Original Research Article

Centralizer of a rhotrix of size 3 over a GCD domain

Abstract

Aims/ objectives: Rhotrix have received a lot of attention in recent years since they may be used as theoretical tools in subjects like engineering, physics, and other closely connected ones(7). The purpose of this paper is to study the form of the centralizer of a rhotrix of size 3 with entries from a GCD domain.

Keywords: Rhotrix; Rhotrix Ring;GCD Domain;Centralizer.

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1 Introduction

A rhotrix is a rhomboidal way of arranging an array of numbers. The idea of a rhotrix, an entity that stands in some respects between 2 by 2 and 3 by 3 matrices, was initially introduced by (1) as an extension of the initiative on matrix-tertions and matrix-noitrets proposed by (2). Ajibade defined the set of rhotrices of size three over reals as follows:

$$R_3(\mathbb{R}) = \left\{ \left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array} \right\rangle : a, b, c, d, e \in \mathbb{R} \right\}$$

The heart of any rhotrix $A_3 \in R_3(\mathbb{R})$ is the entry at the specific intersection of the vertical and horizontal diagonal represented by $h(A_3) = c$. The binary addition $(+)$, scalar multiplication and multiplication $(\circ)$ operations specified in (1) are listed below, respectively:

$$A = \left\langle \begin{array}{ccc} & a & \\ b & h(A) & d \\ & e & \end{array} \right\rangle \text{ and } B = \left\langle \begin{array}{ccc} & f & \\ g & h(B) & j \\ & k & \end{array} \right\rangle$$

$$A + B = \left\langle \begin{array}{ccc} & a+f & \\ b+g & h(A)+h(B) & d+j \\ & e+k & \end{array} \right\rangle$$

For any $k \in \mathbb{R}$

$$kA = \left\langle \begin{matrix} & ka & \\ kb & kh(A) & kd \\ & ke & \end{matrix} \right\rangle$$

and

$$A \circ B = \left\langle \begin{matrix} & ah(B)+fh(A) & \\ bh(B)+gh(A) & h(A)h(B) & dh(B)+jh(A) \\ & eh(B)+kh(A) & \end{matrix} \right\rangle$$

An alternate method of multiplication known as row-column multiplication on rhotices was proposed by (8) . The row-column method of multiplication is described as follows for any two rhotrices with the same size A and B: if

$$A = \left\langle \begin{matrix} & a & \\ b & h(A) & d \\ & e & \end{matrix} \right\rangle \text{ and } B = \left\langle \begin{matrix} & f & \\ g & h(B) & j \\ & k & \end{matrix} \right\rangle$$

Then

$$A \circ B = \left\langle \begin{matrix} & a & \\ b & h(A) & d \\ & e & \end{matrix} \right\rangle \circ \left\langle \begin{matrix} & f & \\ g & h(B) & i \\ & j & \end{matrix} \right\rangle = \left\langle \begin{matrix} & af+dg & \\ bf+eg & h(A)h(B) & ai+dj \\ & bi+ej & \end{matrix} \right\rangle$$

According to (8), row-column based rhotrix multiplication is non-commutative but associative. The identity rhotrix of size three I_3 is given by

$$I_3 = \left\langle \begin{matrix} & 1 & \\ 0 & 1 & 0 \\ & 1 & \end{matrix} \right\rangle$$

2 Preliminaries

A ring is referred to throughout as a ring with $1 \neq 0$. An integral domain R is a commutative ring with nonzero divisors.

The following definition and lemmas are available in (3)

Definition 2.1. Let R be a commutative ring and let $a, b \in R$ with $b \neq 0$.

(1) a is said to be a multiple of b if there exists an element $x \in R$ with $a = bx$. In this case b is said to divide a or be a divisor of a, written $b \mid a$.

(2) A greatest common divisor of a and b is a nonzero element d such that

(i) $d \mid a$ and $d \mid b$, and

(ii) if $d' \mid a$ and $d' \mid b$ then $d' \mid d$.

A greatest common divisor of a and b will be denoted by gcd(a, b)

Definition 2.2. Let R be an integral domain. R is a GCD domain (or R has the GCD property) if every $a, b \in R$ have a gcd in R.

