Height-diameter models for prediction of *Tectona grandis* L.f. stands in the Oluwa forest reserve, Nigeria

ABSTRACT

Tree diameter-height relationship can be used as a key input component in forest growth and yield models, and description of stand dynamics. This study was conducted to fit and evaluate ten existing nonlinear height diameter functions for Tectona grandis in Oluwa forest reserve. A total of 397 trees were measured for their diameter at breast height (D) and height. Considering hetroscadasticity of variance, all functions were fitted using weighted nonlinear least square regression. To evaluate the performance of each model, three fit statistics such as root mean squared error (RMSE), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) were used. Among all the models tested, Logistic 3-parameters H-D function observed to give the best fit based on the model's goodness of fit and predictive ability with values of RMSE, AIC and BIC as 2.8925, 1974 and 1990 respectively. However, Gompertz, Weibull, Chapman-Richards and Michaelis-Menten also provided good fits result comparable to the observed Height-diameter relationship. Therefore, Logistic model with three parameters has been confirmed to provide reliable estimate of total tree height for Tectona grandis in Oluwa Forest Reserve.

Keywords: Logistic-Evaluation statistics-Performance-Variable.

INTRODUCTION

Diameter-height relationships are used to estimate the heights of trees measured for their diameters at breast height. Such relationship describes the correlation between height and diameter of the trees in a stand on a given data, and can be represented by a linear or non-linear mathematical model. However, for diameter-height models, more care is needed, and a representative sample of accurately measured total-height is used as the response variable and dbh as the predictor variable (Mamoun, 2012).

The total height of a tree is the linear distance from ground level to the upper tip of the tree crown, through its main axis (Teo et al. 2017). It is a variable of fundamental importance for quantitative description of a tree and stand. Models describing the relationship between total tree height and diameter at breast height are extremely valuable tools for forest management planning (Uzoh 2017). Because of their importance for a number of forest stand modelling applications, height-diameter (H-D) equations have received considerable attention (Burkhart and Tome 2012). Since height-diameter relationship has a tendency to differ among sites, stand densities, ages and other factors, allowing the relationship to vary by tree stand and growth period may yield more accurate results (Gonda 1998). In addition to these, H-D relationships of tree species vary in geographic region, depending on local climate, soil, altitude and ecological conditions (Peng et al. 2001; Zhang et al. 2002).

Information on tree heights is essential in forest inventories for computing tree volumes. Also, growth and yield simulators usually need information on tree height, either at the individual tree, plot, or stand level, to predict forest dynamics, dominant height, and site index (e.g., Huang et al. 2000). However, field measurement of tree height is rather tedious compared to measuring tree diameter. That is why many forest inventories save time and effort by predicting tree heights using height– diameter (H–D) models instead of direct measurements. Height measurements from a subsample of trees on each sample plot or sample plot cluster may be utilized for improved prediction of the local H–D curve. Predicting total tree-height based on observed diameter at breast height outside bark is routinely in practical management and silvicultural research work (Meyer, 1940). The estimation of tree volume, as well as the description of stands and their development over time, relies heavily on accurate height-diameter functions (Curtis, 1967).

The *Tectona grandis* L.f. is a fast-growing tropical hardwood tree species belonging to the family of Verbenaceae. Teak is one of the most widely cultivated exotic species in Nigeria because of its good anatomical and physical properties (Miranda *et al.* 2011). It is also multipurpose tree species and as such, there is continuous demand for its products (Miranda *et al.* 2011). The teak plantation in Oluwa Forest Reserve is a production forest established for pole and timber. The main aim of the study was to develop diameter-height models and predict heights of the teak species in Oluwa Forest Reserve for the effective management and decision making of the stand.

METHODOLOGY

Study area and data

The data used in this study were from the teak plantation (about 3 ha size) in the Oluwa Forest Reserve (FR) of Southwestern Nigeria. The Oluwa Forest Reserve is situated between 6º 55′ and 7º 20′ N and longitude 3º 45′ and 4º 32′ E, and occupies an area of 87,816 ha (Ogana & Ekpa 2020). Oluwa has an annual rainfall in the range 1700 to 2200 mm, an average annual temperature of 26°C, and a mean elevation of 123 m above sea level (Onyekwelu *et al.* 2006). Establishment of large-scale plantations in the reserve started in early 1960s. *Gmelina arborea* Roxb. and *Tectona grandis* L.f. are the dominant plantation species in Oluwa FR.

