Teacher Moves and Knowledge in Preparing Students for General Certificate of Education Examination

Abstract

This study investigated an experienced teacher's moves in preparing high school students for an end of year certificate examination. Six consecutive lessons taught by this teacher were observed and videotaped in the second term of the 2020/2021 academic year. These lessons were transcribed and coded for teacher moves as well as teacher knowledge deployed in any of the moves. First, the two researchers coded teacher moves independently and agreed on them. Second, interviews were used to obtain justification for teacher moves used. Third, Ball, Thames and Phelps's (2008) categorizations of mathematical knowledge for teaching was used to determine which teacher knowledge motivated the move. Results revealed that this teacher, Ngwa, a pseudonym, deployed the following moves: reading through all the topics to identify key concepts and determine connections between and among them, connecting ideas from one topic to another, returning to previous chapters or units for review of mathematical ideas, reinforce understanding by modifying problems, adoption/adaptations and extension of problems to generate the use of more ideas, withdrawal of the teacher and creation of space for interaction, announcing or advertising what will be learned in the next lesson, and learners designing their own questions for the class to engage with. The findings also revealed that understanding the structure of an examination might be a form of specialized content knowledge as it drives teaching. In addition, teaching and learning was found to be forward and backward to review concepts and reinforce learning in learners. Furthermore, teachers must intentionally create space for student-student interaction which provide opportunities for learners to make sense of what is being taught. The findings of this study might be beneficial to teacher educators and professional development experts to focus training on productive teaching moves. Classroom teachers can also use the outcome of this study for the improvement of their practice of teaching.

Key Words: Teacher moves; Teacher knowledge; adoption and adaptations

Introduction

What teachers do in their classroom to ensure that learners understand the concepts being taught has been of interest to many researchers. Ozgur, Reiten and Ellis (2015) refer to these as "teacher moves." Ozgur, Reiten and Ellis (2015) describe "teacher moves" as "teacher practices aimed at supporting student reasoning" (p. 1062). These practices have been viewed by many researchers from different perspectives such as: how pre-service teachers respond to student thinking (Taşdan & Kabar, 2022), how teachers lead mathematics discussion (Shaughnessy et al., 2021), how teachers open up classroom discussion that is geared towards developing conceptual understanding of mathematics (Michaels, O'Connor & Resnick, 2008), how teachers notice and take up instances of students' mathematical thinking with high potentials to contribute to students' learning of core mathematical ideas (Van Zoest et al., 2017), and "steering" classroom instruction towards the mathematical point of a lesson (Sleep, 2012). Other researchers have focused on tasks and investigated what teachers do to maintain the cognitive demand of tasks at high level in their classrooms (Stein, Grover & Henningsen, 1996) and the type of changes teachers make with questions from textbooks when assigning to learners (Kullberg, Runesson & Märtensson, 2013). Although Ozgur, Reiten and Ellis (2015) described teacher moves and the above cited research studies provided components of teacher moves as investigated by different researchers, we move forward at this point to provide our own definition of this concept. According to us, teacher moves are “whatever a teacher does either before or during a lesson or series of lessons with the sole purpose to engage students for learning.” This definition differs from the description provided by Ozgur, Reiten and Ellis (2015) as it goes way beyond supporting students’ reasoning only to include a broad range of what teachers might do to engage all learners in their lessons. It should be noted that some of the teacher moves are productive, leading students to the intended learning whereas others are not.

Some researchers have documented their findings on moves that teachers make in the classroom during teaching of which some might not be productive. For example, Stigler, Fernandez and Yoshinda (1996) found out that teachers in their study ignored incorrect responses provided by students and selected another student to answer or reinterpret the incorrect answer until the responses fits what the teachers expected. This could be interpreted to mean that the teachers in this study did not follow up on what the students said to ensure that the learners who provided the incorrect answer reflects on the response to modify their thinking. Other researchers identified other teacher moves on a large scale. Some studies have only reported that teachers should build on students’ thinking. These are too big and difficult for teachers to incorporate into their day to day practice. Grossman, Brown, Schuldt, Metz, and Johnson (2013) and Herbel-Eisenmann, Drake, and Cirillo (2009) call for more details in teaching moves that teachers could incorporate into their day to day practice of teaching. Minute-by-minute pedagogical practices that teachers need in order to effectively and efficiently enact their lessons are yet to be elaborated.

This minute-by-minute pedagogical practices however, according to Boaler and Humphreys (2005) depends highly on moment-by-moment decision making. This is because responding to students’ mathematical thinking during whole class instruction has been considered as critical for the growth of students understanding of the mathematics they are to learn (e.g., Kilpatrick et al., 2003; National Council of Teacher of Mathematics [NCTM], 2014; Spangler & Wanko, 2017; Steckero et al., 2019; Steckero et al., 2020). Even more critical to the understanding learners develop in a mathematics classroom is the way teachers respond to the thinking students’ exhibit during enactment (Bishop et al., 2020; Robertson, Scherr, & Hammer, 2016). Van Zoest et al. (2021) and Van Zoest et al. (2019) identified that different teachers can respond differently to the same student thinking making engagement different and productive in different ways. According to Van Zoest et al. (2021), the difference they found could be as a result of the fact that some teachers engage with student response, providing deeper insights to the learners and developing it further while another teacher might mobilize other students in the class to respond to the student response under consideration.