Lemma 2.1. *Let R be a GCD domain. For any $a, b, c \in R$ we have: gcd(gcd$(a, b), c)$ = gcd$(a,$ gcd$(b, c))$.*

Lemma 2.2. *Let R be a GCD domain, $a, b \in R$. If gcd$(a, b) \neq 0$, then*

$$\left(\frac{a}{\text{gcd}(a,b)}, \frac{b}{\text{gcd}(a,b)} \right) = 1$$

Lemma 2.3. *Let R be a GCD domain. For any $a, b, c \in R$, if gcd$(a, b) = 1$ and $a \mid bc$, then $a \mid c$.*

Mohammed,A showed in [(5), p25] that $(R_n(F), +, \circ)$ is a non-commutative rhotrix ring of size n where $+$ and $\circ$ are the operations of rhotrix addition and row-column based multiplication of rhotrices of the same size respectively and F is an arbitrary ring.

In the follow-up to this paper, $\circ$ refers to the row-column based multiplication of rhotrices of the same size. Let R be a ring, since $(R_3(R), +, \circ)$ is a non-commutative ring, given a rhotrix $A \in R_3(R)$ our aim is to find all rhotrices in $R_3(R)$ that commute with A.
Let R be a ring and $A \in R_3(R)$ be a rhotrix of size 3. We denote $Cen_{R_3(R)}(A) = \{B \in R_3(R) \mid B \circ A = A \circ B\}$ its centralizer in $R_3(R)$.

3 Main results

Theorem 3.1. *Let $A = \left\langle \begin{matrix} & a & \\ b & c & d \\ & e & \end{matrix} \right\rangle \in R_3(R)$,where R is a GCD domain. Then*

1. *$Cen_{R_3(R)}(A) = R_3(R)$ if $a = e$ and $b = d = 0$.*
2. *$Cen_{R_3(R)}(A) = \left\{ \left\langle \begin{matrix} & f & \\ bgq^{-1} & h & dgq^{-1} \\ & f-(a-e)gq^{-1} & \end{matrix} \right\rangle \mid f, g, h \in R \right\}$ if*
   - *$b = d = 0$ and $a \neq e$ or*
   - *$b = 0$ and $d \neq 0$ or*
   - *$b \neq 0$*

   where q^{-1} is the inverse of $q = \gcd(a-e, b, d)$ in the quotient field of R.

Definition 3.1 ((4)). Scalar rhotrices are rhotrices of the form kI , where I is the identity rhotrix and k is a non-zero constant

Corollary 3.2. *Let $A \in R_3(R)$, where R is a GCD domain. If A is a scalar rhotrix then*

$$Cen_{R_3(R)}(A) = R_3(R)$$

To prove the theorem 3.1 we need the following lemmas:

Lemma 3.3. *Let $A = \left\langle \begin{matrix} & a & \\ b & c & d \\ & e & \end{matrix} \right\rangle \in R_3(F)$, F a field. Then*

$Cen_{R_3(F)}(A) = R_3(F)$ if $a = e$ and $b = d = 0$

Proof. Assume $a = e$ and $b = d = 0$. Then

$$\left\langle \begin{matrix} & f & \\ g & h & j \\ & k & \end{matrix} \right\rangle \in Cen_{R_3(F)}(A) \Leftrightarrow \left\langle \begin{matrix} & f & \\ g & h & j \\ & k & \end{matrix} \right\rangle \circ \left\langle \begin{matrix} & e & \\ 0 & c & 0 \\ & e & \end{matrix} \right\rangle = \left\langle \begin{matrix} & e & \\ 0 & c & 0 \\ & e & \end{matrix} \right\rangle \circ \left\langle \begin{matrix} & f & \\ g & h & j \\ & k & \end{matrix} \right\rangle$$

$$\Leftrightarrow \left\langle \begin{matrix} & fe & \\ ge & hc & je \\ & ke & \end{matrix} \right\rangle = \left\langle \begin{matrix} & ef & \\ eg & ch & ej \\ & ek & \end{matrix} \right\rangle$$

