
Research progress on trajectory planning of industrial robots

Abstract: The research status of industrial robot trajectory planning is discussed in detail, and the applicable occasions of cubic polynomial interpolation, quintic polynomial interpolation, B-spline curve, mixed polynomial interpolation and other methods commonly used in basic trajectory planning are comprehensively analyzed. At the same time, various methods of optimal trajectory planning for industrial robots are comprehensively reviewed. The advantages and disadvantages of various methods are compared and the important research direction of optimal trajectory planning is prospected.

Keywords: industrial robot ; trajectory planning ; intelligent optimization algorithm ; optimal trajectory

The trajectory planning of the robot is the most basic and important and plays a decisive role in the overall research of the robot [1]. For mobile robots, its trajectory planning is biased towards path trajectory planning [2], which is intended to plan how mobile robots can reach the designated location according to the specified route in a flat working environment or a working environment full of obstacles, or how to avoid various obstacles to reach the target point smoothly and safely. For industrial robots, industrial robots are mostly multi-degree-of-freedom joint manipulators [3], which are used in the processing and manufacturing of intelligent factory automation production lines. The trajectory planning of industrial robots is to plan its curve trajectory for the end effector of its manipulator to reach the target point for work. Therefore, the trajectory planning of the robot is to calculate the curve of displacement, velocity and acceleration in the process of movement through appropriate methods to ensure that the robot can work smoothly and normally. A suitable trajectory planning of a robot can make the robot work more smoothly and normally when it really works. Further optimization of the trajectory can also optimize the working time and energy of the robot.

Industrial robot trajectory planning can be divided into Cartesian space trajectory planning and joint space trajectory planning according to space [4]. Cartesian space trajectory planning can more clearly and intuitively show the path after trajectory planning, and the goal of joint space trajectory planning is the angle, angular velocity and angular acceleration of each joint. Different from the intuitiveness of Cartesian space trajectory planning, the results of joint space trajectory planning are detailed variable data. When entering the robot controller, the complex inverse solution process can be avoided [5-8], and the results of joint space trajectory planning can be directly input, and some robots can be avoided. Singular position during trajectory operation.

Nowadays, with the advent of the Internet of Things era, intelligent 5g factories are also increasingly used in emerging production workshops, in

which industrial robots play an irreplaceable role. In order to further optimize the work efficiency of industrial robots and reduce energy consumption, more and more scholars at home and abroad are also engaged in the research of industrial robot trajectory planning and trajectory optimization. Based on this research trend, this paper will summarize and analyze the advantages and disadvantages of various trajectory planning methods in detail, point out the key problems that may be encountered and look forward to the future development direction of related technologies.

1 Research on general trajectory planning method

According to the different trajectory planning space, the algorithms used are also different. The research methods of trajectory planning commonly used in Cartesian space mainly include linear interpolation and circular interpolation [9-11], while the research methods of joint space trajectory planning include cubic polynomial interpolation, quintic polynomial interpolation, high-order polynomial interpolation, mixed polynomial interpolation, spline curve interpolation [12-13]. Compared with Cartesian space trajectory planning, joint space trajectory planning can only show the value of each joint variable and cannot obtain the trajectory of the end effector. However, considering the factors such as calculation difficulty, different working conditions and path complexity, the trajectory planning methods of the two spaces have their most suitable application scenarios.

1.1 Cartesian space trajectory planning

When the robot is running in the working conditions of processing products or complex paths, we need to understand the trajectory shape of the end effector between two points. [14]. There are many scholars at home and abroad to study the linear acceleration and deceleration, S-curve acceleration and deceleration used in Cartesian space. Guo Xingui [15] and others improved the traditional linear acceleration and deceleration algorithm and proposed an improved linear acceleration and deceleration algorithm. The algorithm is applied to the machine tool to improve the efficiency of the machine tool. Chen Youdong [16-17] and others applied linear acceleration and deceleration and S-curve acceleration and deceleration to the robot and improved it. The improved linear acceleration and deceleration iteration formula and S-curve iteration formula can well predict the deceleration time point in the iteration process to eliminate the low-speed stage and improve the efficiency of robot operation. Kim Doang Nguyen et al. [18] made a more systematic study on the motion contour of S-curves, and proposed a new special strategy. This strategy uses a triangular model to plan the contour of S-curves, and verifies the better application ability of S-curves through a time-optimal S-curve trajectory planning method. Fang Yi and Hu Jie [19] proposed an S-curve trajectory planning method based on piecewise sigmoid function and applied it to industrial robots. The trajectory curve planned by this