The above mentioned body of research were conducted in the context of classroom teaching and learning which is simply natural and not motivated by high stakes testing. However, on the contrary, teaching in Cameroon and perhaps many other countries in Africa is highly focused towards certificate examinations at the end of the academic year and sometimes just regular testing. For example, in Cameroon, the nine months of schooling in an academic year, testing is expected to be done eight times. That means almost every month, learners are tested. This frequent and heavy testing is designed to get students ready for the end of year reporting to parents or Ministries of Basic and Secondary Education and certificate examinations. The end of year certificate examination results are highly awaited by parents and the general public. The performance of each school determines its effectiveness and efficiency. Schools that perform better all the time are considered "good" schools and are highly sort for by parents to enroll their children while those schools that do not perform well seem to suffer in terms of enrolment. As such, teaching in many of the schools in Cameroon is geared towards reporting and certification. This has caused the assessed curriculum to be taken as the teaching curriculum as teachers teach to the examinations.

Therefore, it is worth investigating which teaching moves are productive in preparing learners to be successful in their end of year certificate examinations. This study investigated the teaching moves of an experienced teacher as he focused on getting students ready for the General Certificate of Education (GCE) Advanced Level Examination in Further Mathematics, one of the certificate examinations of the English Subsystem of Education in Cameroon. Our intent here is to identify teaching moves relevant for teaching and preparing learners towards end of year certificate examination and also the kind of knowledge for teaching needed by teachers to prepare students for certificate examinations at end of each academic year. To achieve these objectives, this study addressed the following research questions: (a) what are the teaching moves that an experienced mathematics teacher deploys in the preparation of students for certificate examinations? (b) What mathematical knowledge for teaching does this teacher display during teaching?

Theoretical Framework

Teacher moves are an essential and integral part of the teaching and learning process and usually begin during lesson planning and extends into classroom. The moves teachers take or make before or during a lesson to an extent, drives what students are going to learn. Remillard (2000) found that teachers read through the textbook they use and their “reading was selective and interpretive” (p. 336). According to Remillard, the teachers appropriated and invented tasks. The appropriation and invention of tasks that teachers make after reading a textbook goes a long way to determine what students will learn. In addition to the reading move by Remillard, Sleep (2012) identified seven moves teachers take to “steer” their instruction towards the mathematical point of the lesson. These moves are: attending to and managing multiple purposes, spending instructional time on mathematical work, spending instructional time on the intended mathematics, making sure students are doing the mathematical work, developing and maintaining a mathematical storyline, opening up and emphasizing key mathematical ideas, and keeping a focus on meaning. Other researchers such as Frey and Fisher (2010) see questioning as teacher move to pay attention to while Herbel-Eisenmann, Steele and Cirillo (2013) see classroom discussion as a very important move for teaching. To extend the move on classroom discourse, Ozgur, Reiten and Ellis (2015) identified teacher moves for eliciting student reasoning (eliciting ideas, eliciting understanding, pressing for elaboration), responding to student reasoning (re-presenting and prompting error correction), facilitating student reasoning (providing guidance, building, providing summary explanations, providing conceptual explanation, providing information, encouraging multiple solutions and providing alternative strategies) and extending student reasoning (encouraging reasoning and reflection, pressing for justification and generalization). Effective classroom discourse can only take place when learners have been engaged in solving problems by themselves.

Some researchers have called these efforts action learning and recommended that teachers can use them for productive learning. Abramovich, Grinhpan and Milligan (2019) recommended *Action Learning* as a productive teacher move. According to them, action learning is “self-solving real problems followed up by reflection” (p. 4) and requires teaching using real-world problems that are unfamiliar to learners. Using real-world problems in teaching often leads to classroom discussion. Shaughnessy at al. (2021) decomposed classroom discussion-leading practices into three areas namely *Framing*, *Orchestrating and Recording/representing content*. According to Shaughnessy et al. (2021) *Framing* involves launching and concluding; *Orchestrating* involves eliciting student thinking, probing student thinking, orienting students towards the thinking of others and making contributions; *Recording/representing content* involves keeping accurate public records and choosing and using appropriate representation to convey key ideas. When ideas from students are properly represented and recorded, it is an opportunity for learner to reflect on what others think that might eventually improve their own learning.

Bobis et al. (2021) found that teachers in their studies employed what they call didactic strategies such as contextualizing what students are to learn so that they can make sense of the mathematics, providing opportunities for students to consolidate their learning, make mathematical connections to previous learning, and using multiple tasks for the same concept in each lesson to improve on learners' understanding. Bobis et al. (2021) also found that teachers in their study used consolidation strategies such as repeating important mathematical concept, varying previous tasks (such as changing the numbers for the same problem structure) for learners to engage with, sequencing the lesson so that concepts are gradually built on through implementing a series of tasks, and build connections between what learners learned before and what they are to learn. In varying previous tasks, learners are able to generate their own examples and problems and this is seen as an important aspect of learning. Watson and Shipman (2008) argued that when learners construct their own examples or problems, they are able to gain ownership of the mathematical concepts they are to learn.