Which is always true for all $f, g, h, j, k \in F$. Thus

$$Cen_{R_3(F)}(A) = R_3(F)$$

□

Lemma 3.4. *Let* $A = \left\langle \begin{matrix} & a & \\ b & c & d \\ & e & \end{matrix} \right\rangle \in R_3(F)$ *, F a field. Then*

$$Cen_{R_3(F)}(A) = \left\{ \left\langle \begin{matrix} & f & \\ 0 & h & 0 \\ & k & \end{matrix} \right\rangle \mid f,h,k \in F \right\} \text{ if } a \neq e, \text{and } b = d = 0$$

Proof. Assume that $b = d = 0$ and $a \neq e$. Then

$$\left\langle \begin{matrix} & f & \\ g & h & j \\ & k & \end{matrix} \right\rangle \in Cen_{R_3(F)}(A) \Leftrightarrow \left\langle \begin{matrix} & f & \\ g & h & j \\ & k & \end{matrix} \right\rangle \circ \left\langle \begin{matrix} & a & \\ 0 & c & 0 \\ & e & \end{matrix} \right\rangle = \left\langle \begin{matrix} & a & \\ 0 & c & 0 \\ & e & \end{matrix} \right\rangle \circ \left\langle \begin{matrix} & f & \\ g & h & j \\ & k & \end{matrix} \right\rangle$$

$$\Leftrightarrow \left\langle \begin{matrix} & fa & \\ ga & hc & je \\ & ke & \end{matrix} \right\rangle = \left\langle \begin{matrix} & af & \\ eg & ch & aj \\ & ek & \end{matrix} \right\rangle$$

Hence

$$\begin{cases} ga = eg \\ je = aj \end{cases} \Leftrightarrow \begin{cases} g(a-e) = 0 \\ j(a-e) = 0 \end{cases} \Leftrightarrow \begin{cases} g = 0 \\ j = 0 \end{cases}$$

Thus $Cen_{R_3(F)}(A) \subseteq \left\{ \left\langle \begin{matrix} & f & \\ 0 & h & 0 \\ & k & \end{matrix} \right\rangle : f,h,k \in F \right\}$

Direct calculation shows that for any arbitrary $f,h,k \in F$,

$$\left\{ \left\langle \begin{matrix} & f & \\ 0 & h & 0 \\ & k & \end{matrix} \right\rangle \mid f,h,k \in F \right\} \subseteq Cen_{R_3(F)}(A)$$

We come to the conclusion that $Cen_{R_3(F)}(A) = \left\{ \left\langle \begin{matrix} & f & \\ 0 & h & 0 \\ & k & \end{matrix} \right\rangle \mid f,h,k \in F \right\}$

□

Lemma 3.5. *Let* $A = \left\langle \begin{matrix} & a & \\ b & c & d \\ & e & \end{matrix} \right\rangle \in R_3(F)$ *, F a field. Then*

$$Cen_{R_3(F)}(A) = \left\{ \left\langle \begin{matrix} & f & \\ g & h & 0 \\ & f - b^{-1}(a-e)g & \end{matrix} \right\rangle \mid f,g,h \in F \right\} \text{ if } d = 0 \text{ and } b \neq 0$$

Proof. Assume that $d = 0$ and $b \neq 0$. Then

$$\left\langle \begin{matrix} & f & \\ g & h & j \\ & k & \end{matrix} \right\rangle \in Cen_{R_3(F)}(A) \Leftrightarrow \left\langle \begin{matrix} & a & \\ b & c & 0 \\ & e & \end{matrix} \right\rangle \circ \left\langle \begin{matrix} & f & \\ g & h & j \\ & k & \end{matrix} \right\rangle = \left\langle \begin{matrix} & f & \\ g & h & j \\ & k & \end{matrix} \right\rangle \circ \left\langle \begin{matrix} & a & \\ b & c & 0 \\ & e & \end{matrix} \right\rangle$$

$$\Leftrightarrow \left\langle \begin{matrix} & af & \\ bf+eg & hc & aj \\ & bj+ek & \end{matrix} \right\rangle = \left\langle \begin{matrix} & fa+jb & \\ ga+kb & ch & je \\ & ke & \end{matrix} \right\rangle$$

