (T,Ω) over I is said to be an absolute soft set if $T(\alpha) = I \quad \forall \alpha \in \Omega$. It is denoted by $\tilde{I}$."

Definition 2.3[15]. "A soft set (T,Ω) over I is said to be a soft point if there is exactly one $\alpha \in \Omega$ such that $T(\alpha) = \{i\}$ for some $i \in I$ and $T(\beta) = \phi$ for all $\beta \in \Omega \setminus \{\alpha\}$. Such a soft point is denoted by $\tilde{T}_{\alpha}^{i}$."

NOTE**.** The collection of all soft points of a soft set (T,Ω) is denoted by $SP(T,\Omega)$.

Definition 2.4[15]. "Let $\mathbb{R}$ be the set of real numbers and $B(\mathbb{R})$ be the collection of all non-empty bounded subset of $\mathbb{R}$. Then, the function by $T:\Omega \rightarrow B(\mathbb{R})$ is called a soft real set and is denoted by (T,Ω). If T is a single valued function on Ω taking values in $\mathbb{R}$, then the pair (T,Ω) or simply T is called a soft real number. We denote soft real number and soft constant real number by $\tilde{r}, \tilde{s}, \tilde{t}$ and $\overline{r}, \overline{s}, \overline{t}$ respectively where $\overline{r}$ will denote a particular type of soft real number such that $\overline{r}(\alpha) = r$ for all $\alpha \in \Omega$."

Definition 2.5[15]. "For two soft real numbers $\tilde{p}$ and $\tilde{q}$, the following conditions hold for all

$\alpha \in \Omega$:

(a) $\tilde{p} \tilde{\leq} \tilde{q}$ if $\tilde{p}(\alpha) \lesssim \tilde{q}(\alpha)$;

(b) $\tilde{p} \tilde{\geq} \tilde{q}$ if $\tilde{p}(\alpha) \gtrsim \tilde{q}(\alpha)$;

(c) $\tilde{p} < \tilde{q}$ if $\tilde{p}(\alpha) \tilde{<} \tilde{q}(\alpha)$;

(d) $\tilde{p} > \tilde{q}$ if $\tilde{p}(\alpha) \tilde{>} \tilde{q}(\alpha)$."

Definition 2.6[15]. "A mapping $\tilde{\rho} : SP(\tilde{I}) \times SP(\tilde{I}) \rightarrow R(\Omega)^*$ is a soft metric on the absolute soft set $\tilde{I}$ if $\tilde{\rho}$ satisfies the following conditions:

1. $\tilde{\rho}(\tilde{T}_{\alpha}^{i}, \tilde{T}_{\beta}^{j}) \tilde{\geq} \overline{0}$ for all $\tilde{T}_{\alpha}^{i}, \tilde{T}_{\beta}^{j} \tilde{\in} SP(\tilde{I})$;
2. $\tilde{\rho}(\tilde{T}_{\alpha}^{i}, \tilde{T}_{\beta}^{j}) = \overline{0}$ if and only if $\alpha = \beta$ and $i = j$ for all $\tilde{T}_{\alpha}^{i}, \tilde{T}_{\beta}^{j} \tilde{\in} SP(\tilde{I})$;
3. $\tilde{\rho}(\tilde{T}_{\alpha}^{i}, \tilde{T}_{\beta}^{j}) = \tilde{\rho}(\tilde{T}_{\beta}^{j}, \tilde{T}_{\alpha}^{i})$ for all $\tilde{T}_{\alpha}^{i}, \tilde{T}_{\beta}^{j} \tilde{\in} SP(\tilde{I})$;
4. $\tilde{\rho}(\tilde{T}_{\alpha}^{i}, \tilde{T}_{\gamma}^{k}) \tilde{\leq} \tilde{\rho}(\tilde{T}_{\alpha}^{i}, \tilde{T}_{\beta}^{j}) + \tilde{\rho}(\tilde{T}_{\beta}^{j}, \tilde{T}_{\gamma}^{k})$ for all $\tilde{T}_{\alpha}^{i}, \tilde{T}_{\beta}^{j}, \tilde{T}_{\gamma}^{k} \tilde{\in} SP(\tilde{I})$.

The soft set $\tilde{I}$ together with soft metric $\tilde{\rho}$ is called a soft metric space and is denoted by $(\tilde{I}, \tilde{\rho}, \Omega)$ or simply by $(\tilde{I}, \tilde{\rho})$."

Definition 2.7[1]. "A mapping $\rho^*: I\times I \to \mathbb{R}^*$ is multiplicative metric if d^* satisfies the following conditions:

1. $\rho^*(u,v)\geq 1$ for all $u,v\in I$;
2. $\rho^*(u,v)=1$ if and only if $u=v$ for all $u,v\in I$;
3. $\rho^*(u,v)=\rho^*(v,u)$ for all $u,v\in I$;
4. $\rho^*(u,w)\leq \rho^*(u,v)\cdot\rho^*(v,w)$ for all $u,v,w\in I$.

The pair (I,ρ^*) is called a multiplicative metric space."

Definition 2.8[17]. "A function $\tilde{\rho}^*: SP(\tilde{I})\times SP(\tilde{I})\to R(\Omega)^*$ is soft multiplicative metric on the absolute soft set $\tilde{I}$ if $\tilde{\rho}^*$ meets the following properties:

1. $\tilde{\rho}^*(\tilde{T}^i_\alpha,\tilde{T}^j_\beta)\tilde{\geq}\overline{1}$ for all $\tilde{T}^i_\alpha,\tilde{T}^j_\beta \tilde{\in} SP(\tilde{I})$;
2. $\tilde{\rho}^*(\tilde{T}^i_\alpha,\tilde{T}^j_\beta)=\overline{1}$ if and only if $\alpha=\beta$ and $i=j$ for all $\tilde{T}^i_\alpha,\tilde{T}^j_\beta \tilde{\in} SP(\tilde{I})$;
3. $\tilde{\rho}^*(\tilde{T}^i_\alpha,\tilde{T}^j_\beta)=\tilde{\rho}^*(\tilde{T}^j_\beta,\tilde{T}^i_a)$ for all $\tilde{T}^i_\alpha,\tilde{T}^j_\beta \tilde{\in} SP(\tilde{I})$;
4. $\tilde{\rho}^*(\tilde{T}^i_\alpha,\tilde{T}^k_\gamma)\tilde{\leq}\tilde{\rho}^*(\tilde{T}^i_\alpha,\tilde{T}^j_\beta)\tilde{\rho}^*(\tilde{T}^j_\beta,\tilde{T}^k_\gamma)$ for all $\tilde{T}^i_\alpha,\tilde{T}^j_\beta,\tilde{T}^k_\gamma \tilde{\in} SP(\tilde{I})$.

The soft set $\tilde{I}$ together with soft multiplicative metric $\tilde{\rho}^*$ is called a soft multiplicative metric space and is denoted $(\tilde{I},\tilde{\rho}^*,\Omega)$."

Definition 2.9[17]. "Suppose $(\tilde{I},\tilde{\rho}^*)$ is a soft multiplicative metric space. Then, a sequence $\{\tilde{T}^{i_n}_{\alpha_n}\}$ in $(\tilde{I},\tilde{\rho}^*)$ is soft multiplicative convergent to a soft point $\tilde{T}^j_\beta \tilde{\in} \tilde{I}$ if for given $\tilde{\varepsilon}\tilde{\geq}\overline{1}$, we have a unique positive integer n_0 such that $\tilde{\rho}^*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^j_\beta\right)\tilde{<}\tilde{\varepsilon}$ for all $n\geq n_0$ i.e., $\tilde{\rho}^*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^j_\beta\right)\to\overline{1}$ as $n\to\infty$."

Definition 2.10[17]. "Suppose $(\tilde{I},\tilde{\rho}^*)$ is a soft multiplicative metric space. Then, a sequence $\{\tilde{T}^{i_n}_{\alpha_n}\}$ in $(\tilde{I},\tilde{\rho}^*)$ is soft multiplicative Cauchy sequence if for given $\tilde{\varepsilon}\tilde{\geq}\overline{1}$, we have a unique positive integer n_0 such that $\tilde{\rho}^*(\tilde{T}^{i_m}_{\alpha_m},\tilde{T}^{i_n}_{\alpha_n})\tilde{\leq}\tilde{\varepsilon}$ for all $m,n\geq n_0$ i.e., $\tilde{\rho}^*(\tilde{T}^{i_m}_{\alpha_m},\tilde{T}^{i_n}_{\alpha_n})\to\overline{1}$ as $m,n\to\infty$."

Definition 2.11[17]. "A soft multiplicative metric space $(\tilde{I},\tilde{\rho}^*)$ is complete, if every soft multiplicative Cauchy sequence in $\tilde{I}$ converges to some soft point in $\tilde{I}$."

Definition 2.12[10]. "Let $(\tilde{I},\tilde{\rho},\Omega)$ and $(\tilde{I}',\tilde{\rho}',\Omega')$ be two soft metric spaces. Then, $(h,\psi):(\tilde{I},\tilde{\rho},\Omega)\to(\tilde{I}',\tilde{\rho}',\Omega')$ is a soft mapping where $h: I\to I'$ and $\psi:\Omega\to\Omega'$ are two mappings."

Definition 2.13[17]. "Consider a soft multiplicative metric space $(\tilde{I},\tilde{\rho}^*,\Omega)$. A function $(h,\psi):(\tilde{I},\tilde{\rho}^*,\Omega)\to(\tilde{I},\tilde{\rho}^*,\Omega)$ is said to be soft multiplicative contraction mapping if for every soft point $\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j}\in\tilde{I}$, there exists a soft real number $\bar{\lambda},\ \bar{0}\,\tilde{\leq}\,\bar{\lambda}\,\tilde{<}\,\bar{1}$ such that $\tilde{\rho}^{*}\{(h,\psi)(\tilde{T}_{\alpha}^{i}),(h,\psi)(\tilde{T}_{\beta}^{j})\}\,\tilde{\leq}\,\{\tilde{\rho}^{*}(\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j})\}^{\bar{\lambda}}$."

