# Height-diameter models for prediction of Teak stand in Western Nigeria


## ABSTRACT

The Diameter-height relationship has proven to be an important part in growth and yield models which describe stand changes. Ten existing nonlinear height-diameter models were used to fit and evaluate *Tectona grandis* stand in Oluwa forest reserve (Nigeria) in this study. Three hundred and ninety-seven (397) trees were measured for their stand variables of which diameter at breast height (Dbh) and height (Ht) were paramount. All functions were fitted using weighted nonlinear least square regression (NLLSR), considering hetroscadasticity of variance. Model performance were evaluated using three fit statistics such as root mean squared error (RMSE), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). Logistic 3-parameters H-D function as the best fit based on the model's evaluation statistics and its predictive ability with values of RMSE, AIC and BIC as 2.8925, 1974 and 1990, respectively. Gompertz, Weibull, Chapman-Richards and Michaelis-Menten models also provided good fit results comparable to the observed height-diameter relationship. Logistic function with 3-parameters has been confirmed to provide a secure estimate of total tree height for *Tectona grandis* in Oluwa Forest Reserve.



***Keywords:*** Logistic-Evaluation statistics-Performance-Variable.




## INTRODUCTION

"Diameter-height relationships are useful appraisal for estimating the heights of trees measured for their diameters at breast height (dbh). This relationship describes the measure between height and diameter of sampled trees in a stand on a given data, and it can be represented by a linear or non-linear mathematical model. However, for height-diameter models, more care is needed, and a representative sample of accurately measured total-height is used as the dependent variable while dbh as the independent variable" (Mamoun, 2012).

"The total height of a tree is the linear dimension from ground level to the upper tip of the tree crown, through its main stem" (Teo et al. 2017). "It is a variable of prime importance for quantitative description of trees and stands. Models that describe the correation between total tree height and diameter at breast height are extremely valuable tools for forest management planning" (Uzoh 2017). "Due to their importance for a number of forest stand modelling applications, height-diameter (H-D) equations have received significant attention" (Burkhart and Tome 2012). "Since height-diameter relationship has a tendency to differ among sites, stand densities, ages and other factors, allowing the relationship to vary by tree stand and growth period may yield more accurate results" (Gonda 1998). "Depending on local climate, soil, altitude and ecological conditions, H-D relationships of tree species vary in geographic region", (Peng et al. 2001; Zhang et al. 2002).

"Growth and yield simulators usually need information on tree height, either at the individual tree, plot, or stand level, to predict forest dynamics, dominant height, and site index" (Huang et al. 2000). "Information on tree heights is essential in forest inventories for computing tree volumes.  However, compared to measuring tree diameter, field measurement of tree height is rather tedious. This is why many forest inventories save time and effort by predicting tree heights using height–diameter (H–D) models instead of direct measurements. To improve prediction of the local H–D curve, height measurements from a subsample of trees on each sample plot or sample plot cluster may be utilized. Predicting total tree-height based on observed diameter at breast height outside bark is routinely in practical management and silvicultural research work" (Meyer, 1940). "The description of stands and their development over time, as well as the estimation of tree volume, relies heavily on accurate height-diameter functions" (Curtis, 1967). "Height–diameter models have been conventionally fitted using least squares regression method. However other fitting methods such as neural network (Özçelik et al., 2013 ; Thanh et al., 2019), mixed- effect regression (Kalbi et al., 2018; Corral Rivas et al., 2019; Vanderschaaf, 2020; Bronisz and Mehtätalo, 2020 ), quantile regression (Rust, 2014; Zang, et al., 2016 ; Zhang et al., 2020) and reduced major axis regression (Chen, 2018 ) have been used with good results". "Mixed-effects regression has increasingly taken center stage for height–diameter modeling due to its ability to incorporate tree height–diameter variability arising from different subject (e.g. forest type/sites, sample plots, clusters, species). Mixed-effects models simultaneously estimate both population-averaged (fixed effects) and subject-specific (random effects) parameters, allowing the variability for a given subject to be modeled" (Pinheiro and Bates, 2000).