method can be wirelessly differentiated and adjustable. It is verified by experiments that it has higher efficiency than the triangular model and the solution process is simpler than the traditional high-order complex S-curve, which can better adapt to the operation tasks under different working conditions. The S-curve is smoother and has three stages of uniform speed, acceleration and deceleration. In the acceleration and deceleration stages, the S-curve also has motion states such as deceleration, acceleration and deceleration, which can better avoid the sudden change of speed and acceleration and the impact on the moving robot.

The above research is aimed at the linear trajectory between two points. It is more appropriate to use arc interpolation for complex trajectories with path turning points. The key of arc interpolation is to obtain the arc center and radius of the transition point in the trajectory and ensure the minimum impact of the transition position. Cyrille Froissart et al. [20] summarized and applied Cartesian space trajectory planning with continuous velocity and acceleration on industrial robots, which was a novel method for real-time trajectory planning in Cartesian space at that time. Xiushan LIU et al. [21] established a new solution method for the problem of slope attitude and inclination angle extremum of spatial arc trajectory. Through this method, the trajectory tangent of the minimum and maximum points of inclination angle can be obtained. The control method of spatial arc trajectory in this case is studied and demonstrated, which provides a better research idea for subsequent related research. Chen Guoliang et al. [22] transformed the three-dimensional problem of spatial arc trajectory planning into two-dimensional space for solution, and proposed a controllable precision arc interpolation method to achieve better circular trajectory contour. Wang Shaoxiao [23] applied the artificial life system method to the arc trajectory planning of the robot and played a good role in trajectory optimization. The results of Cartesian space arc trajectory planning are smooth and accurate, which is more suitable for fine continuous machining operations.

1.2 Joint space trajectory planning

1.2.1 Polynomial interpolation trajectory planning

The joint space trajectory planning corresponds to the change of the angle, angular velocity and angular acceleration of each joint of the robot [24]. Although the real-time position change of the end effector cannot be determined, the singular position of the robot can be avoided by the constraint of each joint rotation. For point-to-point joint space trajectory planning, several common polynomial interpolation algorithms and hybrid polynomial interpolation algorithms can be used, and the robot can perform trajectory planning through spline curves in the task of multi-path points under complex working conditions.

The cubic polynomial trajectory planning only needs to know the position

of the two points and ensure that the speed of the starting and ending points is 0, so the solution is relatively simple without other constraints. Ma et al. [25] applied the cubic polynomial interpolation algorithm to a four-degree-of-freedom writing robot and successfully planned the trajectory between two points. Zhou [26] applied cubic polynomial interpolation trajectory planning in the joint space of the four-degree-of-freedom manipulator. For special cases that need to pass through the intermediate path points, multi-segment cubic polynomial interpolation calculation is used. Finally, the joint variables of the intermediate path points are solved by inverse kinematics to make the trajectory smoother, which provides a more diverse application scenario for the industrial four-axis manipulator in processing and production. Although the cubic polynomial interpolation calculation is relatively simple and can obtain the complete trajectory curve of each joint, the lack of acceleration constraints may lead to a large degree of sudden change in acceleration. In this case, it will cause impact to accelerate the wear damage of each joint when the robot is working.

On the basis of cubic polynomial trajectory planning, the fifth-order polynomial adds the constraint condition of acceleration to ensure that the angle, angular velocity and angular acceleration change function of each joint trajectory planning is a smooth curve. In order to test the advantages of the fifth-order polynomial trajectory planning, many scholars have also applied it to various robots for experimental verification. Sun et al. [27] constructed a robot simulation platform through VC++ 6.0 and applied the cubic polynomial and the quintic polynomial to the six-degree-of-freedom chain manipulator for verification. The experimental results show that the trajectory curve of the cubic polynomial meets the general requirements, but the acceleration change curve is a three-segment discontinuous line segment. At this time, it has a great impact on the motor of each joint of the manipulator, and the acceleration change of the quintic polynomial trajectory planning is smooth and continuous at the transition. There is no breakpoint. Although the calculation is troublesome, it avoids the impact of the sudden discontinuous acceleration, which can better ensure the safety of the industrial manipulator and reduce the maintenance cost. If the acceleration of the transition section and the shape of the trajectory set are constrained at the same time, the smoothness of the trajectory can be better guaranteed. For this reason, Liu et al. [28] proposed a double 5-order polynomial trajectory planning algorithm. By giving geometric constraints such as transition radius and maximum acceleration constraints, the coefficients of the transition section curve are calculated to determine the transition section trajectory. The smooth trajectory of the transition stage makes the continuous operation of the industrial manipulator more stable and reliable and improves the accuracy of the trajectory operation. Higher-order interpolation algorithms such as seventh-order polynomials and ninth-order polynomials can further improve the fitting accuracy of curve interpolation. Marek Boryga [29] added multiple constraints to the original basis for