Definition 2.14[18]. "A mapping $(h,\psi):(\tilde{I},\tilde{\rho},\Omega)\to(\tilde{I},\tilde{\rho},\Omega)$ where $(\tilde{I},\tilde{\rho},\Omega)$ is a soft metric space is said to be soft weakly C-contractive or a soft weak contraction if $\forall\ \tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j}\,\tilde{\in}\,SP(\tilde{I})$,

$$\tilde{\rho}\left\{(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right),(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\}\leq\frac{\bar{1}}{2}\left[\tilde{\rho}\left\{\tilde{T}_{\alpha}^{i},(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\}+\tilde{\rho}\left\{\tilde{T}_{\beta}^{j},(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right)\right\}\right]$$
$$-\xi\left[\tilde{\rho}\left\{\tilde{T}_{\alpha}^{i},(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\},\tilde{\rho}\left\{\tilde{T}_{\beta}^{j},(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right)\right\}\right],$$

where $\xi:\left[\bar{0},\infty\right)^{2}\to\left[\bar{0},\infty\right)$ is a continuous mapping such that $\xi\left(\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j}\right)=\bar{0}$ if and only if one of $\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j}=\bar{0}$."

Definition 2.15[18]. "A mapping $(h,\psi):(\tilde{I},\tilde{\rho},\Omega)\to(\tilde{I},\tilde{\rho},\Omega)$ where $(\tilde{I},\tilde{\rho},\Omega)$ is a soft metric space is said to be soft generalized weakly contractive or a soft generalized weak contraction if $\forall\ \tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j}\,\tilde{\in}\,SP(\tilde{I})$,

$$\tilde{\rho}\left\{(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right),(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\}\leq\bar{\eta}\left[\max\left\{\begin{array}{l}\tilde{\rho}\left\{\tilde{T}_{\alpha}^{i},(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right)\right\},\tilde{\rho}\left\{\tilde{T}_{\beta}^{j},(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\},\\ \tilde{\rho}\left\{\tilde{T}_{\alpha}^{i},(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\},\tilde{\rho}\left\{\tilde{T}_{\beta}^{j},(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right)\right\},\tilde{\rho}^{*}\left(\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j}\right)\end{array}\right\}\right]$$
$$-\xi\left(\left\{\begin{array}{l}\tilde{\rho}\left\{\tilde{T}_{\alpha}^{i},(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right)\right\},\tilde{\rho}\left\{\tilde{T}_{\beta}^{j},(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\},\\ \tilde{\rho}\left\{\tilde{T}_{\alpha}^{i},(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\},\tilde{\rho}\left\{\tilde{T}_{\beta}^{j},(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right)\right\},\tilde{\rho}\left(\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j}\right)\end{array}\right\}\right),$$

where $\bar{\eta}\in\left[\bar{0},\frac{\bar{1}}{2}\right)$, $\xi:\left[\bar{0},\infty\right)^{5}\to\left[\bar{0},\infty\right)$ is a continuous mapping such that $\xi\left(\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j},\tilde{T}_{\gamma}^{k},\tilde{T}_{\delta}^{l},\tilde{T}_{\varepsilon}^{m}\right)=\bar{0}$ if and only if one of $\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j},\tilde{T}_{\gamma}^{k},\tilde{T}_{\delta}^{l},\tilde{T}_{\varepsilon}^{m}=\bar{0}$."

3. Main Results

In this section, we define soft multiplicative generalized weakly contractive mappings and prove fixed point results using these mappings.

Theorem 3.1. Let $(\tilde{I},\tilde{\rho}^*,\Omega)$ be a complete soft multiplicative metric space and $(h,\psi):(\tilde{I},\tilde{\rho}^*,\Omega)\rightarrow(\tilde{I},\tilde{\rho}^*,\Omega)$ be a mapping, which satisfies the soft multiplicative generalized weak contractive mapping:

$$\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right),(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\}\leq\frac{\left[\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right),\tilde{T}_{\alpha}^{i}\right\}\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\beta}^{j}\right),\tilde{T}_{\beta}^{j}\right\}\right]^{\overline{p}}\left[\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right),\tilde{T}_{\beta}^{j}\right\}\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\beta}^{j}\right),\tilde{T}_{\alpha}^{i}\right\}\right]^{\overline{q}}}{\xi\begin{bmatrix}\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right),\tilde{T}_{\alpha}^{i}\right\},\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\beta}^{j}\right),\tilde{T}_{\beta}^{j}\right\},\\ \tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right),\tilde{T}_{\beta}^{j}\right\},\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\beta}^{j}\right),\tilde{T}_{\alpha}^{i}\right\}\end{bmatrix}}\ \forall\ \tilde{T}_{\alpha}^{i},\ \tilde{T}_{\beta}^{j}\ \tilde{\in}\ SP\left(\tilde{I}\right),$$

where $\overline{p},\overline{q}$ are non-negative soft real numbers such that $\overline{p}+\overline{q}\ \tilde{<}\ \overline{\frac{1}{2}}$ and $\xi:\left[\overline{1},\infty\right)^2\rightarrow\left[\overline{1},\infty\right)$ is a continuous function such that $\xi\left(\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j},\tilde{T}_{\gamma}^{k},\tilde{T}_{\delta}^{l}\right)=\overline{1}$ iff one of $\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j},\tilde{T}_{\gamma}^{k},\tilde{T}_{\delta}^{l}=\overline{1}$. Then, there exists a unique fixed point of (h,ψ) .

Proof. Let $\tilde{T}_{\alpha_0}^{i_0}$ be any soft point in $SP\left(\tilde{I}\right)$. Fix

$$\begin{aligned}\tilde{T}_{\alpha_1}^{i_1}&=(h,\psi)\tilde{T}_{\alpha_0}^{i_0}\\ \tilde{T}_{\alpha_2}^{i_2}&=(h,\psi)\tilde{T}_{\alpha_1}^{i_1}\\ &\vdots\\ \tilde{T}_{\alpha_{n+1}}^{i_{n+1}}&=(h,\psi)\tilde{T}_{\alpha_n}^{i_n}\end{aligned}$$

Now,

$$\begin{aligned}\tilde{\rho}^*\left(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha_n}^{i_n}\right)&=\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right),(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right)\right\}\\ &\tilde{\leq}\frac{\left[\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{T}_{\alpha_n}^{i_n}\right\}\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right),\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right\}\right]^{\overline{p}}\left[\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right\}\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right),\tilde{T}_{\alpha_n}^{i_n}\right\}\right]^{\overline{q}}}{\xi\begin{bmatrix}\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{T}_{\alpha_n}^{i_n}\right\},\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right),\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right\},\\ \tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right\},\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right),\tilde{T}_{\alpha_n}^{i_n}\right\}\end{bmatrix}}\end{aligned}$$

$$\lesssim \frac{\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\overline{p}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_n}_{\alpha_n}\right)\right\}^{\overline{q}}}{\xi\left\{\begin{matrix}\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),\\ \tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_n}_{\alpha_n}\right)\end{matrix}\right\}}$$

$$\lesssim \frac{\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\overline{p}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_n}_{\alpha_n}\right)\right\}^{\overline{q}}}{\xi\left\{\begin{matrix}\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),\\ \tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),\overline{1}\end{matrix}\right\}}$$

Since x satisfies the given condition, thus

$$\xi\left\{\begin{matrix}\tilde{\rho}*\left(\tilde{T}^{i_{n+!}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),\\ \tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),\overline{1}\end{matrix}\right\}=\overline{1}$$

and thus

$$\begin{aligned}\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right) &\lesssim \left\{\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\overline{p}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_n}_{\alpha_n}\right)\right\}^{\overline{q}}\\ &\lesssim \left\{\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\overline{p}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\overline{q}}\end{aligned}$$

$$\Rightarrow \left\{\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\right\}^{\overline{1}-\overline{p}-\overline{q}} \lesssim \left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\overline{p}+\overline{q}}$$

$$\Rightarrow \tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\lesssim \left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\frac{\overline{p}+\overline{q}}{\overline{1}-\overline{p}-\overline{q}}}$$

$$\Rightarrow \tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\lesssim \left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\overline{\eta}}, \qquad \text{where } \overline{\eta}=\frac{\overline{p}+\overline{q}}{\overline{1}-\overline{p}-\overline{q}}$$

$$\Rightarrow \qquad \tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\lesssim \left\{\tilde{\rho}*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_{n-2}}_{\alpha_{n-2}}\right)\right\}^{\overline{\eta}^2}$$

$$\vdots$$

$$\Rightarrow \qquad \tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\lesssim \left\{\tilde{\rho}*\left(\tilde{T}^{i_1}_{\alpha_1},\tilde{T}^{i_0}_{\alpha_0}\right)\right\}^{\overline{\eta}^n}.$$

