DEVELOPMENT OF TREE VOLUME EQUATIONS FOR *Gmelina arborea Roxb.* STAND IN SOUTHWESTERN, NIGERIA

ABSTRACT

Effective and sustainable forest management is dependent on volume and yield models. This study was conducted to develop 10 different tree volume equations for the sustainable management of the *Gmelina arborea* stand in Oluwa forest reserve. A total of 590 trees were used in this study. Observed volume of the sample trees was calculated by applying Newton's formula. To evaluate the performance of each model, five fit statistics such as r-squared (R^2), root mean squared error (RMSE), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) and relative rank sum were used. All the models in this study performed significantly well in volume estimation as the R^2 were above 75% for all the models. Four models (model 10, 8, 6 and 7) performed best for the data set based on their evaluation statistics and as such were selected for volume estimation of the *Gmelina arborea* stand in Oluwa forest reserve. The evaluation test revealed that model 10 had the least RMSE of 0.2986, hence it was ranked the best model for the study. Scatter plots showed positive correlation between DBH, total height, BA and total volume. The regression residuals were normally distributed, with constant variance and zero mean. The models were therefore efficient from the evaluation standpoint.

Keywords: Volume equations – *Gmelina arborea* - evaluation - Residuals.

INTRODUCTION

Volume equations play a crucial role in forest management. The importance of volume equations is indicated by the existence of numerous such equations and the constant search for their improvement. The objective of any volume equation is to provide accurate estimates with acceptable levels of local bias over the entire diameter range in the data. Equations that provide accurate predictions of volume without local bias over the entire range of diameter are one of the basic building blocks of a forest growth and yield simulation system (Bi & Hamilton 1998). According to Avery and Burkhart (2002), volume equations are used to estimate average content of standing trees of various sizes and species. Volumes of current growing stock and future growth potential are both vital information for forest management.

Tree volume equations are simple methods and tools that can be used to obtain individual tree volume and the volumes of entire stands. An equation is a mathematical statement setting two algebraic expressions equal to each other (Edward, 1992). Volume equation is defined as various Mathematical statements applied to the determination of quantities. Tree volume equations are mathematical expressions which relate tree volume to tree's measurable attributes such as diameter and height.

Sustainable forest management requires estimates of growing stock, flexible and accurate models that can determine the volume of standing trees, individual log and the entire stands. Such information guides forest managers in timber valuation as well as in allocation of forest areas for harvest (Akindele and LeMay, 2006). Little or no substantial work on volume equations for *Gmelina* species in Oluwa Forest Reserve has been done.

Gmelina arborea (Roxb.) belongs to the family Vabenaceae. Gmelina arborea is an unarmed, moderately sized to large deciduous tree with a straight trunk (Roshetko JM et al. 2005). It is wide spreading with numerous branches forming a large shady crown, attains a height of 30 m or more and a diameter of up to 4.5 m. Bark smooth, pale ashy-grey or grey to yellow with black patches and conspicuous corky circular lenticels (Albrecht J. 1993). Inside surface of bark rapidly turns brown on exposure and exfoliates into thick woody plates or scurfy flakes. Blaze pale orange and mottled with a darker orange colour. Leaves opposite-decussate, mostly rather soft and limp; petioles cylindrical, 5-15 cm long, puberulent or glabrous; leaf blades broadly ovate, 10-25 cm x 7-20 cm wide, apically long acuminate or caudate, entire on mature plants but strongly toothed or lobed on young plants, usually cordate or trunkate basally, with a short cuneate attenuation into the petiole, densely tomentose above when young, becoming glabrous above when mature, permanently densely fulvulous-tomentellous with stellate hairs beneath, glanduliferous just above the petiole on the basal attenuation (Nair KSS and Sumardi. 2000). The flowers are abundant, scented, reddish, brown or yellow, in terminal and axillary 1- to-3-

flowered cymes on the panicle branches, which are about 8- 40 cm long (Katende AB et al. 1995). Seeds 1-3, lenticular, exalbuminous. The genus was named after J.C. Gmelin, an 18th-century German botanist. The specific name means tree-like, from the Latin ‘arbor’ (tree). The aim of this study was to develop tree volume equations for the effective management of the *Gmelina arborea* stand in Oluwa Forest Reserve, Ondo State, Nigeria and in other regions with similar vegetation and environmental factors.

METHODOLOGY

Study area and data

The data used in this study were from the *Gmelina* plantation (about 9 ha in size) in the Oluwa Forest Reserve (FR) of Southwestern Nigeria. The Oluwa Forest Reserve is situated between 6º 55′ and 7º 20′ N and longitude 3º 45′ and 4º 32′ E, and occupies an area of 87,816 ha (Ogana & Ekpa 2020). Oluwa has an annual rainfall in the range 1700 to 2200 mm, an average annual temperature of 26°C, and a mean elevation of 123 m above sea level (Onyekwelu *et al.* 2006). Establishment of large-scale plantations in the reserve started in early 1960s. *Gmelina arborea* Roxb. and *Tectona grandis* L.f. are the dominant plantation species in Oluwa FR.