high-order polynomial trajectory planning. Although it ensures the smooth continuity of the trajectory and improves the trajectory accuracy, the calculation is too complex to affect the efficiency and there are still some uncontrollable stages of the trajectory. Therefore, the use of high-order polynomials alone for trajectory planning is not widely used.

Based on the various defects of high-order polynomial trajectory planning, scholars have proposed a hybrid polynomial interpolation algorithm trajectory planning based on cubic polynomials, quintic polynomials, and high-order polynomials. In order to avoid the complexity of the high-order polynomial interpolation algorithm and ensure the continuity of the trajectory, Liu Zhengxiong et al. [30] used the 5-3-5 mixed polynomial interpolation algorithm for the space robot. The first segment of the 5th polynomial interpolation is the acceleration segment, the second segment of the 3rd polynomial interpolation is the middle segment, and the third segment of the 5th polynomial interpolation is the deceleration segment. The 3rd polynomial interpolation smoothly connects the two segments of the 5th polynomial planning trajectory. Cheng LIU et al. [31] applied 4-3-3-4 polynomial trajectory planning in the intelligent packaging Delta manipulator, and also ensured the continuity of the trajectory by using multi-segment polynomial interpolation. The method of 4-segment polynomial interpolation also improves the accuracy of the overall trajectory fitting, and realizes the accurate, fast and controllable high-efficiency packaging operation of the Delta manipulator.

In summary, with the deepening of the research on the joint space trajectory planning of industrial robots, the application of polynomial interpolation algorithm in trajectory planning is becoming more and more extensive. Whether it is cubic polynomial, quintic polynomial or mixed polynomial trajectory planning, the fundamental purpose is to improve the fitting accuracy and ensure the smoothness and continuity of the joint space trajectory curve. However, for the complex trajectory of multi-path points, the continuity of the trajectory cannot be guaranteed by simply using multiple polynomial interpolation. At this time, the continuous planning of multi-path points and multi-segment trajectories can be carried out by spline curve.

1.2.2 Spline curve trajectory planning

Spline curve is more free and widely used in design and calculation because of its controllability of geometric shape. Because of its continuity in the first and second derivative of the value point, it can ensure the smoothness and continuity of the multi-path point trajectory curve in the trajectory planning of industrial robots [32-34]. According to different characteristics such as expression and number of nodes, spline curves can be divided into B-spline curves and Bessel spline curves, uniform spline curves and non-uniform spline curves [35-38].

The B of the B-spline curve is Basic, that is, it is a curve that is a linear combination of each basis function. The curve is controlled by multiple control

points to control the shape of the curve but does not pass through these control points [39-41]. According to the number of B-spline nodes, it can also be subdivided into uniform, quasi-uniform and non-uniform B-spline curves [42]. The B-spline is a special case of the Bezier curve, and the design difficulty of the curve is more complicated than the Bezier curve. The following formula (1.1) (1.2) can simply see some differences from the solution formulas of the two.

$$\text{Bezier : } \overline{P(t)} = \sum_{i=0}^{n-1} \overline{p_i} L_{i,n}(t), t \in [0,1] \quad (1.1)$$

$$\text{B-Spline: } \overline{P(t)} = \sum_{i=0}^{n-1} \overline{p_i} L_{i,d}(t), t_{\min} \leq t \leq t_{\max}, 2 \leq d \leq n \quad (1.2)$$

Which is the coordinate of the control point ; n is the number of control points ; the influence control point polynomial coefficient.

Compared with the Bezier curve, the t value of the B-spline curve and the power selection of the polynomial change, and its value range changes dynamically and the selection is more free. Although the calculation of the B-spline curve is cumbersome, it can be modified locally and can better guarantee the continuity of the curve geometry.