"Teak is indigenous to the Indian peninsula and continental Southeast Asia in a discontinuous or patchy distribution pattern in India, Myanmar, Thailand and Lao PDR at latitudes between $9^0$-$25^0$30'N and longitudes between $73^0$-$104^0$30'E". (Kadambi 1972, Siswamartana 1999). "*Tectona grandis* L.f. is a rapid-growing tropical hardwood tree species which belongs to the family of Verbenaceae. Teak is one of the most commonly cultured exotic species in Nigeria because of its good anatomy and physical properties" (Miranda *et al.* 2011). "It is a multipurpose tree species and as such, its products are in perpetual demand" (Miranda *et al.* 2011). It was introduced into Nigeria in 1902 from India (Ball *et al.*1999). The aim of this study is to develop ten diameter-height models and predict heights of the teak trees in Oluwa Forest Reserve for the effective management and decision making of the forested area.

# METHODOLOGY

## *Study site and data collection*

The teak plantation of about 3 ha size in the Oluwa Forest Reserve (FR) of Southwestern Nigeria was the site where datasets were measured and used in this study. According to Ogana & Ekpa 2020, Oluwa Forest Reserve is located between 6º 55′ and 7º 20′ N and longitude 3º 45′ and 4º 32′ E, and occupies an area of 87,816 ha. The forest site has an annual rainfall in the range 1700 to 2200 mm, an average annual temperature of 26°C, and a mean elevation of 123 m above sea level (Onyekwelu *et al.* 2006). *Gmelina arborea* Roxb. and *Tectona grandis* L.f. are the dominant plantation species in Oluwa Forest Reserve.

Datasets were measured from twelve sample plots of dimension 20 m × 20 m (0.04ha) in the teak stand. Measurements of diameter outside bark of all trees at breast height (1.3 m above ground, DBH) were done using diameter tape. Trees with diameter less than 10cm were not used for the analysis. Height (H) measurements were also taken with relaskop. Basal area ($m^2$) and tree volume ($m^3$) were calculated from the measured variables. The summary statistics of the modelling datasets is presented in table 1 below:

Table 1: Summary of the Modelling Datasets

| **Statistics** | **H(m)** | **DBH (cm)** | **VOL($m^3$)** | **BA($m^2$)** |
|---|---|---|---|---|
| Min | 8.4 | 10.0 | 0.038 | 0.008 |
| Max | 29.5 | 54.0 | 1.410 | 0.229 |
| Mean | 16.1 | 24.1 | 0.375 | 0.051 |
| SD | 3.84 | 8.37 | 0.261 | 0.037 |
| Skewness | 0.57 | 0.73 | 1.156 | 1.747 |
| Kurtosis | 0.23 | 0.51 | 1.091 | 4.384 |
| **N = 397 Trees** | | | | |

Min = Minimum, Max = Maximum and SD = Standard Deviation

## H–D RELATIONSHIP

## The applied nonlinear models

Ten (10) sets of nonlinear models were chosen from past published studies on H–D modeling (Table 2). To avoid problems with back transformation bias, omodels that expressed tree height without transformations were only used. The functions were nonlinear, with either 2-parameters (4 models) or 3-parameters (6 models). The models were parameterized so that parameter $b_0$ defines the scale of the H–D models, which is the major dimension of variability among sample plots. The datasets for this were handled with R-script using package 'nlme' of *R*-environment (R Core Team 2017), to generate parameter estimates.

### Evaluation statistics

The models used in this were assessed based on three evaluation statistics; Root Mean Square Error (RMSE), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The best model were adjudge on the criterion that the smaller the values of the evaluation statistics, the better the model.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n}} \quad [1]$$

$$AIC = n \ln\left(\frac{RSS}{n}\right) + 2p \quad [2]$$

$$BIC = n \ln\left(\frac{\mathrm{RSS}}{\mathrm{n}}\right) + p \ln n \quad [3]$$

Where; n = sample size, RSS = Residual Sum of Square, $p$ = number of parameters; $Yi$ is the observed value and $\hat{Y}_i$ is the theoretical value predicted by the model.

