Effect of Coriolis Force on Modified Eyring Powell Fluid flow

Abstract

Recently, it has been shown that under certain conditions, the Eyring-Powell fluid model can exhibit shear-thickening properties without losing the known Newtonian or shear-thinning properties. The Eyring-Powell and Modified Eyring-Powell fluids have shown great potential in enhancing armour production and reducing the cost of maintenance for bullet proof vests. The Coriolis force, on the other hand, explains natural phenomena such as the direction of cyclones and the true direction of sun rays on earth. In this paper, the shear-thinning properties of the Eyring-Powell fluid is explored under the action of Coriolis force. The flow is modelled as a system of PDEs which are transformed into a system of ODEs by means of similarity variables. A numerical solution of the equations are sought using the Runge-Kutta scheme alongside the Shooting technique. The effects of the Eyring-Powell fluid parameter and Coriolis force are explored on the flow velocity and flow temperature, and the results are illustrated as graphs. The outcomes show that temperature increases with increasing Coriolis force but flow velocity decreases as Coriolis force increases.

Keywords: Eyring-Powell fluid; non-Newtonian fluid; Modified Eyring-Powell fluid;

Table 1: Nomenclature

symbol	Meaning	symbol	Meaning
u, v	velocity in the x, y-directions	q	deformation parameter
T	temperature	ρ	density
T_w, T_∞	temperature at the wall and free stream	f'	dimensionless velocity
κ, α	thermal conductivity and diffusivity	c_p	heat capacity
θ	dimensionless temperature	Pr	Prandtl number
δ, ϵ	dimensionless Eyring-Powell parameters	B, C	Eyring-Powell constants
μ, ν	dynamic and kinematic viscosity	K	Coriolis parameter

Table 2: Abbreviations

symbol	Meaning	symbol	Meaning
EP	Eyring-Powell	MEP	Modified Eyring-Powell
MF	Magnetic field	CNTs	Carbon nanotubes

1 Introduction

The two most general classes of fluids are Newtonian and non-Newtonian fluids. Fluids are Newtonian if they are modelled by Newtonian law while non-Newtonian fluids deviate from the Newtonian law of viscosity. The deviation of the non-Newtonian fluids from the Newtonian law cannot be explained by a single equation and thus, various non-Newtonian fluids are captured into different constitutive equation that can govern them. Eyring-Powell (EP) fluid is an example of non-Newtonian fluids. The constitutive equations of the EP fluid are obtained from the kinetic theory of fluid and this makes it more viable in practice. The model is able to model creep [33] and sudden acceleration dampen [11]. Applications of the Eyring-Powell fluid include body armours [30], crude oil extraction [9], biomedical applications [8], cosmetics and paints [5], and clay slips and greases [29]. See [16, 17, 3, 28, 24] for more studies on viscoelastic fluid.

However, due to the intrinsic alternating shear-thinning/shear-thickening properties of the Eyring-Powell fluid, authors reported two sharp contrast in the behaviours of the Eyring-Powell fluid. Some recorded increased velocity [32, 12, 4, 2, 27] while others recorded decreased velocity [15, 1]. This seeming contradiction was taken care of when Oke [18] introduced the deformation parameter and proposed the Modified Eyring-Powell (MEP) fluid. Initially, Eyring-Powell fluid was considered to exhibit either Newtonian fluid properties or the properties of shear-thinning fluid depending on the chosen conditions. It was remarked by Oke [18] that the shear-thinning characteristics of the EP fluid cause seemingly contradictory results on the effect of the EP parameter on flow velocity". The deformation parameter enabled the MEP fluid to exhibit shear-thinning or shear-thickening characteristics depending on the values of chosen for the deformation parameter.