We believe that all teachers exhibit some kind of knowledge during teaching. We also do believe that any teacher move demonstrates the type of teacher knowledge influencing such a move or vice versa. The research literature is full with forms of knowledge that is useful for teaching. Important to note is the fact that teacher knowledge affect student achievement (Hill, Rowan & Ball, 2005). Shulman (1986) categorized content knowledge for teaching to include subject matter content knowledge, pedagogical content knowledge and curricular knowledge. According to Shulman, subject matter content knowledge consists of teachers knowing facts, concepts of a subject and also why certain facts are true but should also know more than just facts. Shulman added that, teachers should understand why some topics or concepts are critical to a discipline or a sequence of topics. Shulman explained that curricular knowledge involves understanding how a program is designed while pedagogical content knowledge involves how teachers represent a subject and depends on teachers' knowledge of students' conception/misconceptions of the specific topic or content area and what to do to reinforce the conceptions and address the misconceptions.

Ball, Thames and Phelps (2008) derived their mathematical knowledge for teaching from Shulman's three categories. According to Ball, Thames and Phelps, mathematical knowledge for teaching (MKT), is made up of subject matter knowledge (SMK) and pedagogical content knowledge (PCK). Subject Matter Knowledge includes common content knowledge (CCK) and specialized content knowledge (SCK). Pedagogical Content Knowledge (PCK) includes knowledge of content and students (KCS) and knowledge of content and teaching (KCT).

Method

Participant. The teacher we followed is called Ngwa, a pseudonym, and has taught High School Mathematics and Further Mathematics for 15 years. He teaches in a Faith-based school and also has been involved in grading Further Mathematics at the Advanced Level GCE certificate examinations for 13 of the 15 years of teaching. In teaching, Ngwa makes use of the textbook that he used while in high school authored by Bostock and Chandler, from England. The book is titled *Further Mechanics and Probability*.

Data collected. Data was collected from the lessons Ngwa taught. We observed and videotaped six consecutive lessons taught by Ngwa in the second term of the 2020/2021 academic year. These lessons were transcribed and read several times. The intent was to identify teaching moves that were taken by Ngwa in the course of teaching. Ngwa was also interviewed after each lesson in which he was asked to explain why he made certain moves and the potential benefits to the learners. Field notes were taken by both researchers.

Data analysis. The transcripts were read several times by both researchers. Independently, both researchers identified all teacher moves thought to be productive during the lessons that Ngwa taught and agreed upon. From existing literature, codes were identified and used for coding the agreed teacher moves found in the transcripts. Trials coding was done to determine whether the codes we developed captured what it was intended to do. During this trial coding, the researchers got 75% agreement. We now refined the codes in places were greater clarity was needed. Final coding was done according to the following scheme.

Table 1:
Coding scheme

SN	Code	Description
1.	Reading	▪ Reading the chapter or unit or example before teaching ▪ Review of the concepts or examples or key ideas
2.	Relationship	▪ Connection between units ▪ Connection between subunits ▪ Connection between problems
3.	Adaptations	▪ Modifying problems ▪ Extending problems
4.	Space	▪ Providing opportunities for interaction among students ▪ Providing opportunities for students to assists their friends
5.	Emphasizing	▪ Repeating an idea to learners ▪ Highlighting key ideas ▪ Providing more problems of the same kind to learners so that they understand the main idea

After these codes were used, further coding was done independently and the researchers got 95% agreement in coding the categories of teacher moves. Where both researchers disagreed in the coding, a discussion ensued to resolve the disagreement. Interviews were used to obtain justifications from teachers about the moves they took. Then, for each teacher move, we examined the mathematical knowledge for teaching that motivated it. The following codes were used for coding the type of teacher knowledge used by the teacher. *Common content knowledge* (CCK) (knowledge of facts, definitions, recognizing whether an answer is right or wrong, whether textbooks provide a wrong definition or not) and *Specialized content knowledge* (SCK) (knowledge unique to the teaching of mathematics, cannot be exhibited by others not teaching mathematics and it requires an understanding of different interpretations of the operations in ways that students do not readily see or cannot easily distinguish). *Knowledge of content and students (KCS)* **(**anticipate what students are likely to think, think of ways students will approach, know common students' conceptions and misconceptions they carry along) and *Knowledge of content and teaching (KCT)* (teachers must understand the sequencing of topics, carefully choose examples and representations, etc).

Result

We found eight productive teaching moves used by Ngwa as he taught his lessons. These moves are: reading through all the topics to identify key concepts and determine connections between and among them, connecting ideas from one topic to another, returning to previous chapters or units for review of mathematical ideas, reinforce understanding by modifying problems, adoption/adaptations and extension of problems to generate the use of more ideas, withdrawal of the teacher and creation of space for interaction, announcing or advertising what will be learned in the next lesson, and learners designing their own questions for the class to engage in. Below is our explanation and elaboration of the moves that Ngwa took as he taught his lessons. In each case, where possible, we identified the teacher knowledge in action.

