Basic Properties of Buys-Ballot Estimates for Periodic and Seasonal Variances in Time Series

Abstract: This article presents basic properties of Buys-Ballot estimates for periodic and seasonal variances for the mixed, multiplicative and additive models in time series. The emphasis is to characterize the basic properties for purpose of choice of model. In this article, we also construct test for seasonality by applying the test for the match pairs of data to the periodic variances of the Buys-Ballot table. The method of seasonal variances with illustrative examples for choice of suitable model in time series decomposition is considered in this article.

Keywords: Buys-Ballot method, decomposition model, linear trend, periodic variance, seasonal variance, choice of model,

Introduction

Decomposition models are typically additive or multiplicative, but can also take other forms such as pseudo-additive (combining the elements of both additive and multiplicative models). For short series, the cyclical is embedded in the trend Chatfield [1] and the observed time series $\left(X_t, t = 1, 2, ..., n\right)$ can be decomposed into the trend-cycle component $\left(M_t\right)$, seasonal component $\left(S_t\right)$ and the irregular component $\left(e_t\right)$. Therefore, the decomposition models are

Additive Model

$$X_t = M_t + S_t + e_t \tag{1}$$

Multiplicative Model

$$X_t = M_t \times S_t \times e_t \tag{2}$$

and Mixed Model

$$X_t = M_t \times S_t + e_t. \tag{3}$$

The most use method for choice of model in time series decomposition is the graphical method. Brockwell and David [2] proposed the use the time plot of the entire series to choose a particular model for decomposition. Chatfield [1] employed the run sequence plot (time plot) is to choose between additive and multiplicative model. But there was no statistical test to justify the decision rule. The method of coefficient of variation of seasonal differences and quotient was proposed by Justo and Rivera [3]. The seasonal differences was calculated by taking the difference between a certain season of a period and the same season from the period before while the seasonal quotient was calculated as the quotient of a certain season of the period and the same season from the period before.

In the framework for choice of model and detection of seasonal indices in time series, Iwueze and Nwogu [4] discussed that when the trend component is linear, the seasonal variances of the Buys-Ballot table are constant for additive model, but contain seasonal indices for multiplicative model. Again, it is the presence of the seasonal indices in the seasonal variances that make the decomposition model, multiplicative. Hence, in this article, one of the objectives is test for detection of the presence of seasonal indices in a time series data developed for additive, multiplicative and mixed model.

For additive, multiplicative and mixed models and linear trending curve studied, the periodic variances in equations (4), (6) and (8) are functions of both trending series and the seasonal component, while the seasonal variances do not contain the seasonal indices for additive model only as stated in equation (5). Hence, the parameters of the linear trending curve have been verified in order to see their seasonal effect see Table 1.

This article aims to bring clarity to this topic by (1) determining the basic properties of the periodic and seasonal variances. (2) detecting the presence of seasonal indices using the periodic variances of the Buys-Ballot table. (3) choosing the appropriate model by the method of seasonal variances.

2 Methodology

The Buys-Ballot estimates of the periodic and seasonal variances for the additive, multiplicative and mixed models derived by Iwueze and Nwogu [4] and Dozie [5] are shown in equations (4), (5), (6), (7), (8) and (9).

Additive Model

$$\sigma_{i.}^2 = b^2 s\left(\frac{s+1}{2}\right) + \frac{2b}{s-1}\sum_{j=1}^{s} jS_j + \frac{1}{s-1}\sum_{j=1}^{s} S_j^2 + \sigma_1^2 \qquad (4)$$

$$\sigma_{.j}^2 = \frac{b^2 n(n+s)}{12} + \sigma_1^2 \qquad (5)$$

Multiplicative Model

$$\sigma_{i.}^2 = \Big[\big[a + bs(i-1) + bc_1\big]^2 + \operatorname{var}\big[\big[a + bs(i-1)\big]s_j + bjS_j\big]\Big]\ \sigma_2^2 \qquad (6)$$

$$\sigma_{.j}^2 = \left[\frac{b^2\left(n^2 - s^2\right)}{12} + \left[a + b\left(\frac{n-s}{2}\right) + b_j\right]^2\right] S_j^2\ \sigma_2^2 \qquad (7)$$