Coriolis force deflects the direction of a flowing fluid in a rotating frame. The Coriolis force has been studied and verified to have equal order of magnitude as other magnetohydrodynamic forces [6]. Artificial inclusion of Coriolis force in some industrial processes have proven effective in photobioreactor and sewage treatment [7]. Koriko et al. [13] and Koriko et al. [14] expounded the impact provided by Coriolis force on the motion of air over non-uniform surface and deduced that flow velocity and skin friction decreased as Coriolis force increased. Oke et al. [25] investigated the impact of rotation on an MHD Newtonian flow over a non-uniform surface and remarked that temperature increases with Coriolis force and magnetic field strength simultaneously. Oke et al. [26] provided an insight into how Coriolis force affects the dynamics of Casson fluid over a rotating non-uniform surface and remarked that the velocity decrease with increasing Coriolis force. The effects of both Coriolis force and volume fraction on the transport of a suspension of 47nm alumina nanoparticles in water over a nonuniform surface was explored in [22]. The outcome showed that heat source and surface rotation have effects on every flow property. Oke [20] studied the impact of thermal radiation on MHD MEP flow over a nonuniform surface. The study showed that Coriolis force reduces with increasing Coriolis force. By considering different geometry of various nanoparticles, Oke [19] explored the effects of rotation and these properties on the flow of gold-water nanofluid on surfaces with non-uniform thickness. The results showed that Coriolis force reduced the skin friction coefficient. Oke et al. [23] studied a 3D MHD flow of ternary-hybrid nanofluid over a rotating surface. The flow considered the suspension of CNTs, graphene and alumina nanoparticles in water. The outcomes showed that velocity increases with increasing Coriolis force.

The MEP fluid is a relatively new class of fluids and it requires more work to fully understand the properties of the fluid. This study is another step in understanding the flow of MEP fluid. The effects of Coriolis force is considered on the flow MEP fluid. We consider only the shear-thinning characteristic of the MEP fluid in this study.

2 Methodology

2.1 Governing Equations

Relaxation theory uses the additivity of forces in summing the force necessary to destroy a strong bond with the force required for a weak bond. Using the relaxation theory, Powell and Eyring [29] proposed the shear stress for the EP fluid as

$$\tau = \mu \nabla \overrightarrow{v} + B^{-1} \sinh^{-1}\left(C^{-1}\nabla \overrightarrow{v}\right). \tag{1}$$

Gross [10], however, remarked in general that the elastic system does not follow the exponential law of stress decay. Rather, it is composed of a multiplicity of different elementary systems that follow the exponential law individually. This prompted Oke [18] to modify the Eyring-Powell model by introducing a deformation parameter q. The shear stress for the MEP fluid was

$$\tau = \mu \nabla \overrightarrow{v} + \left[B^{-1} \sinh^{-1}\left(C^{-1}\nabla \overrightarrow{v}\right)\right]^{q}.$$

and the deformation parameter $q \in \mathbb{Z}$ produces the characterisation of the MEP fluid as follows; $q = 1$ gives the EP flow, even values of q gives the shear-thinning and odd values of q gives the shear-thickening fluids. By taking the first two terms of the Taylor series and ignoring higher orders, we have

$$\nabla \cdot \tau = \mu \nabla^2 \overrightarrow{v} + (BC)^{-q} (\nabla \overrightarrow{v})^{q-1} q \left(1 - \frac{q+2}{6C^2} (\nabla \overrightarrow{v})^2\right) \nabla^2 \overrightarrow{v},$$

and the momentum equation becomes

$$\rho \frac{D\overrightarrow{v}}{Dt} = -\nabla p + \mu \nabla^2 \overrightarrow{v} + (BC)^{-q} (\nabla \overrightarrow{v})^{q-1} q \left(1 - \frac{q+2}{6C^2} (\nabla \overrightarrow{v})^2\right) \nabla^2 \overrightarrow{v}.$$

