Review Article

About animals, numbers and teaching: linking Mathematics and biology in evolution

Abstract: The implications of Charles Darwin's studies on evolution reach many areas of knowledge. The influence on researchers as Sewall Wright, John Haldane and Ronald Fisher among others, were very important for the selection on husbandry animals. Therefore, the aim of this text is to demonstrate the importance of the relationship between Biology and Mathematics in the construction of new pathways that afford a theoretical framework and the resulting practical applications of these subjects on education.

Keywords: Evolution; Biological Education; Mathematics Education; History of Science

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Introduction

The impact of the Evolution of Charles Darwin (1809 – 1822) stretches across several areas of knowledge, either in biological sciences or human sciences.

In enlarging our knowledge about the species, the association of Darwin's studies to the contribution of others authors has enhanced our understanding of life sciences. This association afforded an expressive progress in the comprehension of the interfaces Biology keeps with other sciences and constitutes fertile soil for the development of techniques that enable the practical applications of such knowledge. These applications assist in the solution of problems faced in practice, in several areas, such as observed in animal breeding, whose techniques make it possible to select animals exhibiting the characteristics desired by breeders and to direct mating systems like inbreeding and crossbreeding.

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But, apart from this, following the pathway opened by Charles Darwin — and going along Sewall Wright, John Haldane and Ronald Fisher, among others, who worked based on the landmarks of variation and natural selection — the quantification of these processes and the applications derived therefrom have exerted decisive influence on the teaching of Biology, due to contributions in several related areas.

Based on these premises, the aim of this text is to demonstrate the importance of the relationship between Biology and Mathematics in the context of the History of Genetics as a discipline and the Charles Darwin's contributions for biological education. This study also addresses the relevance of this process in the construction of new pathways that afforded a theoretical framework and the resulting practical applications of the quantitative methods and the repercussions for Biology teaching. These applications and methods are the outcomes of the intense work of researchers who started with the hypotheses of the Evolution Theory, to arrive at Population Genetics and Quantitative Genetics, in a panorama of mutual contribution between Biology and Mathematics.

From Darwin to Wright, Fisher and Haldane

> "One of the major incentives in the pioneer studies of heredity and the variation which led to modern Genetics was the hope of obtaining a deep insight into the evolutionary process (Wright, 1931, p. 97)."

Starting with this sentence, Wright wrote a careful and very important text with the suggestive title *Evolution in Mendelian Populations*, combining the knowledge developed by two of the most influent researchers in Biology, Darwin and Mendel.

According to Wright (1931, p. 98) "Darwin was the first to present evolution as primarily a statistical process in which random hereditary variation merely furnishes the raw material", and this affirmation had important influence on Sewall's studies.

In the introduction to *Variation of Plants and Animals Under Domestication* published in 1868, Darwin cited that Felix A. Pouchet (1800-1872), in *Plurarity of races*, did not believe that knowledge about domesticated species could help to understand natural modification of species, but Darwin said "I cannot perceive the force of his arguments, or, to speak more accurately, of his assertions to this effect (Darwin, 1868, p. 2)".

Supporting his arguments, Darwin wrote:

> "Man, therefore, may be said to have been trying an experiment on a gigantic scale; and it is an experiment which nature during the long lapse of time has incessantly tried. Hence it follows that the principles of

> domestication are important for us (Darwin, 1868, p. 3)."

In this book, Darwin wrote also about very important subjects influencing contemporary Genetics and animal breeding, as we can see:

> "During this investigation we shall see that the principle of Selection is highly important. Although man does not cause variability and cannot even prevent it, he can select, preserve, and accumulate the variations given to him by the hand of nature almost in any way which he chooses; and thus he can certainly produce a great result. Selection may be followed either methodically and intentionally, or unconsciously and unintentionally. Man may select and preserve each successive variation, with the distinct intention of improving and altering a breed, in accordance with a preconceived idea; and by thus adding up variations,

often so slight as to be imperceptible by an uneducated eye… (Darwin, 1868, p. 3)”.

Darwin’s gradualism and its implications concerning the effects of small variations for selection represents a central point to characters which participate in many genes, as seen in quantitative traits, important aspects to biometrical studies and livestock production.

Darwin, in his autobiography (Darwin, 1892), wrote that the time spent at Cambridge during three years was wasted. He attempted to study Mathematics, including working with a private tutor, but his progress was very slow, “the work was repugnant”, because he could not see any meaning in the early steps in algebra. Francis Darwin points out that Mr. Herbert said Charles Darwin “…had, I imagine, no natural turn for Mathematics…”.

Despite these considerations, we could think that Darwin’s studies — although conducted without mathematical demonstrations — and his thinking about evolution had, to some extent, an intrinsically mathematical view of nature.