So $\begin{cases} af = fa + jb \\ bf + eg = ga + kb \end{cases} \Leftrightarrow \begin{cases} j = 0 \\ k = f - b^{-1}(a-e)g \end{cases}$

Thus $Cen_{R_3(F)}(A) \subseteq \left\{ \left\langle \begin{array}{ccc} & f & \\ g & h & 0 \\ & f-b^{-1}(a-e)g & \end{array} \right\rangle \mid f,g,h \in F \right\}$

Additionally, direct computation demonstrates that for every $f,g,h \in F$,

$$\left\{ \left\langle \begin{array}{ccc} & f & \\ g & h & 0 \\ & f-b^{-1}(a-e)g & \end{array} \right\rangle : f,g,h \in F \right\} \subseteq Cen_{R_3(F)}(A)$$

We get to the conclusion that

$Cen_{R_3(F)}(A) = \left\{ \left\langle \begin{array}{ccc} & f & \\ g & h & 0 \\ & f-b^{-1}(a-e)g & \end{array} \right\rangle \mid f,g,h \in F \right\}$

□

Lemma 3.6. *Let* $A = \left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array} \right\rangle \in R_3(F)$ *, F a field. Then*

$Cen_{R_3(F)}(A) = \left\{ \left\langle \begin{array}{ccc} & f & \\ d^{-1}bj & h & j \\ & f-d^{-1}(a-e)j & \end{array} \right\rangle \mid j,f,h \in F \right\}$ *if* $d \neq 0$

Proof. Assume that $d \neq 0$. Then

$$\left\langle \begin{array}{ccc} & f & \\ g & h & j \\ & k & \end{array} \right\rangle \in Cen_{R_3(F)}(A) \Leftrightarrow \left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array} \right\rangle \circ \left\langle \begin{array}{ccc} & f & \\ g & h & j \\ & k & \end{array} \right\rangle = \left\langle \begin{array}{ccc} & f & \\ g & h & j \\ & k & \end{array} \right\rangle \circ \left\langle \begin{array}{ccc} & a & \\ b & c & d \\ & e & \end{array} \right\rangle$$

$$\Leftrightarrow \left\langle \begin{array}{ccc} & af+dg & \\ bf+eg & hc & aj+dk \\ & bj+ek & \end{array} \right\rangle = \left\langle \begin{array}{ccc} & fa+jb & \\ ga+kb & ch & fd+je \\ & gd+ke & \end{array} \right\rangle$$

So $\begin{cases} af+dg = fa+jb \\ aj+dk = fd+je \end{cases} \Leftrightarrow \begin{cases} g = d^{-1}jb \\ k = f-d^{-1}(a-e)j \end{cases}$

Thus $Cen_{R_3(F)}(A) \subseteq \left\{ \left\langle \begin{array}{ccc} & f & \\ d^{-1}jb & h & j \\ & f-d^{-1}(a-e)j & \end{array} \right\rangle \mid j,f,h \in F \right\}$

Direct verification also demonstrates that for every arbitrary $j,f,h \in F$,

$$\left\{ \left\langle \begin{array}{ccc} & f & \\ d^{-1}jb & h & j \\ & f-d^{-1}(a-e)j & \end{array} \right\rangle \mid j,f,h \in F \right\} \subseteq Cen_{R_3(F)}(A)$$

we conclude that $Cen_{R_3(F)}(A) = \left\{ \left\langle \begin{array}{ccc} & f & \\ d^{-1}jb & h & j \\ & f-d^{-1}(a-e)j & \end{array} \right\rangle \mid j,f,h \in F \right\}$

□

Lemma 3.7. *Let* $A \in R_3(R)$ *,* $R \subset F$*, where F is a field and R is a subring of F. Then*

$$Cen_{R_3(R)}(A) = Cen_{R_3(F)}(A) \cap R$$

Proof.

$$B \in \text{Cen}_{R_3(R)}(A) \Leftrightarrow B \in R \subseteq F \text{ and } B \circ A = A \circ B$$
$$\Leftrightarrow B \in R \cap Cen_F(A)$$

□

Proof of the theorem 3.1:
Let R be a GCD domain and F its field of fractions(quotient field) .