Data were collected from twelve sample plots of 20 m × 20 m in size in the teak stand. Diameter measurements of all trees (over bark) at breast height (1.3 m above ground, DBH) were measured with diameter tape to an accuracy of 0.1 cm. Their corresponding height (H) measurements were also taken with hypsometer. The measured variables were used to calculate the basal area (m^2) and tree volume (m^3). The summary statistics of the modelling data set is presented in table 1 below:

Table 1: Summary of the Modelling Datasets

Statistics	**H(m)**	**DBH (cm)**	**VOL(m^3)**	**BA(m^2)**
Minimum	8.4	10.0	0.038	0.008
Maximum	29.5	54.0	1.410	0.229
Mean	16.1	24.1	0.375	0.051
Standard Deviation	3.84	8.37	0.261	0.037
Skewness	0.57	0.73	1.156	1.747
Kurtosis	0.23	0.51	1.091	4.384
N = 397 Trees				

MODELING THE H–D RELATIONSHIP

The applied nonlinear functions

A set of 10 nonlinear models was selected from previously published studies on H–D modeling (Table 2). To avoid problems with back transformation bias, only models that expressed tree height without transformations was used. All functions were nonlinear, with either 2-parameters (4 models) or 3-parameters (6 models). All models were parameterized so that parameter b_0 defines the scale of the H–D models, which is the major dimension of variability among sample plots. The datasets were handled with R-script using package ‘nlme’ of *R*-environment (R Core Team 2017), to generate parameter estimates.

Evaluation statistics

The models were assessed based on root mean square error (RMSE), Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). Model selection was based on the criterion that the smaller the values of the evaluation statistics, the better the model.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n}} \quad [1]$$

$$AIC = n \ln\left(\frac{RSS}{n}\right) + 2p \quad [2]$$

$$BIC = n \ln\left(\frac{\mathrm{RSS}}{\mathrm{n}}\right) + p \ln n \quad [3]$$

Where: RSS = residual sum of square, n = sample size, p = number of parameters; Yi is the observed value and $\hat{Y}_i$ is the theoretical value predicted by the model.

Table 2: Applied Height-Diameter Models

Number	Model Name	Form	Reference	Eq.
2-parameter functions				
1	Curtis	$H = 1.3 + \frac{b_0 D}{(1+D)^{b_1}}$	Curtis (1967)	[4]
2	Power	$H = 1.3 + b_0 D^{b_1}$	Stoffels and von Soest (1953)	[5]
3	Naslund	$H = 1.3 + \frac{D^2}{(b_0 + b_1 D)^2}$	Naslund (1936)	[6]
4	Michaelis-Menten	$H = 1.3 + \frac{b_0 D}{(b_1 + D)}$	Menten and Michalis (1913), Huang et al. (1992)	[7]
3-parameter functions				
5	Weibull	$H = 1.3 + b_0 \left(1 - e^{b_1 D^{b_2}}\right)$	Yang et al. (1978)	[8]
6	Prodan	$H = 1.3 + \frac{D^2}{b_0 D^2 + b_1 D + b_2}$	Strand (1959)	[9]
7	Logistic	$H = 1.3 + \frac{b_0}{1 + b_1 e^{(-b_2 D)}}$	Pearl and Reed (1920), Huang et al. (1992)	[10]
8	Korf	$H = 1.3 + b_0 e^{(-b_1 d^{-b_2})}$	Lundqvist (1957)	[11]
9	Gompertz	$H = 1.3 \ + b_O \ e(^{-b_1 \ e^{(-b_2 \ D)}})$	Gompertz (1825), Huang et al. (1992)	[12]
10	Chapman-Richards	$H = 1.3 + b_0 (1 - e^{-b_1 D})^{b_2}$	Richards (1959)	[13]

Note: *H*= Total tree height (m); *D* = Diameter at breast height (cm), b_0, b_1, b_2 = model parameter estimates and *e* = base of the natural logarithm.