Method of Data Collection

Data were collected from twenty-five temporal sample plots of 0.04 ha in five stands (aged 19, 24, 29, 34, and 39 years) in the Gmelina stand. Diameter measurements of all trees (over bark) at breast height (1.3 m above ground, DBH) were measured with diameter tape to an accuracy of 0.1 cm. Their corresponding height (H) measurements were also taken with Spiegel relaskop. The measured variables were used to calculate the basal area (m^2) and Newton’s tree volume (m^3). Selective sampling design was adopted and a total of 590 trees with desirable characteristics were purposively selected for enumeration. Healthy trees with more typical growth form and trees that would allow a clear view along most of the stem were selected, dead trees and trees with abnormalities such as limb, bulge, leaning trees, trees that are forked were avoided. From the forest reserve, the following measurements were taken: Stump diameter; Diameter at the breast height; Middle diameter; Top diameter and Total height of all the selected trees in the reserve.

Volume Models Development

Based on reviews of past studies on tree volume models in miombo woodlands (Hofstad 2005; Henry et al. 2011), on the general mensuration literature (e.g. Philip 1983) and on extensive initial testing, ten model forms were selected and tested further. All of the models included dbh and h with the addition of BA and CL in model 10 as the independent variables. The volume models used in this study are comprised of multiple linear regression models (Table 1). The datasets were handled with R-script using package 'lm' of *R*-environment (R Core Team 2017), to generate parameter estimates.

Model Evaluation

The models were evaluated in order to test their plausibility and also recommend them for onward use. The models were assessed based on r-squared (R^2), root mean square error (RMSE), Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and relative rank sum ∑R. Model selection was based on the criterion that the higher the R^2, smaller the values of (RMSE), (AIC), (BIC), the better the model. Residuals were also graphically examined to check for any trend. A rank score between 1 to 10 was assigned to each model based on each criterion (Cao *et al.* 1980). The smaller the rank the better the performance of the model. The evaluation statistics are represented below as:

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}} \qquad [1]$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n}} \qquad [2]$$

$$AIC = n \ln\left(\frac{RSS}{n}\right) + 2p \qquad [3]$$

$$BIC = n \ln\left(\frac{\mathrm{RSS}}{\mathrm{n}}\right) + p \ln n \qquad [4]$$

Where: R^2= Coefficient of determination, SS_{res}= Residual sum of squares, SS_{tot}= Total sum of squares, RSS = residual sum of square, n = sample size, p = number of parameters; Yi is the observed value and $\hat{Y}_i$ is the theoretical value predicted by the model.

Table 1 : Application of the Volume equations

Model No	Model Form	Eq.
1	$V = (\beta_0 d^2 H) + (\beta_1 d^2) + \beta_2$	5
2	$V = \beta_0 + \beta_1 d^2 H$	6
3	$V = \beta_0 (d^2 H)^{\beta_1}$	7
4	$V = \beta_1 d^2 H$	8
5	$V = \beta_0 + \beta_1 d + \beta_2 d^2$	9
6	$V = \beta_1 d^{\beta_2} H^{\beta_3}$	10
7	$V = \beta_0 + \beta_1 H + \beta_2 d + \beta_3 d^2 + \beta_4 d^2 H + \beta_5 dH$	11
8	$V = \beta_0 + \beta_1 d^{\beta_2} H^{\beta_3}$	12
9	$V = \beta_0 + \beta_1 (d) + \beta_2 (dH) + \beta_3 (H)$	13
10	$V = \beta_0 + \beta_1 d^2 H^{\beta_2} + \beta_3 (dH) + \beta_4 (BA) + \beta_5 (CL)$	14

V= Total Volume, d= Dbh at 1.3m from ground level, CL = Crown length, H= Total height, BA = Basal area, β_0, β_1, β_2, β_3, β_4, β_5 regression coefficients

RESULTS

Ten (10) forms of multiple linear regression tree stem volume equations were developed for *Gmelina arborea* trees and stands prediction and estimation in this study. The model were assessed and evaluated to ensure their adequacy for the prediction of total volume of *Gmelina arborea* trees.

The summary statistics of the modelling data set is presented in table 2 below:

Table 2: Summary of the Modelling Datasets

Statistics	DBH (cm)	H (m)	CL (m)	BA (m^2)	VOL (m^3)
Minimum	10.10	5.70	0.80	0.008	0.035
Maximum	52.30	49.70	6.60	0.215	4.685

Mean	24.55	22.08	3.10	0.054	0.674
Standard Deviation	9.05	9.15	1.19	0.039	0.671
Skewness	0.46	0.14	0.57	1.125	1.933
Kurtosis	-0.56	-0.75	-0.24	1.038	5.050
N = 590					

The Ten volume models developed for total volume and their evaluation criteria are thus presented in Table 3. The results of the evaluation criteria shows that all the models performed well in predicting total. However, four (4) models (models 10, 8, 6 and 7) out of the Ten (10) models was rated the best for estimation of total volume of *Gmelina arborea*. The coefficient determination (R^2) for all of the models were greater than of 75%, while for the selected four best models, the R^2 were 80.4%, 80.3%, 80.2% and 80.3% respectively.