By boundary layer analysis, the equations for MEP flow over a rotating surface (as shownin figure (1a)) is

$$u_x + v_y = 0 \tag{2}$$

$$u u_x + v u_y + 2\Omega u = \frac{\mu}{\rho} u_{yy} + \frac{q}{\rho} (BC)^{-q} \left(1 + \frac{q+2}{6C^2} \left(\frac{\partial u}{\partial y}\right)^2\right) \left(\frac{\partial u}{\partial y}\right)^{q-1} \frac{\partial^2 u}{\partial y^2} \tag{3}$$

$$u T_x + v T_y = \alpha T_{yy}, \quad \left(\text{where } \alpha = \frac{\kappa}{\rho c_P}\right) \tag{4}$$

and the conditions

$$\text{at the wall}\ :\quad u = ax,\ v = 0,\ T = T_w \tag{5}$$

$$\text{at the free stream}\ \ u \to 0.\ T \to T_\infty. \tag{6}$$

Quantities of engineering interests are the skin friction coefficient C_f and heat transfer rate Nu;

$$C_f = \frac{\nu u_y\left(y=0\right)}{U_w^2},\ Nu = -\frac{xT_y\left(y=0\right)}{\left(T_w - T_\infty\right)}$$

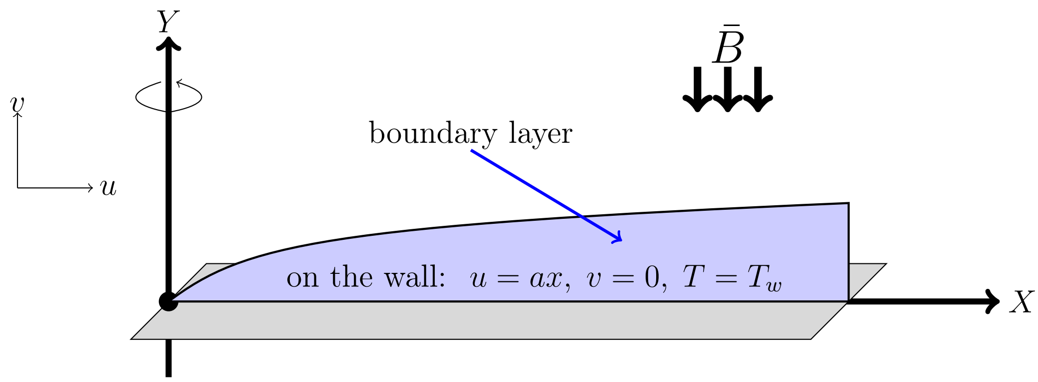

Figure1a: Flow configuration

2.2 Similarity Transformation

The similarity variables adopted for transforming equations (2-4) are

$$\eta = y\sqrt{\frac{a}{\nu}},\ \psi = \sqrt{\nu a}xf(\eta),\ u = \psi_y,\ v = -\psi_x,\ \Theta = \frac{T - T_\infty}{T_w - T_\infty}. \tag{7}$$

The equations reduce to the system of dimensionless ODEs

$$\left(1 + q\epsilon\left(\sqrt{\delta}f''\right)^{q-1}\left(1 - \frac{q+2}{3}\delta\left(f''\right)^2\right)\right)f''' - Kf' - \left(f'\right)^2 + f''f = 0 \tag{8}$$

$$\Theta'' + Prf\Theta' = 0 \tag{9}$$

with the conditions

$$f'\left(0\right) = 1,\ f\left(0\right) = 0,\ \Theta\left(0\right) = 1,\ f'\left(\infty\right) \to 0,\ \Theta\left(\infty\right) \to 0.$$

The Prandtl number Pr, and the EP fluid parameters ϵ and δ are defined as

$$Pr = \frac{\nu}{\alpha},\ \epsilon = \frac{1}{\mu BC},\ \delta = \frac{a^3x^2}{2\nu C^2},$$

The non-dimensional skin-friction coefficient and heat transfer rate are

$$Re^{-1/2}C_f = f''\left(0\right)\left(1 + \epsilon - \frac{\epsilon\delta}{3}f''\left(0\right)\right),\ Re^{1/2}Nu = -\Theta\left(0\right),$$

where

$$Re^{\frac{1}{2}} = \frac{1}{x}\left(\frac{\nu}{a}\right)^{\frac{1}{2}}.$$

2.3 Numerical Solution

Equations (8-9) are rewritten as a system of first order ODEs by setting

$$X_1 = f,\ X_2 = f',\ X_3 = f'',\ X_4 = \Theta,\ X_5 = \Theta'$$

and we have

$$\dot{X}_1 = X_2,\ \dot{X}_2 = X_3, \tag{10}$$

$$\dot{X}_3 = \frac{X_2^2 - X_3 X_1 + K X_2}{1 + q\epsilon\left(\sqrt{\delta}X_3\right)^{q-1}\left(1 - \frac{q+2}{3}\delta X_3^2\right)}, \tag{11}$$