Reading through all the topics to identify key concepts and determine connections between and among them. The syllabus for Advanced Level GCE examination in Further Mathematics requires that the concepts of *rotation*, *moment of inertia*, *rotation about a fixed axis* and *further rotation about a fixed axis* be taught to students. Ngwa read through all these topics in the textbook before he engaged in teaching the students. The reason for reading through these topics is to develop self-understanding of the ideas in each topic. As Ngwa read through, he engaged with solving each exercise under each topic. The reason for solving each exercise in the topic is to understand the depth of reasoning required for each problem and to determine how far each of the subconcepts in the topic should be treated. Now, in reading through each topic and solving the assigned exercises, Ngwa was able to determine which idea the following topic pulls from the previous one and builds on. For example, in the topic of *rotation*, the concept of *moment of inertia* (I), the torque (C= I$\ddot{\theta}$), angular momentum (I$\dot{\theta}$), and rotational kinetic energy $\left(\frac{1}{2}I\dot{\theta}^2\right)$ were all introduced. Then, in the next topic, *calculation of moment inertia* was further developed. Under this next topic, realized that calculation of moment of inertia is emphasized heavily. The basic definition of moment of inertia developed previously is applied to find moment of inertia of some other bodies that usually rotate such as discs and rings. Then, moment of inertias of solid and surfaces of revolutions were found. Thereafter, parallel and perpendicular axis theorems for calculating moments of inertia were treated. This topic ends with many exercises to calculate moment of inertia using the parallel and perpendicular axis theorems. The next topic deals with rotation about a fixed axis. In this topic, rotating bodies were discussed. As a body rotates, rotational kinetic energy is involved, calculating moment of inertia is involved, and using angular velocity and accelerations becomes an integral part of what students ought to know. Reading through all these topics helped Ngwa to have a big picture of the ideas in them and how they are built upon as he explains below:

> I read through and it exposed me to the ideas that students ought to learn and the order in which they have to learn them. It made me see the end from the beginning and how to tie them together so that I support students to have a greater understanding of what is expected of them....Reading through all these topics developed my understanding of each one so that I see them as a whole and

> treating them such that coherence becomes clear to me and also the learners…Reading and solving all problems helped me identify what is tested that is not directly in the notes of the textbook. When I see such ideas, I add them to my notes so that students can have easy access to them when studying.

Connecting ideas from one topic to another. Ngwa beliefs that mathematical ideas do not appear in isolation and are not mutually exclusive of what students learned. He says mathematical ideas are either connected to topics learned earlier or topics to be learned in the future. So, he emphasized that as a teacher one must look for the connections and bring them out clearly so that students can make sense of what is being learned. When these ideas are made visible, students begin to enjoy the subject, he added. According Ngwa, chapter 11 titled *Rotation about a fixed axis* draws a lot from what had been taught in chapters 9 and 10 and even earlier. He said that in rotation, moment of inertia is needed (treated in chapter 10), rotational kinetic energy and potential energy are needed (treated in chapter 9). So, in order to succeed in solving problems in chapter 11, the concepts of chapters 9 and 10 must be understood. This is an indication that Ngwa possess knowledge of content and teaching as he has knowledge of what is taught before a concept as well as after the concept.

As Ngwa began teaching chapter 11 (*Rotation about a fixed axis*) he reviewed key ideas that were taught in chapters 9 and 10. These included calculating moment of inertia rotational kinetic energy and potential energy. He emphasized the idea of initial potential energy and initial rotational kinetic energy. Then he calculated the total initial mechanical energy which is initial potential energy plus initial rotational kinetic energy. When the rod or any rotational object has rotated an angle θ about the pivot, the final potential energy and final rotational kinetic energy are calculated. Ngwa then found the total final mechanical energy which is final potential energy plus final rotational kinetic energy. Now, Ngwa talked about the principle of conservation of mechanical energy which is total initial mechanical energy = total final mechanical energy. This is an indication that Ngwa possess specialized content knowledge as this kind of knowledge is unique to teaching.

Returning to previous chapters or units for review of mathematical ideas. In solving assigned problems, when students are challenged with any of the ideas, Ngwa returns to the topic that taught the ideas and reviewed them, then gives some problems for the students to solve and then return to the problem that is being assigned. For example, when the entire class could not find moment of inertia by applying the parallel and perpendicular axis theorems, Ngwa returned to those theorems and reviewed them. He engaged the learners in the review process by asking questions and calling on students to respond. Then he provided some more problems for them to apply those theorems. Sometimes, when students are challenged, Ngwa asked them to identify the troubling idea such as "In which chapter and under which subtopic or section did we meet this idea?" When students correctly identify the chapter and subtopic or section, Ngwa would ask them to review the idea. After being satisfied, the teacher moved the class back to the main problem and supports them find the moment of inertia using those theorems. Similarly, this happened when students cannot find the potential energy and rotational kinetic energy. The teacher explained why he did this:

> I do this so that learners understand that learning is not forward ever and backward never. I cause them to know that you can return to a previous chapter or unit of your textbook to review an idea that has not well been solidified in your

mind. So, when they are stuck, it is okay to go back and research further the mathematical idea that needs to be used.

Doing this often, Ngwa believes learners open their minds to return and reread a concept that needed to be used yet still challenging to them. This move, according to Ngwa brings in and inculcate the habit of research in the learners. This is an indication that Ngwa possess KCT because of the identification, review and reinforcement of connections across topics. Ngwa was also judged to possess SCK because it is only in teaching that "forward and backward movement" is used to ensure that learners get a solid understanding of the idea and reify their learning.