RESULTS

The estimated parameters b_o, b_1 and b_2 and their corresponding evaluation statistics are presented in table 3. The result showed that 3-parameter logistic model with estimated parameters of 20.6514, -0.0723 and 0.1051 for b_0, b_1, and b_2 respectively, had the smallest evaluation statistics values of RMSE (2.8925), AIC (1974) and BIC (1990). Based on the evaluation statistics criterion, logistic model was selected as the best candidate H-D model for the teak plantation in Oluwa forest Reserve. Closely following logistic model is the 3-parameter Gompertz model which as indicated in the Table 3 had the second smallest values for all the evaluation statistics. Naslund model was the worst H-D model as it consistently produced the highest values of 16.2066 (RMSE), 3342 (AIC) and 3354 (BIC). Values in table 3 further revealed that among the 2-parameter models, Michaelis-Menten model produced the smallest values of 2.9127, 1979 and 1991 for RMSE, AIC and BIC respectively, hence the best suitable 2-parameter H-D function for the data set.

Table 3: Estimated parameters and the evaluation statistics of the ten H-D models

Models	Parameters			Fit Statistics		
	b_o	b_1	b_2	RMSE	AIC	BIC
Curtis	4.6071	0.5940	-	2.9481	1989	2001
Power	4.1437	0.4313	-	2.9528	1990	2002
Naslund	-31.8670	0.7114	-	16.2066	3342	3354
Michaelis-Menten	29.3390	18.5460	-	2.9127	1979	1991
Weibull	21.1382	-0.0507	1.0803	2.9017	1977	1993
Prodan	21.5400	3.0030	−0.6560	2.9111	1980	1995
Logistic	20.6514	-1.0723	0.1051	2.8925*	1974*	1990*
Korf	26.9296	7.9327	0.8833	2.9130	1980	1996
Gompertz	21.0262	1.7141	0.0838	2.8968	1976	1992
Chapman-Richards	21.6274	0.0626	1.0223	2.9130	1977	1993