Table 2: Applied H-D Models

| Number | Model Name | Form | Reference | Eq. |
|---|---|---|---|---|
| **2-parameter functions** | | | | |
| 1 | Curtis | $H = 1.3 + \frac{b_0 D}{(1+D)^{b_1}}$ | Curtis (1967) | [4] |
| 2 | Power | $H = 1.3 + b_0 D^{b_1}$ | Stoffels and von Soest (1953) | [5] |
| 3 | Naslund | $H = 1.3 + \frac{D^2}{(b_0 + b_1 D)^2}$ | Naslund (1936) | [6] |
| 4 | Michaelis-Menten | $H = 1.3 + \frac{b_0 D}{(b_1 + D)}$ | Menten and Michalis (1913), Huang et al. (1992) | [7] |
| **3-parameter functions** | | | | |
| 5 | Weibull | $H = 1.3 + b_0\left(1 - e^{b_1 D^{b_2}}\right)$ | Yang et al. (1978) | [8] |
| 6 | Prodan | $H = 1.3 + \frac{D^2}{b_0 D^2 + b_1 D + b_2}$ | Strand (1959) | [9] |
| 7 | Logistic | $H = 1.3 + \frac{b_0}{1 + b_1 e^{(-b_2 D)}}$ | Pearl and Reed (1920), Huang et al. (1992) | [10] |
| 8 | Korf | $H = 1.3 + b_0 e^{(-b_1 d^{-b_2})}$ | Lundqvist (1957) | [11] |
| 9 | Gompertz | $H = 1.3 + b_o\, e^{(-b_1\, e^{(-b_2 D)})}$ | Gompertz (1825), Huang et al. (1992) | [12] |
| 10 | Chapman-Richards | $H = 1.3 + b_0(1 - e^{-b_1 D})^{b_2}$ | Richards (1959) | [13] |

**Note: *H*= Total tree height (m); *D* = Diameter at breast height (cm), $b_0$, $b_1$, $b_2$ = model parameter estimates and *e* = base of the natural logarithm.**

## RESULTS

The estimated parameters $b_o$, $b_1$ and $b_2$ and their corresponding evaluation statistics are indicated in table 3. The result revealed that 3-parameter logistic model with estimated parameters of 20.6514, -0.0723 and 0.1051 for $b_0$, $b_1$, and $b_2$ respectively, had the smallest evaluation statistics values of RMSE (2.8925), AIC (1974) and BIC (1990). Evaluation statistics criterion revealed that logistic model is selected as the best candidate H-D model for the teak plantation in Oluwa forest Reserve. Closely following logistic model is the 3-parameter Gompertz model which as indicated in the Table 3 had the second smallest values for all the evaluation statistics. Naslund model was the worst H-D model as it consistently produced the highest values of 16.2066 (RMSE), 3342 (AIC) and 3354 (BIC). Values in table 3 further revealed that, among the 2-parameter models, Michaelis-Menten model produced the smallest values of 2.9127, 1979 and 1991 for RMSE, AIC and BIC respectively, hence the best suitable 2-parameter H-D function for the data set.

Table 3: Parameter estimates and the evaluation statistics of the ten H-D models

| Models | Parameters | | | Fit Statistics | | |
|---|---|---|---|---|---|---|
| | $b_o$ | $b_1$ | $b_2$ | RMSE | AIC | BIC |
| Curtis | 4.6071 | 0.5940 | - | 2.9481 | 1989 | 2001 |
| Power | 4.1437 | 0.4313 | - | 2.9528 | 1990 | 2002 |
| Naslund | -31.8670 | 0.7114 | - | 16.2066 | 3342 | 3354 |
| Michaelis-Menten | 29.3390 | 18.5460 | - | 2.9127 | 1979 | 1991 |
| Weibull | 21.1382 | -0.0507 | 1.0803 | 2.9017 | 1977 | 1993 |
| Prodan | 21.5400 | 3.0030 | −0.6560 | 2.9111 | 1980 | 1995 |
| Logistic | 20.6514 | -1.0723 | 0.1051 | 2.8925* | 1974* | 1990* |
| Korf | 26.9296 | 7.9327 | 0.8833 | 2.9130 | 1980 | 1996 |
| Gompertz | 21.0262 | 1.7141 | 0.0838 | 2.8968 | 1976 | 1992 |
| Chapman-Richards | 21.6274 | 0.0626 | 1.0223 | 2.9130 | 1977 | 1993 |