1. If $a = e$ and $b = d = 0$. Then by Lemma 3.3 and Lemma 3.7

$$Cen_{R_3(R)}(A) = Cen_{R_3(F)}(A) \cap R = R_3(F) \cap R = R_3(R)$$

2. (i) If $b = d = 0$ and $a \neq e$, in this case $q := \gcd(a - e, 0, 0) = a - e$. Consequently, it follows from Lemma 3.4 and Lemma 3.7 that

$$\begin{aligned} Cen_{R_3(R)}(A) &= \left\{ \left\langle \begin{matrix} & f & \\ 0 & h & 0 \\ & k & \end{matrix} \right\rangle \mid f, h; k \in F \right\} \cap R_3(R) \\ &= \left\{ \left\langle \begin{matrix} & f & \\ 0 & h & 0 \\ & f - g & \end{matrix} \right\rangle \mid f, g, h \in F \right\} \cap R_3(R) \\ &= \left\{ \left\langle \begin{matrix} & f & \\ 0 & h & 0 \\ & f - g & \end{matrix} \right\rangle \mid f, g, h \in R \right\} \\ &= \left\{ \left\langle \begin{matrix} & f & \\ 0g(a-e)^{-1} & h & 0g(a-e)^{-1} \\ & f - (a-e)(a-e)^{-1}g & \end{matrix} \right\rangle \mid f, g, h \in R \right\} \\ &= \left\{ \left\langle \begin{matrix} & f & \\ bgq^{-1} & h & dgq^{-1} \\ & f - (a-e)(a-e)^{-1}g & \end{matrix} \right\rangle \mid f, g, h \in R \right\}. \end{aligned}$$

(ii) If $d = 0$ and $b \neq 0$, then it follows from Lemma 3.5 and Lemma 3.7 that

$$\begin{aligned} Cen_{R_3(R)}(A) &= \left\{ \left\langle \begin{matrix} & f & \\ g & h & 0 \\ & f - b^{-1}(a-e)g & \end{matrix} \right\rangle \mid f, g, h \in F \right\} \cap R_3(R) \\ &= \left\{ \left\langle \begin{matrix} & f & \\ g & h & 0 \\ & f - b^{-1}(a-e)g & \end{matrix} \right\rangle \mid f, g, h \in R \right\}. \end{aligned}$$

Let B be any element of $Cen_{R_3(R)}(A)$. It follows from above that

$$B = \left\langle \begin{matrix} & f & \\ g & h & 0 \\ & f - b^{-1}(a-e)g & \end{matrix} \right\rangle \in R_3(R)$$

for some $f, g, h \in R$.We will now demonstrate that

$$B = \left\langle \begin{matrix} & f & \\ buq^{-1} & h & 0 \\ & f - (a-e)uq^{-1} & \end{matrix} \right\rangle$$

for some $u \in R$. Since

$$q := \gcd(a-e, d, b) = \gcd(a-e, 0, b) = \gcd(a-e, b)$$

it follows from Lemma2.2 that

$$\begin{cases} b = qb' \\ a-e = ql \end{cases}$$

for some $b', l \in R$ such that $\gcd(b', l) = 1$. Because $g(a-e)b^{-1} \in R$, it follows that

$$g(a-e)b^{-1} = gql\left(qb'\right)^{-1} = \mathrm{gl}\left(b'\right)^{-1} \in R.$$

since $\gcd(b', l) = 1$ it follows from Lemma2.3 that $b' \mid g$, which implies that

$$g = ub'$$

for some $u \in R$. Hence, it follows that

$$\begin{aligned} B &= \left\langle \begin{matrix} ub' & \begin{matrix} f \\ h \\ f - b^{-1}(a-e)g \end{matrix} & 0 \end{matrix} \right\rangle \\ &= \left\langle \begin{matrix} ub' & \begin{matrix} f \\ h \\ f - u(a-e)q^{-1} \end{matrix} & 0 \end{matrix} \right\rangle \\ &= \left\langle \begin{matrix} ubq^{-1} & \begin{matrix} f \\ h \\ f - u(a-e)q^{-1} \end{matrix} & 0 \end{matrix} \right\rangle \end{aligned}$$