Reinforce understanding by modifying problems. Ngwa taught the students and provided problems from the textbook for learners to solve. Once these problems have been solved and their answers verified in the textbook, Ngwa then resulted in providing additional problems for them to solve. These additional problems are modification of existing problems in the textbook. For example, one of the problems in the textbook is given as follows:

> *A flywheel loses kinetic energy amounting to 640J when its angular speed falls from 7 rads^{-1} to 3 rads^{-1}. What is the moment of inertia of the flywheel? (Bostock & Chandler, 1985, p. 181)*

Ngwa modified this problem as follows:

> *A flywheel loses kinetic energy amounting to 1460J when its angular speed falls from 17 rads^{-1} to 12 rads^{-1}. What is the moment of inertia of the flywheel?*

He now assigned this problem to students to solve. When asked why he did this, Ngwa said:

> This is to enable students reinforce their understanding of the concepts being taught in class. I always modify problems in the textbook and give students because in the GCE examinations, they will not see it as exactly as in the textbook. The students should be able to solve unfamiliar but similar problems. (interview on 15/01/2021)

On another day, Ngwa asked students to solve problem 13 of miscellaneous exercise 11. The original problem is as follows:

> *Prove that the moment of inertia of a uniform rod, of length 2a and mass m, about an axis perpendicular to the rod at a distance x from its midpoint is* $\frac{1}{3}m(a^2 + x^2)$.

> *A uniform rod, of weight W and length 6b, is smoothly pivoted to a fixed point at a distance 2b from one end and is free to swing in a vertical plane. The rod is held horizontal and released. Prove that after it has rotated through an angle* θ *its*

angular velocity is $\sqrt{\left(\frac{g\ sin\theta}{2b}\right)}$ *and that when the rod is vertical the reaction in the pivot is* $\frac{3W}{2}$*. (Bostock & Chandler, 1985, p. 251)*

He modified this problem and gave it as follows:

Prove that the moment of inertia of a uniform rod, of length 2a and mass m, about an axis perpendicular to the rod at a distance x from its midpoint is $\frac{1}{3}m(a^2 + x^2)$.

A uniform rod, of weight W and length 6b, is smoothly pivoted to a fixed point at a distance 2b from one end and is free to swing in a vertical plane. The rod is held horizontal and released. The rod rotates with an angular velocity of ω*. After the rod has rotated through an angle of* θ*, show that* $2b\omega^2 = g\ sin\theta$*. Show also that when the rod is vertical the reaction in the pivot is* $\frac{3W}{2}$*.*

We asked him to explain why the new problem has only one modification whereas all other things remained the same. He said:

> It is important for the students to be flexible when solving problems. The students must pull all resources they learned before to arrive at desired proves. This is so because my eye is on the GCE examination and these same problems can be modified in a very simple fashion as this… it has happened in the past. They should not be embarrassed in the examinations. (interview on 18/01/2021)

Ngwa added that he could make the modification even more complex by giving the problem as follows:

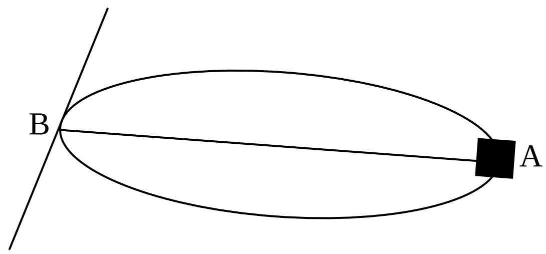
Find the moment of inertia of a uniform rod, of length 2a and mass m, about an axis perpendicular to the rod at a distance x from its midpoint.

A uniform rod, of weight W and length 6b, is smoothly pivoted to a fixed point at a distance 2b from one end and is free to swing in a vertical plane. The rod is held horizontal and released. The rod rotates with an angular velocity of ω*. After the rod has rotated through an angle of* θ*, find the angular velocity in terms of g, b and* θ*. In addition, find the reaction in the pivot when the rod is vertical.*

He explained that the last modification completely hides everything from the learner and makes it difficult. The teacher holds that students' understanding is reinforced by modifying existing problems.

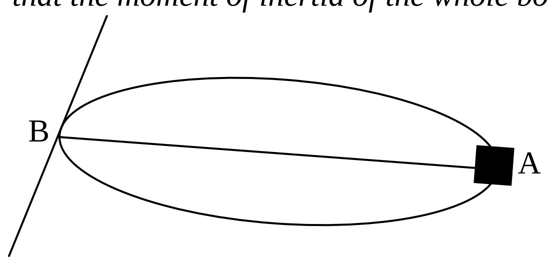
Adoption/Adaptations and extension of problems to generate the use of more ideas. This is an integral part of Ngwa's teaching. He believes a lot in adopting, adapting and extending problems and examples that are in the textbook so that students can be actively engage in their own learning. In adopting the problems in the textbooks, he gave them as they appear in the book. In adapting, the teacher makes significant changes that serious engages the students. For example, example 11c in the textbook is as follow:

> *A ring of mass m and radius r has a particle of mass m attached to it at a point A. The ring can rotate about a smooth horizontal axis which is tangential to the ring at a point B diametrically opposite to A. The ring is released from rest when AB is horizontal. Find the angular velocity and angular acceleration of the body when AB has turned through at angle $\frac{\pi}{3}$. (Bostock & Chandler, 1985, p. 237)*



The teacher adapted the problem and now wrote it as below:

> *A ring of mass m and radius r has a particle of mass m attached to it at a point A. The ring can rotate about a smooth horizontal axis which is tangential to the ring at a point B diametrically opposite to A. The ring is released from rest when AB is horizontal.*
>
> *(i) Show that the moment of inertia of the whole body is $\frac{11}{2}mr^2$.*



> *(ii) Show that when the rod has rotated an angle of θ below the horizontal, angular velocity $\dot{\theta}$ is given by $\dot{\theta}^2 = \frac{12g}{11r}\sin\theta$. Hence, show that the angular velocity of the body when AB has turned through at angle $\frac{\pi}{3}$ is $\sqrt{\frac{6g\sqrt{3}}{11r}}$.*
>
> *(iii) Show that when the rod has rotated an angle of θ below the horizontal, angular acceleration $\ddot{\theta}$ is given by $\ddot{\theta} = \frac{6g}{11r}\cos\theta$. Hence, show that the angular acceleration of the body when AB has turned through at angle $\frac{\pi}{3}$ is $\frac{3g}{11r}$.*

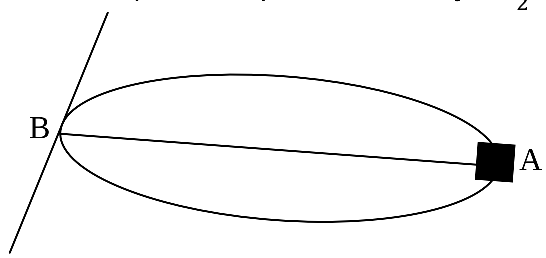
We interviewed him and asked why he changed the problem in the book in this way. He said:

> First, I want to strengthen the capacity of the students to engage with proves because proving things are an integral part of mathematics.... second, proves are very easy problems and when you arrive, you are confident you got it all. That can greatly motivate

the student in the examination hall. Third, in the GCE examination, problems are scaffolded by breaking them into doable parts. So,…. making this problem resemble what they might face in the examination is good.

He further explained that the above problem can be extended to include ideas in differentiation not covered in this topic. According to him, this makes students to think and mobilize all resources for different topics they learned previously when presented with a problem to solve. He stated the extended problem as below:

A ring of mass m and radius r has a particle of mass m attached to it at a point A. The ring can rotate about a smooth horizontal axis which is tangential to the ring at a point B diametrically opposite to A. The ring is released from rest when AB is horizontal.
(i) Show that the moment of inertia of the whole body is $\frac{11}{2}mr^2$.



(ii) Show that when the rod has rotated an angle of θ *below the horizontal, angular velocity is given by* $11r\ cosec\ \theta\left(\frac{d\theta}{dt}\right)^2 = 12g$.
(iii) Show that when the rod has rotated an angle of θ *below the horizontal, angular acceleration is given by* $11r\sec\theta\frac{d^2\theta}{dt^2} = 6g$.

According to Ngwa, additional problems and extension of problems are used to activate the generation of more ideas in mathematics. Seeing the learning of mathematics as mobilizing resources from other topics makes learning of the subject enjoyable.

Withdrawal of the teacher and creation of space for interaction. This forms the core of Ngwa's teaching moves. After focusing students on the critical and key parts of the mathematical ideas to be learned, Ngwa gave opportunities for the students to understand what has just been learned. At this point, the students started talking among themselves sharing ideas of their understanding. As the discussion gets intense, Ngwa withdraws himself and allows the students to make sense of their understanding. When asked why the withdrawal, Ngwa said this:

When you withdraw from talking to remain silent, the students take over and begin to talk to each other…. It is this talking among themselves that cause learners understandings to be developed and shared. It is in the talking to themselves, contributing ideas, refining their ideas that cause learning to be moved to a greater level.

The withdrawal of Ngwa created space for student-student interaction. This space provided opportunities for learners to contribute their own individual understandings to the concept being taught so that each learner leaves the learning arena richer in knowledge. Withdrawal as observed in Ngwa's classes can be characterizes as a time when the teacher is not engaged in any talking and the students are doing the talking in small groups. During this withdrawal time, Ngwa moved around and monitored what each student group was doing, requested for explanations of what they were doing, and asked students to further clarify their thoughts.