For any $m > n,$ where $m, n \in \square$

$$\tilde{\rho}^*(\tilde{T}^{i_n}_{\alpha_n}, \tilde{T}^{i_m}_{\alpha_m}) \tilde{\leq} \{\tilde{\rho}^*(\tilde{T}^{i_n}_{\alpha_n}, \tilde{T}^{i_{n+1}}_{\alpha_{n+1}})\tilde{\rho}^*(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}, \tilde{T}^{i_m}_{\alpha_m})\}$$

$$\tilde{\leq}\left\{\tilde{\rho}^*(\tilde{T}^{i_n}_{\alpha_n}, \tilde{T}^{i_{n+1}}_{\alpha_{n+1}})\right\}\left\{\tilde{\rho}^*(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}, \tilde{T}^{i_{n+2}}_{\alpha_{n+2}})\right\}\left\{\tilde{\rho}^*(\tilde{T}^{i_{n+2}}_{\alpha_{n+2}}, \tilde{T}^{i_m}_{\alpha_m})\right\}$$

$$\tilde{\leq}\left\{\tilde{\rho}^*(\tilde{T}^{i_n}_{\alpha_n}, \tilde{T}^{i_{n+1}}_{\alpha_{n+1}})\right\}\left\{\tilde{\rho}^*(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}, \tilde{T}^{i_{n+2}}_{\alpha_{n+2}})\right\}\left\{\tilde{\rho}^*(\tilde{T}^{i_{n+2}}_{\alpha_{n+2}}, \tilde{T}^{i_{n+3}}_{\alpha_{n+3}})\right\}$$

$$\cdots\{\tilde{\rho}^*(\tilde{T}^{i_{m-1}}_{\alpha_{m-1}}, \tilde{T}^{i_m}_{\alpha_m})\}$$

$$\tilde{\leq}\{\tilde{\rho}^*(\tilde{T}^{i_0}_{\alpha_0}, \tilde{T}^{i_1}_{\alpha_1})\}^{\bar{\eta}^{\,n}}\{\tilde{\rho}^*(\tilde{T}^{i_0}_{\alpha_0}, \tilde{T}^{i_1}_{\alpha_1})\}^{\bar{\eta}^{\,n+1}}\{\tilde{\rho}^*(\tilde{T}^{i_0}_{\alpha_0}, \tilde{T}^{i_1}_{\alpha_1})\}^{\bar{\eta}^{\,n+2}}$$

$$\cdots\{\tilde{\rho}^*(\tilde{T}^{i_0}_{\alpha_0}, \tilde{T}^{i_1}_{\alpha_1})\}^{\bar{\eta}^{\,m-1}}$$

$$\tilde{\leq}\{\tilde{\rho}^*(\tilde{T}^{i_0}_{\alpha_0}, \tilde{T}^{i_1}_{\alpha_1})\}^{\bar{\eta}^{\,n}\left(\bar{1}+\bar{\eta}+{}^2\bar{\eta}^2+\cdots+\bar{\eta}^{\,m-n-1}\right)}$$

$$\tilde{\leq}\{\tilde{\rho}^*(\tilde{T}^{i_0}_{\alpha_0}, \tilde{T}^{i_1}_{\alpha_1})\}^{\frac{\bar{\eta}^{\,n}}{\bar{1}-\bar{\eta}}}.$$

Since $\bar{p}+\bar{q} \,\%\, \frac{\bar{1}}{2},$ thus $\hbar \,\%\, \bar{1}$ and hence $\tilde{\rho}^*(\tilde{T}^{i_n}_{\alpha_n}, \tilde{T}^{i_m}_{\alpha_m}) \to \bar{1}$ as $m, n \to \infty.$ So, the soft sequence $\{\tilde{T}^{i_n}_{\alpha_n}\}$ is soft multiplicative Cauchy sequence in $\tilde{I}$. Being the completeness of $(\tilde{I}, \tilde{\rho}^*, \Omega)$, there exists a soft point $\tilde{T}^{i^*}_{\alpha^*} \tilde{\in} \tilde{I}$ such that $\tilde{T}^{i_n}_{\alpha_n} \to \tilde{T}^{i^*}_{\alpha^*}$ as $n \to \infty.$

Also,

$$\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i^*}_{\alpha^*}\right), \tilde{T}^{i^*}_{\alpha^*}\right\} \tilde{\leq}\left[\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i^*}_{\alpha^*}\right), (h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\}\left\{\tilde{\rho}^*(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right), \tilde{T}^{i^*}_{\alpha^*}\right\}\right]$$

$$\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i^*}_{\alpha^*}\right), \tilde{T}^{i^*}_{\alpha^*}\right\} \tilde{\leq} \frac{\left[\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i^*}_{\alpha^*}\right), \tilde{T}^{i^*}_{\alpha^*}\right\}\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right), \tilde{T}^{i_n}_{\alpha_n}\right\}\right]^{\bar{p}} \left[\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i^*}_{\alpha^*}\right), \tilde{T}^{i_n}_{\alpha_n}\right\}\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right), \tilde{T}^{i^*}_{\alpha^*}\right\}\right]^{\bar{q}} \left[\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right), \tilde{T}^{i^*}_{\alpha^*}\right\}\right]}{\xi\begin{bmatrix}\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i^*}_{\alpha^*}\right), \tilde{T}^{i^*}_{\alpha^*}\right\}, \tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right), \tilde{T}^{i_n}_{\alpha_n}\right\}, \\ \tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i^*}_{\alpha^*}\right), \tilde{T}^{i_n}_{\alpha_n}\right\}, \tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right), \tilde{T}^{i^*}_{\alpha^*}\right\}\end{bmatrix}}$$

$$\tilde{\leq} \frac{\left\{\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i^*}_{\alpha^*}\right), \tilde{T}^{i^*}_{\alpha^*}\right\}\tilde{\rho}^*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}, \tilde{T}^{i_n}_{\alpha_n}\right)\right\}^{\bar{p}} \left\{\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i^*}_{\alpha^*}\right), \tilde{T}^{i_n}_{\alpha_n}\right\}\tilde{\rho}^*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}, \tilde{T}^{i^*}_{\alpha^*}\right)\right\}^{\bar{q}} \tilde{\rho}^*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}, \tilde{T}^{i^*}_{\alpha^*}\right)}{\xi\begin{bmatrix}\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i^*}_{\alpha^*}\right), \tilde{T}^{i^*}_{\alpha^*}\right\}, \tilde{\rho}^*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}, \tilde{T}^{i_n}_{\alpha_n}\right), \\ \tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i^*}_{\alpha^*}\right), \tilde{T}^{i_n}_{\alpha_n}\right\}, \tilde{\rho}^*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}, \tilde{T}^{i^*}_{\alpha^*}\right)\end{bmatrix}}$$

$$\left[\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right),\tilde{T}_{\alpha*}^{i*}\right\}\right]^{\bar{1}-\bar{p}} \tilde{\leq} \frac{\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\bar{p}} \left\{\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right),\tilde{T}_{\alpha_n}^{i_n}\right\}\tilde{\rho}*\left(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha*}^{i*}\right)\right\}^{\bar{q}}\tilde{\rho}*\left(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha*}^{i*}\right)}{\xi\left[\begin{array}{l}\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right),\tilde{T}_{\alpha*}^{i*}\right\},\tilde{\rho}*\left(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha_n}^{i_n}\right),\\ \tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right),\tilde{T}_{\alpha_n}^{i_n}\right\},\tilde{\rho}*\left(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha*}^{i*}\right)\end{array}\right]}$$

$$\left[\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right),\tilde{T}_{\alpha*}^{i*}\right\}\right]^{\bar{1}-\bar{p}-\bar{q}} \tilde{\leq} \bar{1} \ as \quad n\to\infty.$$

Since $\bar{p}+\bar{q}<\dfrac{\bar{1}}{2}$, thus $\bar{1}-\bar{p}-\bar{q}>\bar{0}$ and hence $\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right),\tilde{T}_{\alpha*}^{i*}\right\}=\bar{1}$. This signifies that $\tilde{T}_{\alpha*}^{i*}$ is a “soft fixed point” of (h,ψ).

Now, if $\tilde{T}_{\alpha'}^{i'}$ be another “soft fixed point” of (h,ψ). Then,

$$\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)=\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right),(h,\psi)\left(\tilde{T}_{\alpha'}^{i'}\right)\right\}$$

$$\tilde{\leq}\frac{\left[\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right),\tilde{T}_{\alpha*}^{i*}\right\}\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha'}^{i'}\right),\tilde{T}_{\alpha'}^{i'}\right\}\right]^{\bar{p}}\left[\left\{\tilde{\rho}*\left((h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right),\tilde{T}_{\alpha'}^{i'}\right)\right\}\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha'}^{i'}\right),\tilde{T}_{\alpha*}^{i*}\right\}\right]^{\bar{q}}}{\xi\left[\begin{array}{l}\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right),\tilde{T}_{\alpha*}^{i*}\right\},\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha'}^{i'}\right),\tilde{T}_{\alpha'}^{i'}\right\},\\ \left\{\tilde{\rho}*\left((h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right),\tilde{T}_{\alpha'}^{i'}\right)\right\},\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha'}^{i'}\right),\tilde{T}_{\alpha*}^{i*}\right\}\end{array}\right]}$$

$$=\frac{\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha*}^{i*}\right)\tilde{\rho}*\left(\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha'}^{i'}\right)\right\}^{\bar{p}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right\}^{\bar{2}\bar{q}}}{\xi\left[\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha*}^{i*}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha'}^{i'}\right),\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right\},\tilde{\rho}*\left\{\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha*}^{i*}\right\}\right]}$$

$$\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right\}^{\bar{1}-\bar{2}\bar{q}}\tilde{\leq}\bar{1}$$

Since $\bar{p}+\bar{q}$ %$\dfrac{\bar{1}}{2}$ and $\bar{p}>\bar{0}$, thus $\bar{1}-\bar{2}\bar{q}>\bar{0}$ and hence $\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)=\bar{1}\Rightarrow\tilde{T}_{\alpha*}^{i*}=\tilde{T}_{\alpha'}^{i'}$.

Hence, there is one and only one soft fixed point of (h,ψ).

Theorem 3.2. Let $(\tilde{I},\tilde{\rho}*,\Omega)$ be a complete soft multiplicative metric space and $(h,\psi):(\tilde{I},\tilde{\rho}*,\Omega)\to(\tilde{I},\tilde{\rho}*,\Omega)$ be a mapping, which satisfies the soft multiplicative generalized weak contractive mapping $\forall\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j}\tilde{\in}SP\left(\tilde{I}\right)$,

$$\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right),(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\} \leq \frac{\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j}\right)\right\}^{\overline{p}}\left[\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right),\tilde{T}_{\alpha}^{i}\right\}\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\beta}^{j}\right),\tilde{T}_{\beta}^{j}\right\}\right]^{\overline{q}}\left[\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right),\tilde{T}_{\beta}^{j}\right\}\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\beta}^{j}\right),\tilde{T}_{\alpha}^{i}\right\}\right]^{\overline{r}}}{\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j}\right),\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right),\tilde{T}_{\alpha}^{i}\right\},\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\beta}^{j}\right),\tilde{T}_{\beta}^{j}\right\},\\ \tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right),\tilde{T}_{\beta}^{j}\right\},\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\beta}^{j}\right),\tilde{T}_{\alpha}^{i}\right\}\end{bmatrix}},$$

where $\overline{p}$, $\overline{q}$ and $\overline{r}$ are non-negative soft real number such that $\overline{p}+\overline{2}\overline{q}+\overline{2}\overline{r} \tilde{<} \overline{1}$ and $\xi:\left[\overline{1},\infty\right)^{2} \to \left[\overline{1},\infty\right)$ is a continuous function such that $\xi\left(\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j},\tilde{T}_{\gamma}^{k},\tilde{T}_{\delta}^{l},\tilde{T}_{\kappa}^{m}\right)=\overline{1}$ iff one of $\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j},\tilde{T}_{\gamma}^{k},\tilde{T}_{\delta}^{l},\tilde{T}_{\kappa}^{m}=\overline{1}$. Then, there exists a unique fixed point of (h,ψ).