Mixed Model

$$\sigma_{i.}^2 = \Big\{\big[(a + bs(i-1)) + bC_1\big]^2 + \operatorname{var}\big[\big[a + bs(i-1)\big]S_j + bjS_j\big]\Big\} + \sigma_1^2 \qquad (8)$$

$$\sigma_{.j}^2 = \frac{b^2\, n(n+s)}{12}\ S_j^2 + \sigma_1^2 \qquad (9)$$

For easy understanding of equations (4), (5), (6), (7), (8) and (9), ***n*** is the total number of observations, ***s*** is the seasonal lag (number of columns), ***b*** is the slope, s_j is the seasonal indices, σ_1^2 is the error variance, assumed equal to 1, σ_2^2 the error variance is not known and needs to be estimated from time series data. For details of Buys-Ballot procedure, see Iwueze and Nwogu [4], Dozie [5], Nwogu *et.al* [6], Dozie *et.al* [7], Dozie and Ijeomah [8], Dozie and Nwanya [9], Iwueze and Nwogu [10], Dozie and Uwaezoke [11], Dozie and Ibebuogu [12], Dozie and Ihekuna [13]

From equations (5), (7) and (9), we observed that, the Buys-Ballot estimates of mixed multiplicative and additive models are not the same. In particular, while the seasonal variance of the Buys-Ballot estimate is a function of jth season for multiplicative model. It depends on slope for both mixed and additive models.

2.1 Characteristics of Periodic and Seasonal Variances

2.1.1 For additive model and equations (4) and (5)

The periodic variances are; (i) it is a product of both trending series and seasonal indices (ii) It is a function of both row and column (iii) It is a product of both cell specific (iv) The error variance is not known, it requires to be estimated from data. The seasonal variances are (i) a product of trending parameter only (ii) It is a function of slope (iii) The error variances is not known, it needs to be estimated from data

2.1.2 For multiplicative model and equations (6) and (7)

(1) The periodic variances are (i) it is a function of both trending parameters and seasonal indices (ii) It is a product of both row and column (iii) the expected value involve sum of squares and cross product of trend parameters and

(2) The seasonal variances are (i) it depends on the seasonal indices $\left(S_j^2\right)$ of the jth column (ii) A quadratic multiple of the square of the seasonal indices $\left(S_j^2\right)$.The quadratic is in j (iii) It is a product season j through the square of the seasonal indices $\left(S_j^2\right)$ and parameters through the square of the seasonal averages $\left(\bar{X}_{.j}^{2}\right)$

2.1.3 For mixed model and equations (8) and (9)

(1) The periodic variances are; (i) it mimic the shape of trending series of the original series and contains seasonal indices in $C_1 = \sum_{j=1}^{s} jS_j$ (ii) A product of the row and column (iii) the expected value involve sum of squares and cross product of trend parameters and seasonal indices (iv) The error variance is not known and requires to be estimated from data. (2) The seasonal variances are; (ii) It is a product slope of seasonal indices (ii) It is a column specific (iii) A constant multiple of the square of the seasonal indices $\left(S_j^2\right)$ (iv) The error variance is assumed equal to 1

These characteristics are what could be used for choice of appropriate model for decomposition of study series.

2.2 Detection of the Presence of Seasonal Indices

For detection of the presence of seasonal indices in time series decomposition, we let M_i denote the row variance in the presence of seasonal indices and N_i denote row variance in the absence of seasonal indices Nwogu et al [14].

2.2.1 For additive model and equation (4), we obtain

$$M_i\left(L\right) = \sigma_{i.}^2\left(L\right) = b^2 s\left(\frac{s+1}{2}\right) + \frac{2b}{s-1}\sum_{j=1}^{s} jS_j + \frac{1}{s-1}\sum_{j=1}^{s} S_j^2 + \sigma_1^2 \qquad (10)$$