Francis Galton (1822-1911) was responsible for fundamental innovations in the analysis of individual differences, utilizing his biometrical studies related to heredity (Sober, 1997) and his aim was to verify the influence of heredity on the variation in populations. Francis Galton and Karl Pearson (1857-1936) developed an approach, connecting Genetics and Statistics to explain biological variation in the characters applied to biometrical variables, and developing correlation and regression coefficients as important methods to analyze data in many sciences.

With rediscovery of Mendel's researches in 1900 by Karl Correns (1864–1933), Hugo de Vries (1848–1935), and Erich Tschermak von Seysenegg (1871–1962) the science faced temporary controversies that place particulate and continuous theories of heredity against each other.

In the midst of debate between Mendelians and biometricians, the applications of Galton's work will contribute for Genetics and Biology, but also for Human Sciences, Statistics and animal sciences, mainly animal breeding of livestock animals.

Independently, Wilhelm Weinberg (1862-1937) and G. Hardy (1877-1947) developed the genetic equilibrium, making it possible to understand the maintenance of genes and genotypes frequencies in populations, and paving the way for subsequent studies on the effects of selection, mutation and migration.

It advanced the application of Mendel's laws of heredity from individuals to populations, and by applying Mendelian Genetics to Darwin's theory of evolution, it improved the understanding of the role of mutations and how natural selection works in hereditary adaptations. The Hardy-Weinberg Equilibrium nowadays enables to determine whether evolution is occurring in populations or not.

Sewall Wright (1889-1988), with Ronald Fisher (1890-1926) and John Haldane (1892-1962) were the founders of Population Genetics, and gave a quantitative basis to artificial selection, although Wright said that his point of view about evolution was different in the 1920's, comparing with that of Fisher and of Haldane (Wright, 1978).

The presuppositions that Darwin's evolution is mainly due to mass selection of quantitative variability was given a mathematical explanation with the work of Ronald Fisher and his theorem of natural selection: "the rate of increase in fitness of an organism at any time is equal to its genetic variance in fitness at that time" (Wright, 1978, p. 1194). Therefore, evolution takes place by changes in genetic variation in function of alterations in environmental conditions, favoring some alleles.

Another important subject was the definition of genetic variance having additive and non-additive components and its implications afforded an easy way to understand the role additive genes play in polygenic traits, and considering that the majority of economic aspects in livestock production are determined by actuation of many genes, the sum of the effects of each gene is the breeding value of the animal (Wright, 1978).

In talking about Wright, it is interesting to know that he learned Mathematics with his father and studying by himself, and he never took any advanced course (Crow, 1988). Maybe these facts have favoured him in developing applications for other subjects like Genetics and livestock animals. His thesis about coat color in guinea pigs anticipated the work of Beadle and Tatum on the foundation of biochemical Genetics coat colors in guinea pigs, and his work about inbreeding has afforded the important union of Biology and Mathematics, the central point in animal breeding.

James Crow, in a sensitive and short biography published in journal of Genetics, wrote that Wright, who died in 1988, was the last survivor belonging to a group that "established Genetics as a solid science" jointly with Sturtevant, Muller, Bridges, Stadler, Fisher and Haldane in the beginning of twentieth century (Crow, 1988, p. 1).

Wright's contributions for animal breeding are also in his works about systems of mating and provide the theoretical basis for plant and animal breeding, and the student of evolution theory knows that stochastic changes and the subdivision of populations have evolutionary consequences because of him. For Wright, the balance of many factors acting simultaneously changes gene frequency (Wright, 1978), and these factors included mutation, selection, inbreeding and crossbreeding. The equilibrium among these factors could be the key to the maintenance of life in all levels of organization based on the statistical consequence of

Mendelian heredity (Wright, 1932).

It is very interesting that Wright studied coat color of guinea pigs, and this character is a good example of discrete occurrence of some kinds of coat color type, mixed with some continuous variation in the quantities of pigmentation. So, in a narrow sense, his study was a kind of mosaic of Mendelian and Darwinian approaches; in other words, he could help science to build a bridge between these important contemporaneous researchers.

Wright said that the interactions occurring between the genes related to coat color in guinea pigs had complex patterns, and indicated the multifactor aspects enrolled in those traits (Wright, 1978). But his studies of inbreeding had a great value to animal breeding, helping breeders in choosing sires and dams to mating, trying to decrease the deleterious effects related to inbreeding.

Concerning evolution, Wright said that he learned the efficacy of mass selection in changing a character due to quantitative variability. So, at first, he accepted Darwin's contention related to evolution as being dependent on quantitative evolution rather than caused by favorable major mutations, and he assumed that species have differences in few loci, in a situation maintained "in a continually shifting state of near-equilibrium by the opposing pressures of mutation, diffusion and weak selection" (Wright, 1978, p. 1197).