Scatter Plots

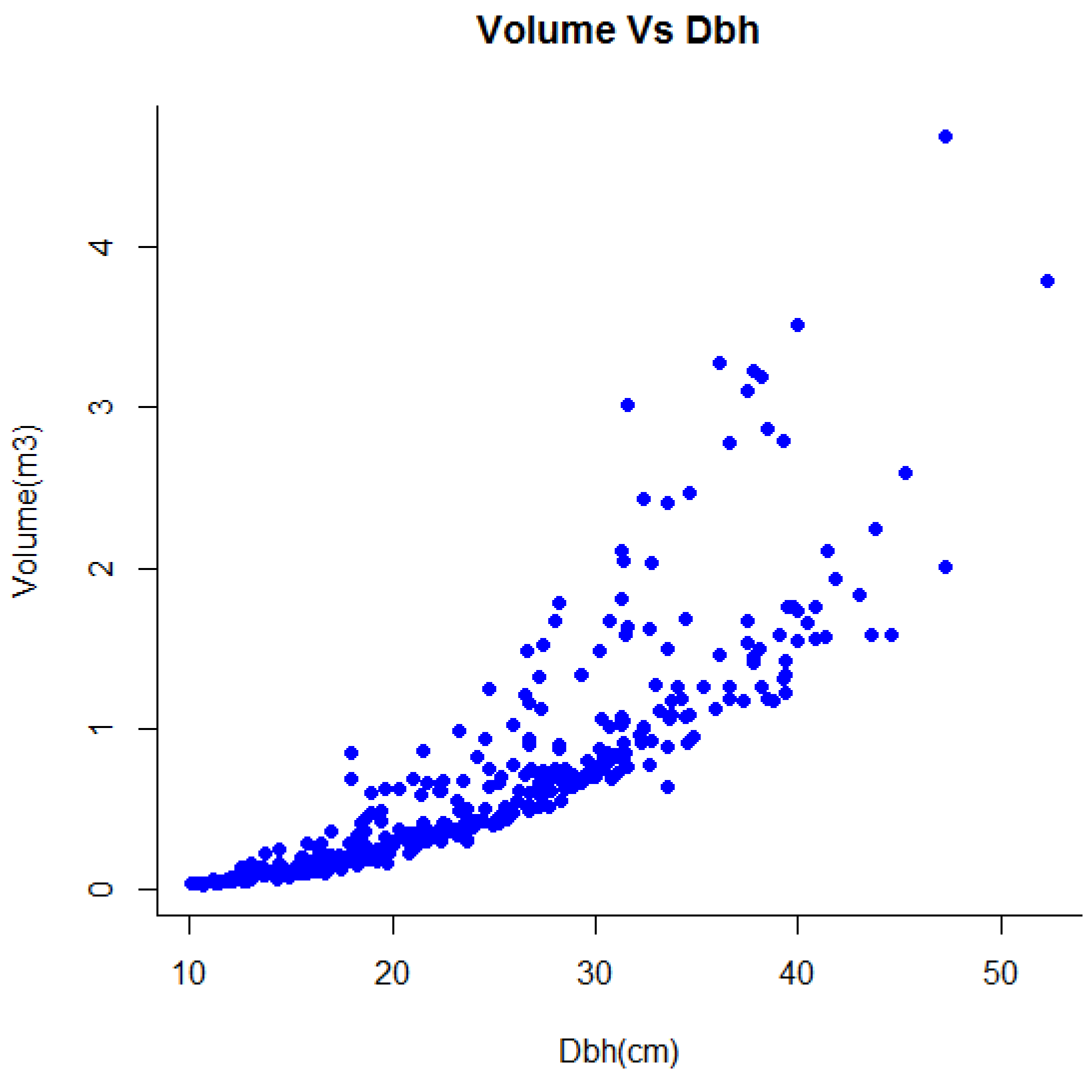

Fig 1: Scatter plot of Volume plotted against diameter at breast height (DBH) for Gmelina trees