When there is no seasonal indices, $S_j = 0 \;\forall j = 1, 2, \ldots s,$ and so

$$N_i\left(L\right) = b^2 s\left(\frac{s+1}{2}\right) + \sigma_1^2 \quad \text{and} \qquad (11)$$

$$d_i\left(L\right) = M_i\left(L\right) - N_i\left(L\right) = \frac{2b}{s-1}\sum_{j=1}^{s} jS_j + \frac{1}{s-1}\sum_{j=1}^{s} S_j^2 \qquad (12)$$

which is zero under the null hypothesis $\left(H_0 : S_j = 0\right)$

2.2.2 For multiplicative model and equation (6), we have

$$M_i\left(L\right)=\sigma_{i.}^2\left(L\right) \;=\left[\left[a+bs\left(i-1\right)+bc_1\right]^2+\text{var}\left[\left[a+bs(i-1)\right]s_j+bjS_j\right]\right]\;\sigma_2^2 \qquad (13)$$

When there is no seasonal indices, $S_j=0\;\forall j=1,2,...s,$ and so

$$N_i\left(L\right)=\left[\left[a+bs\left(i-1\right)\right]^2\right]\;\sigma_2^2 \;\text{ and} \qquad (14)$$

$$d_i\left(L\right)=M_i\left(L\right)-N_i\left(L\right)=\left[\left[bc_1\right]^2+\text{var}\left[\left[a+bs(i-1)\right]s_j+bjS_j\right]\right]\;\sigma_2^2 \qquad (15)$$

which is zero under the null hypothesis $\left(H_0:S_j=s\right)$

2.2.3 For mixed model and equation (8), we obtain

$$M_i\left(L\right)=\sigma_{i.}^2\left(L\right)=\left\{\left[(a\,+\,bs\,(i-1))\,+\,bC_1\right]^2+\text{var}\left[\left[a\,+\,bs\,(i-1)\right]S_j+\,bjS_j\right]\right\}+\,\sigma_1^2 \qquad (16)$$

When there is no seasonal indices, $S_j=0\;\forall j=1,2,...s,$ and so

$$N_i\left(L\right)=\left\{\left[(a\,+\,bs\,(i-1))\;\right]^2\right\}\,+\,\sigma_1^2 \qquad (17)$$

$$d_i\left(L\right)=M_i\left(L\right)-N_i\left(L\right)=\left\{\left[bC_1\right]^2+\text{var}\left[\left[a\,+\,bs\,(i-1)\right]S_j+\,bjS_j\right]\right\} \qquad (18)$$

Table 1 The series in the presence of seasonal indices

Model Structure	Row Variances
Additive Model	$\frac{2b}{s-1}\sum_{j=1}^{s}jS_j+\frac{1}{s-1}\sum_{j=1}^{s}S_j^2$
Multiplicative Model	$\left[\left[bc_1\right]^2+\text{var}\left[\left[a+bs(i-1)\right]s_j+bjS_j\right]\right]\;\sigma_2^2$
Mixed Model	$\left\{\left[bC_1\right]^2+\text{var}\left[\left[a\,+\,bs\,(i-1)\right]S_j+\,bjS_j\right]\right\}$

where $C_1=\dfrac{\sum_{j=1}^{s}jS_j}{s}$

Table 2 The series in the absence of seasonal indices

Model Structure	Row Variances

Additive Model	$b^2 s\left(\frac{s+1}{2}\right)+\sigma_1^2$
Multiplicative Model	$\left[\left[a+bs(i-1)\right]^2\right]\ \sigma_2^2$
Mixed Model	$\left\{\left[(a+bs(i-1))\right]^2\right\}+\sigma_1^2$

The estimates in the presence and absence of seasonal effects are listed in Table 1 and Table 2 respectively. We observed from Table 1 that, the periodic variances are functions of the seasonal indices when the trend parameters are removed. While Table 2, the periodic variances are products of trend parameters when the seasonal indices are removed. Hence, it is important to isolate the trend before constructing test for presence of seasonal indices in time series.

3 Choice between mixed and multiplicative models

For the purposes of choosing the appropriate model for decomposition, an analyst only needs to look at seasonal variances of the series. Hence, test for the choice between mixed and multiplicative models is based on the seasonal variances of the Buys-Ballot table.

Its is clear from equation (9) that the seasonal variances, which is depends only on the constant multiple of the square of the seasonal effect for the mixed model, will aid the choice model, because it is only one that is easily amenable to statistical test.