In order to improve some defects of Bezier curves, Chen Xiaoyan et al. [43] verified whether two adjacent quasi-cubic Bezier curves can be smoothly connected together, and proposed a C ~ 2 quasi-cubic Bezier curve that can adjust the shape of the curve locally. By changing the two shape parameters of the curve, the curve can be controlled to achieve the desired target geometry. The flexible and controllable Bezier curve also greatly improves its applicability. However, in the application of trajectory planning, it is not only necessary to ensure the arbitrary controllability of the spline curve shape, but also to make the trajectory curve continuous and smooth. Therefore, many scholars use B-spline curves in the trajectory planning of multi-path points in joint space. Younsung Choi et al. [44] applied the B-spline curve trajectory planning algorithm to the dual-arm robot to achieve a smooth and collision-free continuous trajectory. Simulation experiments show that the trajectory planned by the B-spline curve can ensure that the dual-arm robot safely and correctly performs assembly operations. For the problem of trajectory deviation of the manipulator, Liu et al. [45] used the characteristic that the shape of the cubic B-spline curve can be partially modified to improve the problem of trajectory deviation caused by the hysteresis of each joint of the manipulator. The experimental results show that the cubic B-spline trajectory planning can reduce the error of joint torque, joint velocity and acceleration, and effectively improve the trajectory accuracy and safety of the manipulator. For the problem that the joint axis of the manipulator may have residual vibration during operation, Liu Bao et al. [46] added a fifth-order correction function and a sixth-order correction function near the piecewise point of the cubic B-spline

curve. The addition of the correction factor makes the third-order derivative of the manipulator trajectory smooth and continuous as a whole and the acceleration of the starting and ending points is 0, which can well reduce the vibration impact of the manipulator.

The construction of uniform spline trajectory planning is relatively simple, but when the spline curve is divided into more segments, the calculation steps will become extremely cumbersome, and there is also the problem of unstable starting point and end point velocity [47-49]. Li Hong et al. [50] used cubic non-uniform spline curve and quintic non-uniform spline curve for trajectory planning of excavator robots on the basis of the defects and limitations of cubic uniform spline curve trajectory planning. The applicability of cubic and quintic non-uniform spline curves was verified and their advantages and disadvantages were compared. Under the premise that both can ensure the smoothness of angle, angular velocity and angular acceleration, the more suitable cubic non-uniform spline can effectively increase the service life of excavator robots. In summary, the trajectory planning of industrial robots can be classified as follows in Table 1.

Table 1 Comparison of trajectory planning methods

| space | method | Advantages and disadvantages are applicable occasions |
|-----------------|--|--|
| Descartes space | Linear trajectory planning | Acceleration discontinuous, suitable for point to point simple straight path . |
| | Arc trajectory planning | High precision but complex calculation, suitable for fine and complex continuous path. |
| joint space | Cubic polynomial trajectory planning | Acceleration impact, suitable for point-to-point simple motion The acceleration is stable and continuous, but the calculation is relatively complex, which is suitable for fine motion. |
| | Fifth-order polynomial trajectory planning | The fitting precision is better. The curve is stable and continuous, but the calculation degree is extremely complex, which is suitable for high precision motion. |
| | High-order polynomial trajectory planning | The polynomial piecewise interval connection is smooth but the calculation amount is increased when the segmentation is too much, which is suitable for the |
| | Mixed polynomial trajectory planning | |

| | | |
|--|------------------------------------|---|
| | B-spline curve trajectory planning | <p>motion trajectory with more interpolation points.</p> <p>The shape of the curve is controllable and can be modified locally. When there are many segments, the calculation amount is too large, which is suitable for complex continuous trajectories of multipath points.</p> |
|--|------------------------------------|---|

2 Optimal trajectory planning research

In order to improve the working efficiency and service life of industrial robots, the basic trajectory planning has been unable to meet these requirements. For this reason, scholars have improved or constrained variables such as speed, acceleration, and torque in various basic trajectory planning methods, and finally optimized the trajectory to achieve the purpose of reducing the running time, joint impact, and energy consumption of the robot. According to the number of optimization objectives, the optimal trajectory planning can be divided into single-objective trajectory optimization and multi-objective trajectory optimization [51].