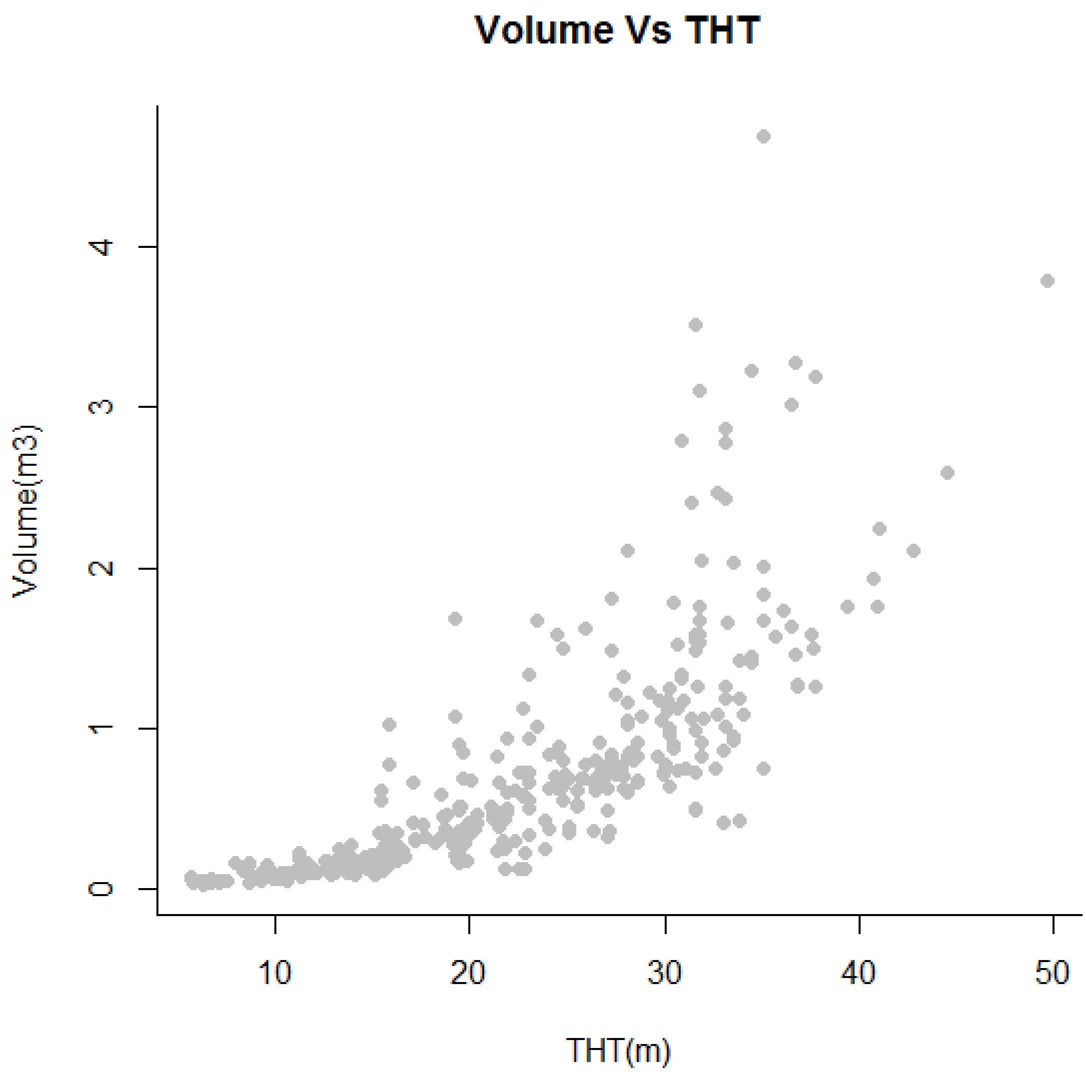

Fig 2: Scatter plot of Volume plotted against total height for Gmelina trees

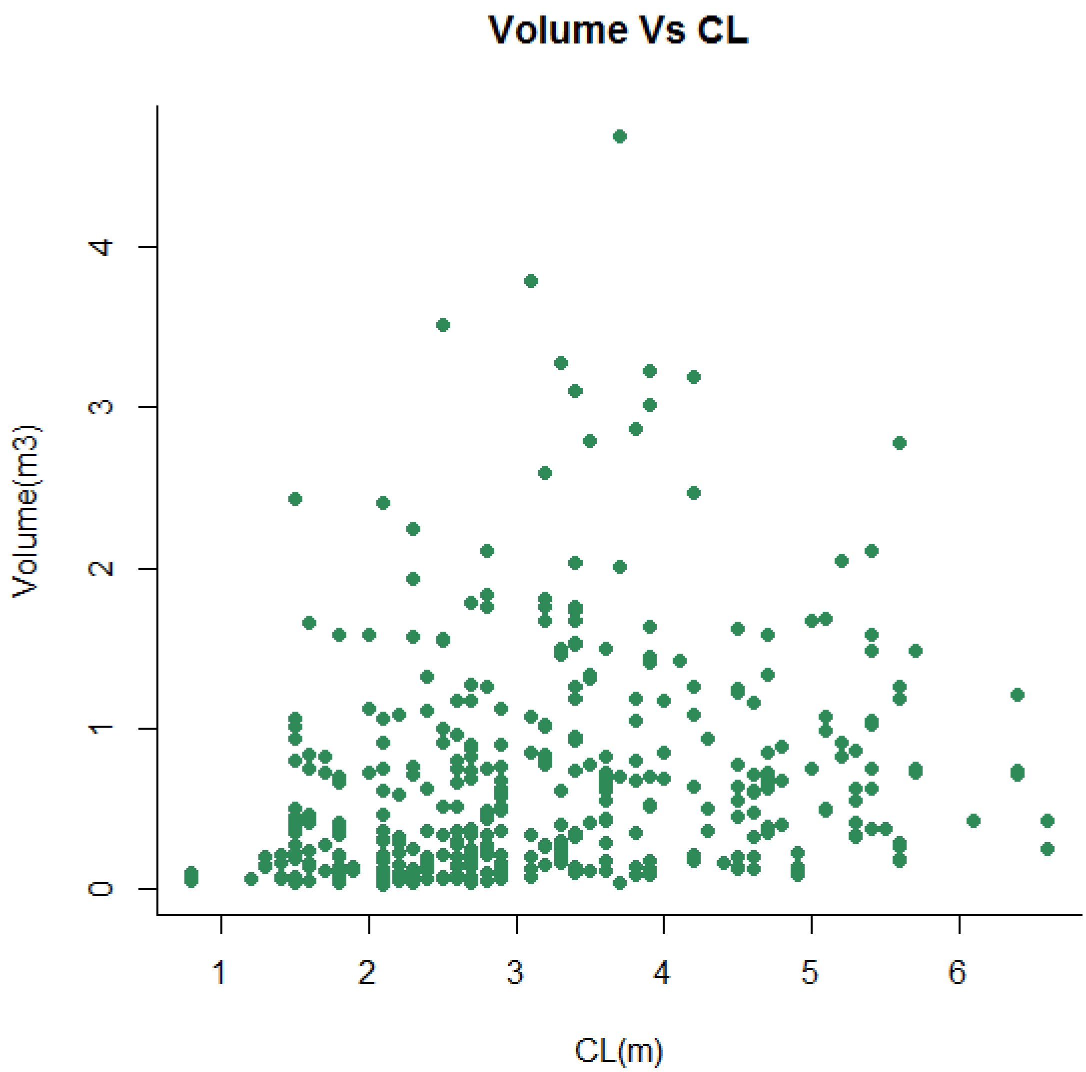

Fig 3: Scatter plot of Volume plotted against crown length for Gmelina trees

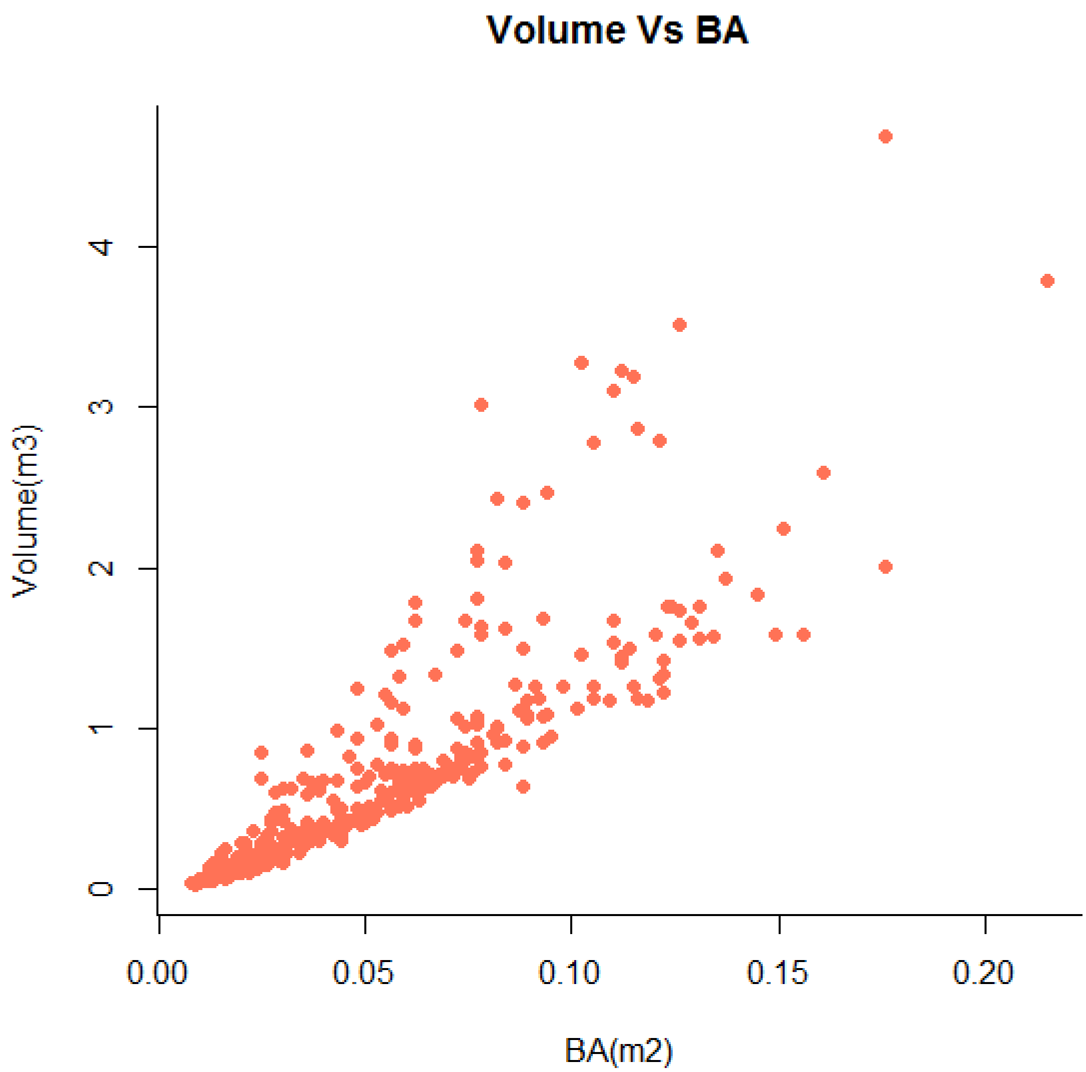

Fig 4: Scatter plot of Volume plotted against basal area for Gmelina trees

Results of fitted plots of volume against diameter at breast height (Dbh), total height (THT) and basal area (BA) reveals a positive correlation as indicated in the upward spread (Figs. 1, 2 & 4), while crown length showed a negative correlation (Fig. 3) with no coherent spread in its correlation suggesting that it is a poor variable in estimating volume for the Oluwa Forest reserve data.

Volume Models Generated and the Output Summary

The Ten volume models developed for total volume and their assessment criteria are thus presented in Table 3. The results of the model assessment criteria shows that four (4) models (models 10, 6, 8 and 7) out of the Ten (10) models was rated the best for estimation of total volume of Gmelina arborea.