*Selected model based on evaluation statistics criterion

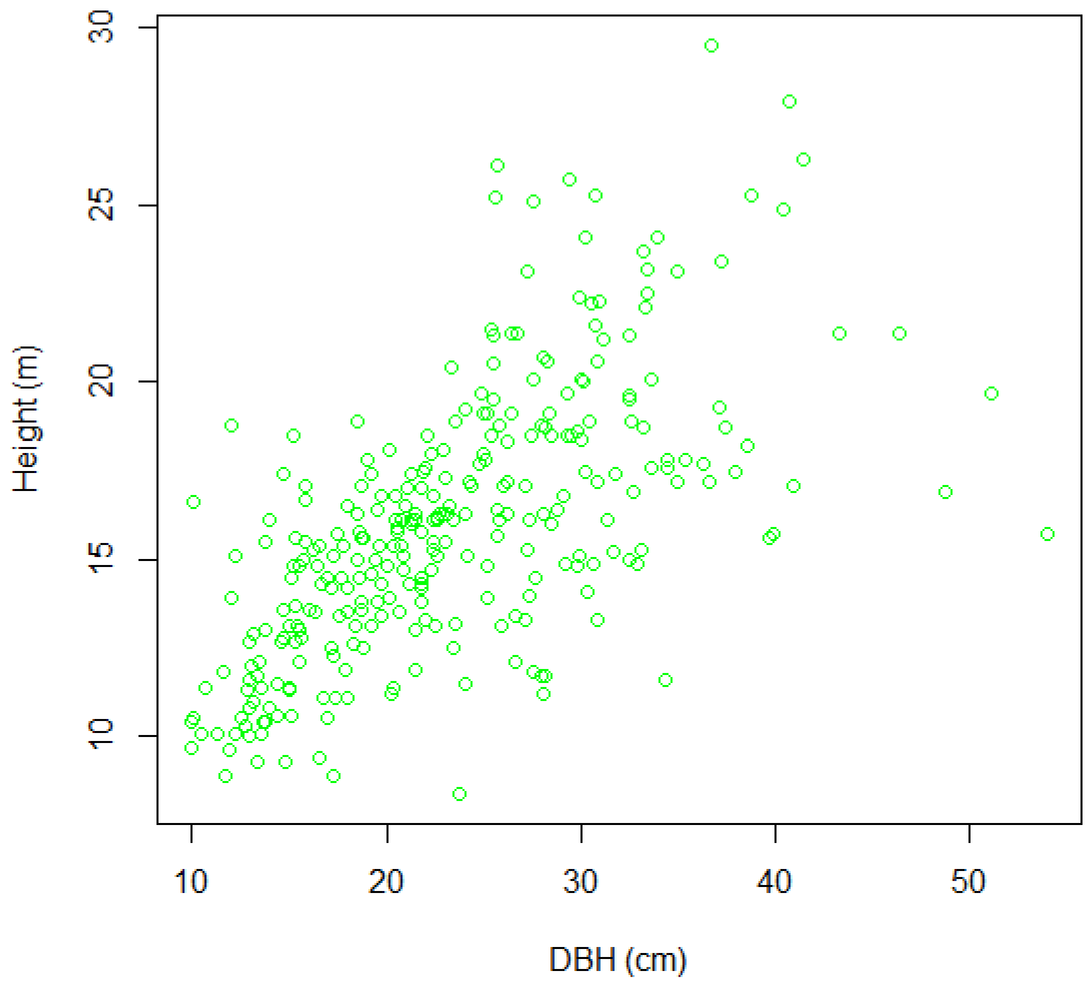

Fig 1: Scatter plot of Total height plotted against diameter at breast height (DBH) for teak trees