2.1 Single objective optimal trajectory planning

2.1.1 Time optimal trajectory planning

When the industrial robot carries out tasks such as handling and welding, the robot will continuously carry out round-by-round circular operations, and shortening the time of one round of operations will further improve the production efficiency of the factory production line. Therefore, under the premise of satisfying the kinematic constraints of industrial robots, the trajectory running time should be shortened as much as possible through the time-optimal single-objective trajectory planning, and the end effector of the robot should be ensured to reach the working area normally.

The optimal running time of the trajectory can be calculated and solved by intelligent optimization algorithms such as PSO algorithm, GA algorithm and CS algorithm. Firstly, the optimization objective function is constructed according to the trajectory time, and the relevant kinematic constraints are added. Finally, the optimal value of the objective function is solved according to the iterative optimization process of the method. As early as 1998, Martinez-Alfaro et al. [52] used simulated annealing algorithm to obtain the optimal trajectory based on the B-spline curve trajectory planning of Nomadic 200 robot. In order to study the advantages and disadvantages of various algorithms, Miao Jianwei et al. [53] used simulated annealing algorithm and ant colony algorithm to plan the optimal trajectory of six-degree-of-freedom manipulator. The experimental results verify the excellent global optimization ability of simulated annealing algorithm, while the ant colony algorithm has higher solution accuracy and more stable optimization process. In order to

determine the optimal time trajectory of the climbing robot, Yao et al. [54] used a quantum-behaved particle swarm optimization algorithm based on 3-5-3 polynomial trajectory planning to optimize the transition gait of the robot. The optimization results were compared with the basic particle swarm optimization algorithm and genetic algorithm, which proved the superiority of the optimization ability of the algorithm. In order to further improve the optimization degree of time, Miao et al. [55] combined genetic algorithm and particle swarm optimization to improve the particle swarm optimization algorithm, which improved the convergence speed and avoided the emergence of local optimal solution. The experimental results show that the trajectory optimized by the improved algorithm ensures the continuous and smooth change of angle, angular velocity and angular acceleration under the premise of finding the optimal time.

Nowadays, various intelligent optimization algorithms are also constantly being optimized and improved. In the application of trajectory planning, various optimization algorithms have their own advantages and disadvantages for different types and different occasions of use of robots. Some of them will lead to local optimal problems due to insufficient global optimization ability. Some are too cumbersome because of the optimization conditions, and the overall iterative optimization process is too long to calculate and solve. The research on the improvement of algorithm defects is also deepening, and the research on robot time-optimal trajectory planning is also constantly developing.

2.1.2 Impact optimal trajectory planning

The safety of industrial robot trajectory is also a key issue in actual production. Reducing the impact of each joint of the robot can improve the stability of the robot's work, prevent the sudden change of joint speed and acceleration from causing the joint to run outside the safe range, and effectively extend the service life of the industrial robot. The optimal trajectory planning of the impact is intuitively reflected in the smoothness of the trajectory curve and the change of the joint torque. The fundamental goal is to ensure that the trajectory curve is smooth and continuous without mutation, and to optimize the joint acceleration and torque [56-57].

Zhang [58] optimized the maximum static torque of the shoulder joint of the four-degree-of-freedom manipulator by minimizing the maximum value method, analyzed and studied various situations of maximum torque, and ensured the optimal torque of each joint after trajectory planning. In order to solve the problems of vibration and mechanical wear of industrial manipulators, Yang et al. [59] used S-shaped curve instead of trapezoidal curve in the interpolation process of B-spline curve trajectory planning. The optimized trajectory curve ensures the continuity of acceleration and reduces joint impact. However, for the arc trajectory curve with large curvature, the optimization method is not universal. The premise of this method is to constrain the bending speed at the arc trajectory. Wu et al. [60] proposed a PTPA trajectory planning algorithm to

solve the impact optimal trajectory planning problem. The velocity threshold and displacement threshold calculated according to kinematic constraints and impact constraints are used as performance evaluation, so as to establish an impact optimal trajectory planning model with continuous acceleration.

In the study of impact optimal trajectory planning, scholars' research direction is how to make the acceleration more stable and continuous. In the optimization of robot joint speed and acceleration, the optimization of trajectory running time is often included. In order to further improve the operation efficiency of industrial robots and reduce production costs, the current research on impact optimal trajectory planning is mostly based on time-impact multi-objective optimal trajectory planning [61].