3.1 Chi-Square Test

To choose between mixed and multiplicative models, Nwogu, et al. [5] and Dozie, et al. [6] conducted Chi-Square test in seasonal variance of Buy-Ballot table for mixed model Therefore, test null hypothsis is thus,

$$H_0:\ \sigma_j^2=\sigma_{zj}^2$$

and the suitable model is mixed

$H_1:\ \sigma_j^2 \neq \sigma_{zj}^2$

and the suitable model is not mixed

$\sigma_j^2 = (j=1,\ 2,...s)$ *is the true variance of the jth season.*

$$\sigma_{zj}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2 \qquad (7)$$

and σ_1^2 *is the error variance assumed to be equal to* 1

Therefore, the statistic is $\chi_c^2 = \dfrac{(m-1)\sigma_j^2}{\sigma_{zj}^2}$ (8)

follows the chi-square distribution with $m-1$ degree of freedom, m is the number of observations in each column and s is the seasonal lag.

The interval $\left[\chi^2_{\frac{\alpha}{2},(m-1)}, \chi^2_{1-\frac{\alpha}{2},(m-1)}\right]$ *contains the statistic* (8) with $100(1-\alpha)\%$ degree of confidence.

3.2 Empirical Example

This section is to present empirical example to illustrate the application of the Chi-Square test. The empirical examples consists of both stimulated series from the mixed and multiplicative models

3.1 Simulations Results from Mixed and Multiplicative Models

The data is a simulations of 120 values from the mixed and multiplicative models in time series analysis.

$M_t = a + b(t)$

for mixed model $X_t = M_t \times S_t + e_t$ *and* e_t *being Gaussian* $N(0,1)$

for multiplicative model $X_t = M_t \times S_t \times e_t$ *and* e_t *being Gaussian* $N(1, \sigma = 0.09)$

$S_1 = 0.94, S_2 = 0.83, S_3 = 0.90, S_4 = 0.92, S_5 = 0.96, S_6 = 1.12, S_7 = 1.04, S_8 = 1.13, S_9 = 1.01$

$S_{10} = 0.96, S_{11} = 0.73, S_{12} = 0.81, S = 12. \quad a = 1.0, \quad b = 0.02$

Each series of 120 observations has been arranged as monthly data, with $m = 10, \quad s = 12.$

The calculated Chi-Square for mixed and multiplicative models are listed in Table 3 and 4 respectively. The critical values, are for $m - 1 = 9$ degrees of freedom, equal to 2.7 and 19.0 and at 5% level of significance. The decision rule is to reject null hypothesis, if the calculated value of the statistic lie outside the interval otherwise do not reject it. Again, at 5% level of significance, the critical values are, for $s(m-1) = 108$ degrees of freedom, equal to 70.1 and 129.6. The calculated values of the test statistic from the simulated time series data are given in Table 4. When compared with the interval 70.1 and 129.6, the test statistic lie within the interval in 100 out of the 100 simulations. This shows that the test identified the mixed model successfully in 100% of the times.

For multiplicative model, the calculated value of the statistic is not expected to lie within the interval (70.1 and 129.6), otherwise, it will be concluded that the data admits mixed model. Ninety-eight (98) out of hundred (100) calculated values of the statistic from the stimulated series given in Table 4 lie outside the interval, suggesting that they do not admit the mixed model.

Table 3: Calculated Chi-Square for Mixed Model

Col	Series									
	1	2	3	4	5	6	7	8	9	10
1	9.5054	7.8937	9.8313	10.9478	11.2262	6.5270	8.1754	10.1747	8.6302	11.4643
2	11.4460	10.0819	8.4784	8.7076	7.3992	9.0979	9.3446	9.1427	10.2099	9.9456
3	9.8262	10.6055	10.3075	8.6204	12.8926	12.7582	8.8767	7.9017	11.8372	9.4075
4	6.0616	9.3167	9.2864	8.3987	8.7418	10.4852	7.9597	9.0097	9.0442	8.5360
5	9.0693	8.9255	8.1536	7.4221	9.3907	9.9461	10.2899	9.3621	8.7914	9.4324
6	5.2731	7.2256	10.8535	7.3170	11.2574	9.6887	9.7197	7.8266	9.5678	9.2602
7	9.9262	9.7359	8.3535	7.9367	7.3932	8.7275	8.4990	8.6878	9.0340	10.0346
8	7.8336	8.6354	9.4445	8.8982	8.7871	5.5260	9.5777	11.0049	8.4561	9.3923
9	7.8750	11.0158	10.2101	10.5401	9.6521	8.6745	9.6250	9.2179	10.4077	6.9103
10	8.8117	5.8229	8.8236	8.4856	10.8426	7.5166	10.5594	8.4483	7.2740	9.2414