Proof. Let $\tilde{T}_{\alpha_0}^{i_0}$ be any soft point in $SP\left(\tilde{I}\right)$. Fix

$$\begin{aligned}\tilde{T}_{\alpha_1}^{i_1} &= (h,\psi)\tilde{T}_{\alpha_0}^{i_0}\\ \tilde{T}_{\alpha_2}^{i_2} &= (h,\psi)\tilde{T}_{\alpha_1}^{i_1}\\ &\vdots\\ \tilde{T}_{\alpha_{n+1}}^{i_{n+1}} &= (h,\psi)\tilde{T}_{\alpha_n}^{i_n}\end{aligned}$$

Now,

$$\tilde{\rho}*\left(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha_n}^{i_n}\right)= \tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right),(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right)\right\}$$

$$\tilde{\leq}\frac{\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right)\right\}^{\overline{p}}\left[\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{T}_{\alpha_n}^{i_n}\right\}\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right),\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right\}\right]^{\overline{q}}\left[\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right\}\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right),\tilde{T}_{\alpha_n}^{i_n}\right\}\right]^{\overline{r}}}{\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right),\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{T}_{\alpha_n}^{i_n}\right\},\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right),\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right\},\\ \tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right\},\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right),\tilde{T}_{\alpha_n}^{i_n}\right\}\end{bmatrix}}$$

$$\tilde{\leq}\frac{\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right)\right\}^{\overline{p}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha_n}^{i_n}\right)\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right)\right\}^{\overline{q}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right)\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\overline{r}}}{\xi\begin{Bmatrix}\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right),\\ \tilde{\rho}*\left(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_n}^{i_n}\right)\end{Bmatrix}}$$

$$\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\overline{p}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\overline{q}}$$

$$\tilde{\leq}\frac{\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\overline{r}}}{\xi\left\{\begin{array}{l}\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),\\ \tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),\overline{1}\end{array}\right\}}$$

Since x satisfies the given condition, thus

$$\xi\left\{\begin{array}{l}\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),\\ \tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),\overline{1}\end{array}\right\}=\overline{1}$$

and thus

$$\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\right\}^{\overline{1}-\overline{q}-\overline{r}}\ \tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\overline{p}+\overline{q}+\overline{r}}$$

$$\Rightarrow\qquad \tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\frac{\overline{p}+\overline{q}+\overline{r}}{\overline{1}-\overline{q}-\overline{r}}}$$

$$\Rightarrow\qquad \tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\overline{\eta}},\qquad where\ \overline{\eta}=\frac{\overline{p}+\overline{q}+\overline{r}}{\overline{1}-\overline{q}-\overline{r}}$$

$$\Rightarrow\qquad \tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_{n-2}}_{\alpha_{n-2}}\right)\right\}^{\overline{\eta}^2}$$

$$\vdots$$

$$\Rightarrow\qquad \tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}^{i_1}_{\alpha_1},\tilde{T}^{i_0}_{\alpha_0}\right)\right\}^{\overline{\eta}^n}.$$

For any $m>n,$ where $m,n\in$ □

$$\begin{aligned}\tilde{\rho}*(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_m}_{\alpha_m})&\tilde{\leq}\{\tilde{\rho}*(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}})\tilde{\rho}*(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_m}_{\alpha_m})\}\\ &\tilde{\leq}\left\{\tilde{\rho}*(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}})\right\}\left\{\tilde{\rho}*(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_{n+2}}_{\alpha_{n+2}})\right\}\left\{\tilde{\rho}*(\tilde{T}^{i_{n+2}}_{\alpha_{n+2}},\tilde{T}^{i_m}_{\alpha_m})\right\}\\ &\tilde{\leq}\left\{\tilde{\rho}*(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}})\right\}\left\{\tilde{\rho}*(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_{n+2}}_{\alpha_{n+2}})\right\}\left\{\tilde{\rho}*(\tilde{T}^{i_{n+2}}_{\alpha_{n+2}},\tilde{T}^{i_{n+3}}_{\alpha_{n+3}})\right\}\\ &\qquad\cdots\{\tilde{\rho}*(\tilde{T}^{i_{m-1}}_{\alpha_{m-1}},\tilde{T}^{i_m}_{\alpha_m})\}\\ &\tilde{\leq}\{\tilde{\rho}*(\tilde{T}^{i_0}_{\alpha_0},\tilde{T}^{i_1}_{\alpha_1})\}^{\overline{\eta}^n}\{\tilde{\rho}*(\tilde{T}^{i_0}_{\alpha_0},\tilde{T}^{i_1}_{\alpha_1})\}^{\overline{\eta}^{n+1}}\{\tilde{\rho}*(\tilde{T}^{i_0}_{\alpha_0},\tilde{T}^{i_1}_{\alpha_1})\}^{\overline{\eta}^{n+2}}\\ &\qquad\cdots\{\tilde{\rho}*(\tilde{T}^{i_0}_{\alpha_0},\tilde{T}^{i_1}_{\alpha_1})\}^{\overline{\eta}^{m-1}}\end{aligned}$$

$$\tilde{\leq}\{\tilde{\rho}*(\tilde{T}^{i_0}_{\alpha_0},\tilde{T}^{i_1}_{\alpha_1})\}^{\bar{\eta}^{n}\left(\bar{1}+\bar{\eta}+{}^{2}\bar{\eta}^{2}+\cdots+\bar{\eta}^{m-n-1}\right)}$$

$$\tilde{\leq}\{\tilde{\rho}*(\tilde{T}^{i_0}_{\alpha_0},\tilde{T}^{i_1}_{\alpha_1})\}^{\frac{\bar{\eta}^{n}}{\bar{1}-\bar{\eta}}}.$$

Since $\bar{p}+\bar{2}\bar{q}+\bar{2}\bar{r}\,\tilde{<}\,\bar{1}$, thus $\hbar\,\tilde{<}\,\bar{1}$. Therefore, $\tilde{\rho}*(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_m}_{\alpha_m})\to\bar{1}$ as $m,n\to\infty$. So, the soft sequence $\{\tilde{T}^{i_n}_{\alpha_n}\}$ is soft multiplicative Cauchy sequence in $\tilde{I}$. Being the completeness of $(\tilde{I},\tilde{\rho}*,\Omega)$, there exists a soft point $\tilde{T}^{i*}_{\alpha*}\,\tilde{\in}\,\tilde{I}$ such that $\tilde{T}^{i_n}_{\alpha_n}\to\tilde{T}^{i*}_{\alpha*}$ as $n\to\infty$.

Also,

$$\begin{aligned}
\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),\tilde{T}^{i*}_{\alpha*}\right\} \tilde{\leq} & \left[\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\}\left\{\tilde{\rho}*(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{T}^{i*}_{\alpha*}\right\}\right] \\
& \left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_n}_{\alpha_n}\right)\right\}^{\bar{p}}\left[\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),\tilde{T}^{i*}_{\alpha*}\right\}\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{T}^{i_n}_{\alpha_n}\right\}\right]^{\bar{q}} \\
& \left[\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),\tilde{T}^{i_n}_{\alpha_n}\right\}\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{T}^{i*}_{\alpha*}\right\}\right]^{\bar{r}} \\
\tilde{\leq} & \frac{\left[\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{T}^{i*}_{\alpha*}\right\}\right]}{\xi\left[\begin{array}{l}\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),\tilde{T}^{i*}_{\alpha*}\right\},\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{T}^{i_n}_{\alpha_n}\right\}, \\ \tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),\tilde{T}^{i_n}_{\alpha_n}\right\},\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{T}^{i*}_{\alpha*}\right\}\end{array}\right]} \\
& \left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_n}_{\alpha_n}\right)\right\}^{\bar{p}}\left[\tilde{\rho}*\left((h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),\tilde{T}^{i*}_{\alpha*}\right)\right]^{\bar{q}}\left[\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\right]^{\bar{q}} \\
& \left[\tilde{\rho}*\left((h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),\tilde{T}^{i_n}_{\alpha_n}\right)\right]^{\bar{r}}\left[\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i*}_{\alpha*}\right)\right]^{\bar{r}} \\
\tilde{\leq} & \frac{\left[\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i*}_{\alpha*}\right)\right]}{\xi\left[\begin{array}{l}\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}*\left((h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),\tilde{T}^{i*}_{\alpha*}\right),\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right), \\ \tilde{\rho}*\left((h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i*}_{\alpha*}\right)\end{array}\right]} \\
& \left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_n}_{\alpha_n}\right)\right\}^{\bar{p}}\left[\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right)\right]^{\bar{q}} \\
& \left[\tilde{\rho}*\left((h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),\tilde{T}^{i_n}_{\alpha_n}\right)\right]^{\bar{r}}\left[\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i*}_{\alpha*}\right)\right]^{\bar{r}}
\end{aligned}$$

$$\left[\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),\tilde{T}^{i*}_{\alpha*}\right\}\right]^{\bar{1}-\bar{q}} \tilde{\leq} \frac{\left[\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i*}_{\alpha*}\right)\right]}{\xi\left[\begin{array}{l}\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}*\left((h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),\tilde{T}^{i*}_{\alpha*}\right),\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i_n}_{\alpha_n}\right), \\ \tilde{\rho}*\left((h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}*\left(\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},\tilde{T}^{i*}_{\alpha*}\right)\end{array}\right]}$$

$$\left[\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right),\tilde{T}_{\alpha*}^{i*}\right\}\right]^{\bar{1}-\bar{q}-\bar{r}} \tilde{\leq} \quad \bar{1} \quad as \quad n\to\infty.$$