2.1.3 Energy optimal trajectory planning

The research on the energy consumption of robots is not very deep at first, and the demand for energy in the production of ordinary industrial robots is not very large. However, with the development of high-precision fields such as military industry and aerospace, such as Jiaolong underwater robot and Tiangong space manipulator, the research on the energy consumption of robots is becoming more and more extensive. How to ensure the minimum energy consumption in the same running time, so that the robot can complete more tasks under the limited energy storage equipment is the fundamental goal of energy-efficient trajectory planning.

The way to reduce energy consumption can achieve the optimal energy distribution of each joint by optimizing the power system of the robot. Secondly, the speed and acceleration can be optimized by the relevant energy consumption optimization algorithm, so that the trajectory of each joint runs smoothly and continuously to reduce energy consumption. Li Baiya et al. [62] derived the optimal energy consumption function based on the velocity curve and acceleration curve of 4-3-4 polynomial trajectory planning, and used PSO algorithm combined with energy consumption function to optimize the trajectory. The simulation results verify the correctness of the optimal energy consumption function, and effectively reduce the energy consumption of the manipulator after trajectory optimization. Luo et al. [63] used the Lagrange interpolation method to plan the optimal trajectory of energy consumption. At the same time, the Chebyshev method was used to effectively avoid the Runge phenomenon in the trajectory planning process of the Lagrange interpolation method. The established angular velocity function and angular acceleration function were optimized by the improved Lagrange interpolation method, and the optimized function was substituted into the optimal energy consumption index to finally obtain the industrial manipulator trajectory with the optimal energy consumption. Zhang et al. [64] used asymmetric fifth-order and sixth-order polynomials to plan the trajectory of Par4 parallel robot to ensure the smooth operation of the end effector, and used GWO algorithm to optimize the trajectory with the goal of optimal energy consumption, and successfully verified the effectiveness of the

new intelligent optimization algorithm in the optimal energy consumption trajectory planning. On the basis of five B-spline trajectory planning, Jia Wenyong et al. [65] used dragonfly algorithm to optimize. At the same time, quantum behavior and differential evolution were used to improve the dragonfly algorithm, and the dynamic model and energy consumption optimal function model were constructed as the fitness function of the improved dragonfly algorithm. The superiority of the improved algorithm was verified by several sets of simulation experiments, and the energy consumption of industrial robots was further optimized.

Constructing the optimal function of energy consumption and using intelligent algorithms for trajectory planning can effectively reduce energy consumption. However, various studies at present are aimed at the robot's simple rigid body motion. In the face of complex industrial production environment, various problems such as friction loss and flexible changes of robot joints will have more or less impact on energy consumption. Simply using algorithms to optimize energy consumption alone cannot achieve the expected goal. Most scholars study energy consumption while also optimizing the joint impact and trajectory running time of the robot. Multi-objective optimal trajectory planning research can better solve the related problems of industrial robot energy consumption.

2.2 Multi-objective optimal trajectory planning

In the complex actual production work of industrial robots, the individual impact optimization or energy consumption optimization often cannot meet the actual production requirements. The joint optimization of time-impact, time-energy consumption, time-energy consumption-impact can better solve the problems of impact and energy consumption. For multi-objective optimization methods, the constructed optimization objective function can be weighted, or after the main optimization objective is selected, the remaining optimization objectives can be constrained to ensure that they are within a reasonable range, but it is impossible to achieve the optimal solution for each objective [66-68].

Abe et al. [69] proposed a trajectory planning method with minimum energy consumption and residual vibration. The residual vibration and energy consumption are taken as the objective function and optimized by VEPSO algorithm, which realizes the manipulator to suppress the residual vibration with less energy consumption. Wang et al. [70] added the optimization objective function of trajectory smoothness on the basis of time-energy optimization. Firstly, a smooth and continuous joint trajectory was established by B-spline curve, and INSGA-II algorithm was used to optimize multiple objectives at the same time. A set of well-distributed Pareto optimal solutions was obtained, and a set of solutions with optimal time, energy consumption and pulsation was selected from the obtained multiple sets of solutions. In order to solve the problems of wear and impact vibration of industrial manipulator, Lu et al. [71]

optimized the trajectory running time and impact of the manipulator at the same time, and effectively reduced the joint impact through the five-order non-uniform B-spline curve. Finally, the trajectory running time and jerk were used as the optimization objective function and optimized by NSGA-II algorithm. Under the premise of optimal trajectory time, the controllable, smooth and continuous joint velocity and acceleration were ensured, and the joint impact accumulation was effectively reduced. Wu et al. [72] used a more novel IBOA optimization algorithm to solve the problem of time-impact optimization. The algorithm introduced the initial population and fractional differential method based on chaotic sequence on the basis of BOA algorithm. The convergence speed of the improved optimization algorithm has been significantly improved, and its application is verified in Delta robot trajectory planning. The experimental data show that the IBOA algorithm can effectively reduce the trajectory running time and the vibration acceleration of the end effector, and achieve the expected goal of time-impact optimal trajectory planning.