Announcing or advertising what will be learned in the next lesson. Ngwa ended his lesson each day making a review of what has been learned that day in class. He asked learners to state and explain what they have learned in that lesson. After learners have shared what they learned in that lesson and the teacher is satisfied, he picked up and linked to what learners will experience during the next lesson. He explained how what will be learned in the next lesson connects to what has been learned today. He further added how knowledge of the present lesson will be used in the next lesson and extend their understanding further. He also tells the learners other mathematical ideas that will be deployed and mobilized to achieve the task of the next lesson. We call this move announcing or advertising the next lesson. When we asked the teacher why he did this, he said:

> I want my students to all attend my next lesson as I intent to build on their knowledge systematically. I motivate them to come and develop their capacities. I push them to read and refresh their understanding on mathematical ideas that had been learned earlier so that they can be reactivated for use in the next lesson. I try to situate the lesson between and among other lessons so that the learners don't see it as a standalone lesson. I cause my learner to see connections among topics in mathematics and learn as such
> (Interview on 23/01/2021)

Ngwa has a clear view here of what he wants to achieve with his learners. Causing them to review what needs to be used in upcoming lessons can actually save tremendous amount of classroom time from being used on activating knowledge. In this case, classroom time will be used more for building and extending learners' knowledge in productive ways. This move removes mathematical obstacles that can hinder learning of learners and calls for proper, efficient and effective management of classroom time. In this way, more can be achieved in a lesson.

Learners designing their own questions for the class to engage in. At the end of a lesson in which this teacher believed he has achieved much, he always ask learners to design their own questions, one per learner, solve it and then exchange with a partner to solve in the next lesson. In a class of 40 students, he had 40 problems. The next lesson was dedicated only for these problems. During this lesson, Ngwa was less visible while the students were more visible. At the start of the lesson, the teacher welcomes all the learners and asked if they were ready for the class of the day. Then he moved further to ask for what they achieved in the last lesson and if there is any home work to do. The students reviewed the main ideas of the previous lesson and then explained the task they were to do at home, "design your own question and solve it" and then bring to class. After this, the teacher asked each person to keep his or her solution, exchange your problem with the person sitting next to you, and then engage in solving. After both of you have finished solving the task given to you by your partner, then the solutions are discussed. If there are disagreements, see whether you can resolve it just between the two of you. If the disagreements persists, then hold it on to present to the whole class. At this point, the teacher asked whether the instructions are clear before they began working. The class agreed the instructions were very clear. Then he asked them to work for 10 minutes and discuss the solutions for another 5 minutes.

The learners got to work with great silence, focusing on the task given by their partners. After 8 minutes, the teacher asked if any person is still working and some indicated. He then asked those who have finished to begin discussing and sharing their solutions. At this point there was productive noise in the class as the learners discussed, disagreed and finally agreed. In some cases, the problems could not be solved because the given information was incomplete. The discussion identified missing data, adjusted the problem and then together as pairs, they solved them. In other cases, the problems had all information but partners couldn't solve them. The discussion focused on helping partners understand the problem and hence develop further the way of thinking about such problems. When each pair had worked and fixed their problems, it was agreed that all students copy each of the problems and solve those you had not attempted. In addition, when they find it difficult to solve, they can consult the person who wrote the problem for the solution. The person who wrote the problem is expected to walk their peers through the solution and not just hand you the solution. After this lesson, we had an interaction with Ngwa and he said this.

Interviewer: We realized that this lesson saw about 40 different questions appearing in class. Why do you do this?

Teacher: I do this for three reasons...um.. First, the students are exposed to a variety of questions and that prepare them for the end of course examination very well. Secondly, by the students constructing their own questions tells me they have understood the ideas. For those who struggled, their friends helped them and not me. Thirdly, this gives the students independence to struggle and sort out themselves.

Interviewer: You insisted that when someone cannot solve someone's problem, the designer of the problem walks the person through the solution and not just give the solution. Why did you say that?

Teacher: I think when the person explains, he or she makes his or her understanding even stronger. When both sit, the interaction creates more learning opportunities between the two. Also, they become great friends.

Interviewer: You have just talked about the benefits to the student. Is this move beneficial to you the teacher?

Teacher: Ohhh yes. It is beneficial to me the teacher. First, I have a sense of where each student is and I can know how to help further. Second, I also determine the kind of human resources I have in the class in terms of students who can assist others. Third, my personal knowledge about the topic is enriched because some of the questions the students designed are challenging. Lastly, I use these questions generated by my students to teach students in subsequent years.

Ngwa clearly sees this move as one that cause learners solidify their understanding of the concept being taught. First, it generates enormous problems for classroom practice. Second, it gives a sense of understanding to the teacher of where each learner is and what they are still struggling with and possible support that can be received. Third, this is a possible style to share classroom authority with the learner. On this day, it was noted that the learners were more active and visible than the teacher. The learners made decisions as they progressed with each other during the interaction. The teacher's role now was that of monitoring to ensure that the correct ideas are learned by each students. Fourth, this move generated a rich question bank for students. Fifth, the teacher clearly expressed that learning takes place for both the learners and teacher.

Discussion and conclusion

This study has illuminated our understanding of moves a teacher might engage with in preparing learners for high stake testing. In addition, each teacher move is motivated by teacher knowledge for teaching. We now discuss five teacher moves, highlighting the teacher knowledge for teaching that motivated each. In our discussion, we do not claim that the teacher knowledge identified is the only one responsible for that move. We believe there are many involved but we choose to identify and highlight the most significant teacher knowledge in that move.