Since $\bar{p}+\bar{2}\bar{q}+\bar{2}\bar{r} \tilde{<} \bar{1}$ and $\bar{p}$ %$\bar{0}$, therefore $\bar{1}$- $\bar{q}$- $\bar{r}$ %$\bar{0}$ and hence $\tilde{T}_{\alpha*}^{i*}$ is a "soft fixed point" of (h,ψ) Now, if $\tilde{T}_{\alpha'}^{i'}$ be another "soft fixed point" of (h,ψ) . Then,

$$\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)=\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right),(h,\psi)\left(\tilde{T}_{\alpha'}^{i'}\right)\right\}$$

$$\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right\}^{\bar{p}}\left[\tilde{\rho}*\left\{(h,\psi)\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha*}^{i*}\right\}\tilde{\rho}*\left\{(h,\psi)\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha'}^{i'}\right\}\right]^{\bar{q}}$$

$$\leq\frac{\left[\left\{\tilde{\rho}*\left((h,\psi)\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right\}\tilde{\rho}*\left\{(h,\psi)\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha*}^{i*}\right\}\right]^{\bar{r}}}{\xi\begin{bmatrix}\tilde{\rho}*\left\{(h,\psi)\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha*}^{i*}\right\},\tilde{\rho}*\left\{(h,\psi)\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha'}^{i'}\right\},\\ \left\{\tilde{\rho}*\left((h,\psi)\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right\},\tilde{\rho}*\left\{(h,\psi)\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha*}^{i*}\right\}\end{bmatrix}}$$

$$\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right\}^{\bar{p}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha*}^{i*}\right)\tilde{\rho}*\left(\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha'}^{i'}\right)\right\}^{\bar{q}}$$

$$=\frac{\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right\}^{\bar{2}\bar{r}}}{\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha*}^{i*}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha'}^{i'}\right),\\ \left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right\},\tilde{\rho}*\left\{\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha*}^{i*}\right\}\end{bmatrix}}$$

$$\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right\}^{\bar{1}-\bar{p}-\bar{2}\bar{r}}\tilde{\leq}\bar{1}$$

Since $\bar{p}+\bar{2}\bar{q}+\bar{2}\bar{r} \tilde{<} \bar{1}$ and $\bar{q}$ %$\bar{0}$, therefore $\bar{1}$- $\bar{p}$- $\bar{r}$ %$\bar{0}$. Thus,

$$\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)=\bar{1}\Rightarrow\tilde{T}_{\alpha*}^{i*}=\tilde{T}_{\alpha'}^{i'}.$$

Hence, there is one and only one soft fixed point of (h,ψ).

Theorem 3.3. Let $(\tilde{I},\tilde{\rho}*,\Omega)$ be a complete soft multiplicative metric space and $(h,\psi):(\tilde{I},\tilde{\rho}*,\Omega)\to(\tilde{I},\tilde{\rho}*,\Omega)$ be a mapping, which satisfies the soft multiplicative generalized weak contractive mapping:

$$\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^i_\alpha\right),(h,\psi)\left(\tilde{T}^j_\beta\right)\right\} \leq \frac{\left[\max\left[\tilde{\rho}^*\left\{\tilde{T}^i_\alpha,(h,\psi)\left(\tilde{T}^i_\alpha\right)\right\},\tilde{\rho}\left\{\tilde{T}^j_\beta,(h,\psi)\left(\tilde{T}^j_\beta\right)\right\},\tilde{\rho}^*\left(\tilde{T}^i_\alpha,\tilde{T}^j_\beta\right)\right]\right]^{\overline{p}}\left[\tilde{\rho}^*\left\{\tilde{T}^i_\alpha,(h,\psi)\left(\tilde{T}^j_\beta\right)\right\}\tilde{\rho}^*\left\{\left(\tilde{T}^j_\beta\right),(h,\psi)\left(\tilde{T}^i_\alpha\right)\right\}\right]^{\overline{q}}}{\xi\left[\begin{array}{l}\tilde{\rho}^*\left\{\tilde{T}^i_\alpha,(h,\psi)\left(\tilde{T}^i_\alpha\right)\right\},\tilde{\rho}\left\{\tilde{T}^j_\beta,(h,\psi)\left(\tilde{T}^j_\beta\right)\right\},\tilde{\rho}^*\left(\tilde{T}^i_\alpha,\tilde{T}^j_\beta\right),\\ \tilde{\rho}^*\left\{\tilde{T}^i_\alpha,(h,\psi)\left(\tilde{T}^j_\beta\right)\right\},\tilde{\rho}^*\left\{\left(\tilde{T}^j_\beta\right),(h,\psi)\left(\tilde{T}^i_\alpha\right)\right\}\end{array}\right]}$$

$$\forall\ \tilde{T}^i_\alpha,\ \tilde{T}^j_\beta \tilde{\in} SP\left(\tilde{I}\right),$$

where $\overline{p},\overline{q}$ are non-negative soft real numbers such that $\overline{p}+\overline{2}\overline{q}\,\tilde{<}\,\overline{1}$ and $\xi:\left[\overline{1},\infty\right)^2\rightarrow\left[\overline{1},\infty\right)$ is a continuous function such that $\xi\left(\tilde{T}^i_\alpha,\tilde{T}^j_\beta,\tilde{T}^k_\gamma,\tilde{T}^l_\delta,\tilde{T}^m_\kappa\right)=\overline{1}$ iff one of $\tilde{T}^i_\alpha,\tilde{T}^j_\beta,\tilde{T}^k_\gamma,\tilde{T}^l_\delta,\tilde{T}^m_\kappa=\overline{1}$. Then, there exists a unique fixed point of (h,ψ) .

Proof. Let $\tilde{T}^{i_0}_{\alpha_0}$ be any soft point in $SP\left(\tilde{I}\right)$. Fix

$$\begin{aligned}\tilde{T}^{i_1}_{\alpha_1}&=(h,\psi)\tilde{T}^{i_0}_{\alpha_0}\\ \tilde{T}^{i_2}_{\alpha_2}&=(h,\psi)\tilde{T}^{i_1}_{\alpha_1}\\ &\vdots\\ \tilde{T}^{i_{n+1}}_{\alpha_{n+1}}&=(h,\psi)\tilde{T}^{i_n}_{\alpha_n}\end{aligned}$$

Now,

$$\tilde{\rho}^*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)=\tilde{\rho}^*\left\{(h,\psi)\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\}$$

$$\tilde{\leq}\frac{\left[\max\left\{\tilde{\rho}^*\left\{\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),(h,\psi)\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\},\tilde{\rho}^*\left\{\left(\tilde{T}^{i_n}_{\alpha_n}\right),(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\},\tilde{\rho}^*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_n}_{\alpha_n}\right)\right\}\right]^{\overline{p}}\left[\tilde{\rho}^*\left\{\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\}\tilde{\rho}^*\left\{\left(\tilde{T}^{i_n}_{\alpha_n}\right),(h,\psi)\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}\right]^{\overline{q}}}{\xi\left[\begin{array}{l}\tilde{\rho}^*\left\{\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),(h,\psi)\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\},\tilde{\rho}^*\left\{\left(\tilde{T}^{i_n}_{\alpha_n}\right),(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\},\tilde{\rho}^*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_n}_{\alpha_n}\right),\\ \tilde{\rho}^*\left\{\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right),(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\},\tilde{\rho}^*\left\{\left(\tilde{T}^{i_n}_{\alpha_n}\right),(h,\psi)\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}\end{array}\right]}$$

$$\tilde{\leq}\frac{\left[\max\left\{\tilde{\rho}^*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}^*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}^*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_n}_{\alpha_n}\right)\right\}\right]^{\overline{p}}\left\{\tilde{\rho}^*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\ \tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\tilde{\rho}^*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_n}_{\alpha_n}\right)\right\}^{\overline{q}}}{\xi\left\{\begin{array}{l}\tilde{\rho}^*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}^*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}^*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_n}_{\alpha_n}\right),\\ \tilde{\rho}^*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\ \tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}^*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_n}_{\alpha_n}\right)\end{array}\right\}}$$

$$\left[\max\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}\right]^{\overline{p}}$$

$$\tilde{\leq}\frac{\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}^{\overline{q}}}{\xi\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\overline{1}\right\}}$$

Since x satisfies the given condition, thus

$$\xi\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\overline{1}\right\}=\overline{1}$$

and thus

$$\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\tilde{\leq}M^{\overline{p}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}^{\overline{q}}$$

where M= $\max\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_n}_{\alpha_n}\right),\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}$

Now, two cases will be arised:

CASE 1. If M = $\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)$, then

$$\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}^{\overline{p}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_n}_{\alpha_n}\right)\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}^{\overline{q}}$$

$$\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}^{\overline{1}-\overline{p}-\overline{q}}\tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_n}_{\alpha_n}\right)\right\}^{\overline{q}}$$

$$\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_n}_{\alpha_n}\right)\right\}^{\frac{\overline{q}}{\overline{1}-\overline{p}-\overline{q}}}$$

$$\Rightarrow\quad\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n-1}}_{\alpha_{n-1}},\tilde{T}^{i_n}_{\alpha_n}\right)\right\}^{\overline{\eta}},\qquad where\ \overline{\eta}=\frac{\overline{q}}{\overline{1}-\overline{p}-\overline{q}}$$

$$\Rightarrow\quad\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}^{i_{n-2}}_{\alpha_{n-2}},\tilde{T}^{i_{n-1}}_{\alpha_{n-1}}\right)\right\}^{\overline{\eta}^2}$$

$$\vdots$$

$$\Rightarrow\quad\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}^{i_0}_{\alpha_0},\tilde{T}^{i_1}_{\alpha 1}\right)\right\}^{\overline{\eta}^n}.$$

CASE 2. If M = $\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)$, then

$$\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\ \tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\overline{p}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\right\}^{\overline{q}}$$

$$\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\right\}^{1-\overline{q}}\ \tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\overline{p}+\overline{q}}$$

$$\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\frac{\overline{p}+\overline{q}}{1-\overline{q}}}$$

$$\Rightarrow\ \tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\overline{\eta}},\qquad where\ \ \overline{\eta}=\frac{\overline{p}+\overline{q}}{1-\overline{q}}$$

$$\Rightarrow\ \tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-2}}^{i_{n-2}},\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right)\right\}^{\overline{\eta}^2}$$

$$\vdots$$

$$\Rightarrow\ \tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\tilde{\leq}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_0}^{i_0},\tilde{T}_{\alpha 1}^{i_1}\right)\right\}^{\overline{\eta}^n}.$$