*Selected model based on evaluation statistics criterion

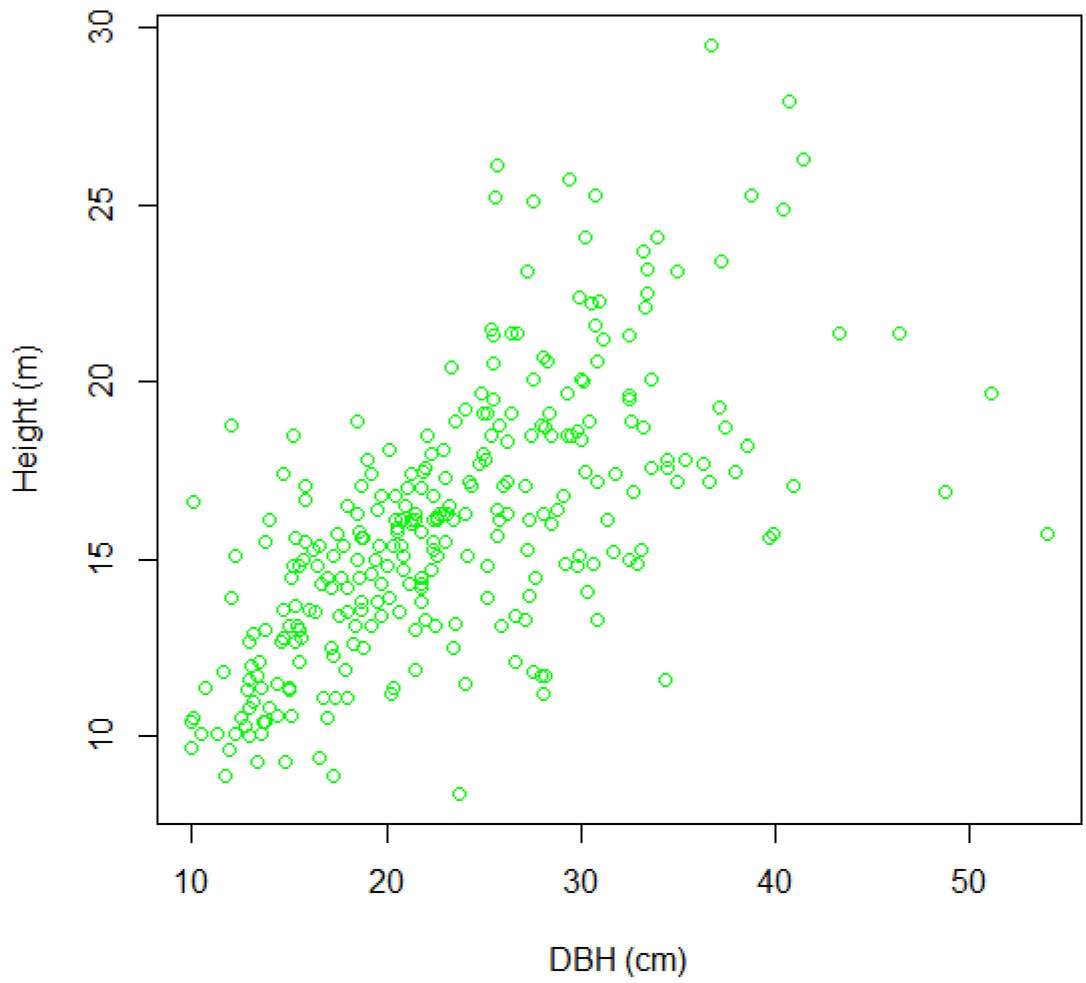


Fig 1: Scatter plot of Total height against diameter at breast height (DBH)