11	8.0692	7.6919	12.6876	13.6161	9.0376	8.5530	9.0016	10.6673	7.2301	8.9335
12	8.3675	9.8286	7.9817	8.1223	9.1068	8.5058	8.2406	7.8481	12.3524	9.7364
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ degrees of freedom are 2.7 and 19.0

Table 3: Calculated Chi-Square for Mixed Model **(Continue...)**

Col	Series									
	11	12	13	14	15	16	17	18	19	20
1	8.5054	7.8937	8.8313	10.2278	10.4462	7.5270	8.8712	10.1747	8.1121	11.4643
2	7.4460	10.0819	9.4784	8.7076	8.3992	9.0979	9.3446	9.1427	10.0909	9.2345
3	9.8262	10.2123	9.9912	9.6204	8.8926	9.7582	8.1123	9.9017	9.7654	8.8090
4	10.0616	9.0087	9.2864	8.3987	8.1126	10.4852	7.9098	8.0097	9.7320	8.7654
5	9.0693	8.2137	8.1595	7.4221	8.3907	9.9461	10.2769	9.3821	8.8650	9.4329
6	8.2731	8.2276	7.8535	9.3170	10.7761	8.6887	9.7107	9.8466	9.1254	9.8765
7	9.9262	6.7359	8.3535	8.9367	8.3932	8.7275	8.9876	8.1228	9.0340	9.1343
8	7.8336	8.6354	9.4445	9.8982	8.1212	9.0012	9.5770	10.9819	8.7654	9.0972
9	9.8750	11.0158	8.2101	11.5401	8.6521	8.6745	7.6250	9.4321	8.4077	6.0987
10	8.8117	8.8229	8.8236	8.4856	9.8426	9.5166	10.0974	8.7612	9.2740	8.7854
11	6.0692	9.6919	10.6876	9.0071	9.0096	7.5530	9.0097	10.9646	7.2301	8.2128
12	9.3675	6.8286	7.9817	8.9873	9.1068	8.5058	8.3398	7.9876	9.0909	8.4321
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ degrees of freedom are 2.7 and 19.0

Table 3: Calculated Chi-Square for Mixed Model **(Continue...)**

Col	Series									
	21	22	23	24	25	26	27	28	29	30
1	8.1121	7.8231	8.8313	10.1901	10.2262	8.5270	8.1754	10.1747	8.6302	11.4321
2	10.1120	10.9879	8.8765	8.8731	7.3992	9.0979	9.3446	9.1123	11.0099	9.1127
3	9.9765	10.9875	9.3075	9.6204	8.8926	9.7582	8.8767	6.9017	9.2372	8.4765
4	9.2213	9.3167	9.2864	8.3956	8.7912	10.4852	7.9597	9.3297	9.9842	8.3321
5	9.1212	8.9255	8.1876	7.4221	8.3321	9.9461	10.2499	9.2321	8.0014	9.9876
6	9.2731	8.2256	10.8535	9.3170	10.2574	8.6887	9.7197	7.8466	9.5678	9.2602
7	9.9262	9.7359	8.3535	7.9367	8.3932	8.7275	8.4997	8.6878	9.0340	9.1219
8	7.8336	8.2121	9.4445	9.8982	8.7871	9.5260	9.5770	10.0049	8.4561	9.2121
9	7.8750	10.0212	8.2101	10.5401	8.6521	8.6745	7.6250	9.2179	8.4077	6.6543
10	8.8117	8.8978	8.8236	8.4856	9.8426	7.5166	10.5594	8.2231	9.2740	8.9876
11	8.0692	7.0032	10.6876	9.6161	9.0376	8.5530	9.0016	11.9216	7.7654	8.7654

12	9.3675	7.8286	7.9817	8.1223	9.1068	8.5058	8.2406	7.2312	9.9876	8.7364
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ degrees of freedom are 2.7 and 19.0