Table 3: Summary of volume models for *Gmelina arborea* in Oluwa Forest Reserve, Ondo State, Nigeria

Models	Parameters						Evaluation Statistics				
	b_0	*b_1*	*b_2*	*b_3*	*b_4*	*b_5*	R^2	RMSE	AIC	BIC	∑R
1	0.00001966	0.0004395	-0.0004023				0.799	0.3013	263.9203	281.4408	6
2	0.09821	0.00003031					0.794	0.3050	276.9101	290.0505	7
3	-0.1542	0.001205					0.800	0.3133	308.8864	322.0268	10
4		0.00003280					0.783	0.3128	305.928	314.6883	9
5	0.0258258	-0.0148589	0.0014793				0.784	0.3126	307.3287	324.8492	8
6		0.0001926	1.69300	0.8135			0.802	0.2989	254.3176	271.8381	2
7	0.1237	-0.008821	-0.01558	0.0005707	0.000001384	0.00115	0.803	0.2991	257.9798	288.6406	4
8	-0.0338402	0.0002741	1.6401629	0.7745170			0.803	0.2989	255.3672	277.2678	2
9	0.0736044	0.0013815	0.0018024	-0.024671			0.802	0.2994	257.2119	279.1125	5
10	-0.1168	0.000003733	1.29600	0.0004067	5.31100	0.01972	0.804	0.2986	256.0509	286.7117	1*

b0, b1, b2, b3, b4 & b5 are regression coefficients, R^2- R Squared, RMSE – Root Mean Square Error, AIC – Aikaike Information Criterion, BIC – Bayesian Information Criterion, ∑R – Relative Rank, *Selected Model