To sum up, for the multi-objective optimal trajectory planning method, on the one hand, the high-order B-spline curve or NURBS curve is used for trajectory planning to obtain a smooth and non-impact curve. On the other hand, the intelligent optimization algorithm can be used to optimize the specific objective function, or the different optimization objectives are optimized twice successively [73]. With the deepening of the research on multi-objective optimization algorithm, PSO, GA, CS and other optimization algorithms are constantly improving the defects to improve the optimization efficiency. However, there is no clear standard for the objective optimization function of multi-objective optimal trajectory planning. Whether it is sequential quadratic optimization or unified optimization with weight coefficients, most of them are in the theoretical stage, and there is no clear standard calculation method. In the face of the actual industrial robot working environment, the details that need to be optimized are more numerous and complex. In order to achieve more effective practical application results, more in-depth research combining theory with practice is needed in the future.

3 Summary and Outlook

Whether it is the basic trajectory planning of Cartesian space or joint space, the relevant theoretical methods have gradually matured. Polynomial interpolation, mixed polynomial interpolation, and B-spline curves also have their own applicable occasions. According to the expected planning objectives and computational difficulty, choose whether to use high-order polynomials or high-order B-spline curves for trajectory planning. The related research on optimal trajectory planning is still in continuous development. Among them, there are many common methods for time-optimal single-objective trajectory planning and many optimization algorithms also have good versatility. However, when considering energy consumption, impact and other factors, how to better apply the optimization results to the actual work tasks of industrial robots is one

of the important directions for future research. At present, there is no uniform standard for solving multi-objective optimization methods, and the research entry points of various scholars are also different. Many optimization algorithms are still at the theoretical level. In summary, the optimal trajectory planning can be studied in depth from the following aspects in the future.

(1) Reliability optimal trajectory planning for actual working conditions

Most of the optimal trajectory planning only achieves the optimal theoretical value by optimizing the smoothness, speed and acceleration of the trajectory, but in the actual production task, the working environment, workpiece type, obstacle avoidance mode and other specific conditions need to be considered. For different types of workpieces, industrial robots need to control a specific speed for handling, welding and so on. Reducing impact at high speed and controlling acceleration at low speed to reduce wear can not only reduce the loss of the manipulator but also ensure the accuracy of the workpiece. In order to ensure the safety of industrial robots, it is necessary to increase research on real-time dynamic obstacle avoidance. Compared with the static obstacle avoidance method of industrial robots, real-time dynamic obstacle avoidance requires faster reaction time to plan the trajectory in advance. At present, several real-time dynamic obstacle avoidance methods are commonly used. DWA algorithm, A^{*} algorithm, etc. [74-75].

(2) Real-time robot trajectory planning

In order to further increase the processing accuracy of industrial robots and improve the degree of intelligence, most industrial robots have been equipped with visual sensors, infrared sensors, etc., and timely trajectory correction can be achieved through real-time information transmission. In addition, the development of machine learning has also made industrial robots gradually get rid of fixed mathematical models, and the expected models can be continuously trained through reinforcement learning to enhance the trajectory tracking performance of industrial robot systems.

(3) Trajectory planning based on virtual reality

Before the trajectory planning results are actually applied to the robot, a lot of debugging and correction are needed. In order to reduce the related economic loss and time cost, there are already many virtual workstations developed and used in the network, such as Robotstudio, Roboguide and so on. The trajectory simulation through the virtual workstation reduces the difficulty of debugging, and can also observe the trajectory shape more intuitively after the robot trajectory planning. With the widespread popularization of the 5G Internet era, 5G intelligent factory human-computer interaction is the core trend of future intelligent development. The real-time connection between the simulation system and the industrial robot can carry out remote trajectory control and parameter debugging, and the intelligent factory will move forward steadily on the road of development.

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