First, announcing or advertising at the end of each lesson, what will be learned in the next lesson can be highlighted as a move that sets the stage and motivate learners to be present for the next lesson. This move completely presents what comes up next and relates to what has been learned so as to cause students not to intentionally create a gap in their study. Ball, Thames and Phelps (2008) identified such a move as knowledge of content and teaching. This knowledge of content and teaching intentionally creates connections between topics and ideas in mathematics, thereby exposing to learners that learning is reinforced by connectivity.

Second, students creating their own questions and giving to their peers to solve. This is an extension of the findings of Watson and Shipman (2008). According to Watson and Shipman (2008), learners learn a lot from creating their own problems in class. This study goes further to show that other learners in class also learn tremendously from task created by their peers. In addition, we see that this move brings about the creation of space for learners to interact between and among themselves. This interaction is supported by Vygotsky's (1978) learning theory that higher order learning exists outside the learner and can only begin to be internalized after social interaction in which the learner is an active participant. Ngwa ensures this interaction takes place among learners. Most often, teachers hold strongly onto being at the centre of action which only reduces opportunities of learners' interaction. We see here that Ngwa withdraws from the action to allow learners take central stage to interact, shape and refine their learning by themselves and thus making them independent learners. Boaler (2008) and Ingram (2008) described this move as students learning together. This move has the advantage that it makes students depend on themselves for success and not overly dependent on the teacher.

Third, this study also revealed that preparing for high stake testing is not just enough. Understanding the structure of the concepts learners are to learn and the examination they are to write is critical and provides greater insights into teaching. Ngwa has a great understanding of the General Certificate of Education (GCE) Examination and this greatly guides his teaching. His adaptations, modifications, connections from one lesson to another seem to be informed by his knowledge and structure of the GCE examination he is preparing his students to take. This knowledge about the GCE examination exhibited severally by Ngwa during his lessons seems to suggest that it is specialized content knowledge since it is unique to teaching and in preparing learners for particular certificate examinations. As such, we posit that knowledge about the high stake testing system could be seen as an aspect of specialized content knowledge. We use "content" here to refer to the mathematical content students are to learn as they prepare for high stake testing and the structure of the examination, an extension of our understanding of Ball et al.'s (2008) classification of specialized content knowledge.

Fourth, this study further reveal that Ngwa modifies problems in the textbooks from simple to complex. In its simplest form, this could just be altering given figures in the original problem and leaving the context unchanged. In other cases, the context might change, making the problems complex such as introducing proofs to cause learners to move into deep reasoning, reflection and stimulating creativity in solving problems. According to Bobis et al. (2021), modifying problems for students to solve successfully, provides opportunities for learners to consolidate and extend their learning and creating the desire to receive further learning. We believe this is a form of knowledge of content and students as these modifications enable learners to dig deeper into the concept to they are learning. We note further that Ngwa's modifications were on a student to student basis, addressing particular needs and supporting them to develop deep mathematical understanding of the concept being taught.

Fifth, teaching can go forward and backward many times to establish needed connection between or across topics or units. This forward and backward movement is helpful for learners to review concepts not fully mastered so as to solidify understanding and building on this can lead learners to even greater understanding. Many researchers have advocated for connections to be made by teachers so that students see concepts as interwoven rather than disjoint. Anghileri (2006) charge teachers to make these connections by setting the stage for learners to engage with complex mathematical task. Anghileri's recommendation assumes that by just giving learners complex mathematical tasks, connections will be made. In this study, we see that Ngwa brings forth a move that makes the attainment of connections even more realistic. Ngwa asked his student to reread notes from the topics or subtopics that have ideas they are struggling with. This rereading of the notes multiple times enable learners get deep reflection on the ideas they are struggling with and eventually make the needed connections. Therefore, teaching and learning goes forward and backward to establish needed connections and reinforce student learning. The forward and backward movement suggests that the teacher is fully conscious of connections between and among topics. This, we think is a form of knowledge of content and teaching as the teacher is fully aware of what in previous topics will support the learners and direct them to reread.

The findings of this study can be used by professional development experts, teacher trainers and teachers themselves. During professional development programs, focus can be placed on multidimensional productive modification and adaptations of existing problems in the textbooks to make them more challenging and problematic for learners, thereby providing them with great opportunities to extend their learning and make needed connections across concepts and topics with mathematics. Teacher trainers might use the results of this study to enrich their training outcomes by supporting pre-service teachers create their own problems, examine their appropriateness and workability. This is so that pre-service teachers can develop the skills of modifying and creating problems for their learners to engage with. This might lead to creation of problems that might have cultural and contextual realities, thereby motivating learning in the learners. Teachers might use the moves enacted by Ngwa in this study to incorporate into their practice for better students' performance at high stake examinations. In addition, teacher trainers might emphasize the need for pre-service teachers to support students reread notes in textbooks several times reflectively to establish connections that will support their understanding of future concepts.

Although the outcomes of this study are quite valuable, they cannot be generalized because only one teacher was investigated and only one school used. Further research can be engaged with many experienced teachers of mathematics preparing students for high stakes testing and from different schools (private, public and faith-based) to determine general productive teaching moves employed. In addition, a comparison can be made with a number of novice teachers preparing students for certificate examinations. This might help us know further what expert teachers do when preparing students for certificate examinations that novice teachers do not. Furthermore, following the performances of the students at the certificate examination might give the research community the impact of these moves.

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