Wright's work in the Animal Husbandry Division plays a crucial role in animal science, mainly regarding livestock production, because it represents a powerful link between theoretical Genetics and animal breeding, as well as its applications in society, concerning food production.

The discordance between Wright and Fisher was observed in the role played by drift and selection, because Wright believed that genetic drift was very important in the changes in gene frequency, and Fisher believed in the power of selection.

Wright wrote that his paper on evolution tried to introduce parameters for the factors related to gene frequency; besides, he points out that Haldane derived expressions for change of gene frequency under selection, obtaining curves that explained what could happen after fixation of a favorable allele under various conditions, while Fisher, as explained by Wright, studied the progress according to the decreasing additive genetic variance as long as conditions remain unchanged. After describing the work of Fisher and Haldane, Wright said that in his work emphasis was placed on the description of the momentary states of balance, and stressed the great importance of subdivision into partially isolated populations. Thus, the shifting balance implies a state of balance lightly held at each moment, with a continual readiness to shift to another state of balance. The analysis of cattle herds was a revelation to Wright, in that even though there were many herds at any given time, only a few were distinctly superior, and these were the principal sources of sires for the breed (Wright, 1978).

As a boy, Ronald Fisher (1890-1962) was undecided between Biology and Mathematics, and the combination of these disciplines was the central point in his work. In the preface of his book *The Genetical Theory of Natural Selection*, Fisher wrote that mathematical approach and imagination were complementary to a biologist's work.

This Fisher's book published in 1930 is a careful demonstration of his approach about blending theory of inheritance:

> "I hope to make clear the logical consequences of the blending theory and to show their influence not only on the development of Darwin's view but on the change of attitudes toward these, and other

> suppositions, necessitated by the acceptance of the opposite theory of particulate inheritance theory" (Fisher, 1930, p. 1).

Fisher demonstrated a system which considers the effects of many genes, each one participating in a part of variation, the polygenic inheritance, which makes it possible to conciliate biometrical characters with Mendelian principles. In 1918 Fisher produced the paper *The Correlation Between Relatives on the Supposition of Mendelian Inheritance* which showed how the continuous variation measured by biometricians could be the result of the action of many discrete genes, and in 1930, with his book *The Genetical Theory of Natural Selection* Fisher was able to show how Mendelian Genetics was consistent with the main elements of Neo-Darwinism. According to Mayr (1988), Fisher promoted the extreme phenotypic gradualism, because "all evolutionary change was due to mutations with very small phenotypic effects".

J. B. S. Haldane studied many different subjects, mainly the relationship of Mendelian genetics and evolution. The Darwinian variation, the influences of mutations, the relationship between enzymes and genes were important topics in his work, but the mathematical analysis applied to natural selection showed that this phenomenon was the most efficacious source for change in the genetic composition of a population, because mutation without support of selection was not strong enough a factor to change gene frequency (Haldane, 1932). Alongside Wright and Fisher, he contributed for Population Genetics and for a basis for applications in herds on artificial selection.

Haldane (1932) wrote in the book *The causes of evolution* that the union of Biology by Mathematics is only beginning, but unless the history of science is an inadequate guide, it will continue, and the studies presented in this book represent the beginning of a new branch of applied Mathematics.

Haldane's contributions for animal breeding can be demonstrated also by his studies about the interaction of nature and nurture, a very important subject mainly in tropical countries that do not have native cattle, like Brazil, and are faced with the need to import animals to form their herds.

Animal breeding and Biology teaching

In a biography of Jay Lush, Chapman (1991) wrote:

> "Dr. Lush was unique in combining the work of both Fisher and Wright to solve animal breeding problems. Many of Dr. Lush's papers from 1926 to 1930 could be described as developing and using more accurate ways to measure quantitative traits." (Chapman, 1991, p. 2673)

Jay Lush (1896-1982) worked at Iowa State University and is considered the father of scientific animal breeding. He defines his studies with these words: “All of my work has hinged around finding ways to apply Genetics more efficiently in improving animals and plants” (Chapman, 1991, p. 2672).

The influence of Sewall Wright on Lush was strong, and it is shown by his travels to the University of Chicago to watch the courses of Wright and to consult with him about his work. Once Lush said to a friend that “Those were by far the most fruitful ten weeks I ever had” (Chapman, 1991, p. 2673).

According to Chapman (1991), the studies of Ronald Fisher exerted influence on Lush, because until 1930 the statistical methods utilized in animal breeding were correlation and regression, and Fisher’s lectures in Iowa State between 1931 and 1936 played an important role in increasing the utilization of Statistics in animal science. Lush was unique in his combination of Fisher's and Wright's works, aiming to solve the problems occurred in the application of these studies in animal breeding. Lush's research involved data of swine, dairy cattle, beef cattle, sheep, goats, poultry and honeybees, and in 1930 began a experiment about closed-herd selection in dairy cattle and another about swine selection and inbreeding.