Using both the cases, for any $m>n,$ where $m,n\in$ □ , we have

$$\begin{aligned}
\tilde{\rho}*(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_m}^{i_m})\ &\tilde{\leq}\{\tilde{\rho}*(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}})\tilde{\rho}*(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha_m}^{i_m})\}\\
&\tilde{\leq}\left\{\tilde{\rho}*(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}})\right\}\left\{\tilde{\rho}*(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha_{n+2}}^{i_{n+2}})\right\}\left\{\tilde{\rho}*(\tilde{T}_{\alpha_{n+2}}^{i_{n+2}},\tilde{T}_{\alpha_m}^{i_m})\right\}\\
&\tilde{\leq}\left\{\tilde{\rho}*(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}})\right\}\left\{\tilde{\rho}*(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha_{n+2}}^{i_{n+2}})\right\}\left\{\tilde{\rho}*(\tilde{T}_{\alpha_{n+2}}^{i_{n+2}},\tilde{T}_{\alpha_{n+3}}^{i_{n+3}})\right\}\\
&\qquad\cdots\{\tilde{\rho}*(\tilde{T}_{\alpha_{m-1}}^{i_{m-1}},\tilde{T}_{\alpha_m}^{i_m})\}\\
&\tilde{\leq}\{\tilde{\rho}*(\tilde{T}_{\alpha_0}^{i_0},\tilde{T}_{\alpha_1}^{i_1})\}^{\overline{\eta}^n}\{\tilde{\rho}*(\tilde{T}_{\alpha_0}^{i_0},\tilde{T}_{\alpha_1}^{i_1})\}^{\overline{\eta}^{n+1}}\{\tilde{\rho}*(\tilde{T}_{\alpha_0}^{i_0},\tilde{T}_{\alpha_1}^{i_1})\}^{\overline{\eta}^{n+2}}\\
&\qquad\qquad\cdots\{\tilde{\rho}*(\tilde{T}_{\alpha_0}^{i_0},\tilde{T}_{\alpha_1}^{i_1})\}^{\overline{\eta}^{m-1}}\\
&\tilde{\leq}\{\tilde{\rho}*(\tilde{T}_{\alpha_0}^{i_0},\tilde{T}_{\alpha_1}^{i_1})\}^{\overline{\eta}^{n\left(\overline{1}+\overline{\eta}+{}^2\overline{\eta}^2+\cdots+\overline{\eta}^{m-n-1}\right)}}\\
&\tilde{\leq}\{\tilde{\rho}*(\tilde{T}_{\alpha_0}^{i_0},\tilde{T}_{\alpha_1}^{i_1})\}^{\frac{\overline{\eta}^n}{\overline{1}-\overline{\eta}}}.
\end{aligned}$$

Since $\overline{p}+\overline{2}\overline{q}$ %$\overline{1}$, therefore $\hbar$ %$\overline{1}$ and hence $\tilde{\rho}*(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_m}^{i_m})\to\overline{1}$ as $m,n\to\infty$. So, the soft sequence $\{\tilde{T}_{\alpha_n}^{i_n}\}$ is soft multiplicative Cauchy sequence in $\tilde{I}$. Being the completeness of $(\tilde{I},\tilde{\rho}*,\Omega)$, there exists a soft point $\tilde{T}_{\alpha*}^{i*}\tilde{\in}\tilde{I}$ such that $\tilde{T}_{\alpha_n}^{i_n}\to\tilde{T}_{\alpha*}^{i*}$ as $n\to\infty$.

Also,

$$\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\} \tilde{\le} \tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\tilde{\rho}*\left\{\tilde{T}^{i_{n+1}}_{\alpha_{n+1}},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}$$

$$\tilde{\le} \tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right),(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}$$

$$\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\max\left[\tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\},\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right]^{\overline{p}}$$

$$\tilde{\lesssim}\frac{\left[\tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\}\right]^{\overline{q}}}{\xi\left[\begin{array}{l}\tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\},\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\\ \tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\}\end{array}\right]}$$

$$\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\max\left[\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right]^{\overline{p}}$$

$$\tilde{\lesssim}\frac{\left[\tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right]^{\overline{q}}}{\xi\left[\begin{array}{l}\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\\ \tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\end{array}\right]}$$

$$\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\max\left[\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right]^{\overline{p}}$$

$$\tilde{\lesssim}\frac{\left[\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right]^{\overline{q}}}{\xi\left[\begin{array}{l}\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\\ \tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\end{array}\right]}$$

$$\left\{\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}\right\}^{\overline{1}-\overline{q}} \tilde{\lesssim}\frac{\left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}^{\overline{1}+\overline{q}}(M_2)^{\overline{p}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right\}^{\overline{q}}}{\xi\left[\begin{array}{l}\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\\ \tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\end{array}\right]} \quad ...(3.1)$$

where $M_2 = \max\left[\tilde{\rho}*(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}),\tilde{\rho}*\{\tilde{T}^{i*}_{\alpha*},(h,\psi)(\tilde{T}^{i*}_{\alpha*})\},\tilde{\rho}*(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*})\right]$.

Suppose $M_2 = \tilde{\rho}*(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}})$. Then, from (3.1), we have

$$\left[\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}\right]^{\bar{1}-\bar{q}} \tilde{\leq} \frac{\left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}^{\bar{1}+\bar{q}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}^{\bar{p}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right\}^{\bar{q}}}{\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\\ \tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\end{bmatrix}}$$

$$\tilde{\leq} \frac{\left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}^{\bar{1}+\bar{q}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right\}^{\bar{p}}\left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}^{\bar{p}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right\}^{\bar{q}}}{\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\\ \tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\end{bmatrix}}$$

$$\tilde{\leq} \frac{\left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}^{\bar{1}+\bar{p}+\bar{q}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right\}^{\bar{p}+\bar{q}}}{\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\\ \tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\end{bmatrix}}$$

$$\to \bar{1} \quad as \quad n\to\infty.$$

Since $\bar{p}+\bar{2}\bar{q} \lesssim \bar{1}$ and $\bar{p} \gtrsim \bar{0}$, therefore $\bar{1}-\bar{q} \gtrsim \bar{0}$ which indicates that $\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}=\bar{1}$ and hence $\tilde{T}^{i*}_{\alpha*}$ is a “soft fixed point” of (h,ψ).

Now, assume $M_2 = \tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)$. Then, from (3.1), we have

$$\left[\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}\right]^{\bar{1}-\bar{q}} \tilde{\leq} \frac{\left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}^{\bar{1}+\bar{q}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right\}^{\bar{p}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right\}^{\bar{q}}}{\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\\ \tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\end{bmatrix}}$$

$$\tilde{\leq} \frac{\left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}^{\bar{1}+\bar{q}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right\}^{\bar{p}+\bar{q}}}{\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\\ \tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\end{bmatrix}}$$

$$\to \bar{1} \quad as \quad n\to\infty.$$

Since $\bar{p}+\bar{2}\bar{q} \lesssim \bar{1}$ and $\bar{p} \gtrsim \bar{0}$, therefore $\bar{1}-\bar{q} \gtrsim \bar{0}$ which indicates that $\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}=\bar{1}$ and hence $\tilde{T}^{i*}_{\alpha*}$ is a “soft fixed point” of (h,ψ).

Now, if $M_2 = \tilde{\rho}*\{\tilde{T}^{i*}_{\alpha*},(h,y)(\tilde{T}^{i*}_{\alpha*})\}$. Then, from (3.1), we have

$$\left[\tilde{\rho}*\left\{\tilde{T}_{\alpha*}^{i*},(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right)\right\}\right]^{\bar{1}-\bar{q}} \precsim \frac{\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\right\}^{\bar{1}+\bar{q}}\left[\tilde{\rho}*\left\{\tilde{T}_{\alpha*}^{i*},(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right)\right\}\right]^{\bar{p}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha*}^{i*}\right)\right\}^{\bar{q}}}{\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}_{\alpha*}^{i*},(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right)\right\},\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha*}^{i*}\right)\\ \tilde{\rho}*\left\{\tilde{T}_{\alpha_n}^{i_n},(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right)\right\},\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\end{bmatrix}}$$

$$\left[\tilde{\rho}*\left\{\tilde{T}_{\alpha*}^{i*},(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right)\right\}\right]^{\bar{1}-\bar{q}-\bar{p}} \precsim \frac{\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\right\}^{\bar{1}+\bar{q}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha*}^{i*}\right)\right\}^{\bar{q}}}{\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}_{\alpha*}^{i*},(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right)\right\},\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha*}^{i*}\right)\\ \tilde{\rho}*\left\{\tilde{T}_{\alpha_n}^{i_n},(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right)\right\},\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\end{bmatrix}}$$

$$\to\bar{1}\quad as\quad n\to\infty.$$

Since $\bar{p}+\bar{2}\bar{q}\prec\bar{1}$, therefore $\bar{1}-\bar{q}-\bar{p}\succ\bar{0}$ which indicates that $\tilde{\rho}*\left\{\tilde{T}_{\alpha*}^{i*},(h,\psi)\left(\tilde{T}_{\alpha*}^{i*}\right)\right\}=\bar{1}$ and hence $\tilde{T}_{\alpha*}^{i*}$ is a "soft fixed point" of (h,ψ).