$$\dot{X}_1 = X_2,\ \dot{X}_1 = -Pr \cdot X_1 X_5, \tag{12}$$

with the boundary conditions

$$X_1 = 0,\ X_2 = 1,\ X_4 = 1,\ \text{at}\ \eta = 0;\ X_1 = 0,\ X_4 = 0\ \ \text{as}\ \eta \to \infty. \tag{13}$$

The system of equations (10-12), alongside the conditions (13), are solved using the 3-stage Lobatto IIIa by coding it into MATLAB [31, 21]. The absolute tolerance is taken to be 10^{-8} while the relative tolerance is taken as 10^{-8}. When the parameters are not the subject of discussion, the following values are used

$$q = 2; \epsilon = 0.1; \delta = 0.3; Pr = 7; K = 1.$$

3 Results and Discussion

The MEP flow over a rotating flat plate is analysed and discussed. Coriolis force and MEP parameter ϵ are investigated and the outcomes are outputted in form of graphs and tables.

The flow of a fluid over a rotating surface is often affected by the rotation of the surface. The flow trajectories on the rotating surface often appears deflected (although it is not) when the speed of rotation is proportional to the flow velocity. The inertia force responsible for the fictitious deflection of the flow path is the Coriolis force. The Coriolis force is proportional to the velocity of surface rotation and hence, the magnitude of the Coriolis force increases as a consequence of increasing velocity of the surface ro- tation. Rotation, on the other hand, builds up more thermal energy in the system and consequently, increases the thermal boundary layer. This explains why temperature pro- file has risen, as seen in figure (2a). Figure (2a) shows that temperature profiles increase with increasing Coriolis force. However, increasing the rotation counter-clockwise, and consequently increasing Coriolis force, forces the motion in the direction of rotation and thereby opposing motion in the primary direction. Due to this opposition, the viscous boundary layer thicknesses increases and thus reduces the velocity. Figure (2b) shows that increasing rotation, and consequently the Coriolis force, brings about a reduction in the flow velocity.

The deformation parameter q has been shown in [20, 18] to be play an important role in characterising the behaviour of the resulting MEP fluid. Taking the positive even values of q produces the shear-thinning effect in the MEP fluid. In this study, the the deformation parameter q is chosen as the simplest even number, 2, to represent a shinning MEP fluid. By increasing the shear-thinning MEP fluid parameter ϵ, the fluid gradually becomes the Newtonian. Figures (2c) and (2d) show the effect of increasing the shear- thinning MEP fluid parameter on the flow velocity and flow temperature. As the MEP fluid parameter increases, the shear thinning property becomes stronger, and both the thermal and the viscous boundary layer reduce. Hence, as $\epsilon \rightarrow \infty$, the viscous boundary layer thickness reduces and the flow velocity is increased (as shown in figure (2c)) while the thermal boundary layer declines, leading to a decline in the temperature profiles (asin figure (2d)).