The book *Animal Breeding Plans*, published in 1937, is a careful effort about the applications of theoretical population Genetics in the selection of herds. The rigorous methods utilized by the researchers occupied with evolution found a way to help the "blind selection" work done by the breeders trying to choose the sires and dams to improve their animals. The applications of scientific concepts in performance quantification of individuals and progenies, and his contribution for the estimation of breeding values separated from non-genetic factors represent a kind of "turning point"

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in the teaching of animal science.

Lush’s work also brought progress to heritability as an expression of the phenotypic variation pertaining to additive genetic variation (Bell, 1977). But Charles Henderson (1911-1989), Lush’s student, in using the Mixed Model Methodology — such as other researchers — obtained an efficacious way to estimate breeding value and EPD (Expected Progeny Differences) for important traits, supporting the fruitful animal science taught in all courses related to husbandry, and providing many kinds of examples utilized in Genetics courses.

All high school teachers have used cattle, sheep, poultry or other examples to explain the basis of inheritance and to show students how this theoretical lesson can help us to understand the role played by genes role and phenomena observed in nature.

When teaching biometrical variables and talking about the methods to estimate breeding values, we resort to this way of thinking, which is almost identical to that of the founders of Population Genetics, and we bring in our classes the demonstrations utilized by those scholars to make these subjects understandable in our courses.

Broadly speaking, the proximity between Biology and Mathematics occurred in research on Genetics brought a way to think Biology, genes and organisms in a Mathematical way, and made possible many kinds of demonstrations to teach Biology. The applications on Ecology, Zoology and Botany are just examples of our thankfulness to all the people quoted in this article, and to many others also.

When studying History and Epistemology, all of us learned that the linear thought approach, linking personal achievements as a single way of individual contributions, has not been a fruitful way to understand the History of Science. Yet, in affording an easier understanding of animal breeding as a discipline as a branch of the “generous tree” named Evolutive Biology, Darwin as a great spreader deserves the expression of our gratitude, because he is quoted by all characters mentioned in this text. To build a bridge linking him to Wright, Fisher and Haldane, and then linking these with Lush is just a way to construct a didactic timeline for animal breeding.

Walking on British grass, looking at the landscapes, and thinking all of time in variation and selection, Darwin desired to know evolution as it occurs in the natural world. His careful work helped other researchers think about domestic animals, whenever they remember Darwin’s conclusions about species living in natural environment.

Conclusion

Beginning with Darwin’s observations about animals and plants, and moving through the innovating and brilliant works of Galton and Pearson on biometry, the world of science watched the conjunction of Darwinian evolution and Mendel's experiments after studies of Fisher, Haldane e Wright and after the applications in selection of livestock animals and plants in agricultural production.

Logically, this paper is not a simple account of the linear and pacific accumulation of knowledge; rather, it is a summary of many lives dedicated to Science and Education and to the building of one of the bridges between Mathematics and Biology that helped both fields.

To *Mathematize* Biology and to *Biologize* Mathematics made it possible to understand and, of course, teach many kinds of branches pertaining to Biology. When the teachers go to the blackboard, or when they talk with their students, trying to explain how Mathematics can help us to link biological phenomena in the abstract way with which we perceive the relationships between animals, plants and their

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environments, it is like to history happen again. But this is not all, because, as a consequence of knowing the factors related to organisms, we learn about the complex adjustments, interactions in particles, enzymes and many kinds of substances. Such knowledge and wisdom come as a consequence of the studies conducted by people who were just imagining what could be happening inside cells, because in those times the double helix had not yet been discovered, and studies on the role played by many molecules were taking their first steps.

Bringing to us a successive closeness between these subjects, the science that came to assistance was Mathematics. Nevertheless, the usefulness of Mathematics is not restricted to the numbers required to explain animals and plants: it also encompasses the thought about biological aspects in mathematical terms and, in a complementary way, to think Mathematics in a biological sense, discerning the most useful ways to construct the lines between disciplines that we isolated with didactic purposes, and that are yet intrinsically united in all of things we can observe. Charles Darwin was not a mathematician, but his approach to variation is a good example of thinking mathematically, when he said that small variations could represent an advantage in natural selection and when demonstrating the many forms through which selection may happen. This lesson was learned, with some differences by many other researchers who came after, as Wright, Fisher and Haldane, and all of us are grateful for this.

Nowadays, when teachers like me go into a classroom to tell students about variation, we remember Darwin. When we talk about selection, we remember Darwin too, and when we think about a simple form to understand Biology, we likewise cannot help bringing Darwin in our reflections.

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