Residual Plots of the Selected Models

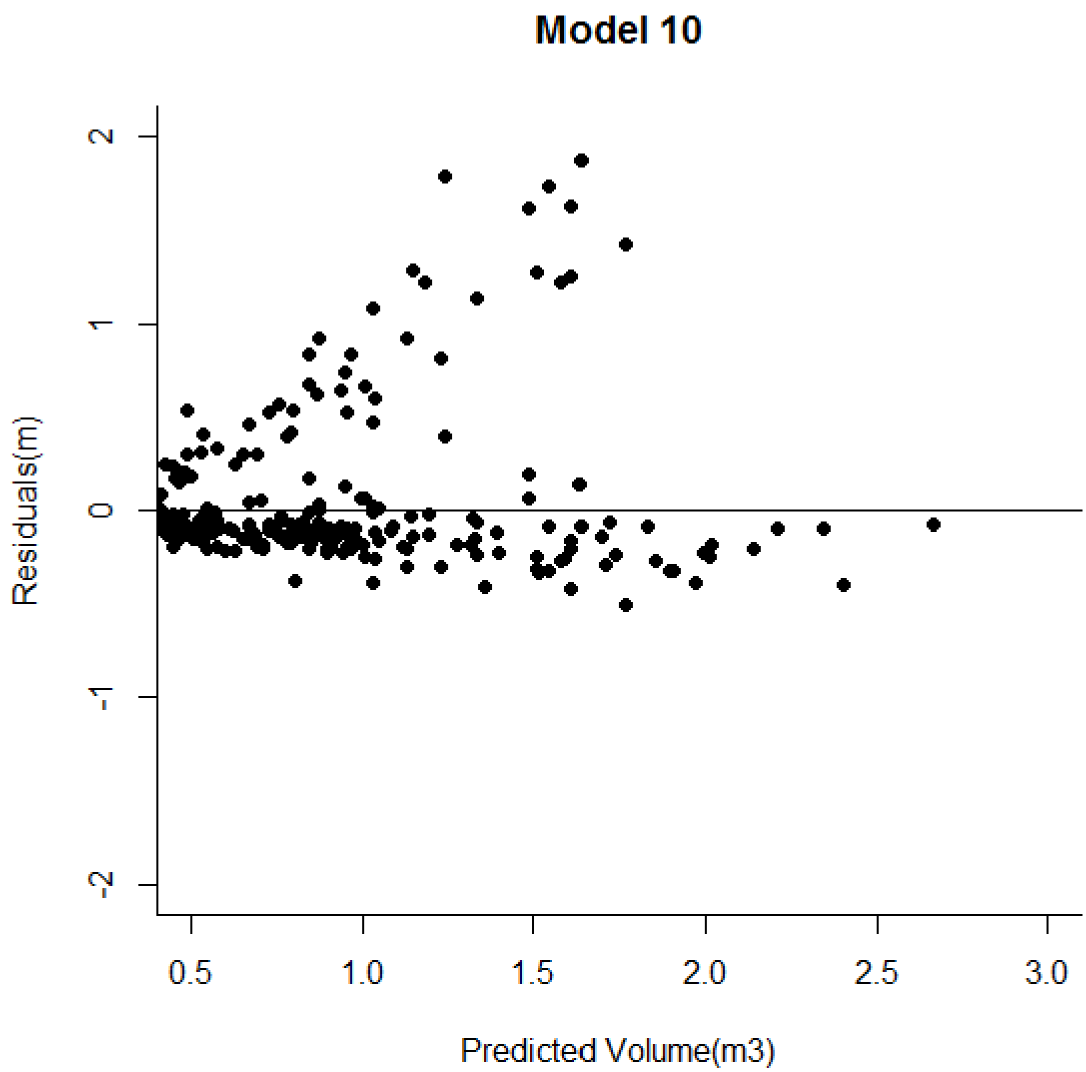

Fig. 5: Residual plot of Model 10

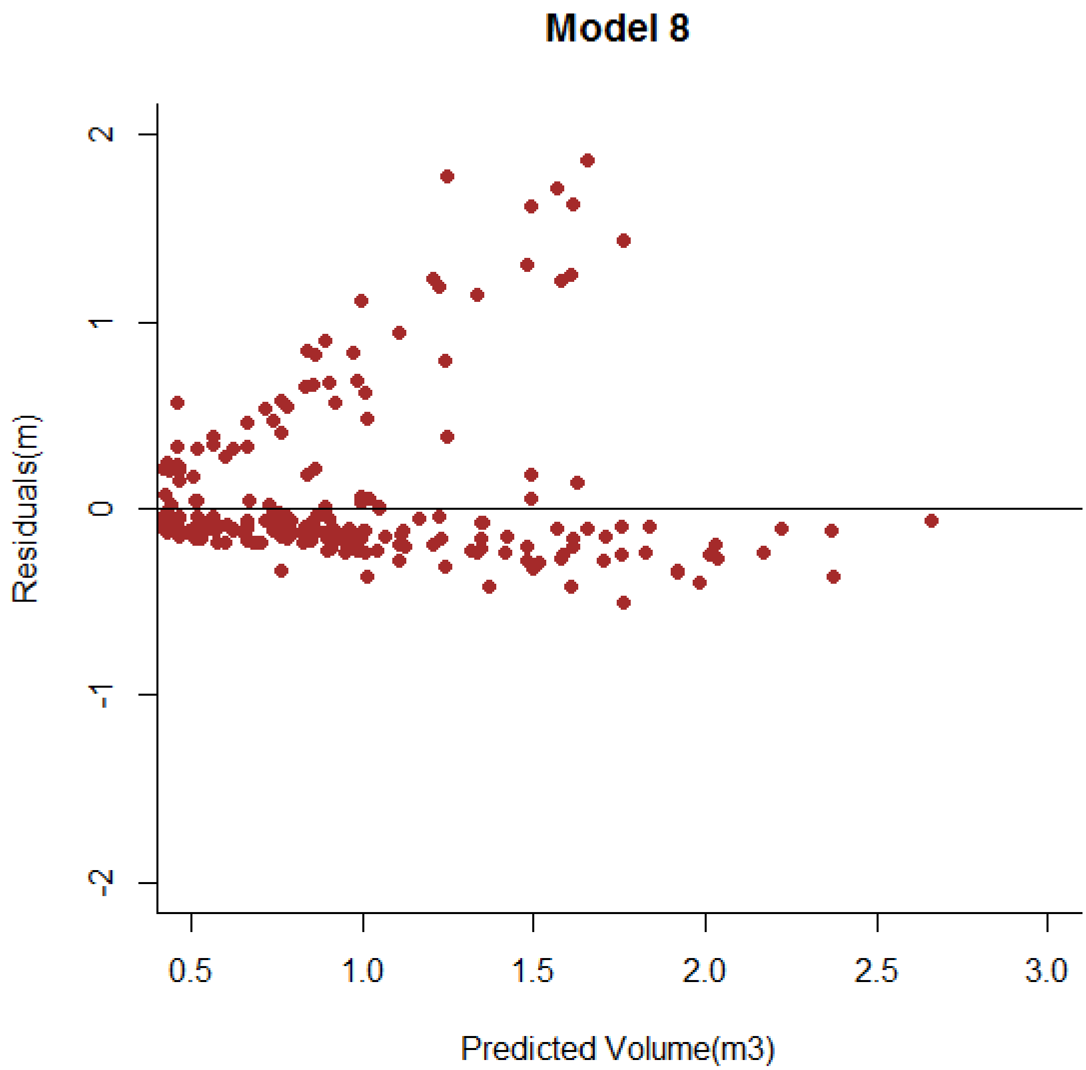

Fig. 6: Residual plot of Model 8

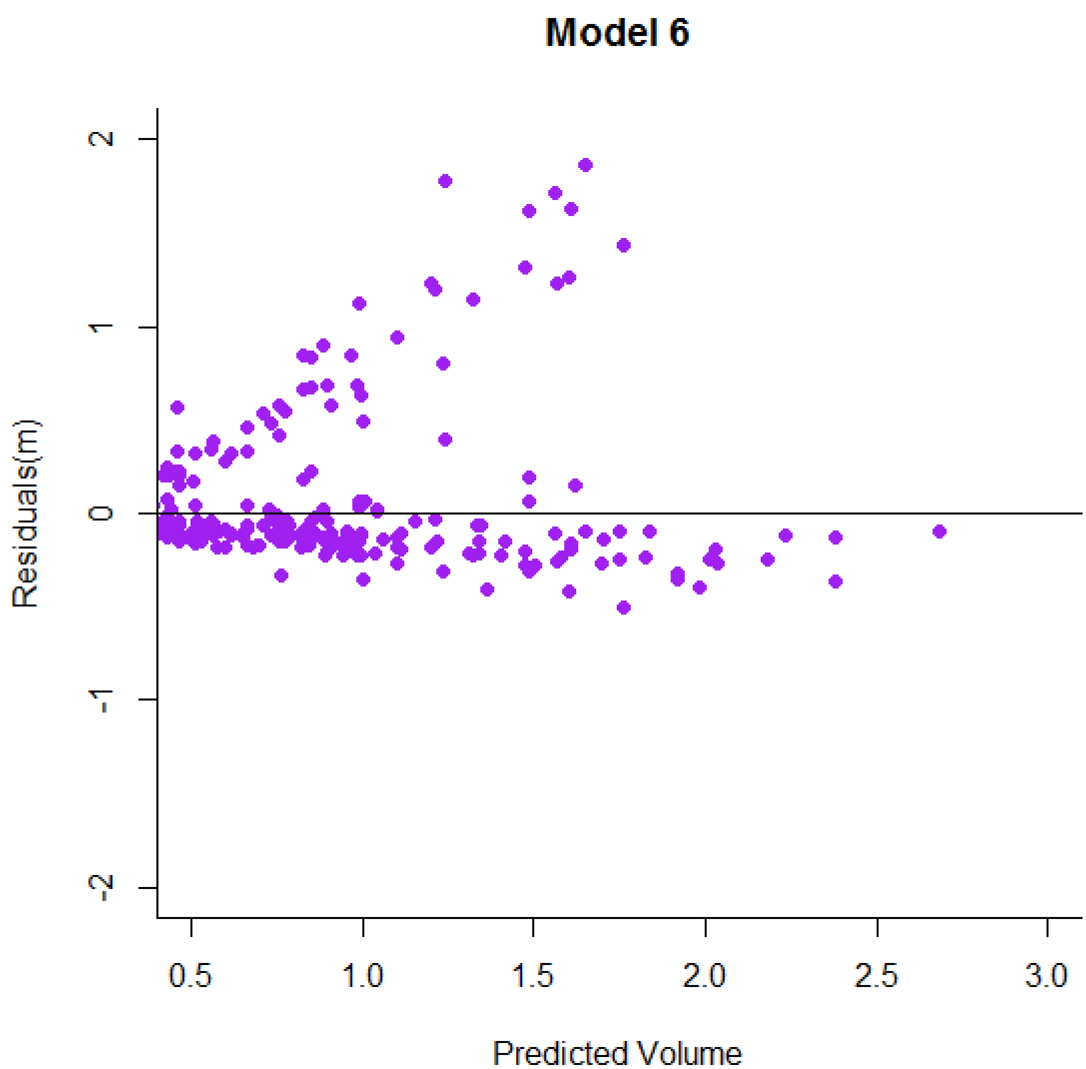

Fig. 7: Residual plot of Model 6

DISCUSSION

In this study the observed volume of the sample trees was calculated by applying Newton‘s formula: $Volume = \frac{\pi h}{24}(Db^2 + 4Dm^2 + Dt^2)$. Ten (10) tree volume models that could be used for *Gmelina arborea* trees or stands volume estimation were developed and tested. In order to assess the accuracy of various models developed, R^2, RMSE, AIC, and BIC were calculated for each model and relative ranking (∑R) of the complete values of the fit statistics were also included to facilitate comparison of the models. The single-entry models with crown length was the only variable that did not perform well with the data set, but the single entry Models with diameter at breast height; total height and basal area performed very well with the data set and the findings is in line with the combined variable equations of Spurr (1952) and the logarithmic of Schamacher and Hall (1933) which are classic volume models and commonly used, often without question, when developing stem volume equations.

Most models tested performed similarly to each other. All the models in this study performed significantly well in volume estimation as the R^2 were above 75% for all the models. However, four models (model 10, 8, 6 and 7) performed best for the data set based on their evaluation statistics and as such are selected for volume estimation of the *Gmelina arborea* stand in Oluwa forest reserve. Residual analysis showed little differences in magnitudes of bias and precision amongst many models. Residual plots for the best volume equations generally indicated an even spread of residuals above and below the zero line, with systematic trend (Fig. 5-7). The R^2 result suggests that a very large proportion of the variation in tree volume is explained by diameter at breast height and total height for the stands. The result of RMSE, AIC and BIC shows that the models have good fit and are therefore recommended for tree volume estimation for *Gmelina arborea*. The values for the evaluation statistics are similar to what were obtained by Sonmez *et al.* (2009) and Adam and Csalovics (2010).