Thus, in all the cases, we get $\tilde{T}_{\alpha*}^{i*}$ as a "soft fixed point" of (h,ψ).Now, if $\tilde{T}_{\alpha'}^{i'}$ be another "soft fixed point" of (h,ψ) . Then,

$$\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)=\tilde{\rho}*\left\{\left(h,\psi\right)\left(\tilde{T}_{\alpha*}^{i*}\right),\left(h,\psi\right)\left(\tilde{T}_{\alpha'}^{i'}\right)\right\}$$

$$\left[\max\left[\tilde{\rho}*\left\{\tilde{T}_{\alpha*}^{i*},\left(h,\psi\right)\left(\tilde{T}_{\alpha*}^{i*}\right)\right\},\tilde{\rho}\left\{\tilde{T}_{\alpha'}^{i'},\left(h,\psi\right)\left(\tilde{T}_{\alpha'}^{i'}\right)\right\},\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right]\right]^{\overline{p}}$$

$$\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\leq\frac{\left[\tilde{\rho}*\left\{\tilde{T}_{\alpha*}^{i*},\left(h,\psi\right)\left(\tilde{T}_{\alpha'}^{i'}\right)\right\}\tilde{\rho}*\left\{\tilde{T}_{\alpha'}^{i'},\left(h,\psi\right)\left(\tilde{T}_{\alpha*}^{i*}\right)\right\}\right]^{\overline{q}}}{\xi\begin{bmatrix}\tilde{\rho}*\left\{\tilde{T}_{\alpha*}^{i*},\left(h,\psi\right)\left(\tilde{T}_{\alpha*}^{i*}\right)\right\},\tilde{\rho}\left\{\tilde{T}_{\alpha'}^{i'},\left(h,\psi\right)\left(\tilde{T}_{\alpha'}^{i'}\right)\right\},\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right),\\\tilde{\rho}*\left\{\tilde{T}_{\alpha*}^{i*},\left(h,\psi\right)\left(\tilde{T}_{\alpha'}^{i'}\right)\right\},\tilde{\rho}*\left\{\tilde{T}_{\alpha'}^{i'},\left(h,\psi\right)\left(\tilde{T}_{\alpha*}^{i*}\right)\right\}\end{bmatrix}}$$

$$\left[\max\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha*}^{i*}\right),\tilde{\rho}\left(\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha'}^{i'}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right\}\right]^{\overline{p}}$$

$$\leq\frac{\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\tilde{\rho}*\left(\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha*}^{i*}\right)\right\}^{\overline{q}}}{\xi\begin{Bmatrix}\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha*}^{i*}\right),\tilde{\rho}\left(\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha'}^{i'}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right),\\\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\tilde{\rho}*\left(\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha*}^{i*}\right)\end{Bmatrix}}$$

$$\leq\frac{\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right\}^{\overline{p}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\tilde{\rho}*\left(\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha*}^{i*}\right)\right\}^{\overline{q}}}{\xi\begin{Bmatrix}\overline{1},\overline{1}\,\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right),\\\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\tilde{\rho}*\left(\tilde{T}_{\alpha'}^{i'},\tilde{T}_{\alpha*}^{i*}\right)\end{Bmatrix}}$$

$$\leq\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right\}^{\overline{p}+2\overline{q}}$$

$$\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},\tilde{T}_{\alpha'}^{i'}\right)\right\}^{\overline{1}-\overline{p}-2\overline{q}}\leq\overline{1}$$

Since $\overline{p}+\overline{2}\overline{q}$ %$\overline{1}$, therefore $\overline{1}-\overline{p}-\overline{2}\overline{q}$ %$\overline{0}$ Thus, $\tilde{\rho}*\left(\tilde{T}_{\alpha*}^{i*},T_{\alpha'}^{i'}\right)=\overline{1}\Rightarrow\tilde{T}_{\alpha*}^{*}=\tilde{T}_{\alpha'}^{'}$.

Hence, there is one and only one soft fixed point of $\left(h,\psi\right)$.

Theorem 3.4. Let $(\tilde{I},\tilde{\rho}*,\Omega)$ be a complete soft multiplicative metric space and $\left(h,\psi\right):(\tilde{I},\tilde{\rho}*,\Omega)\to(\tilde{I},\tilde{\rho}*,\Omega)$ be a mapping, which satisfies the soft multiplicative generalized weak contractive mapping

$$\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right),(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\} \leq \frac{\left[\tilde{\rho}*\left\{\tilde{T}_{\alpha}^{i},(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right)\right\}\tilde{\rho}*\left\{\tilde{T}_{\beta}^{j},(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\}\right]^{\frac{\overline{p}}{2}}\left[\tilde{\rho}*\left\{\tilde{T}_{\alpha}^{i},(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\}\tilde{\rho}*\left\{\tilde{T}_{\beta}^{j},(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right)\right\}\right]^{\frac{\overline{q}}{2}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j}\right)\right\}^{\overline{r}}}{\xi\begin{bmatrix}\tilde{\rho}*\left\{\tilde{T}_{\alpha}^{i},(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right)\right\},\tilde{\rho}*\left\{\tilde{T}_{\beta}^{j},(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\}, \\ \tilde{\rho}*\left\{\tilde{T}_{\alpha}^{i},(h,\psi)\left(\tilde{T}_{\beta}^{j}\right)\right\},\tilde{\rho}*\left\{\tilde{T}_{\beta}^{j},(h,\psi)\left(\tilde{T}_{\alpha}^{i}\right)\right\},\tilde{\rho}*\left(\tilde{T}_{\alpha}^{i},\tilde{T}_{\beta}^{j}\right)\end{bmatrix}}$$

$$\forall\ \tilde{T}_{\alpha}^{i},\ \tilde{T}_{\beta}^{j}\ \tilde{\in}\ SP\left(\tilde{I}\right),$$

where $\overline{p}$, $\overline{q}$ and $\overline{r}$ are non-negative soft real numbers such that $\overline{p}+\overline{q}+\overline{r}\ \tilde{<}\ \overline{1}$ and $\xi:\left[\overline{1},\infty\right)^{2}\rightarrow\left[\overline{1},\infty\right)$ is a continuous function such that $\xi\left(\tilde{T}_{\alpha}^{i},\ \tilde{T}_{\beta}^{j},\tilde{T}_{\gamma}^{k},\tilde{T}_{\delta}^{l}\right)=\overline{1}$ iff one of $\tilde{T}_{\alpha}^{i},\ \tilde{T}_{\beta}^{j},\tilde{T}_{\gamma}^{k},\tilde{T}_{\delta}^{l}=\overline{1}$. Then, there exists a unique fixed point of (h,ψ).

Proof. Let $\tilde{T}_{\alpha_0}^{i_0}$ be any soft point in $SP\left(\tilde{I}\right)$. Fix

$$\begin{aligned}\tilde{T}_{\alpha_1}^{i_1}&=(h,\psi)\tilde{T}_{\alpha_0}^{i_0}\\ \tilde{T}_{\alpha_2}^{i_2}&=(h,\psi)\tilde{T}_{\alpha_1}^{i_1}\\ &\vdots\\ \tilde{T}_{\alpha_{n+1}}^{i_{n+1}}&=(h,\psi)\tilde{T}_{\alpha_n}^{i_n}\end{aligned}$$

Now,

$$\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)= \tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right),(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right)\right\}$$

$$\tilde{\precsim}\frac{\left[\tilde{\rho}*\left\{\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right)\right\}\tilde{\rho}*\left\{\tilde{T}_{\alpha_n}^{i_n},(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right)\right\}\right]^{\frac{\bar{p}}{2}}\left[\tilde{\rho}*\left\{\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right)\right\}\tilde{\rho}*\left\{\tilde{T}_{\alpha_n}^{i_n},(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right)\right\}\right]^{\frac{\bar{q}}{2}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\bar{r}}}{\xi\begin{bmatrix}\tilde{\rho}*\left\{\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right)\right\},\tilde{\rho}*\left\{\tilde{T}_{\alpha_n}^{i_n},(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right)\right\},\\ \tilde{\rho}*\left\{\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},(h,\psi)\left(\tilde{T}_{\alpha_n}^{i_n}\right)\right\},\tilde{\rho}*\left\{\tilde{T}_{\alpha_n}^{i_n},(h,\psi)\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right)\right\},\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\end{bmatrix}}$$

$$\tilde{\precsim}\frac{\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\right\}^{\frac{\bar{p}}{2}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\frac{\bar{q}}{2}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\bar{r}}}{\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right),\\ \tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\end{bmatrix}}$$

$$\tilde{\precsim}\frac{\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\right\}^{\frac{\bar{p}}{2}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\right\}^{\frac{\bar{q}}{2}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\bar{r}}}{\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right),\\ \tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\end{bmatrix}}$$

$$\tilde{\precsim}\frac{\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\frac{\bar{p}}{2}+\frac{\bar{q}}{2}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\right\}^{\frac{\bar{p}}{2}+\frac{\bar{q}}{2}}\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\bar{r}}}{\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right),\\ \tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\end{bmatrix}}$$

$$\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\right\}^{\bar{1}-\frac{\bar{p}}{2}-\frac{\bar{q}}{2}}\leq\frac{\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\frac{\bar{p}}{2}+\frac{\bar{q}}{2}+\bar{r}}}{\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right),\\ \tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right),\bar{1},\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\end{bmatrix}}$$

Since x satisfies the given condition, thus

$$\xi\begin{bmatrix}\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right),\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right),\\ \tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right),\bar{1},\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\end{bmatrix}=\bar{1}.$$

Thus, we have

$$\left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\right\}^{1-\frac{\bar{p}}{2}-\frac{\bar{q}}{2}} \le \left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\frac{\bar{p}}{2}+\frac{\bar{q}}{2}+\bar{r}}$$

$$\Rightarrow \quad \left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\right\} \le \left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\frac{\frac{\bar{p}}{2}+\frac{\bar{q}}{2}+\bar{r}}{1-\frac{\bar{p}}{2}-\frac{\bar{q}}{2}}}$$

$$\Rightarrow \quad \left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}}\right)\right\} \le \left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-1}}^{i_{n-1}},\tilde{T}_{\alpha_n}^{i_n}\right)\right\}^{\eta}, \qquad \eta = \frac{\frac{\bar{p}}{2}+\frac{\bar{q}}{2}+\bar{r}}{1-\frac{\bar{p}}{2}-\frac{\bar{q}}{2}}$$

$$\le \left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_{n-2}}^{i_{n-2}},\tilde{T}_{\alpha_{n-1}}^{i_{n-1}}\right)\right\}^{\eta^2}$$

$$\vdots$$

$$\le \left\{\tilde{\rho}*\left(\tilde{T}_{\alpha_0}^{i_0},\tilde{T}_{\alpha_1}^{i_1}\right)\right\}^{\eta^n}.$$