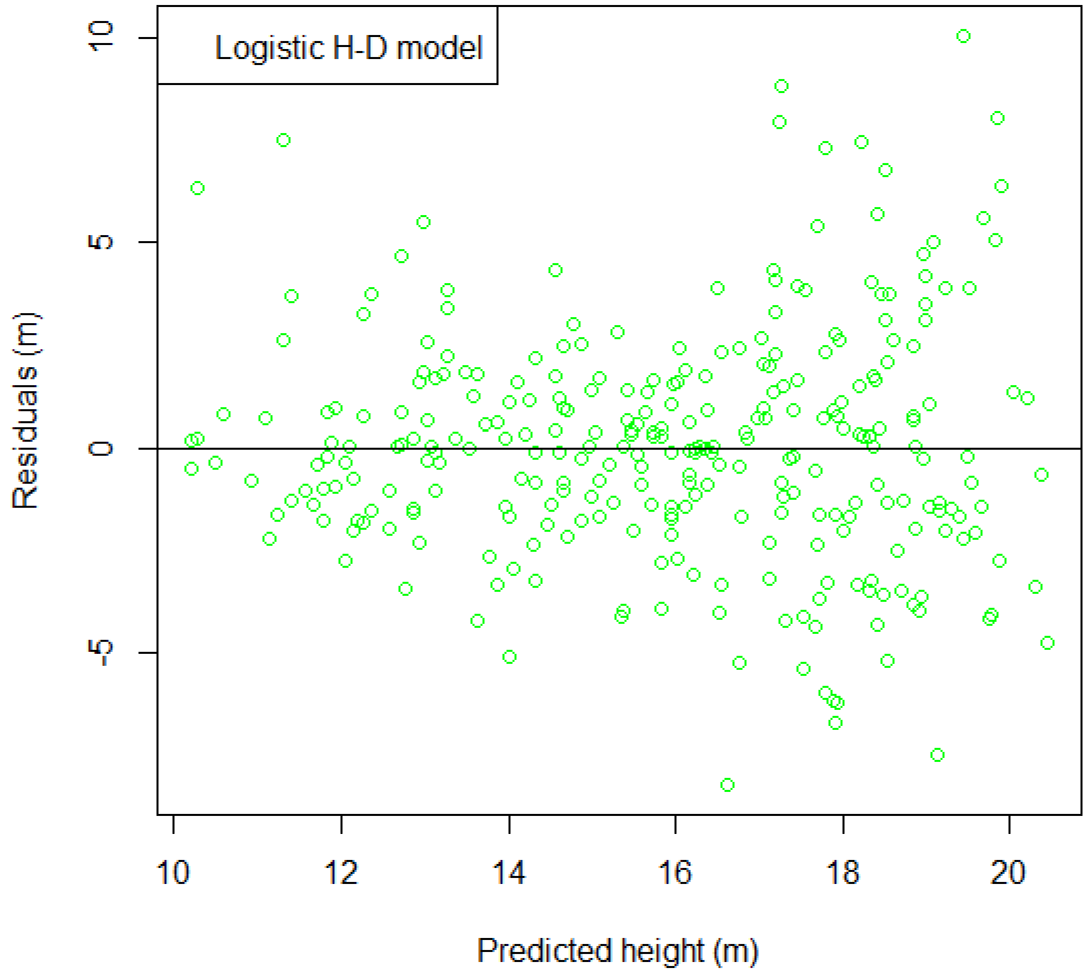


Fig 2: Residual of the selected height-diameter model

## DISCUSSION

"Tree height and diameter relationship is one of the most important elements of forest structure" (Zhang et al. 2014). Ten (10) non-linear functions were used in describing the relationships between the diameters and the heights of the Teak species in Oluwa forest reserve. Logistic model is the most suitable non-linear model for the teak stand among the functions evaluated.

Gompertz, Weibull, Chapman-Richards and Michaelis-Menten provided good fit result when compared to the observed Height-diameter relationship.

"Findings from previous studies states that for any appropriate H-D model, the asymptotic t-statistic for each coefficient has to be significant, the model RMSE has to be small and the standardized residual plot should show approximately homogeneous variance over the full range of predicted values" (Huang et al. 1992). Selection of the most accurate models should be based on appropriate criteria (Zhang 1997) such as AIC, BIC, RMSE, ME and $R^2_{adj}$ which were among the evaluation criteria used in this study.