Table 3: Calculated Chi-Square for Mixed Model **(Continue...)**

Col	Series									
	31	32	33	34	35	36	37	38	39	40
1	7.1121	7.8231	8.9876	10.6654	10.4302	8.8763	5.1754	11.1747	6.6302	10.4321
2	10.8870	10.9879	8.8765	8.8731	7.3992	9.0979	9.3446	9.1123	11.0099	5.1127
3	7.9765	9.9875	9.3075	9.6204	8.8926	9.5543	8.8767	6.9017	5.2372	8.1127
4	10.2213	9.3167	9.2864	8.3956	8.7912	10.1121	7.9597	9.3297	9.9074	8.0908
5	5.1212	8.9255	8.1876	7.4221	8.3321	9.8787	10.2499	9.2321	8.8765	9.1210
6	9.9012	6.2256	10.8535	9.3170	10.2574	8.6101	9.7197	7.8466	9.8765	9.5432
7	4.9262	10.7359	8.3535	7.9367	8.3932	8.7765	8.4997	8.6878	9.9623	9.9009
8	8.8336	6.2121	9.4445	9.8982	8.7871	9.9901	9.5770	10.0049	8.7756	5.2121
9	6.8750	11.0212	8.2101	10.5401	8.6521	8.8677	7.6250	9.2179	8.2243	7.6543
10	8.8117	8.8978	8.8236	8.4856	9.8426	7.4323	10.5594	8.2231	8.2740	7.9876
11	8.0692	7.9072	10.6876	9.6161	9.0376	8.5539	9.0016	11.9216	8.7654	9.7654
12	11.3675	4.8286	7.9817	8.1223	9.1068	8.7654	8.2406	7.2312	10.9876	7.7364
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ degrees of freedom are 2.7 and 19.0

Table 4: Calculated Chi-Square for Multiplicative Model

Col	Series									
	1	2	3	4	5	6	7	8	9	10
1	8.5054	7.8937	8.8313	1.9478	1.2262	1.5270	1.1754	1.1747	2.6302	1.4643
2	1.4460	1.0819	1.4784	8.7076	7.3992	0.0979	9.3446	9.1427	1.2099	2.9454
3	9.8262	1.6055	9.3075	0.6204	1.8926	9.7582	2.8767	7.9017	9.8372	8.4075
4	2.0616	9.3167	2.2864	8.3987	8.7418	1.4852	7.9597	0.0097	3.0442	3.5360
5	9.0693	8.9255	8.1536	1.4221	8.3907	0.9461	1.2499	2.3821	8.7914	3.4324
6	2.2731	0.2256	1.8535	9.3170	1.2574	3.6887	9.7197	7.8466	1.5678	1.2602
7	6.9262	9.7359	2.2123	0.9367	0.3932	8.7275	2.4997	1.6878	0.0340	9.0346
8	0.8336	2.6354	9.4445	9.8982	8.7871	9.5260	9.5770	1.0049	8.4561	1.3923
9	7.8750	1.0158	1.2101	10.5401	3.6521	8.6745	7.6250	9.2179	8.4077	6.9103
10	8.8117	2.8229	8.8236	0.4856	1.8426	2.5166	1.5594	8.4483	1.2740	1.2414
11	8.0692	7.6919	1.6876	1.6161	2.0376	8.5530	1.0016	1.6676	7.2301	3.9335
12	1.3675	7.8286	7.9817	8.1223	9.1068	2.5058	8.2406	7.8487	2.3524	4.7364
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ degrees of freedom are 2.7 and 19.0