Since

$$\begin{aligned} g(a-e)b^{-1} &= gql\left(qb'\right)^{-1} \\ &= gl\left(b'\right)^{-1} \\ &= ub'l\left(b'\right)^{-1} \\ &= ul \\ &= u(a-e)q^{-1} \end{aligned}$$

Thus,

$$Cen_{R_3(R)}(A) \subseteq \left\{ \left\langle \begin{matrix} bgq^{-1} & \begin{matrix} f \\ h \\ f - g(a-e)q^{-1} \end{matrix} & 0 \end{matrix} \right\rangle \mid f, g, h \in R \right\}$$

and by letting $c = bgq^{-1}$ then $f - g(a-e)q^{-1} = f - (a-e)b^{-1}c$, hence

$$\begin{aligned} \left\{ \left\langle \begin{matrix} bgd^{-1} & \begin{matrix} f \\ \alpha \\ f - g(a-e)q^{-1} \end{matrix} & 0 \end{matrix} \right\rangle \mid f, g, h \in R \right\} &\subseteq \left\{ \left\langle \begin{matrix} c & \begin{matrix} f \\ h \\ f - (a-e)b^{-1}c \end{matrix} & 0 \end{matrix} \right\rangle \mid f, c, h \in R \right\} \\ &= Cen_{R_3(R)}(A) \end{aligned}$$

Therefore, we conclude that

$$\begin{aligned} Cen_{R_3(R)}(A) &= \left\{ \left\langle \begin{matrix} gbq^{-1} & \begin{matrix} f \\ h \\ f - (a-e)gq^{-1} \end{matrix} & 0 \end{matrix} \right\rangle \mid f, g, h \in R \right\} \\ &= \left\{ \left\langle \begin{matrix} bgq^{-1} & \begin{matrix} a \\ h \\ f - (a-e)gq^{-1} \end{matrix} & dgq^{-1} \end{matrix} \right\rangle \mid f, g, h \in R \right\} \end{aligned}$$

(iii) If $d \neq 0$ then, it follows from Lemma 3.6 and Lemma 3.7 that

$$Cen_{R_3(R)}(A) = \left\{ \left\langle \begin{array}{ccc} & a & \\ d^{-1}bj & h & j \\ & f - d^{-1}(a-e)j & \end{array} \right\rangle \mid f, h, j \in F \right\} \cap R_3(R)$$

$$= \left\{ \left\langle \begin{array}{ccc} & f & \\ d^{-1}bj & h & j \\ & f - d^{-1}(a-e)j & \end{array} \right\rangle \mid f, h, j \in R \right\}.$$

Let B be an arbitrary element of $Cen_{R_3(R)}(A)$. Then it follows that

$$B = \left\langle \begin{array}{ccc} & f & \\ d^{-1}bj & h & j \\ & f - (a-e)jd^{-1} & \end{array} \right\rangle$$

for some $f, h, j \in R$. We now show that

$$B = \left\langle \begin{array}{ccc} & f & \\ bgq^{-1} & h & dgq^{-1} \\ & f - (a-e)gq^{-1} & \end{array} \right\rangle$$

for some $f, g, h \in R$. Now, let

$$d_1 := \gcd(b, d).$$

Then by Lemma 2.2

$$\begin{cases} b = d_1 b' \\ d = d_1 d' \end{cases}$$

for some $b', d' \in R$ such that $\gcd(b', d') = 1$. Since,

$$bjd^{-1} = d_1 b' j \left(d_1 d'\right)^{-1} = b' j \left(d'\right)^{-1} \in R$$

hence, $b'j = \alpha d', \alpha \in R$ and since $\gcd(b', d') = 1$, it follows by Lemma2.3 that $d' \mid j$. Thus

$$j = d'j'$$

for some $j' \in R$. Hence, it follows that

$$bjd^{-1} = d_1 b' d' j' \left(d_1 d'\right)^{-1} = b'j'.$$

Furthermore, it follows that

$$(a-e)jd^{-1} = (a-e)j'd' \left(d_1 d'\right)^{-1} = (a-e)j' d_1^{-1}$$

and so, that

$$B = \left\langle \begin{array}{ccc} & f & \\ d^{-1}bj & h & j \\ & f - (a-e)jd^{-1} & \end{array} \right\rangle$$

$$= \left\langle \begin{array}{ccc} & f & \\ b'j' & h & d'j' \\ & f - (a-e)j'd_1^{-1} & \end{array} \right\rangle$$