The findings from this study were consistent with the works of Ogana (2020), Egonmwan & Ogana (2020) who reported a similar result in which Logistic function was selected as the best candidate H-D model based on evaluation statistics criterion. Mehtätalo *et al*. (2015) reported RMSE values of 2.20 and 2.79 for *Eucalyptus globulus* and mixed Tropical species, respectively for Logistic H-D model. But, Mengesha Tsega (2018) who in his work found out that "Weibull H-D model ranked overall best and was the best model in predicting total height of Cupressus lusitanica in Gergeda forest". Also, Huang et al. (1992) compared "nonlinear H-D functions for major Alberta tree species (16 species in 9 groups) and found that the Weibull and the generalized logistic functions were the functions which gave the most satisfactory results". While Naslund had RMSE of 2.29 and 2.81, respectively, for the two species in their study. Also, Shamaki *et al*. (2016) reported $R^2$ and RMSE of 0.336 and 0.995 and 0.335 and 0.995 for Chapman-Richards and Weibull H-D models, respectively, for *Tectona grandis* in Nimbia Forest Reserve. There is no basis for comparing the fit indices values of the study at hand and others reported in forestry literatures. As the parameters of the models are locations specific and more importantly heavily depends on the quality of data used in fitting the models.

The analysis of the plot of residual and predicted values demonstrated that there was little or no systematic bias towards over-or underestimation of the tree total height (Fig 2). The assumption of ordinary nonlinear least squares regression was not violated in the models as the residual analysis showed almost homogenous variance (homoscedasticity) over the range of the predicted values and there were no systematic patterns. The residual plot of Logistic H-D model were only presented.

Huang *et al*. (1992) asserted that "for a good H-D model, the asymptotic t-statistic for each coefficient should be significant, the MSE should be small and the residual plot should approximate homogeneous variance over the full range of predicted values". The Logistic H-D model met these criteria relative to other established H-D functions evaluated in this study.

"The majority of the models used in this study have shown to be very flexible and have been used extensively in growth and yield studies for describing height-age, diameter-age, and volume-age relationships" (Somers, Farrar 1991). "However, the relationship between diameter and height varies within a region, depending on local environmental conditions, and also varies within a geographic region" (Özçelik et al. 2014).

## CONCLUSION

In this study, ten existing H-D models were used to assess the H-D relationships of *Tectona grandis* stand in Oluwa forest Reserve. Based on the different statistical evaluation criterion used for this study, Logistic model is selected as the best H-D model as it provided the best satisfactory results as compared to the other H-D functions. It was also noted that Gompertz, Weibull, Chapman-Richards and Michaelis-Menten also provided good fit results when compared to the observed Height-diameter relationship. The local H-D functions which are only dependent on tree diameter can normally be applied to the stand where the data were gathered. Therefore, the models developed and evaluated in this study could provide more reliable estimate of height for *Tectona grandis* stand in Oluwa forest Reserve by reducing cost and time spent on forest inventory. The H-D models are site-specific; hence their use is limited to the prediction of tree heights in the study forest. It is recommended that height-diameter models should be surveyed by eco-region for reflecting the regional differences. This is because ecoregion-based models provide useful tools to forest resource managers in forest management practices and decision making. The models assessed in this study are an economical tool for tree height estimation during field inventories and represent an important step in furnishing forest managers with decision options for the effective management of the teak trees in Oluwa forest reserve.

## REFERENCES


Bronisz, K., Mehtätalo, L., 2020. Mixed-effects generalized height–diameter model for young silver birch stands on post-agricultural lands. For. Ecol. Manag. 460, 117901. doi: 10.1016/j.foreco.2020.117901.

Burkhart HE, Tome M. 2012. Modeling Forest Trees and Stands. New York: Springer.

Chen, X., 2018. Diverse scaling relationships of tree height and diameter in five tree species. Plant Ecol. Divers 11, 147–155. doi: 10.1080/17550874.2018.1445128

Corral Rivas, S., Silva Antuna, A.M., Quiñonez Barraza, G., 2019. Generalized non-linear height-diameter model with mixed effects for seven Pinus species in Durango, Mexico. Rev. Mex. Cienc. For. 10. doi: 10.29298/rmcf.v10i53.500.

Curtis, R. O. 1967. Height-diameter and height-diameter-age equations for second-growth Douglas-fir. *Forest Science* 13**:** 365–375.