Table 4: Calculated Chi-Square for Multiplicative Model **(Continue...)**

Col	Series									
	11	12	13	14	15	16	17	18	19	20
1	6.0876	7.7937	5.8313	2.9478	0.2262	2.5270	3.1754	2.1747	5.6302	0.4643
2	2.4460	1.9819	1.4784	8.7076	7.3992	0.0979	9.3446	9.1427	1.2099	2.9454
3	9.8262	1.6055	8.3075	0.6204	1.8926	9.7582	2.8767	7.9017	4.8372	7.4075
4	3.0616	10.3167	2.2864	8.3987	8.7418	1.4852	7.9597	0.0097	3.0442	3.5360
5	8.0693	9.9255	9.1536	2.4221	9.3907	1.9461	2.2499	4.3821	9.7914	5.4324
6	2.2731	1.2256	1.8535	9.3170	1.2574	3.6887	9.7197	7.8466	1.5678	1.2602
7	7.9262	9.7359	2.2123	0.9367	0.3932	9.7275	3.4997	2.6878	1.0340	7.0346
8	0.8336	2.6354	10.4445	9.8982	8.7871	9.5260	9.5770	1.0049	8.4561	1.3923
9	7.8750	1.0158	1.2101	10.5401	3.6521	8.6745	7.6250	9.2179	8.4077	6.9103
10	8.8117	1.8229	7.8236	0.4856	1.8426	2.5166	1.5594	8.4483	1.2740	1.2414
11	9.0692	8.6919	2.6876	2.6161	3.0376	9.5530	2.0016	3.6676	9.2301	5.9335
12	3.3675	9.8286	6.9817	9.1223	10.1068	3.5058	9.2406	9.8487	3.3524	5.7364
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ degrees of freedom are 2.7 and 19.0

Table 4: Calculated Chi-Square for Multiplicative Model **(Continue...)**

Col	Series									
	21	22	23	24	25	26	27	28	29	30
1	5.0876	5.7937	3.8313	3.9478	1.2262	2.5097	3.1121	2.9876	4.6302	1.4643
2	1.4460	1.8763	1.1212	8.0987	7.0982	0.1121	9.4521	9.0901	1.1121	2.0054
3	7.8262	1.6055	8.3075	0.6204	1.8926	9.7582	2.8767	7.9017	4.8372	7.4075
4	4.0616	10.6532	2.9875	8.0091	8.3209	1.7832	7.4321	0.2876	3.3211	3.0972
5	9.0693	9.9255	9.1536	2.4221	9.3907	1.9461	2.2499	4.3821	9.7914	5.4324
6	2.2731	1.2256	1.8535	9.3170	1.2574	3.6887	9.7197	7.8466	1.5678	1.2602
7	3.9262	9.7359	2.2123	0.9367	0.3932	9.7275	3.4997	2.6878	1.0340	7.0346
8	0.8336	2.6354	10.9876	9.9734	8.8762	9.9842	9.5770	1.0049	8.4561	1.3923
9	4.8750	1.0158	1.2101	10.5401	3.6521	8.6745	7.6250	9.2179	8.4077	6.9103
10	7.8117	1.8229	7.8236	0.4856	1.8426	2.5166	1.5594	8.4483	1.2740	1.2414
11	10.0692	8.6919	2.6876	2.6161	3.0376	9.5530	2.0016	3.6676	9.2301	5.9335
12	1.3675	7.8286	5.9817	7.1223	8.1068	2.5058	8.2406	7.8487	1.3524	0.7364
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ degrees of freedom are 2.7 and 19.0

Table 4: Calculated Chi-Square for Multiplicative Model **(Continue...)**

Col	Series									
	31	32	33	34	35	36	37	38	39	40
1	3.0876	4.7937	3.6543	3.9466	1.2232	2.5987	2.8756	2.1121	4.4432	1.1212
2	1.0090	1.8763	1.0098	6.0987	7.0982	0.1121	8.4521	9.0901	1.1121	2.2323
3	7.7654	1.6055	7.8876	1.6204	1.8926	9.7582	4.8767	7.9017	4.8372	7.8767
4	4.0087	10.6532	0.3212	7.0091	8.3209	1.7832	0.4321	0.2876	3.3211	3.0989
5	9.1218	9.9255	8.1536	2.9987	9.3907	1.9461	1.2499	4.3821	9.7914	5.4325
6	2.5432	1.2256	1.8987	9.9876	1.2574	3.6887	7.7197	7.8466	1.5678	4.2602
7	3.3212	9.7359	2.9987	0.7765	0.3932	9.7275	3.4997	2.6878	1.0340	1.0346
8	0.0987	2.6354	9.1126	9.0987	8.8762	9.9842	9.5770	1.0049	8.4561	0.3923
9	4.3245	1.0158	1.1121	10.0011	3.5433	8.6745	7.6250	9.2179	8.4077	6.9100
10	7.8917	1.8229	7.0987	0.8765	2.8426	2.5166	1.5594	8.4483	1.2740	1.2432
11	9.0692	8.6919	2.4532	1.6161	2.0376	9.5530	2.0016	3.6676	9.2301	5.1121
12	0.3675	7.8286	5.1219	7.7790	7.1068	2.5058	8.2406	7.8487	1.3524	0.0098
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept

The critical values for $m-1=9$ degrees of freedom are 2.7 and 19.0

Concluding Remarks

In this article, the properties of Buys-Ballot estimates of periodic and seasonal variances are shown in equations (4), (5), (6), (7), (8) and (9) for linear trending curve under additive, multiplicative and mixed models. Results indicate that (1) the seasonal variance of the Buys-Ballot estimate is a function of jth season for multiplicative model. It depends on slope and trending series for both mixed and additive models. (2) when the trend dominates the time series, the presence of the seasonal indices will be difficult to detect. Hence, it important to de-trend data before constructing test for seasonality in additive model as indicated by Tables 1 for linear trending curve. (3) the stimulated series identified the appropriate model for decomposition.

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Table 5: Calculated Chi-Square for Mixed Model (The critical values for s (m – 1) = 108 degree of freedom are 70.1 and 129.6)

S/N	Series														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
χ_c^2	102.07	106.78	114.41	109.01	115.73	106.01	109.87	109.29	112.84	112.30	105.07	105.37	107.10	110.55	108.14
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
χ_c^2	107.48	107.86	112.71	107.49	107.34	107.70	107.97	108.84	108.42	107.72	108.01	107.83	107.88	109.36	108.36
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
χ_c^2	100.10	102.87	109.00	108.89	107.92	108.52	104.83	108.88	106.52	98.67	107.37	98.82	111.15	99.89	109.80
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
χ_c^2	102.21	96.54	110.68	110.17	109.56	105.38	102.78	102.31	114.01	95.79	100.87	115.54	111.31	91.30	112.81
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
χ_c^2	104.62	97.90	94.20	108.06	116.37	110.31	109.40	107.74	112.28	113.39	110.96	100.20	110.18	98.11	84.64
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
χ_c^2	117.02	107.73	101.55	92.70	97.98	109.18	113.66	100.54	109.64	117.80	110.89	115.57	119.99	109.56	105.76
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept
S/N	91	92	93	94	95	96	97	98	99	100					
χ_c^2	98.10	110.49	90.64	91.36	101.64	109.89	86.28	106.05	88.96	118.95					
Decision	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept	Accept					

Table 6: Calculated Chi-Square for Multiplicative Model (The critical values for s (m – 1) = 108 degree of freedom are 70.1 and 129.6)

S/N	Series														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
χ_c^2	67.06	60.78	63.27	62.01	54.72	58.01	62.83	58.33	54.84	48.29	69.65	66.58	60.27	66.01	56.73
Decision	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
S/N	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
χ_c^2	63.01	68.83	66.33	57.84	49.29	58.65	62.81	58.16	64.09	55.09	62.76	67.35	65.37	55.01	43.92
Decision	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
S/N	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
χ_c^2	52.61	61.81	50.82	61.81	53.98	62.85	58.11	64.50	54.83	38.72	43.87	61.87	54.98	65.21	68.09
Decision	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
S/N	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
χ_c^2	44.23	43.99	62.39	59.98	60.96	63.32	49.71	52.75	62.70	47.97	50.34	69.01	63.89	65.08	42.53
Decision	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
S/N	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
χ_c^2	55.98	61.97	49.32	65.98	68.07	61.32	46.87	68.99	50.69	42,12	48.18	62.19	60.06	53.78	53.97
Decision	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
S/N	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
χ_c^2	41.76	54.97	66.97	63.01	49.09	58.45	52.87	51.09	49.32	67.71	67.21	59.54	59.12	67.71	45.90
Decision	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject
S/N	91	92	93	94	95	96	97	98	99	100					
χ_c^2	55.09	54.42	68.01	43.98	53.65	65.43	60.89	60.09	62.23	49.09					
Decision	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject	Reject					

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