For any $m>n,$ where $m,n\in$ □

$$\tilde{\rho}*(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_m}^{i_m}) \tilde{\le} \{\tilde{\rho}*(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}})\tilde{\rho}*(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha_m}^{i_m})\}$$

$$\tilde{\le} \left\{\tilde{\rho}*(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}})\right\}\left\{\tilde{\rho}*(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha_{n+2}}^{i_{n+2}})\right\}\left\{\tilde{\rho}*(\tilde{T}_{\alpha_{n+2}}^{i_{n+2}},\tilde{T}_{\alpha_m}^{i_m})\right\}$$

$$\tilde{\le} \left\{\tilde{\rho}*(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_{n+1}}^{i_{n+1}})\right\}\left\{\tilde{\rho}*(\tilde{T}_{\alpha_{n+1}}^{i_{n+1}},\tilde{T}_{\alpha_{n+2}}^{i_{n+2}})\right\} \ldots \{\tilde{\rho}*(\tilde{T}_{\alpha_{m-1}}^{i_{m-1}},\tilde{T}_{\alpha_m}^{i_m})\}$$

$$\tilde{\le} \{\tilde{\rho}*(\tilde{T}_{\alpha_0}^{i_0},\tilde{T}_{\alpha_1}^{i_1})\}^{\bar{\eta}^{\,n}} \{\tilde{\rho}*(\tilde{T}_{\alpha_0}^{i_0},\tilde{T}_{\alpha_1}^{i_1})\}^{\bar{\eta}^{\,n+1}} \ldots \{\tilde{\rho}*(\tilde{T}_{\alpha_0}^{i_0},\tilde{T}_{\alpha_1}^{i_1})\}^{\bar{\eta}^{\,m-1}}$$

$$\tilde{\le} \{\tilde{\rho}*(\tilde{T}_{\alpha_0}^{i_0},\tilde{T}_{\alpha_1}^{i_1})\}^{\bar{\eta}^{\,n}\left(\bar{1}+\bar{\eta}+{}^{2}\bar{\eta}^{2}+\cdots+\bar{\eta}^{\,m-n-1}\right)}$$

$$\tilde{\le} \{\tilde{\rho}*(\tilde{T}_{\alpha_0}^{i_0},\tilde{T}_{\alpha_1}^{i_1})\}^{\frac{\bar{\eta}^{\,n}}{\bar{1}-\bar{\eta}}}.$$

Since $\bar{p}+\bar{q}+\bar{r}$ %$\bar{1}$, therefore ℏ %$\bar{1}$ which indicates that $\tilde{\rho}*(\tilde{T}_{\alpha_n}^{i_n},\tilde{T}_{\alpha_m}^{i_m}) \to \bar{1}$ as $m,n\to\infty.$ So, the soft sequence $\{\tilde{T}_{\alpha_n}^{i_n}\}$ is soft multiplicative Cauchy sequence in $\tilde{I}$. Being the completeness of $(\tilde{I},\tilde{\rho}*,\Omega)$, there exists a soft point $\tilde{T}_{\alpha^*}^{i^*} \tilde{\in} \tilde{I}$ such that $\tilde{T}_{\alpha_n}^{i_n} \to \tilde{T}_{\alpha^*}^{i^*}$ as $n\to\infty.$

Also,

$$\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}\tilde{\leq}\left[\tilde{\rho}*\left\{\left(\tilde{T}^{i*}_{\alpha*}\right),(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\}\left\{\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right),(h,\psi)\tilde{T}^{i*}_{\alpha*}\right\}\right\}\right]$$

$$\left[\tilde{\rho}*\left\{T^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\}\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}\right]^{\frac{\bar{p}}{2}}$$

$$\leq\ \tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\frac{\left[\tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\}\right]^{\frac{\bar{q}}{2}}\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right\}^{\bar{r}}}{\xi\left[\begin{array}{l}\tilde{\rho}*\left\{T^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\},\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\\ \tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i_n}_{\alpha_n}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\end{array}\right]}$$

$$\left[\tilde{\rho}*\left(T^{i_n}_{\alpha_n},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}\right]^{\frac{\bar{p}}{2}}$$

$$\left[\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right]^{\frac{\bar{q}}{2}}$$

$$\leq\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\frac{\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right\}^{\bar{r}}}{\xi\left[\begin{array}{l}\tilde{\rho}*\left(T^{i_n}_{\alpha_n},T^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\\ \tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},T^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\end{array}\right]}$$

$$\left[\tilde{\rho}*\left(T^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}\right]^{\frac{\bar{p}}{2}}$$

$$\left[\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right]^{\frac{\bar{q}}{2}}$$

$$\leq\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\frac{\left\{\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right\}^{\bar{r}}}{\xi\left[\begin{array}{l}\tilde{\rho}*\left(T^{i_n}_{\alpha_n},T^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\\ \tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},T^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\end{array}\right]}$$

$$\left[\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}\right]^{\bar{1}-\frac{\bar{p}}{2}-\frac{\bar{q}}{2}}\leq\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\frac{\left\{\tilde{\rho}*\left(T^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\right\}^{\frac{\bar{p}}{2}+\frac{\bar{q}}{2}+\bar{r}}\left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i_{n+1}}_{\alpha_{n+1}}\right)\right\}^{\frac{\bar{p}}{2}+\frac{\bar{q}}{2}}}{\xi\left[\begin{array}{l}\tilde{\rho}*\left(T^{i_n}_{\alpha_n},T^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\\ \tilde{\rho}*\left\{\tilde{T}^{i_n}_{\alpha_n},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},T^{i_{n+1}}_{\alpha_{n+1}}\right),\tilde{\rho}*\left(\tilde{T}^{i_n}_{\alpha_n},\tilde{T}^{i*}_{\alpha*}\right)\end{array}\right]}$$

Since $\bar{p}+\bar{q}+\bar{r}\ \%\ \bar{1}$ and $\bar{r}>\bar{0}$, therefore $\bar{1}-\frac{\bar{p}}{2}-\frac{\bar{q}}{2}\ \%\ \bar{0}$ and hence

$\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)\right\}=\bar{1}\quad as\quad n\to\infty$. This shows that $(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right)=\tilde{T}^{i*}_{\alpha*}$ and hence $\tilde{T}^{i*}_{\alpha*}$ is a "soft fixed point" of (h,ψ).

Now, if $\tilde{T}^{i'}_{\alpha'}$ be another "soft fixed point" of (h,ψ). Then,

$$\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i'}_{\alpha'}\right)=\tilde{\rho}*\left\{(h,\psi)\left(\tilde{T}^{i*}_{\alpha*}\right),(h,\psi)\left(\tilde{T}^{i'}_{\alpha'}\right)\right\}$$

$$\leq\frac{\left[\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\tilde{T}^{i*}_{\alpha*}\right\}\tilde{\rho}*\left\{\tilde{T}^{i'}_{\alpha'},(h,\psi)\tilde{T}^{i'}_{\alpha'}\right\}\right]^{\frac{\bar{p}}{2}}\left[\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\tilde{T}^{i'}_{\alpha'}\right\}\left\{\tilde{\rho}*\left(\tilde{T}^{i'}_{\alpha'},(h,\psi)\tilde{T}^{i*}_{\alpha*}\right)\right\}\right]^{\frac{\bar{q}}{2}}\left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i'}_{\alpha'}\right)\right\}^{\bar{r}}}{\xi\left[\begin{array}{l}\tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\tilde{T}^{i*}_{\alpha*}\right\},\tilde{\rho}*\left\{\tilde{T}^{i'}_{\alpha'},(h,\psi)\tilde{T}^{i'}_{\alpha'}\right\},\\ \tilde{\rho}*\left\{\tilde{T}^{i*}_{\alpha*},(h,\psi)\tilde{T}^{i'}_{\alpha'}\right\},\left\{\tilde{\rho}*\left(\tilde{T}^{i'}_{\alpha'},(h,\psi)\tilde{T}^{i*}_{\alpha*}\right)\right\},\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i'}_{\alpha'}\right)\end{array}\right]}$$

$$=\frac{\left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i*}_{\alpha*}\right)\tilde{\rho}*\left(\tilde{T}^{i'}_{\alpha'},\tilde{T}^{i'}_{\alpha'}\right)\right\}^{\bar{p}}\left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i'}_{\alpha'}\right)\tilde{\rho}*\left(\tilde{T}^{i'}_{\alpha'},\tilde{T}^{i*}_{\alpha*}\right)\right\}^{\bar{q}}\left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i'}_{\alpha'}\right)\right\}^{\bar{r}}}{\xi\left[\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i*}_{\alpha*}\right),\tilde{\rho}*\left(\tilde{T}^{i'}_{\alpha'},\tilde{T}^{i'}_{\alpha'}\right),\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i'}_{\alpha'}\right)\tilde{\rho}*\left(\tilde{T}^{i'}_{\alpha'},\tilde{T}^{i*}_{\alpha*}\right),\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i'}_{\alpha'}\right)\right]}$$

$$\left\{\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i'}_{\alpha'}\right)\right\}^{\bar{1}-\bar{q}-\bar{r}}\,\tilde{\leq}\,\bar{1}$$

Since $\bar{p}+\bar{q}+\bar{r}$ %$\bar{1}$ and $\bar{p}>\bar{0}$, therefore $\bar{1}-\bar{q}-\bar{r}$ %$\bar{0}$ and hence

$$\tilde{\rho}*\left(\tilde{T}^{i*}_{\alpha*},\tilde{T}^{i'}_{\alpha'}\right)=\bar{1}\Rightarrow\tilde{T}^{i*}_{\alpha*}=\tilde{T}^{i'}_{\alpha'}.$$

Hence, there is one and only one soft fixed point of (h,ψ).

4. Conclusion

"Soft set theory" is a wide mathematical aid for handling vagueness and uncertainty. In this paper, some basic concepts of soft set and soft metric spaces are considered. We proved fixed point theorem for mappings satisfying generalized weak contractive conditions in soft multiplicative metric space.

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