Since $q := \gcd(b, d, a-e)$ and by Lemma 2.1 $q = \gcd(\gcd(b, d), a-e) = \gcd(d_1, a-e)$, it follows by Lemma 2.2 that

$$\begin{cases} d_1 = d_1' q \\ (a-e) = lq \end{cases}$$

for some $d_1', l \in R$ such that $\gcd(d_1', l) = 1$. Since,

$$(a-e)j'd_1^{-1} = (a-e)j'\left(d_1'q\right)^{-1} = lqj'q^{-1}(d_1')^{-1} = lj'\left(d_1'\right)^{-1} \in R$$

hence,

$$lj' = d_1'\alpha$$

for some $\alpha \in R$ thus $d_1' \mid lj'$ and since $\gcd(d_1', l) = 1$, it follows by Lemma2.3 that $d_1' \mid j'$. Therefore

$$j' = cd_1'$$

for some $c \in R$. Thus,

$$(a-e)j'd_1^{-1} = lqj'\left(d_1'\right)^{-1} = lqcd_1'\left(d_1'q\right)^{-1} = lc.$$

since $\begin{cases} d_1 = d_1'q \\ a-e = lq \end{cases}$ implies $\begin{cases} d_1' = d_1q^{-1} \\ l = (a-e)q^{-1} \end{cases}$ it follows that

$$\begin{aligned} B &= \left\langle \begin{matrix} b'cd_1' & \begin{matrix} f \\ h \\ f-lc \end{matrix} & d'cd_1' \end{matrix} \right\rangle \\ &= \left\langle \begin{matrix} b'cq^{-1}d_1 & \begin{matrix} f \\ h \\ f-(a-e)cq^{-1} \end{matrix} & d'cq^{-1}d_1 \end{matrix} \right\rangle \\ &= \left\langle \begin{matrix} (b'd_1)cq^{-1} & \begin{matrix} f \\ h \\ f-(a-e)cq^{-1} \end{matrix} & (d'd_1)cq^{-1} \end{matrix} \right\rangle \\ &= \left\langle \begin{matrix} bcq^{-1} & \begin{matrix} f \\ h \\ f-(a-e)cq^{-1} \end{matrix} & dcq^{-1} \end{matrix} \right\rangle \end{aligned}$$

Thus,

$$Cen_{R_3(R)}(A) \subseteq \left\{ \left\langle \begin{matrix} bgq^{-1} & \begin{matrix} f \\ h \\ f-(a-e)gq^{-1} \end{matrix} & dgq^{-1} \end{matrix} \right\rangle \mid f, g, h \in R \right\}$$

and

$$\left\{ \left\langle \begin{matrix} bgq^{-1} & \begin{matrix} f \\ h \\ f-(a-e)gq^{-1} \end{matrix} & dgq^{-1} \end{matrix} \right\rangle \mid f, g, h \in R \right\} =$$

$$\left\{ \left\langle \begin{matrix} bd^{-1}(dgq^{-1}) & \begin{matrix} f \\ h \\ f-(a-e)gq^{-1} \end{matrix} & dgq^{-1} \end{matrix} \right\rangle \mid f, g, h \in R \right\} \subseteq$$

$$\left\{ \left\langle \begin{matrix} bd^{-1}j & \begin{matrix} f \\ h \\ f-(a-e)d^{-1}j \end{matrix} & j \end{matrix} \right\rangle \mid f, h, j \in R \right\} = Cen_{R_3(R)}(A)$$

Hence we conclude that

$$Cen_{R_3(R)}(A) = \left\{ \left\langle \begin{matrix} gbq^{-1} & \begin{matrix} f \\ h \\ f-(a-e)gq^{-1} \end{matrix} & dgq^{-1} \end{matrix} \right\rangle \mid f, g, h \in R \right\}$$

4 Conclusion

In this paper, we have extended the idea of centralizer of a matrix to a rhotrix. A characteization of the centralizer of a rhotrix of size 3 with entries from a field and from a gcd domain are presented.

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