The scatter-plots were consistent with the results of other statistical indices for validation. This shows that the regression assumptions were not violated. Other tree growth modeling studies, including Andreassen and Tomter (2003), Mabvurira and Miina (2002), Zhao *et al.* (2004), Trasobares and Pukkala (2004), and Sonmez *et al* (2009) also observed no constant variance of residuals, an inevitable phenomenon for forest populations due to the nature of the growth process.

The evaluation test revealed that model 10 had the least RMSE of 0.2986, hence it was ranked the best model for the study (Table 3). This equation is therefore, suitable when there is a need for high precision and accuracy. Model 3 had the highest RMSE and were ranked the highest in this study. However, all the models performed statistically well in predicting total volume for the *Gmelina arborea* stand as they had R^2 greater than 75%.

CONCLUSION

We have developed models for estimating volume of *Gmelina arborea* trees. Models that use dbh and *h* as independent variables were most suitable for tree volume estimation. Evaluations revealed that the general total tree models can be applied over a wide range of geographical and biophysical conditions in Nigeria with an appropriate accuracy in predictions. Findings of this study confirmed that there was a strong positive correlation between the total volume as dependent variable and diameter at breast height (dbh) and total height as independent variables. Therefore, using dbh and total height can give precise estimates of total volume of *Gmelina* trees. The criteria used to predict the accuracy of the models developed were R^2, RMSE, AIC, BIC and rank sum. The regression residuals were normally distributed, with constant variance and zero mean. The models were therefore efficient from the evaluation standpoint.

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