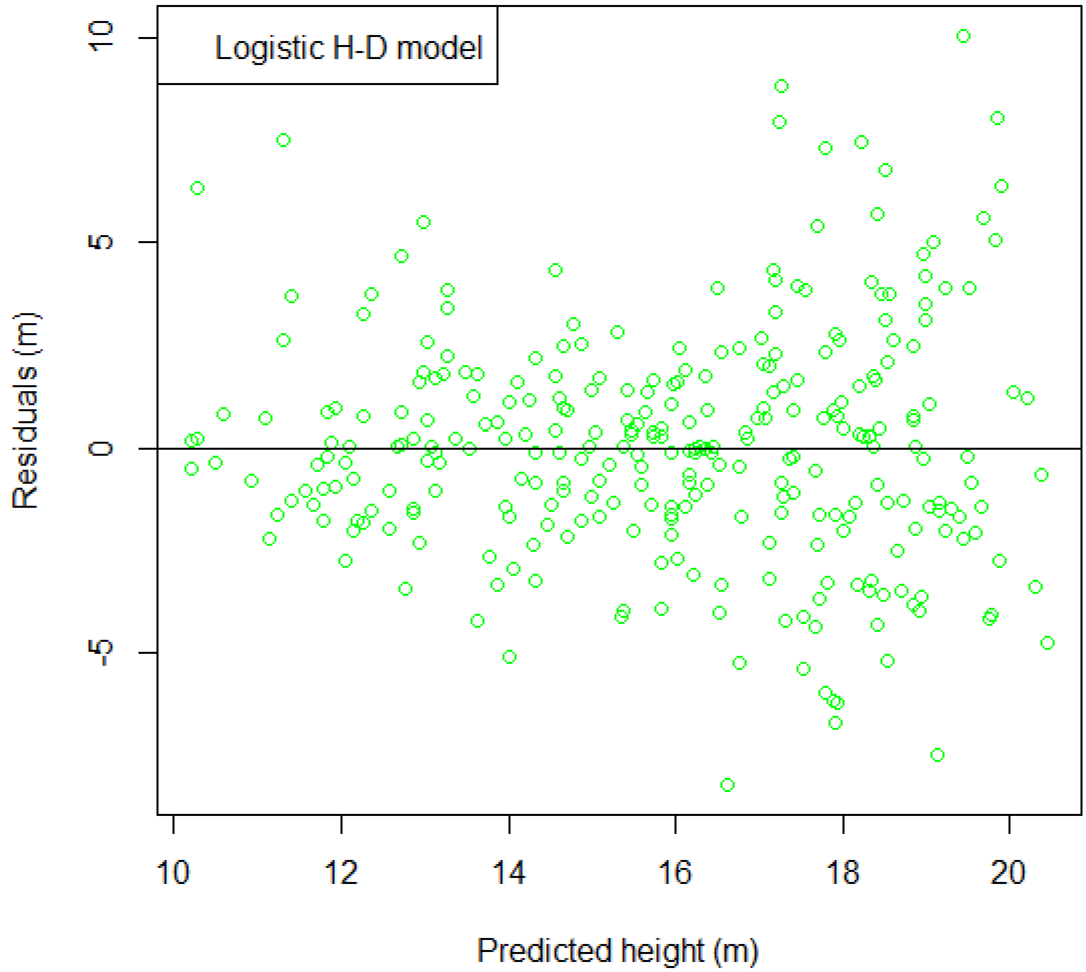

Fig 2: Residual of the selected height-diameter model

DISCUSSION

The relationship between tree height and diameter is one of the most important elements of forest structure (Zhang et al. 2014). Based on the results of this study, ten different non-linear functions were used in describing the relationships between the diameters and the heights of the Teak species in Oluwa forest reserve. Logistic model was the most suitable non-linear model for the teak stand among the functions evaluated. Gompertz, Weibull, Chapman-Richards and

Michaelis-Menten also provided good fits result comparable to the observed Height-diameter relationship.

Previous studies stated that for any appropriate H-D model, the asymptotic t-statistic for each coefficient has to be significant, the model RMSE has to be small and the standardized residual plot should show approximately homogeneous variance over the full range of predicted values (Huang et al. 1992). Selection of the best and most accurate models should be based on appropriate criteria (Zhang 1997) such as AIC, BIC, RMSE, ME and R^2_{adj} which were among the evaluation criteria used in this study.

The findings from this study were consistent with the works of Ogana (2020), Egonmwan & Ogana (2020) who reported a similar result in which Logistic model was selected as the best candidate H-D model based on evaluation statistics criterion. Mehtätalo *et al*. (2015) reported RMSE values of 2.20 and 2.79 for *Eucalyptus globulus* and mixed Tropical species, respectively for Logistic H-D model. But, Mengesha Tsega (2018) who in his work found out that Weibull H-D function was the first in overall ranking and provided the best model in predicting total height of Cupressus lusitanica in Gergeda forest. Also, Huang et al. (1992) compared nonlinear H-D functions for major Alberta tree species (16 species in 9 groups) and found that the Weibull and the generalized logistic functions were the models which gave most satisfactory results. While Naslund had RMSE of 2.29 and 2.81, respectively, for the two species in their study. Also, Shamaki *et al*. (2016) reported R2 and RMSE of 0.336 and 0.995 and 0.335 and 0.995 for Chapman-Richards and Weibull H-D models, respectively, for *Tectona grandis* in Nimbia Forest Reserve. There is no basis for comparing the fit indices values of the study at hand and others reported in forestry literatures. As the parameters of the models are locations specific and more importantly heavily depends on the quality of data used in fitting the models.

The analysis of the plot of residual and predicted values demonstrated that there was little or no systematic bias towards over-or underestimation of the tree total height (Fig 2). The assumption of ordinary nonlinear least squares regression was not violated in the models as the residual analysis showed almost homogenous variance (homoscedasticity) over the range of the predicted values and no systematic patterns. Only the residual plot of Logistic H-D model was presented. Huang *et al*. (1992) asserted that "for a good H-D model, the asymptotic t-statistic for each coefficient should be significant, the MSE should be small and the residual plot should approximate homogeneous variance over the full range of predicted values". The Logistic H-D model met these criteria relative to other established H-D functions evaluated in this study.

The majority of the models used in this study have shown to be very flexible and have been used extensively in growth and yield studies for describing height-age, diameter-age, and volume-age relationships (Somers, Farrar 1991). However, the relationship between diameter and height varies within a region, depending on local environmental conditions, and also varies within a geographic region (Özçelik et al. 2014). It is recommended that height-diameter models should be surveyed by eco-region for reflecting the regional differences. The ecoregion-based models can provide useful tools to forest resource managers in forest management practices and decision making.

CONCLUSION

This study presents an assessment of H-D relationships with ten existing H-D models using the *Tectona grandis* stand in Oluwa forest Reserve. Based on the different statistical evaluation criterion used for this study, Logistic function was selected as the best candidate H-D model as it provided more satisfactory results as compared to the other H-D functions. It was also noted that Gompertz, Weibull, Chapman-Richards and Michaelis-Menten also provided good fits result comparable to the observed Height-diameter relationship. The local H-D functions which are only dependent on tree diameter can normally be applied to the stand where the data were gathered. Therefore the models developed and evaluated could provide more reliable estimate of height for *Tectona grandis* stand in Oluwa forest Reserve by optimizing cost and time spent on forest inventory.

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