Egonmwan IY & Ogana FN (2020) Application of diameter distribution model for volume estimation in *Tectona grandis* L.f. stands in the Oluwa forest reserve, Nigeria. *Tropical Plant Research* 7(3): 573–580

Gomperz, B. 1825. On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies. Philos. Trans. R. Soc. Lond. B Biol. Sci. 115: 513–585.

Gonda HE. 1998. Height-Diameter and Volume Equations, Growth Intercept and Needle Length Site Quality Indicators, and Yield Equations for Young Ponderosa Pine Plantations in Neuquen, Patagonia, Argentina [PHD Thesis]. Corvallis, Oregon: Oregon State University.

Huang, S., Price, D., and Titus, S.J. 2000. Development of ecoregion-based height diameter models for white spruce in boreal forests. For. Ecol. Manage. 129: 125–141. doi:10.1016/S0378-1127(99)00151-6.

Huang, S., Titus, S.J., and Wiens, D.P. 1992. Comparison of nonlinear height–diameter functions for major Alberta tree species. Can. J. For. Res. 22: 1297–1304. doi:10.1139/x92-172.

Kadambi, K. 1972. Silviculture and management of teak. Stephen F. Austin State University, School of Forestry, Bulletin 24. 37p.

Kalbi, S., Fallah, A., Bettinger, P., et al., 2018. Mixed-effects modeling for tree height prediction models of oriental beech in the Hyrcanian forests. J. For. Res. 29, 1195–1204. doi: 10.1007/s11676-017-0551-z.

Lundqvist, B. 1957. On the height growth in cultivated stands of pine and spruce in Northern Sweden. Medd. frstatens skogforsk 133.

Mamoun, E. I. and Osman, H. 2012. Modelling height-diameter relationships of selected economically important natural forests species. *Journal of Forest products and industries* 2 (1): 34–42.

Mehtätalo, L.,de-Miguel, S. & Gregoire, T.G. 2015. Modelling height-diameter curves for prediction. *Canadian Journal of Forest Research*, 45: 826-837

Mengesha Tsega, Awoke Guadie, Zebene Lakew Teffera, Yigez Belayneh & Dongjie Niu (2018) Development and Validation of Height-Diameter Models for *Cupressus lusitanica* in Gergeda Forest, Ethiopia, Forest Science and Technology, 14:3, 138-144, DOI: 10.1080/21580103.2018.1482794

Menten, L., and Michaelis, M.I. 1913. Die Kinetik der Invertinwirkung. Biochem. Z. 49: 333–369.

Meyer, H. A. 1940. A mathematical expression for height curves. *Journal of Forestry* 38: 415–420.

Miranda I, Sousa V & Pereira H (2011) Wood properties of teak (*Tectona grandis* L.f.) from a mature unmanaged stand in East Timor. *Journal of Wood Science* 57: 171–178.

Näslund, M. 1937. Skogsförsöksanstaltens gallringsförsök i tallskog (Forest research intitute's thinning experiments in Scots pine forests). Meddelanden frstatens skogsförsöksanstalt Häfte 29 (In Swedish).

Ogana FN & Ekpa NE (2020) Modeling the non-spatial structure of *Gmelina arborea* Roxb Stands in the Oluwa Forest Reserve, Nigeria. *Forestist* 70(2): 133–140.

Ogana FN (2020) Comparison of a Modified Log-Logistic Distribution with established models for tree height prediction. JOURNAL OF RESEARCH IN FORESTRY, WILDLIFE AND ENVIRONMENT VOLUME 10, No. 2, JUNE, 2018.

Onyekwelu JC, Mosandl R & Stimm B (2006) Productivity, site evaluation and state of nutrition of *Gmelina arborea* plantation in tropical rainforest zone in south-western Nigeria. *Forest Ecology and Management* 229: 214–227.

Özçelik R., Yavuz H., Karatepe Y., Gürlevik N., Kiriş R. (2014): Development of ecoregion-based height–diameter models for 3 economically important tree species of southern Turkey. Turkish Journal of Agriculture and Forestry, 38: 399–412.

Özçelik, R., Diamantopoulou, M.J., Crecente-Campo, F., Eler, U., 2013. Estimating Crimean juniper tree height using nonlinear regression and artificial neural network models. For. Ecol. Manag. 306, 52–60. doi: 10.1016/j.foreco.2013.06.009.