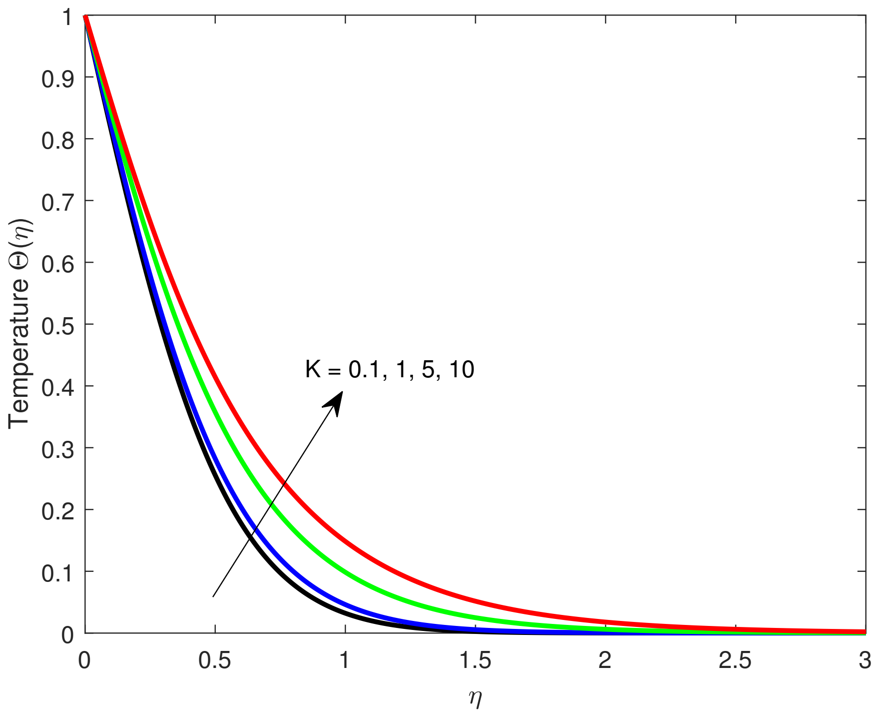

Figure 2a: variation of temperature profiles with Coriolis force

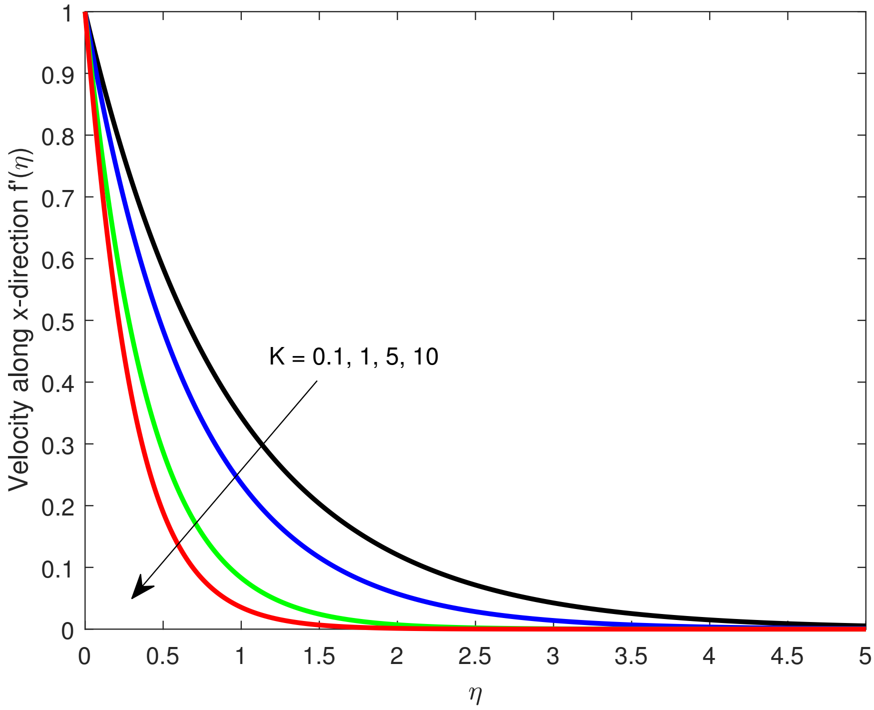

Figure 2b: velocity profiles with Coriolis force

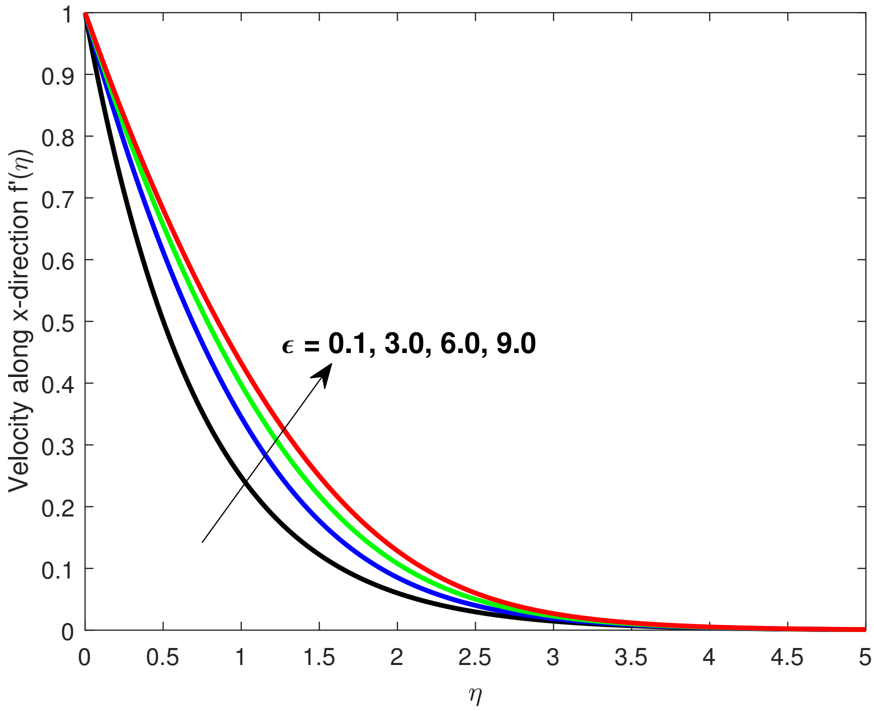

Figure 2c: velocity profiles with MEP parameter

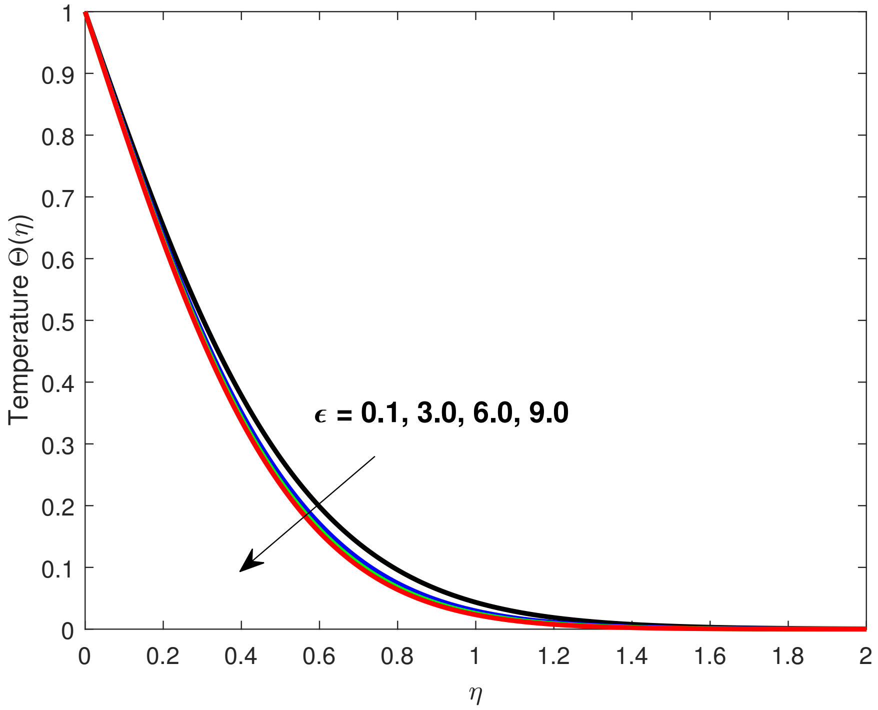

Figure 2d: temperature profiles with MEP parameter

4 Conclusion

This paper investigates the MEP fluid flow on a rotating surface. The shear-thinning MEP fluid is considered in this study by setting the deformation parameter q to 2. The governing PDEs are nondimensionalised and the resulting system of ODEs is numerically solved. The outcomes of this study shows that;

1. Raising Coriolis force inadvertently raises temperature but drops the velocity.
2. Eyring-Powell parameter ϵ increases velocity but decreases temperature.

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