Pearl, R., and Reed, L.J. 1920. On the rate of growth of the population of the united states since 1790 and its mathematical representation. Proc. Natl. Acad. Sci. U.S.A. 6: 275–288. doi:10.1073/pnas.6.6.275.

Peng C, Zhang L, Huang S, Zhou X, Parton J, Woods M. 2001. Developing Ecoregion-Based Height-Diameter Models for Jack Pine and Black Spruce in Ontario. Ontorio Forest Resource Institute (OFRI report 159: 1–10).

Pinheiro, J.C. , Bates, D.M. , 2000. Mixed-Effects Models in S and S-PLUS. Springer-Verlag, New York.

Richards, F.J. 1959. A flexible growth function for empirical use. J. Exp. Bot. 10(29): 290–300. doi:10.1093/jxb/10.2.290.

Rust, S., 2014. Analysis of regional variation of height growth and slenderness in popula- tions of six urban tree species using a quantile regression approach. Urban For. Urban Green 13, 336–343. doi: 10.1016/j.ufug.2013.12.003.

Shamaki, S.B., Akindele, S.O., Isah, A.D. & Mohammed, I. 2016. Height-diameter relationship models for Teak (*Tectona grandis*) plantation in Nimbia Forest Reserve, Nigeria. *Asian Journal of Environment and Ecology*, 1(1): 1-7.

Siswamartana, S. 1999. Teak plantation productivity in Indonesia. Paper to Regional seminar on site, technology and productivity of teak plantation, Chiangmai, Thailand. 9p.

Somers G.L., Farrar R.M. (1991): Bio-mathematical growth equations for natural longleaf pine stands. Forest Science, 37: 227–244.

Stoffels, A., and van Soest, J. 1953. The main problems in sample plots. Ned.Boschb. Tijdschr. 25: 190–199.

Strand, L. 1959. The accuracy of some methods for estimating volume and increment on sample plots. Medd. Norske Skogfors. 15(4): 284–392 (In Norwegian with English summary).

Teo SJ, Machado SdA, Figueiredo Filho A, Tome M. 2017. General height- diameter equation with biological attributes for Pinus taeda L. stands. Cerne. 23:403–411.

Thanh, T.N., Tien, T.D., Shen, H.L., 2019. Height-diameter relationship for Pinus ko- raiensis in Mengjiagang forest farm of Northeast China using nonlinear regres- sions and artificial neural network models. J. For. Sci. 65 (2019), 134–143. doi: 10.17221/5/2019-JFS.

Uzoh FCC. 2017. Height-diameter model for managed even-aged stands of Ponderosa Pine for the Western United States using hierarchical nonlinear mixed-effects model. Aust J Basic Appl Sci. 11:69–87.

Vanderschaaf, C.L., 2020. Predictive ability of mixed-effects height–diameter mod- els fit using one species but calibrated for another species. For. Sci. 66, 14–24. doi: 10.1093/forsci/fxz058.

Yang RC, Kozak A & Smith JHG (1978) The potential of Weibull-type functions as a flexible growth curves. *Canadian Journal of Forest Research* 8: 424–431.

Zang, H., Lei, X., Zeng, W., 2016. Height–diameter equations for larch plantations in northern and northeastern China: a comparison of the mixed-effects, quantile regression and generalized additive models. For. Int. J. For. Res. 89, 434–445. doi: 10.1093/forestry/cpw022.

Zhang L, Peng C, Huang S. 2002. Development and evaluation of ecoregion-based jack pine height-diameter models for Ontario. For Chron. 78:530–538.

Zhang L. (1997): Cross-validation of non-linear growth functions for modeling tree height-diameter relationships. Annals of Botany, 79: 251–257.

Zhang X, Duan A, Zhang J, Xiang C. 2014. Estimating tree height-diameter models with the Bayesian method. Sci J. 2014:1–9.

Zhang, B., Sajjad, S., Chen, K., et al., 2020. Predicting tree height-diameter relation- ship from relative competition levels using quantile regression models for Chi- nese Fir (Cunninghamia lanceolata) in Fujian Province, China. Forests 11, 183. doi: 10.3390/f11020183.